PREFACE

Welcome to UDAAN- a program to give wings to the girl students!

The UDAAN programme has been initiated with the primary objective of increasing the enrollment of girl students in leading engineering institutions. The program is designed to provide a platform to deserving girls who aspire to pursue higher education in engineering and assist them in preparing for the entrance examinations.

The study material covers the syllabus of the Joint Entrance Examinations. It has been designed to ensure that it is easy to understand. The progression of the content moves from simpler to complex concepts as the Unit/Chapter proceeds. Each topic and subtopic is followed by some practice questions for you to attempt. Answers to these questions are given at the end. In case you have difficulty in solving any of them, please email us at udaanhelpdesk2017@gmail.com or contact at 011-23231070.

Best of luck and happy learning!

Chairperson, CBSE

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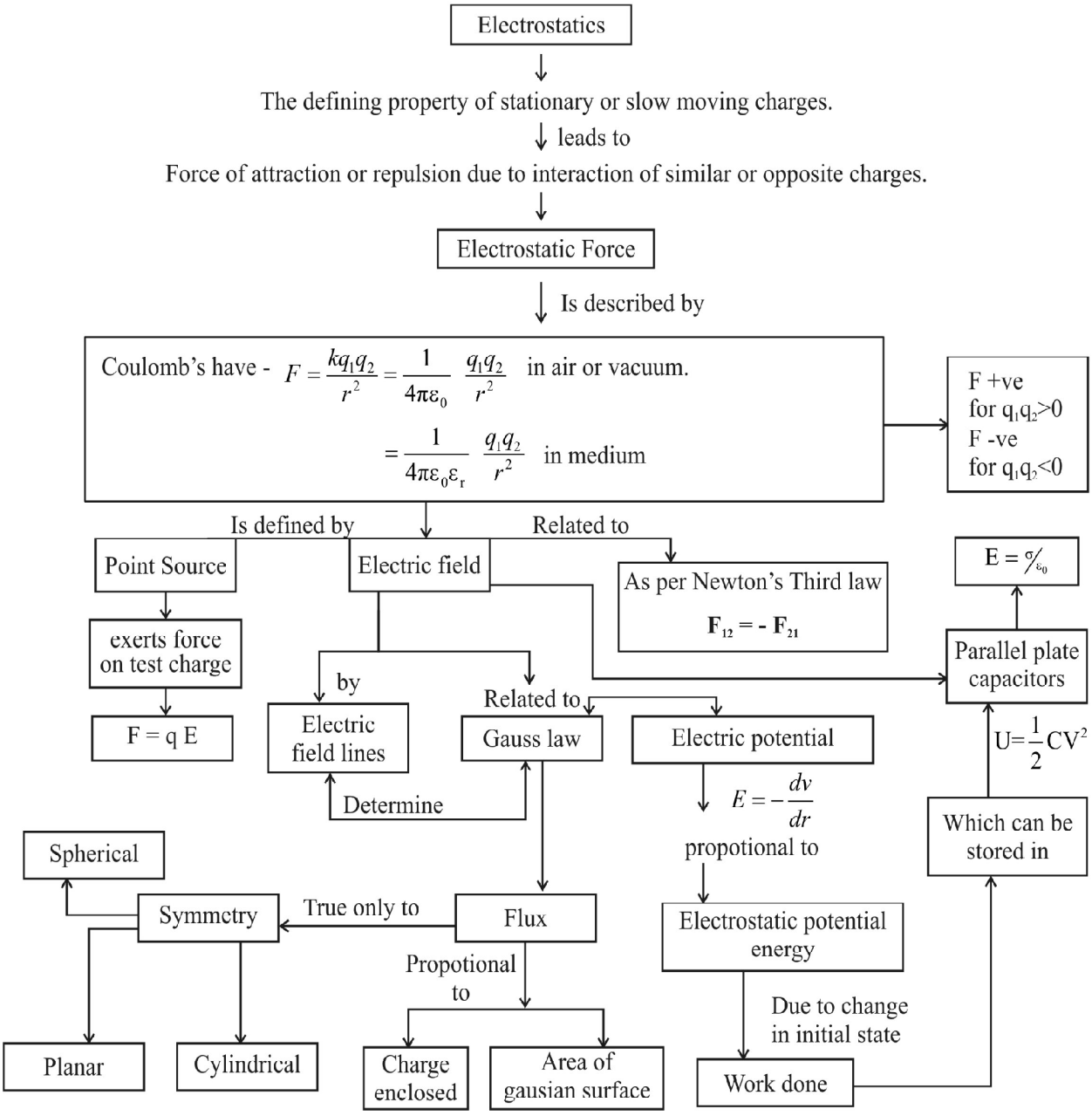
UNIT–11 : ELECTROSTATICS

Learning Objectives

After going through this, unit you will be able to understand, appreciate and apply the following concept:

- Identify different type of charges.
- Determine the electric force between two point charges using Coulomb's Law.
- Apply Coulambl's law to a system of point charges to determine the net force on a charge.
- Calculate the electric field due to a point charge or system of point charges.
- Determine the electric flux through a surface.
- Use Guass law to find electric field of various charge distribution.
- Determine the torque experienced by an electric dipole in a uniform electric field.
- Distinguish between electric potential and electric potential energy.
- Calculate the electric potential of a configuration of point charges.
- Determine how electric field strength is related to electric potential and electrostatic potential energy.
- Calculate the capacitance of a given electric network, energy stored in a parallel plate capacitor with and without dielectric.

Concept Map of Electrostatics



Electrostatics

Electrostatics is the branch of physics that deals with the phenomena and properties associated with charges at rest.

Electric Charge

Electric charge is an intrinsic characterstics of the fundamental particles of matter giving rise to all electric and magnetic forces and interactions.

From simple experiments on frictional electricity, it has been known that some materials, such as amber or ebonite, on being rubbed with, wool get 'electrically charged'. Similarly, when a glass rod is rubbed with silk cloth, the glass rod gets ' electrically charged'. However, the charges of these two rods are of different types. One can infer that there are two types, of charges in nature: 'glass rod type' and 'ebonite rod type'. The first type has been called as 'positive charge' while the second type has been called the 'negative charge'. The changes were named as 'positive' and 'negative' by the American scientist, Benjamin Fraklin, in 1750. When the 'positive' and 'negative' charges on an object, are not in equal amounts, there is a net charge on the object, that can interact with other objects. We regard such an object as an 'electrified' or ' 'charged' object. When an object has no net charge, it is said to be electrically neutral.

It has been observed that: "Like charges repel each other and unlike charges attract each other."

Basic Properties of Electric Charge

1. **Charge is quantised :** Electric charge can have only discrete values. The smallest discrete value that can exist in nature is the charge on the electron, 'e'. Charge on any body always exists in integral multiples of this fundamental 'unit' of electric charge.

   $$q = \pm n\, e \text{ where } n = \pm 1, \pm 2.....$$

2. **Charge is conserved :** For an isolated system, its total charge remains constant. " charge is neither created nor distroyed", it can only be tansferred from one object to the other.

3. **Charge is additive :** If a system contains 'n' charges; q_1, q_2, q_3.... q_n; the total charge of the sytsem is:

   $$q_{net} = (q_1 + q_2 + q_3 + ... + q_n)$$

   Charges thus add up like 'real numbers', they are scalars like mass.

4. **Charge is invariant :** The numerical value of the charge on an isolated object is independent of its state of rest or motion.

The S.I. Unit of charge is the 'Coulomb'. It is defined in terms of 'electric current'. 'One Coulomb' equals the charge, transferred in one second, across the cross section of a wire carrying a current of one ampere.

Coulomb's Law

We have noted that two like charges repel each other while two unlike charges attract each other. This force of interaction, between two charges is known as 'electric force' or 'coulomb force'. On the basis of experiments, coulomb gave a law for finding the magnitude of the force acting between two point charges.

According to Coulomb's law; two stationary point charges, q_1 and q_2, repel, attract, each other with a force ('F') which is directly proportional to the product (q_1q_2) of the two of charges and inversely propotional to the square of the distance ('r') between them

$$\therefore\ F \alpha \left(\frac{q_1 q_2}{r} \right)$$

or $F = K\frac{q_1 q_2}{r^2}$ where K is a proportionality constant; its value depends upon the nature of the medium separating the two charges. When the charges are placed in vacuum (or air) we take this proportionality constant as $\frac{1}{4\pi\varepsilon_0}$. Thus $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$.

The constant ε_0 is called permittivity of free space or vacuum. In SI Units, its value is

$$K\left(= \frac{1}{4\pi\varepsilon_0} \right) = 9 \times 10^9\ Nm^2 / C^2.$$

$$\therefore\ \varepsilon_0 \simeq 8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$$

For mediums other than vacuum (or air) the force between the charges is given by:

$$F = \frac{1}{4\pi\varepsilon_m}\frac{q_1 q_2}{r^2}$$

Here ε_m is called the permittivity of the medium concerned. For materials other than vaccum (or air), we can put

$$\varepsilon_m = k\varepsilon_0$$

Here k is a dimensionless constant. It is called as the 'dielectric constant' or 'relative permittivity', of the medium.

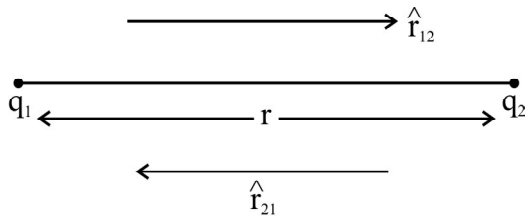
The above expression can, therefore, be written as

$$F = \frac{1}{k}\left(\frac{1}{4\pi\varepsilon_0} \right)\frac{q_1 q_2}{r^2}$$

Important Points Regarding Coulomb's Law

1. The Coulomb force, acts along the straight line, connecting the ponits of location of the two 'point' charges.
2. The law can be used to define the SI unit of charge the 'Coulomb'.
3. For a point charge, the coulomb force due to it is 'central' and has a spherical symmetry.
5. For materials media the value of 'k' is greater than 1. Thus, if two charges kept a given distance apart, have a medium other than vacuum (or air), between them the force, between them, decreases by a factor k.

Coulomb's Law in Vector Form



It q_1 and q_2 be two point charges separated by a distance r in vacuum, the force on q_2 due to q_1 $\left(\mathbf{F}_{21}\right)$ is given by:

$$\mathbf{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

Where $\hat{r}_{12}$ is a unit vector, pointing from q_1 to q_2.

The force, on q_1 due to q_2, is given by

$$\mathbf{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{21}$$

Here $\hat{r}_{21}$ is a unit vector along the direction of the line directed from q_2 to q_1

$\hat{r}_{12}$ and $\hat{r}_{21}$ are clearly unit vectors in opposite directions. We can therefore, write

$$\mathbf{F}_{21} = -\mathbf{F}_{21}$$

In general, one can write:

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^3}\,\hat{r}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^3}\left(x\hat{i} + y\hat{j} + z\hat{k}\right)$$

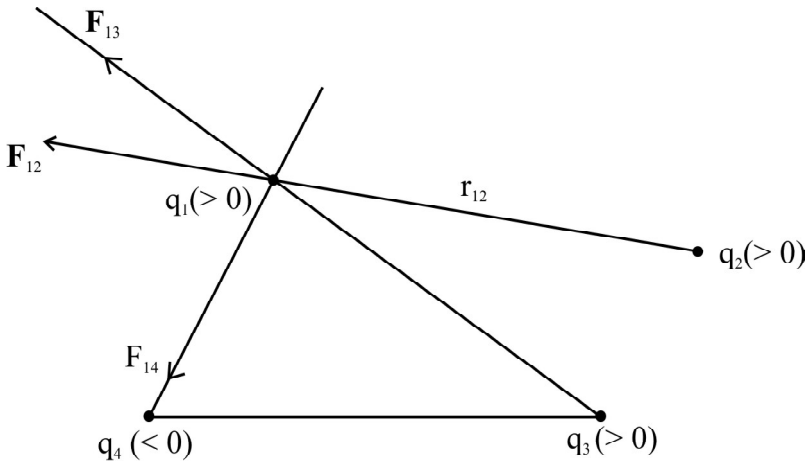
Here $\hat{r}$ is the unit vector that would have 'its origin' at the source of the force. A positive magnitude of $|\mathbf{F}|$, implies repulsion while a negative value of $|\mathbf{F}|$, implies attraction between the two charges.

How to Solve Problems using Coulomb's Law

1. Find the magnitude of force, using Coulomb's law, ignoring the signs of charges. Signs only indicate the direction of force.
2. Decide whether the force, due to a given charge, is attractive or repulsive, express it by drawing vectors.

Principle of Superposition of Charges

When more than two charges are present in space, Coulomb's law holds for every pair of charges. Thus, the net force on any charge is the 'vector' sum of the forces exerted on it by all other charges, taking one charge at a time.



This is known as the principle of super position for charges.

Thus if there are (n+1) point charges, the force, one of them, say q_1 due to all other charges is given by.

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + ... + \mathbf{F}_n$$

Here the 'individual forces' are unaffected due to the presence of other charges.

$$\text{or } \therefore\ \mathbf{F}_1 = \left(\frac{1}{4\pi\varepsilon_0} q_1\right)\sum_{i=2}^{n}\frac{q_i}{r_{i1}^2}\,\mathbf{r}_{i1}$$

Here $\vec{r}_{i1}$ is the unit vector along the direction joining the charge q_i to the charge q_1.

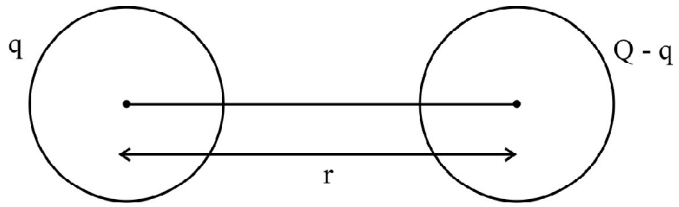
Example–1:

A given charge Q is to be shared between two (fixed) objects kept at given distance apart. Find the values of the charges, on the two objects, so that the force between them has its maximum values.

Solution:

Let charge on one object be 'q'; The charge on other, will then be $Q-q$

$$\therefore\ F=\frac{1}{4\pi\varepsilon_0}\frac{q(Q-q)}{r^2}$$



For F to be maximum, we need to have

$$\frac{dF}{dq}=0$$

$$\therefore\ \frac{d}{dq}\left(\frac{1}{4\pi\varepsilon_0}\frac{(qQ)}{r^2}-\frac{1}{4\pi\varepsilon_0}\frac{(q^2)}{r^2}\right)=0$$

$$=\frac{Q}{4\pi\varepsilon_0 r^2}-\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}=0$$

$$=q=\frac{Q}{2}$$

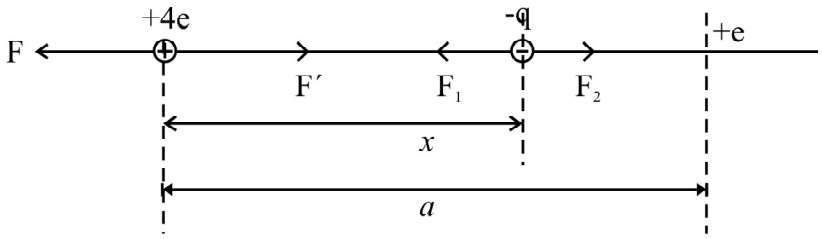
Thus, the charge should equally divided between the two objects.

Example–2:

Two 'free' point charges, (+4e) and (+e), are placed a distance 'a' apart. Where should a third charge, (say q), be placed, between them, such that the entire system may be in equilibrium? What should be, the magnitude and sign of q? State the 'type of equilibrium' of the system.

Solution:

Let charges be placed as shown in the figure.



The charge +e exerts a repulsive force on charge +4e. Hence for the equilibrium of charge +4e the third charge 'q' must exert an attractive force F' on +4e. This requires cahrge 'q' to be a negative charge. For equilibrium of charge (+4e).

$$\mathbf{F} = -\mathbf{F}'$$

$$= |\mathbf{F}| = |-\mathbf{F}'|$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{(4e \times e)}{a^2} = \frac{1}{4\pi\varepsilon_0}\frac{(4eq)}{x^2}$$

$$\therefore\ q = \frac{ex^2}{a^2}$$

For equilibrium of charge ('–q)

$$\mathbf{F}_1 = -\mathbf{F}_2$$

$$|\mathbf{F}_1| = |-\mathbf{F}_2|$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\left(\frac{4eq}{x^2}\right) = \frac{1}{4\pi\varepsilon_0}\frac{eq}{(a-x)^2}$$

$$\Rightarrow x = \frac{2a}{3}$$

Thus $|q| = \frac{ex^2}{a^2} = \frac{4e}{9}$

$$\therefore\ q = \left(-\frac{4e}{9}\right)$$

This charge has to be kept at a distance $\left(\frac{2a}{3}\right)$ from the charge (+4e).

It is easy to see that the equilibrium of the system is an unstable equilibrium.

Example–3:

Two identical spheres, having unequal opposite charges, are placed at a distance 0.9 m apart from each other. After touching them mutually, they are again put at the same distance apart. They are now observed

to repel each other with a force of 0.025N. Calculate the final charge on each of the two spheres.

Solution:

Since the spheres are identical after touching them they will acquire similar and equal charges. Suppose the charge on each is 'q' then by Coulomb's law;

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$$

$$\therefore\ 0.025 = 9\times10^9 \times \frac{q^2}{(0.9)^2}$$

$$q^2 = 2.25\times10^{-12}\,C^2$$

$$\text{or } q = \sqrt{2.25\times10^{-12}\,C^2}$$

$$\text{or } q = \pm1.5\times10^{-6}\,C = \pm1.5\,\mu C$$

Concept of Electric Field

Let us consider an electric change q located in space. It we now bring another change (Δq_0), near this charge q, the charge (Δq_0) expriences a force of attraction or repulsion. The space, around an electric charge, q, in which another charge, (Δq_0) experiences its (electrostatic) force of interaction, may be thought of as the electric field of charge q. The charge 'q' is known as the 'source' charge and charge (Δq_0), used to 'detect' the electric field (of charge q) is known as the 'test' charge.

To measure the strength or intensity of an electric field of any point, we may place a small positive charge (Δq_0) at that point and measure the electrostatic force (due to the source charge q') experienced by this charge Δq_0. This force, per unit charge, is taken as a measure of the strength of the electric field, at that point.

That, if $(\Delta\mathbf{F})$ be the fore acting, on a test charge (Δq_0), placed at a given point in an elctric field, the electric field intensity, or strength, $\mathbf{E}$, of the field at that point in given by.

$$\mathbf{E} = \lim_{(\Delta q_0 - 0)}\left(\frac{\Delta\mathbf{F}}{\Delta q_0}\right)$$

The test charge must be negligibly small so that the source charge, q, is not disturbed from its position and its electric field at a given point, remains unchanged. The SI unit of $\mathbf{E}$ is, therefore, $\frac{N}{C}$.

The electric field, due to a single point charge, kept in vacuum (or air) is given by

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

$$\therefore \ |\mathbf{E}| = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Important Points Regarding the Electric Field

1. Once the field strenght $(\mathbf{E})$ is known, the force, on any charge 'q', can be obtained from

   $$\mathbf{F} = q\mathbf{E}$$

   $$\therefore |\mathbf{F}| = q|\mathbf{E}| \text{ or } F = qE$$

   A positive charge would experience a force in the direction of the electric field while a negative charge would do so in a direction opposite to that of field.

2. Electric field, due to a system of charges, at a point, is the vector sum of the electric fields at that point due to the individual charges.

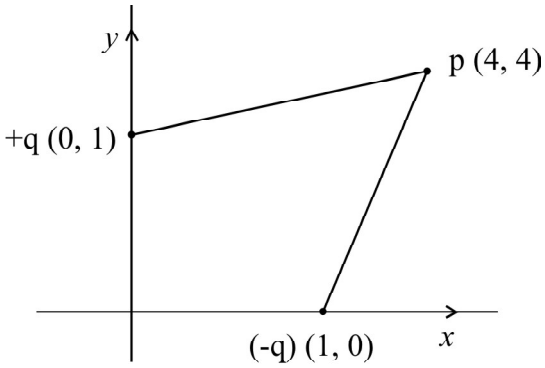
   $$\mathbf{E}_{net} = \mathbf{E}_1 + \mathbf{E}_2 + ...\mathbf{E}_n$$

3. Electric field, due to a point charge, points away from the charge if it is a positive charge and towards it if it is negative charge.

4. Electric field, of a point charge, has a spherical summetry.

Example–4:

The figure shows system of two equal (in magnitude) point charges. Charge + q is at (0, 1) and charge –q is at (1, 0). Find the resultant field intensity at a point P having co-ordinates (4, 4).



Solution:

We have,

$$\mathbf{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q\left(4\hat{i}+3\hat{j}\right)}{(5)^3} \text{ units}$$

$$\mathbf{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{-q\left(4\hat{i}+3\hat{j}\right)}{(5)^3} \text{ units}$$

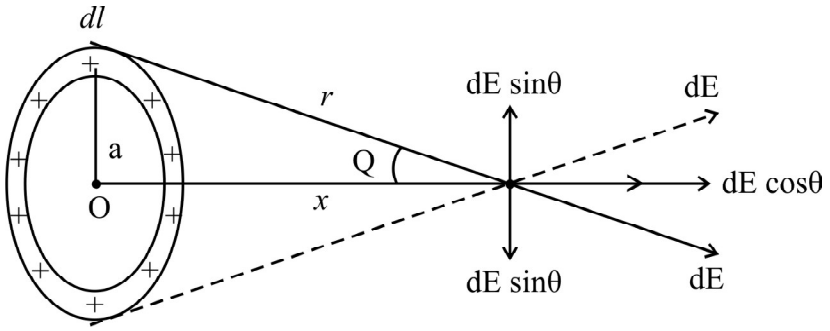
$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \frac{1}{4\pi 6_0}\frac{(q)}{25\sqrt{5}}\left(\hat{i}-\hat{j}\right) \text{ units}$$

Continuous Charge Distributions

A system of closely spaced charges may be thought of as forming a continuous charge distribution. There are three kinds of continuous charge distributions– linear charge distribution, surface charge distribution and volume charge distribution. The electric field due to a continuous charge distribution, can be obtained in the same way as for a system of discrete charges using Coulomb's law and the principle of super position. We first find the electric field due to small element 'dq' and then intergrate the expression over the whole continuous charge distribution.

Example–5:

Axial electric field due to a circular loop of charge : (charged ring)



Let us consider a uniformly charged ring of radius 'a' having a linear charge density $'\lambda'$. The total charge 'q', on the ring, then equals $2\pi a\lambda$.

Consider a small (line) element '$d\ell$'. It carries a charge $dq = \frac{q}{2\pi a}d\ell$.

The magnitude of, $\mathbf{E}$, at an axial pont P, due todq, is $dE = \frac{1}{4\pi 6_0}\frac{dq}{r^2}$

Now consider another element $d\ell$; diagonally opposite to $d\ell$; we can see that vertical components of $d\mathbf{E}$ counter balance each other in pairs. Hence the net electric field, is only due to the horizontal components of $(d\mathbf{E})$'s.

$$\therefore \ \mathbf{E} = \left(\hat{i}\right)\int dE\cos\theta$$

$$\therefore \ \mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{qx\,\hat{i}}{2\pi a\left(a^2+x^2\right)^{3/2}}$$

Now, $\cos\theta = \frac{x}{r}$ and $r = \frac{r}{\sqrt{a^2+x^2}}$

$$\mathbf{E} = \left[\frac{1}{4\pi\varepsilon_0}\frac{qx}{\left(a^2+x^2\right)^{3/2}}\right]\hat{i}$$

This $\vec{E}$ is directed along the axial line OP.

If 'P' lies at centre $|\mathbf{E}| = 0$

If P is a far off axial point, i.e. if $x >> a$,

$$|\mathbf{E}| \simeq \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$$

Thus, for points on the axis, at distances much larger than the radius of the ring, the ring bahaves like a point charge.

Some Important Results for $\mathbf{E}$:

- Super position of electric field - electric field at a point due to a system of charges is the vector sum of the electric fields at that point due to individual charges.

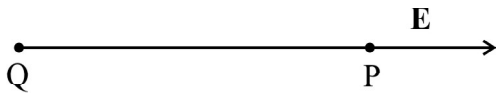
$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + ... + \mathbf{E}_n$$

- The electric field due to continuous charge distribution can be obtained in much the same way as for a system of discrete charges using Coulomb's law and the super position principle. First we find the electric field due to small element dq and then integrate the expression for whole body.

Electric field due to various (discrete / continous) charge distribution

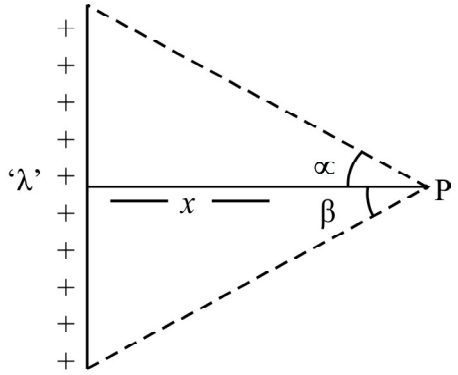
We write below some important results for the electric field due to some charge configurations.

(i) Electric field strength due to a point charge at any point 'p'.



$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

(ii) Electric field strength due to a finite line charge with charge density 'λ' at a general 'P'.

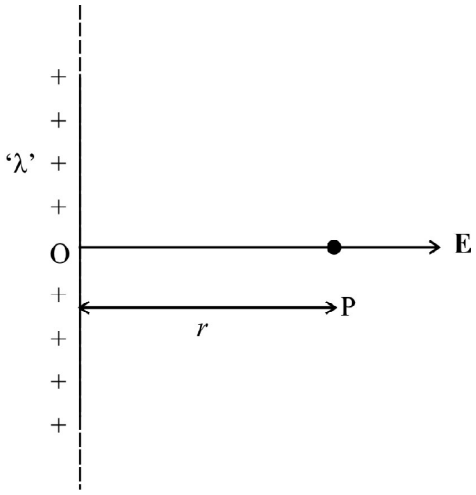


$$E = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{x}(\sin\alpha + \sin\beta)$$

where $\lambda = \frac{q}{L}$, Total charge on line = q, length = L

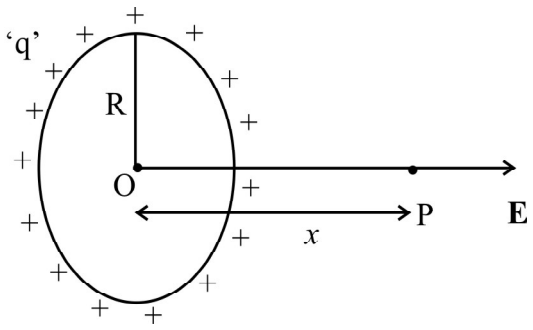
(iii) Electric field strength due to a infinite charge with charge density 'λ' at any general point 'P'.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r}$$



(iv) Electric field strength due to a circular ring of radius 'R' with uniform distribution of charge 'q' at any general point 'P' on the axis of the ring.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{qx}{\left(R^2 + x^2\right)^{3/2}}$$



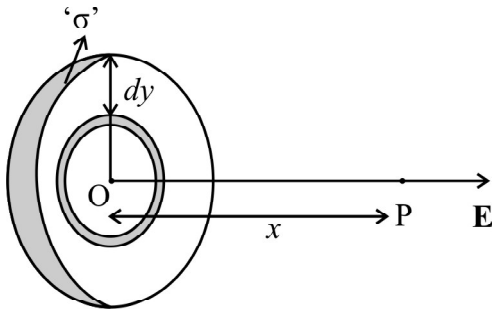
If $x >> R$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$$

Behaves like point charge.

(v) Electric field strength due to a uniformly surface charged, disc of radius 'R' any general point 'P' on its axis.

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$



If $x >> R$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\theta}{x^2}$$

Disc behaves like point charge.

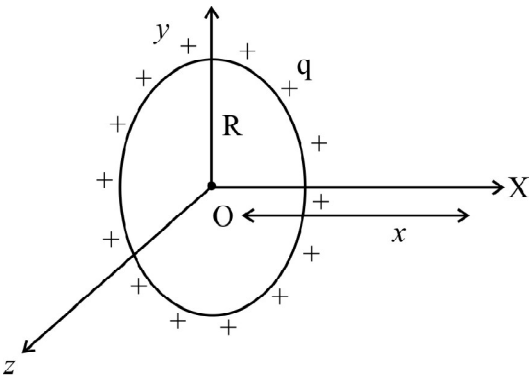
If $x >> R$

$$E = \frac{\sigma}{2\varepsilon_0}$$

Disc behaves like then plane sheet.

Example–6:

A ring of radius R with uniformly distributed charge q is placed in the y–z plane with its centre at the origin. Plot the variation of E along the axis of the ring and obtain the maximum value of E and identify the position of the corresponding point.



Solution:

We have,

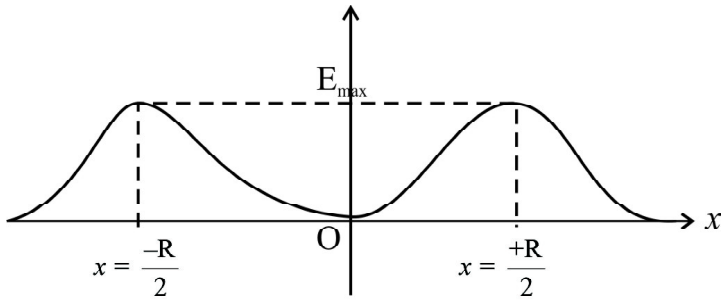
$$E = \frac{1}{4\pi\varepsilon_0}\frac{qx}{\left(R^2 + x^2\right)^{3/2}}$$

For maximum field intensity $\frac{dE}{dx} = 0$.

$$\Rightarrow x = \pm\frac{R}{2}$$

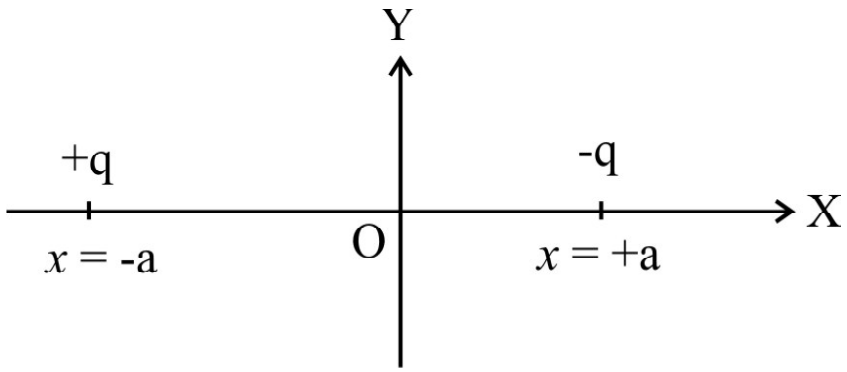
$$\Rightarrow E_{max} = \frac{q}{6\sqrt{3}\pi\varepsilon_0 R^3} \qquad \left(\text{Putting } x = \frac{R}{2}\right)$$

The plot is an shown.



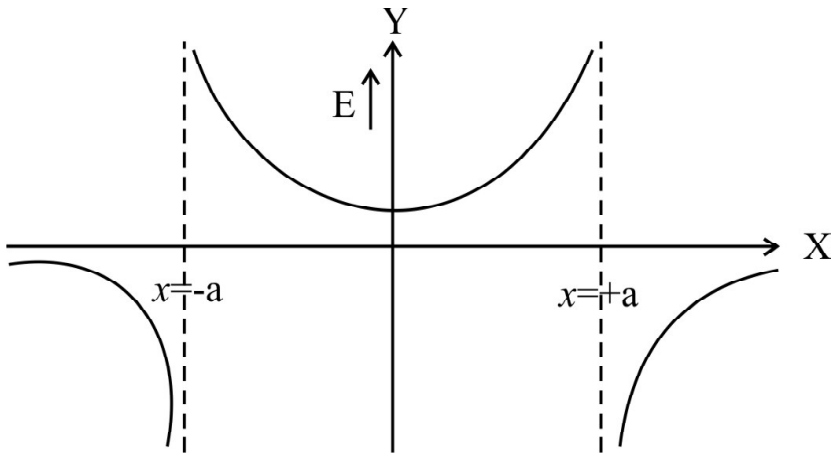
Example–7:

A pair of charges, +q and –q, are placed at $x = -a$ and $x = +a$ respectively. Plot the veriation of E along the x–axis.



Solution:

The variation is as shown.



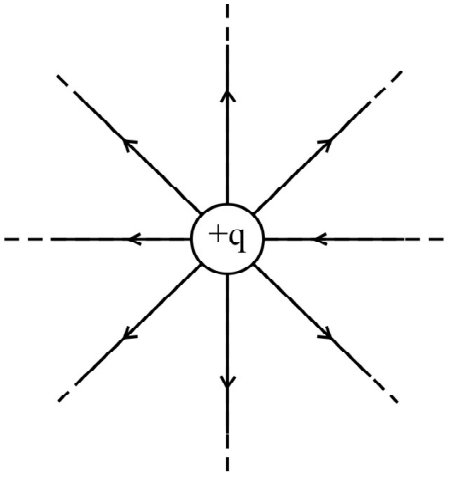
Electric Field Lines

An electric field line is a curve drawn in three dimensions in such a way that the tangent at each ponit on the curve gives the direction of electric field at that point.

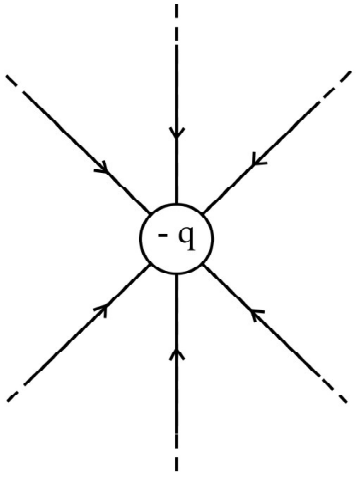
The idea of electric lines of force, or the electric field lines, introduced by Micheal Faraday, is a way to visualize electrislatic field geometrically.

The properties of electric field lnies are:

(i) Electric field lines are continuous curves in an electric field; starting from a positive charge and ending on a negative charge.



Electric field lines due to a positive charge.



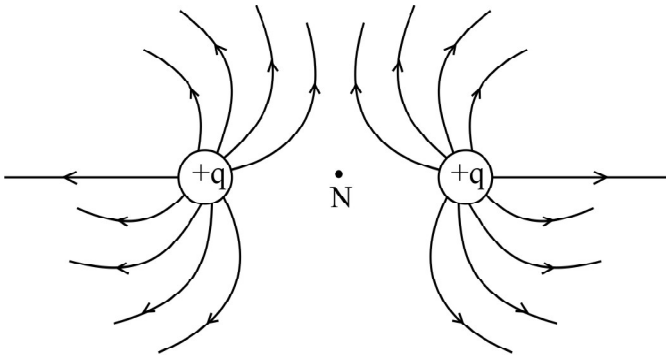
Electric field lines due to a negative charge.

(ii) Two electric field lines never intersect.

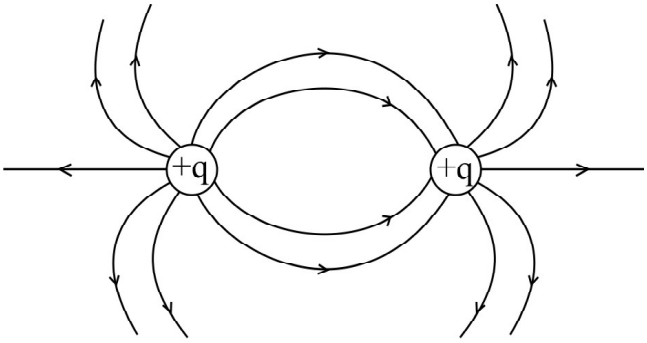
(iii) Electric field lines do not pass through, they leave, or end, on a charged conductor normally.

(iv) The relative closeness of field lines indicates the relative strength of electric field at different points; crowded lines represent strong field regions while distant field lines correspond to a weak field. Further, if the field lines are uniformly spaced parallel straight lines, the field is uniform.

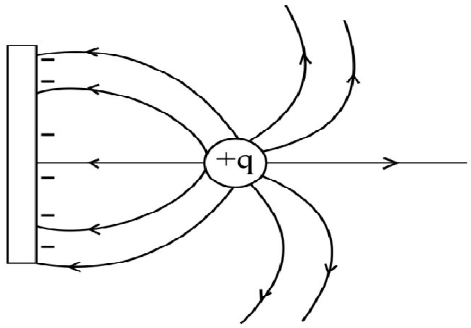
(v) It is not necessary that a small test charge released at a point in an electric field will move along the field line passing through that ponit. The field lines give the direction of acceleration they do not essentialy give the direction of motion of charge.



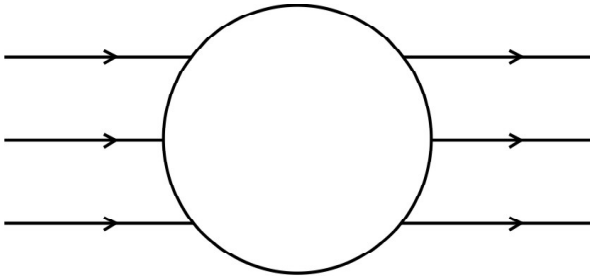
Electric field lines due to two equal like charges.



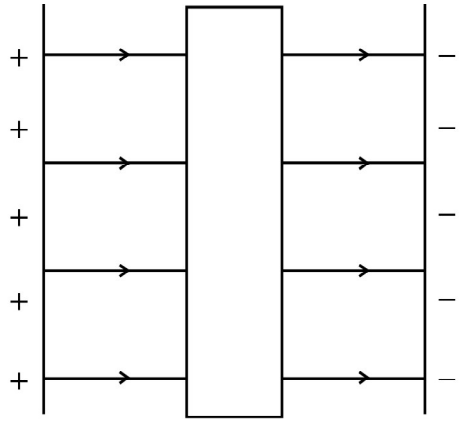
Electric field lines due to two equal and opposite charges.



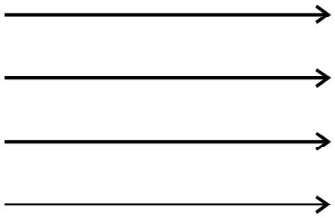
Electric field lines due to a fixed charge +q placed near an infinite metal sheet



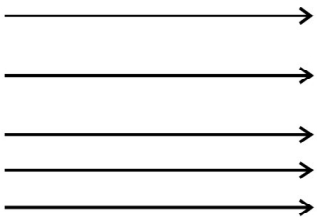
A solid metal sphere placed in
uniform electric field



A metallic slab placed between two oppositely
charged metal parallel plates.



Field lines of a uniform electric field.
(Both magnitude and direction constant)



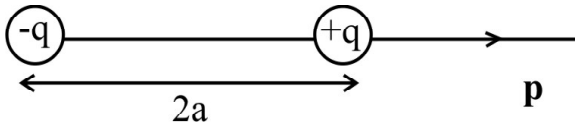
Field lines of a non-uniform electric field
(constant in direction, but varries in magnitude)

Electric Dipole

An electric dipole is a pair of equal and opposite charges [(+q) and (–q)], seperated by a (small) distance '2a'. Any molecule in which the centre of the positive charges does not coincide with the centre of negative charges, may be approximated as an electric dipole.

The basic characterstics of a dipole is its electric dipole moment $(\mathbf{p})$. It is defined as the product of the magnitude of either charge and the separation between the charges and is given by

$$\mathbf{p} = q(2\mathbf{a})$$

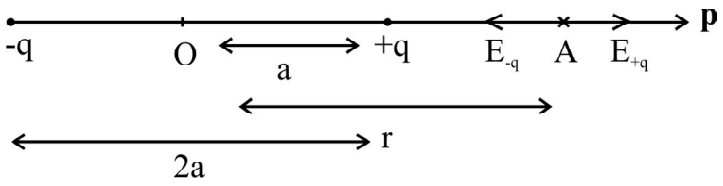


Dipole moment is thus a vector quantity; its direction is the direction of the dipole axis, from (–q) to (+q).

Its SI Unit in Cm.

Electric field Intensity due to an Electric Dipole

(i) Axial position : The field, on the axis, at a distance 'r' from the centre:



The electric field on axial position 'A' can be calculated by vector addition of electric field due to +q and –q at point 'A'

$$\mathbf{E}_{\text{axial}} = \mathbf{E}_{+q} + \mathbf{E}_{-q}$$

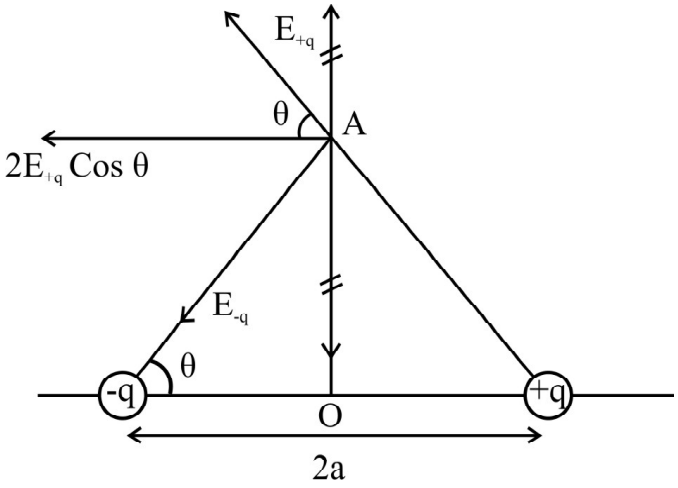
$$= \frac{1}{4\pi\varepsilon_0}\frac{2p\mathbf{r}}{\left(r^2 - a^2\right)}$$

For an ideal electric dipole $r >> 2a$

$$\therefore \mathbf{E} \cong \frac{1}{4\pi\varepsilon_0}\frac{2\mathbf{p}}{r^3}$$

Thus, Electric field is along the direction of $\mathbf{p}$, for all axial points.

(ii) Equitorial position : The field on the equitorial plane (i.e. plane perperndicular to the axis and passing through its centre) at a distance 'r' from centre is:



As seen in the diagram above, electric field on equitorial position is only due to the horizontal components of electric fields due to (+q) and (–q).

$$\therefore \ \mathbf{E}_{eq} = 2\,\mathbf{E}_{+q}\cos\theta$$

$$\mathbf{E}_{eq} = \frac{1}{4\pi\varepsilon_0}\frac{-\mathbf{p}}{\left(r^2+a^2\right)^{3/2}}$$

For an ideal dipole : r >> 2a

$$\therefore \ \mathbf{E}_{eq} \simeq -\frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}}{r^3}$$

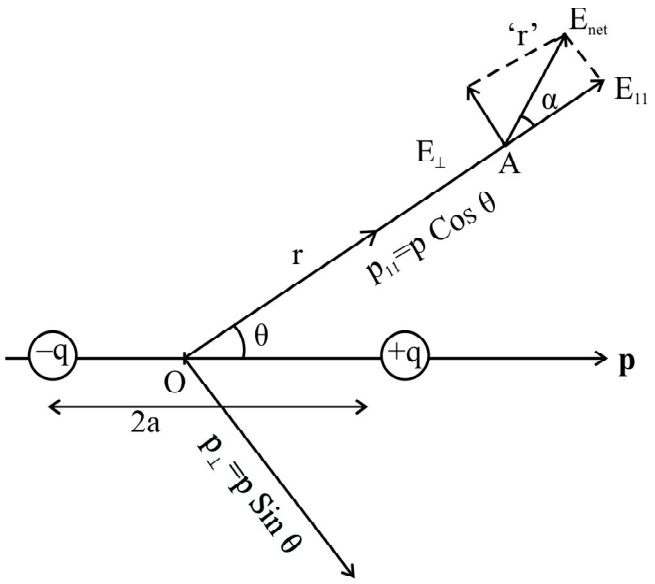
Thus, electric field is in the direction opposite to $\vec{P}$ for points on the equitorial plane.

For the same value of the distance 'r'.

$$E_{axial} = 2E_{eq} \qquad \text{(For a small dipole)}$$

(iii) Field at any general ponit 'r'.

Let 'θ' be the angle between $\mathbf{p}$ and $\mathbf{r}$. We can now resolve $\mathbf{P}$ into two components; $\mathbf{p}_{11}$ and $\mathbf{p}_{\perp}$, i.e. parallel to $\mathbf{r}$ and perpendicular to $\mathbf{r}$; point 'r' then lies on the axial line of $\mathbf{p}_{11}$ and lies on the equitorial line of $\mathbf{p}_{\perp}$.



Hence, $E_{net} = \sqrt{E_{11}^2 + E_\perp^2}$ where

$$E_{11} \simeq \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3} \qquad \left(\because\ p_{11} = p\cos\theta\right)$$

and $$E_\perp = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^3} \qquad \left(\because\ p_\perp = p\sin\theta\right)$$

$$\therefore\ E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}r\sqrt{1+3\cos^2\theta}$$

This (general) electric field, at any point 'r', makes an angle α w.r.t. $\mathbf{E}_{11}$; where $\alpha = \tan^{-1}\left(\frac{1}{2}\tan\theta\right)$

Example–8:

The electric field, due to a short dipole, at a distance r, on the axial line, from its mid point is the same as the electric field at a distance r' on the equitorial line from its mid point. Determine the ratio $\left(\frac{r}{r'}\right)$.

Solution:

We are given that

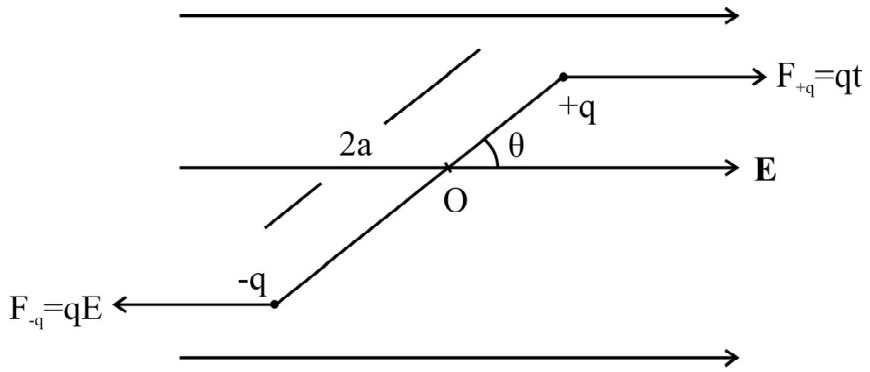
$$E_{axial(r)} = E_{eq(r')}$$

$$\therefore \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3} = \frac{1}{4\pi\varepsilon_0}\frac{p}{(r')^3}$$

$$\Rightarrow \frac{2}{r^3} = \frac{1}{r'^3}$$

$$\therefore \left(\frac{r}{r'}\right) = 2^{1/2}$$

Electric dipole placed in a uniform external field



When a dipole is placed in uniform external field, the net translation motion is zero as:

$$\mathbf{F}_{+q} = -\mathbf{F}_{-q}$$

$$\therefore \mathbf{F}_{net} = 0$$

However, the forces $\mathbf{F}_{+q}$ and $\mathbf{F}_{-q}$ form a couple. Their turning effect, or torque, is given by

$$\tau = (qE)(2a\sin\theta)$$

$$= pE\sin\theta$$

where q2a = p (electric dipole moment)

The direction of $\boldsymbol{\tau}$ is given by

$$\boldsymbol{\tau} = \mathbf{p}\times\mathbf{E}$$

Hence, due to this torque, the dipole tries to allign itself parallel to the direction of electric field. This is its position of stable equilibrium.

Special Case : When the dipole is placed in a non-uniform external electric field, the charges of dipole experience unequal forces; therefore, net force on the dipole is not zero. Now dipole has both translation as well as rotational motion.

Example–9:

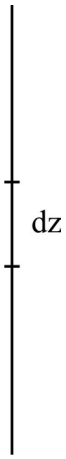
In a certain region of space, electric field is along the +z direction throughout. The magnitude of electric field is, however, not cnostant but increases uniformly along the +z direction; at the rate of (10^5 N/C), per metre. Find the forces and torque, expreirenced by a system having a total dipole moment equal to 10^{-7} cm along the negative direction.

The dipole moment, for a segment of length dz, is

$$dp = qdz$$

The net force, on this elemental dipole, is

$$dF = q\frac{dE}{dz}dz$$



$\therefore$ The total force is

$$F = \int q\frac{dE}{dz}dz$$

$$= 10^{+5}\int qdz$$

$$= 10^{+5}(p) = 10^{+5}\times 10^{-7}\,N - 10^{-2}\,N$$

This force is directed along the direction of the dipole moment, i.e., the (–ve) direction of z–axis.

$$\therefore\ \mathbf{F} = 10^{-2}\left(-\hat{k}\right)N$$

Work done in Rotation of a Dipole

Suppose the dipole is initially kept in a uniform electric field at an angle θ_i ; Now to turn it to a position at an anode θ_f , work done by the field is $W = \int_{\theta_i}^{\theta_f}\tau\, d\theta$.

This work done is stored in the form of electrostatic potential energy. Thus, potential energy of a dipole in uniform electric field will be

$$w = \int_{\theta_i}^{\theta_f}\tau\, d\theta$$

For conservative forces, $W = -\Delta U$

$\therefore\ \Delta U = pE\cos\theta_i - pE\cos\theta_f$

We take $\theta_i = \frac{\pi}{2}$, as the reference position. If $\theta_f = \theta$, we then have

$$U = -pE\cos\theta$$

$$U = -\mathbf{p}.\mathbf{E}$$

$\therefore$ At $\theta = 0^0$, $U = -pE = U_{min}$

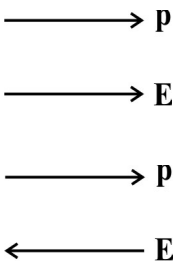
and $\tau = 0$

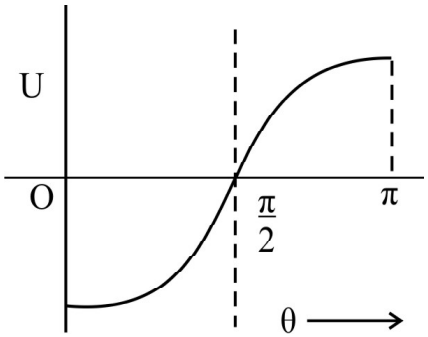
The dipole is in stable equilibrium.

At $\theta = 180^0, U = U_{max} = +pE$

and $\tau = 0$

Here dipole is in unstable equilibrium.





Special Case:

If a dipole is given a small angular displacement 'θ' about its quilibrium (stable equilibrium) position, the dipole will execute angular S.H.M. with time period.

$$T = 2\pi\sqrt{\frac{I}{PE}}$$

Where I is the moment of inertia of the dipole about an axis passing through its centre. This is because the torque will act like a restoring torque:

$$\tau \simeq -pE(\theta) = \frac{Id^2\theta}{dt^2}$$

Electric Flux

The electric flux $\Delta\phi$, of an electric field $\mathbf{E}$, through a small area element $\Delta\mathbf{s}$, is given by

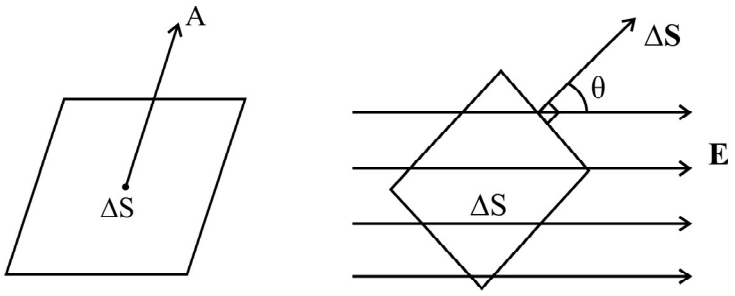
$$\Delta\phi = \mathbf{E}.\Delta\mathbf{s}$$

The vector area element $\Delta\mathbf{s}$ is giveny by

$$\Delta \mathbf{s} = \Delta s\, \hat{n}$$

Δs is the magnitude of the area element and $\hat{n}$ is the (outward) normal to the area element.

For an area element of a closed surface, $\hat{n}$ is taken to be the direction of outward normal. For a closed surface, outward flux is taken as positive while inward flux is taken as negative.



The total flux, ' ϕ ', therefore linking with surface 's' is

$$\phi = \int_s \mathbf{E}.\mathbf{ds}$$

The SI unit of electric flux, is Vm or $NC^{-1}m$.

Gauss Law

The electric flux, through any closed surface 's', is $\frac{1}{\varepsilon_0}$ times the total charge enclosed by 's'.

$$\phi = \oint \mathbf{E}.\mathbf{ds} = \frac{q}{\varepsilon_0} \qquad \text{[For a given 'enclosed' charge]}$$

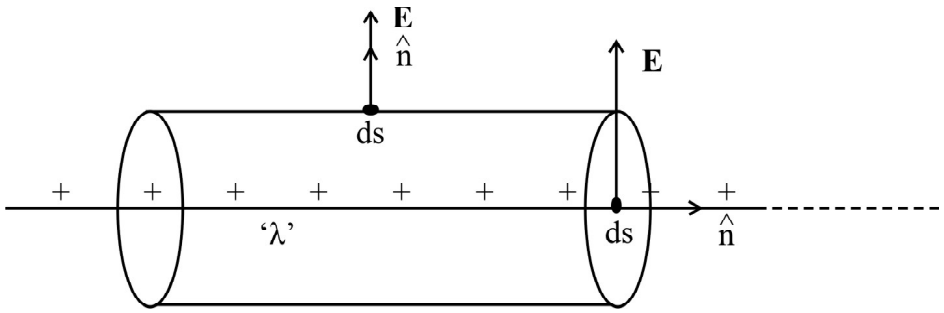
The integral $\oint \mathbf{E}.\mathbf{ds}$, known as surface intergral of electric field over a closed surface, is independent of the shape and size of the surface and position of the charge inside it.

- The term q on the right side of Gauss's law includes the sum of the charges enclosed by the surface. The charges my be located anywhere inside the surface.
- The surface that we choose for application of Gauss's law is called as the Gaussian surface.
- On the left side of gauss law, for calculation of $\mathbf{E}$, all the charges contribute whether they lie inside or outside the Gaussian surface.
- Gauss law is based on the inverse square dependence on distance contained in the Coulomb's law.

Any violation of Gauss's law will indicate departure from the inverse square law.

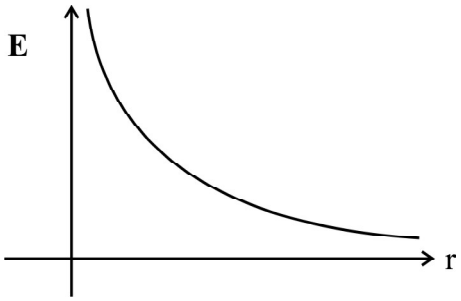
- Gauss law is expecially useful in determining electric field E, when the source distribution has a simple symmetry : like the spherical or planar or cylindrical charge distribution.

(a) Thin infinitely long straight wire of uniform linear charge density 'λ'.

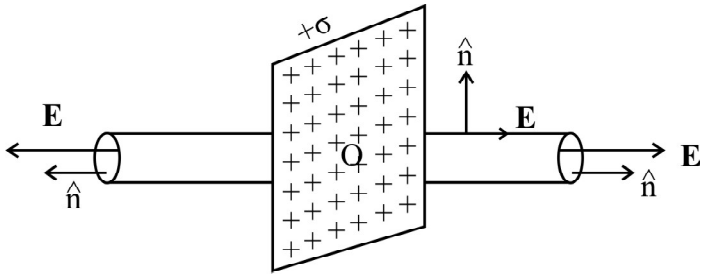


$$\mathbf{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{n}$$

where 'r' is the perpendicular distance of the poinnt from the wire and $\hat{n}$ is the radial unit vector in the plane normal to the wire passing through the point.

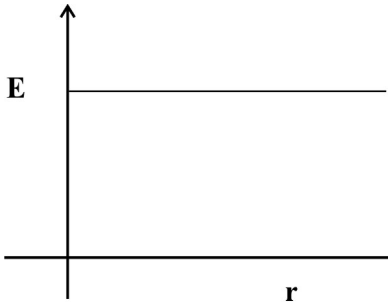


(b) Infinite thin non-conducting plane sheet, of uniform surface charge density 'σ'. (Non-conducting uniformaly charged plane sheet).

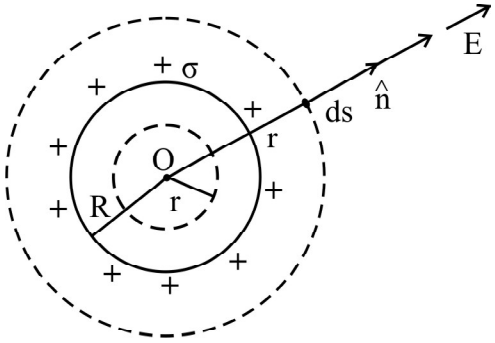


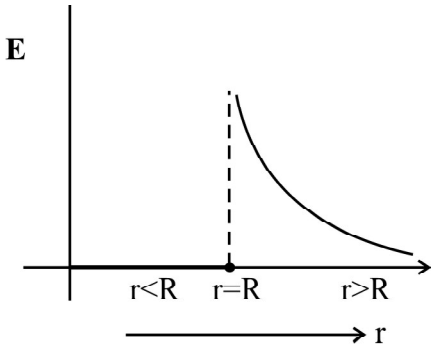
$$\mathbf{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$$

where $\hat{n}$ is a unit vector normal to the plane, outward on either side. For conducting sheet $\mathbf{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$.



(c) Thin shperical shell of uniform surface charge density 'σ'.



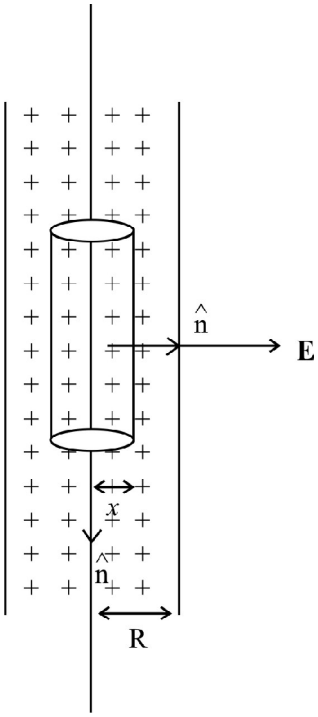


$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} = \frac{\sigma R^2}{\varepsilon_0 r^2}\hat{r}\,(r \geq R)$$

$$= 0\,(r < R)$$

where r is the distance of the point from the centre of the shell and R the radius of the shell.

The electric field inside the shell is zero and outside it behaves as that of a point charge.

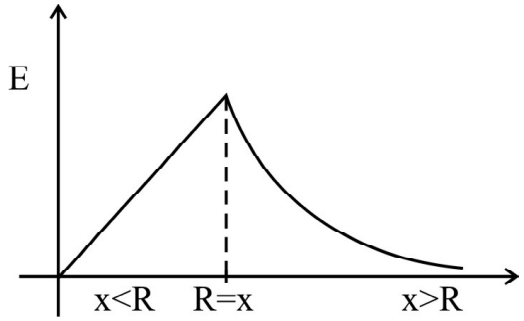


(d) Long (Non-conducting) uniformly charged rod/cylinder having uniform volume charged density 'ρ'.

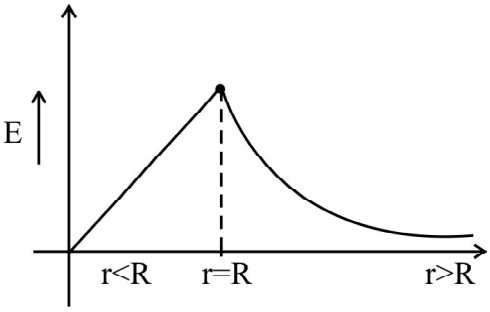
$$E = \frac{\rho R^2}{2x\varepsilon_0}\,(x \geq R)$$

$$= \frac{\rho x}{2\varepsilon_0}(x < R)$$

Where x is the distance of point from the axis of rod / cylinder.



(e) Solid (Non-conducting) sphere of total charge 'q' having volume charge density 'f'.



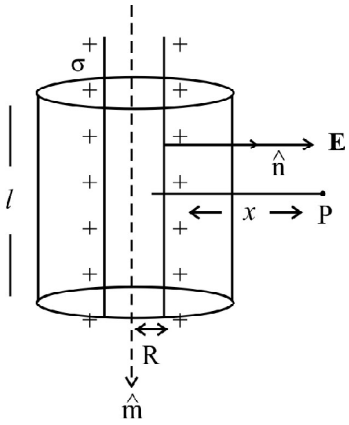
$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} = \frac{\rho R^3}{3\varepsilon_0 r^2}\hat{r}(r \geq R)$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{qr}{R^3}\hat{r} = \frac{\rho r}{3\varepsilon_0}(r < R)$$

For a solid conductor E = 0 (r < R) (conducting solid sphere) inside.

Hence, A conducting solid sphere acts like a spherical shell.

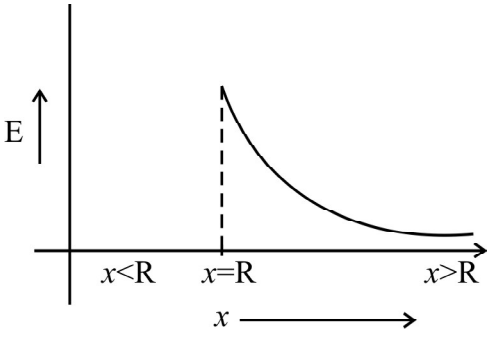
(f) Long uniformaly charged conducting cylinder / rod.



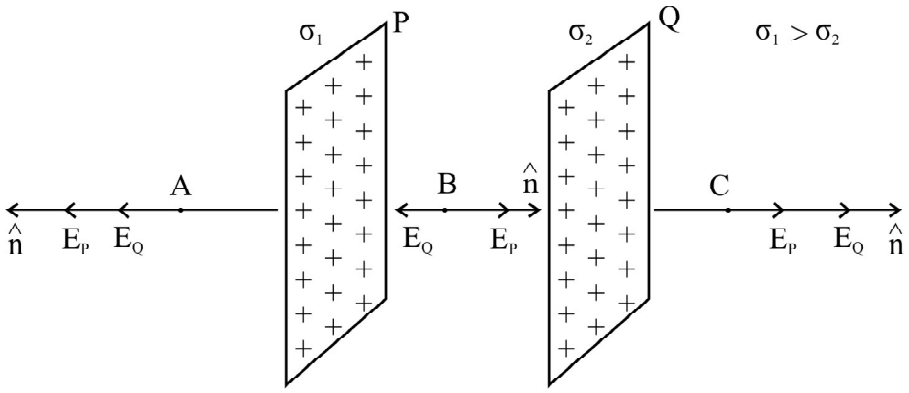
$$E = \frac{\sigma R}{\varepsilon_0 x}(x \geq R)$$

$= 0$ $(x < R)$

For a conductor, electric field strength at interior points is zero.



(g) Two infinite thin plates having uniform surface charge density held parallel to each other.



$$\mathbf{E}_A = -\left(\frac{\sigma_1 + \sigma_2}{2\varepsilon_0}\right)\hat{n}$$

$$\mathbf{E}_C = -\left(\frac{\sigma_1 + \sigma_2}{2\varepsilon_0}\right)\hat{n}$$

$$\mathbf{E}_B = -\left(\frac{\sigma_1 + \sigma_2}{2\varepsilon_0}\right)\hat{n}$$

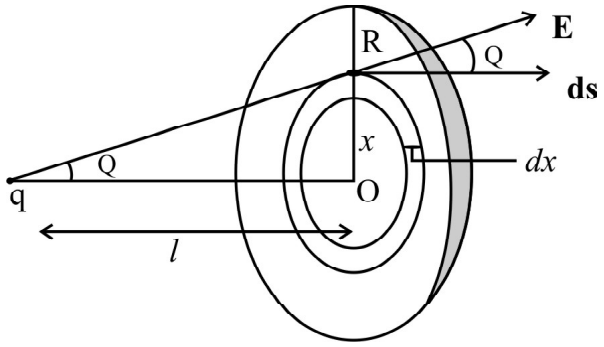
Special Case : For two plane sheets with equal and opposite charge densities.

$E = \frac{\sigma}{2\varepsilon_0}$ directed from 'positive' plate to negative plate, between the plane sheets.

E = 0 outside the plane sheets.

Example–10:

A point charge q is placed at a distance ℓ from a disc of radius R. Find the electric flux through the surface of disc due to this point charge.



Solution:

Consider an elemental ring on the disc surface; of radius x and width dx as shown in the figure.

Area of this strip = $2\pi x dx$

Electric field 'E', due to q, at this ring is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(x^2+\ell^2\right)}$$

If $d\phi$ is the flux passing through this ring, we have,

$$d\phi = E ds\cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(x^2+\ell^2\right)} 2\pi x\, dx \times \frac{\ell}{\left(x^2+\ell^2\right)}$$

$$= \frac{xq\ell dx}{2\varepsilon_0\left(\ell^2+x^2\right)^{3/2}}$$

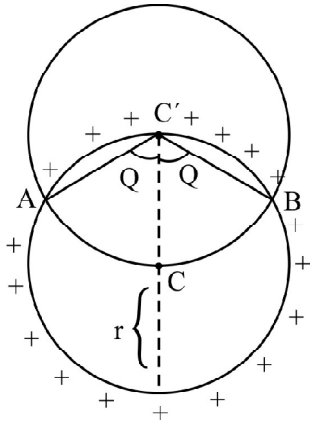
Total flux through the disc surface,

$$\phi = \int_0^R d\phi = \frac{q\ell}{2\varepsilon_0}\int_0^R \frac{x dx}{\left(\ell^2+x^2\right)^{3/2}}$$

$$\Rightarrow \phi = \frac{q\ell}{2\varepsilon_0}\left[\frac{1}{\ell} - \frac{1}{\ell^2+R^2}\right]$$

Example–11:

Consider a ring of radius 'r' on which a charge q has been uniformly distributed. If a shepre of radius r is centred on the circumference of the ring. Find the electric flux passing through the surface of the spere.



Solution:

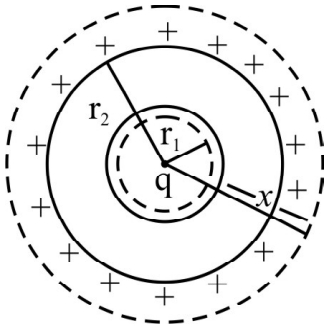
It is clear from the geometry of the figure that $\theta = 60^0$. So one third part of the ring lies inside the sphere

hence, enclosed charge is $\frac{q}{3}$ only.

Using Gauss law, electric flux passing through the surface of sphere.

$$\phi = \frac{1}{3}\frac{q}{\varepsilon_0}$$

Example–12:



A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge 'Q'. A charge of q is placed at the centre of the shell.

(a) What is the surface charge density on the (i) inner surface (ii) outer surface of the shell?

(b) Write the expression for the electric field at a point $x > r_2$ from the centre of the shell.

Solution:

(a) (i) Surface charge density on the inner surface of the shell.

$$\sigma_1 = \frac{-q}{4\pi r_1^2}$$

(ii) Surface charge density on the outer surface of the shell.

$$\sigma_2 = \frac{Q+q}{4\pi r_2^2}$$

(b) Using Gauss's law

$$E4\pi x^2 = \frac{Q+q}{\varepsilon_0}$$

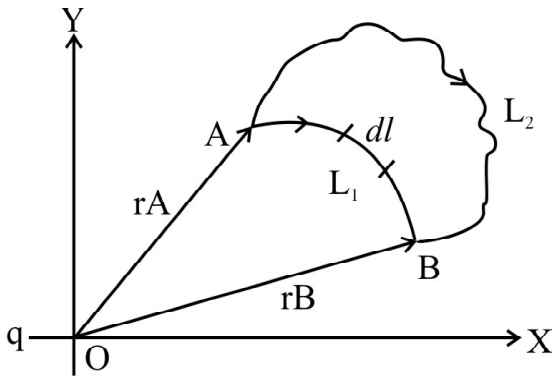
$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q+q}{x^2}$$

The Conservation Nature of the Electrostatic Field

It is an important property of electrostatic fields that the line integral of electric field,

$$\int_A^B \mathbf{E}.\mathbf{d}\ell$$

depends only on the initial and final position and is independent of the path taken in going from A to B.



Consider two points A and B in the electrostatic field of a charge 'q' kept at origin.

We then get: $\int_A^B \mathbf{E}.\mathbf{d}\ell = \frac{1}{4\pi\varepsilon_0} q\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$

The line integral is, therefore, seem to be independent of path connecting A and B. We can conclude that line integral of electrostatic field (does not depends upon the curve L_1 or L_2) depends only on the position vectors of the initial and final points.

For a closed curve the initial and final positions are the same. Hence

$$\oint \mathbf{E}.\mathbf{d}\ell = 0$$

The line integral of electrostatic field over a closed path is zero; the electrostatic field is conservative in nature.

Consider the work done in displacing a unit positive charge through a distance '$\mathrm{d}\ell$' in the field;

$$\mathrm{d}\omega = \mathbf{F}.\mathbf{d}\ell = q\mathbf{E}.\mathbf{d}\ell$$

$$= 1 \times \mathbf{E}.\mathbf{d}\ell = \mathbf{E}.\mathbf{d}\ell$$

Total work done, in moving a unit positive charge from A to B, then is

$$w = \int_A^B dw = \int_A^B \mathbf{E}.\mathbf{d}\ell$$

Thus line intergral of electric field represents the amount of work done is moving a unit positive charge, between two given points, in an electrostatic field.

This work done by the elctrostatic force, is path independent, as electrostatic force is conservative in nature. The concept of potential energy exists only in the case of conservative forces. When electrostatic force exists, between two or more point charges within system, we can assign a potential energy U to the system (due to two interaction of the point charges).

Electrostatic Potential Energy

Consider an electrostatic field 'E' due to a charge Q placed at the origin. Now, imagine that we brieng a test charge q from a point R to a point P against the repulsive force on it due to charge 'Q', without any acceleration. (i.e. we apply an external force. F_{ext} just enough to counter the electrostatic repulsive force F).

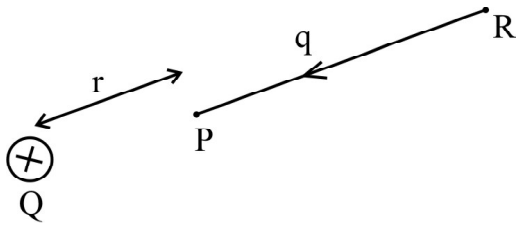
Thus, work done by external force in moving a charge q from R to P.

$$w_{RP} = \int_R^P \mathbf{F}_{ext}.\mathbf{dr} = \int_R^P F dr$$

This work done is against electrostatic repulsive force F and is path independent. At every point, test charge q possesses certain potential energy, this work done increases its potential energy.

Thus, potential energy difference –

$$\Delta U = U_P - U_R = w_{RP}$$



Therefore we can define electric potential energy difference between two points as the work required to be done by an external force in moving (without acceleration) charge q from one point to another for electric field of any arbitary charge configuration.

A convenient choice is to have electrostatic potential energy zero at infinity with his choice if we take point 'R' at infinity, we can write.

$$W_{\infty P} = U_P - U_\infty = U_P = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}$$

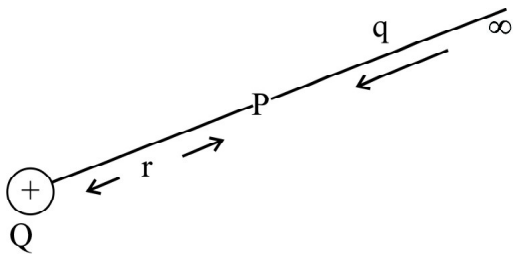
Thus, potential energy of a charge q at a point (in the presence of field due to any charge configuration) is the work done by the external force (equal and opposite to electrostatic force) in bringing the charge q from infinity to that point.

Electric Potential at a Point

Electric / electrostatic potential 'V' at any point in a region with electrostatic field is the work done in bringing a unit positive charge (without acceleration) from infinity to that point.

Thus, $V = \frac{\Delta w}{\Delta q} = \frac{U}{q}$

$$= \int_{\infty}^{r} \mathbf{E}.\mathbf{d}\ell$$



Hence potential V at 'P' due to the charge Q is given by

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

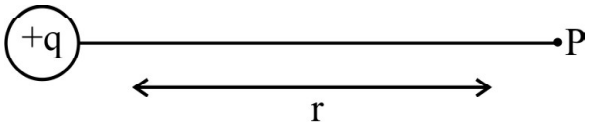
Important Points

- Electric potential is a scalar quantity, its value may be positive, negative or zero.
- SI Unit of potential is $= \frac{1\,Joule}{1\,Coulomb}$ or $1V = \frac{1J}{1C}$.
- Electric potential at a point is also equal to the negative of the work done by the field in taking the unit positive charge from infinity to that point i.e. $V_A - V_B$ = work done in moving a unit positive charge from B to A, or $\int_A^B \mathbf{E}.\mathbf{d}\ell = V_A - V_B$.

  The above result is valid only for electric potential of a point charge. If we wish to find the electric potential due to a continuous charge distribution (charged extended body), we first find the potential due to an element charge 'dq' on the body and then intergrate the expression over the whole charge distribution.

Electric Potential due to Various Charge Distribution

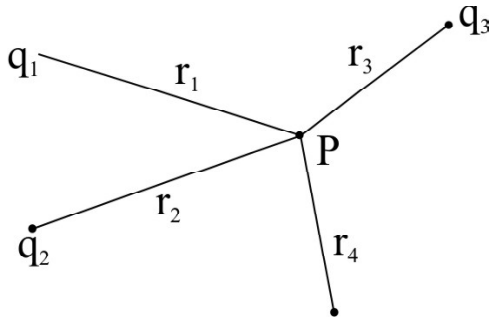
(i) Electric potential due to an isolated point charge at a point 'P'.



$$V = \frac{1}{4\pi\varepsilon_0}$$

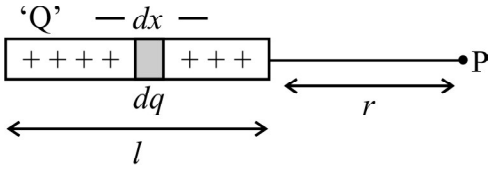
for q > 0, V + ve for q < 0, V – ve

(ii) Electric potential due to a system of 'n' point charges at a point 'P'.



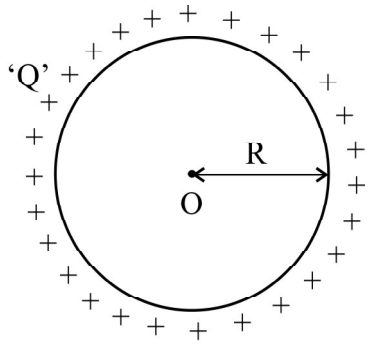
$$V = V_1 + V_2 + V_3 .... + V_n$$

(iii) Electrical potential due to a charged rod of length 'ℓ' at a point P having uniform charge 'θ'.



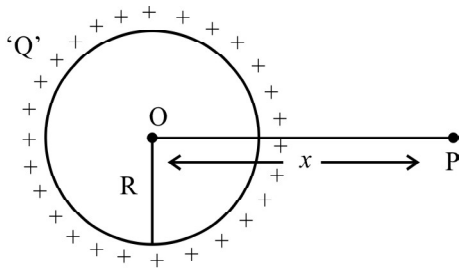
$$V = \frac{1}{4\pi\varepsilon_0}\frac{\theta}{\ell}\,\ell n\,\frac{(r+\ell)}{r}$$

(iv) Electric potential due to a charged ring having uniform charge 'Q' at its centre having radius R.



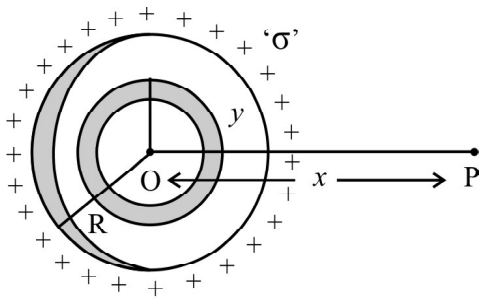
$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

(v) Electric potential due to a uniformally charged sing on the axis of the ring at a point 'P'.



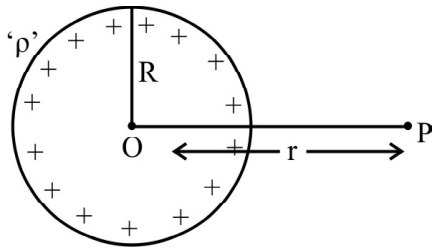
$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2 + x^2}}$$

(vi) Electric field due to a circular disc of radius R with uniform charge density 'σ' at a point 'P'.



$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{x^2 + R^2} - x\right)$$

(vii) Electric field due to a solid spere of radius 'R' with volume charge density 'ρ' and $V = \frac{\rho R^3}{3\varepsilon_0}\frac{1}{r}$ for, $r > R$.



$$V = \frac{\rho R^2}{6\varepsilon_0}\left(\frac{3 - r^2}{R^2}\right) \text{ for } r \le R$$

Example–13:

A metal sphere 0.30m in radius is positively charged with 2 μC. Find the potential at (a) 1m from the centre of sphere (b) a position on the sphere (c) the centre of sphere.

Solution:

(a) $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

$= \frac{9\times10^{9}\times2\times10^{-6}}{(1)}$

$= 18$ KV

(b) $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

$= \frac{9\times10^{9}\times2\times10^{-6}}{0.3}$

$= 60$ KV

(c) Potential at the centre of sphere is same as on the surface.

Example–14:

Eight changed water droplets, each with a radius 1mm and a charge of 10^{-10}C, coalesce to form a single drop. Calculate the potential of the bigger drop.

Solution:

Let R and r be the radii of the bigger and small droplet.

$\Rightarrow$ Volume of 8 droplets = Volume of bigger drop

$= 8\frac{4}{3}\pi r^3 = \frac{4}{3}\pi r^3$

$\Rightarrow R = 2r$

$= 2\times10^{-3}$ m

As charge is conserved total charge $Q = 8\times10^{-10}$ C

Potential of bigger drop $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$

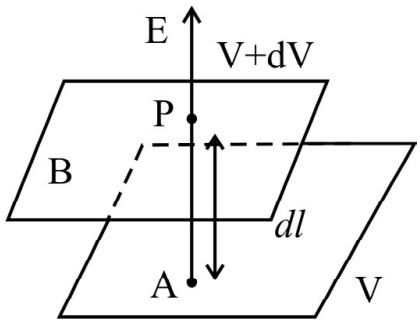
$V = \frac{9\times10^{9}\times8\times10^{-10}}{2\times10^{-3}}$

$= 3.6 \times 10^4$V

Relation between Electric Field Strength and Electric Potential

Consider two surfaces A and B having potential V and V+dV, where dV is the change in potential in the direction of $\mathbf{E}$.

Let a unit positive charge is moved from A to P in the direction of $\mathbf{E}$; then work done in this process is $|\mathbf{E}|\,d\ell$.



Thus, $|\mathbf{E}|\,d\ell = V-(V+dV)$

or $\quad |\mathbf{E}| = -\dfrac{dV}{d\ell} = -\dfrac{dV}{dr}$ as $(d\ell = dr)$

or $\quad \int dV = -\int E dr$

$$\Rightarrow V_B - V_A = -\int_{r_A}^{r_B} E dr$$

Important Conclusion

- Electric field always pionts in the direction in which the potenital decreases steepest.
- Magnitude of the component of electric field (in any direction) is given by the change in the manitude of potential per unit displacement. In otehr words magnitude of component of electric field is numerically equal to the potential gradient.

$$\text{i.e. } \frac{N}{C} = \frac{V}{m}$$

Equipotential Surface

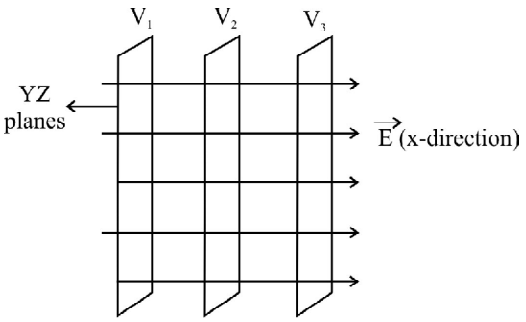
Equipotential surface is a surface over which potential has a constant value.

Hence, no work is required to move a test charge on the equipotential surface. Thus, electric field, at every point, is normal to the equipotential surafce passing through that point.

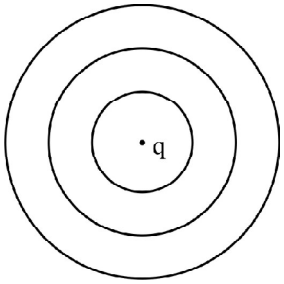
Qualitatively, we can determine the magnitude and direction of electric field by understanding the following points about equipotential surfaces:

(i) The density of equipotential surfaces gives an idea about the magnitude of electric field. Higher the density, larger the field strength.

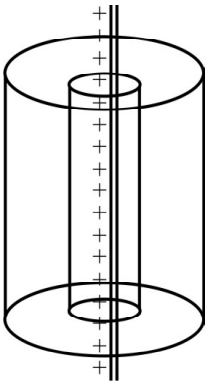
(ii) The direction of electric field is normal to equipotential at every point and points in the direction of decreasing potential.



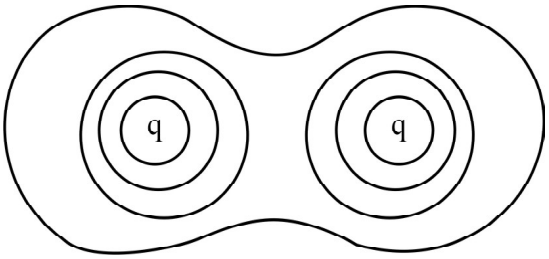
In uniform $\vec{E}$ equipotential surfaces are equally spaced.



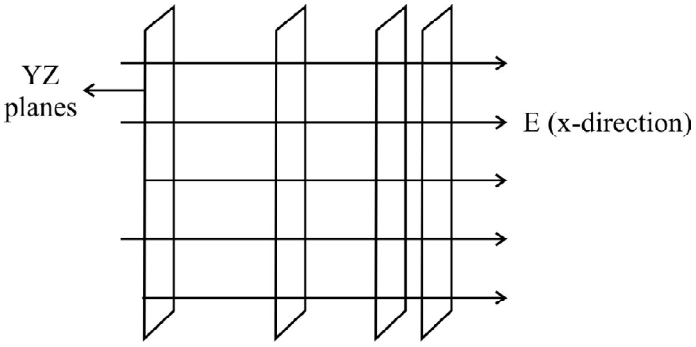
Equipotential surfaces for isolated charge are concentric spherical surfaces



Equipotential surfaces are coaxial cylindrical surfaces for a line charge/uniform charged wire with axis lying at line charge



Equipotential surfaces for two identical charges



Equipotential surfaces for an electric field which increases in magnitude but remains constant in +*x*–direction

Example–15:

The potential of an electrostatic field is given by $V = 2x^2$. Determine the field strength at the point (2m, 0, 3m).

Solution:

We known that $E_x = -\frac{\delta v}{\delta x}; \; E_y = -\frac{\delta v}{\delta y}; \; E_z = -\frac{\delta v}{\delta z}$

$\therefore \; Ex = -\frac{d}{dx}\left(2x^2\right) = -4x$

Also $E_y = E_z = 0$

$\therefore$ In vector form $\mathbf{E} = -4x\hat{i}$

Hence $\mathbf{E}$ at (2m, 0, 3m) is given by

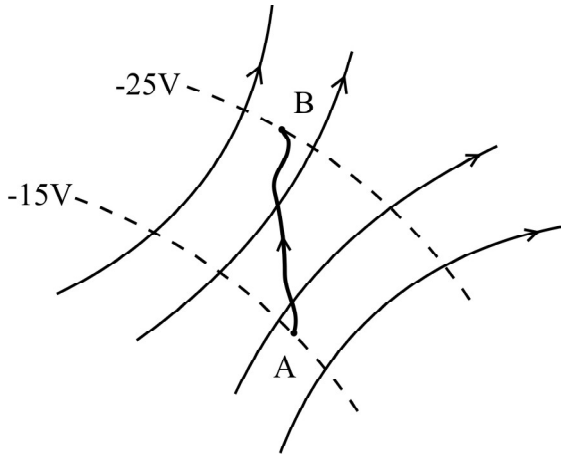
$$E = -4 \times 2\hat{i} = -\left(8\hat{i}\right)\frac{N}{C}$$

Example–16:

The figure shows field lines and equipotentials (dashed lines)

(i) Find the external work done to move a $-2\,\mu C$ charge, at constant speed from A to B, along the path shown.

(ii) Also, determine the work done by the electric field.



Solution:

Since electric field is conservative, therefore, work done by electric force is independent of path.

Using work-energy theorem, we have

$$W_{ext} + W_{electric} = \Delta k$$

The there is no change in speed, $\Delta k = 0$

$$W_{ext} = -W_{electric}$$

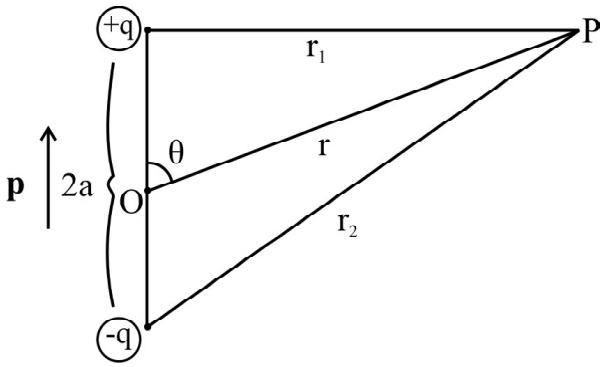
$$W_{ext} = q(V_B - V_A) = -2\times10^{-6}\,(-25-(-15))$$

$$= 20\,\mu J$$

$$\Rightarrow W_{electric} = -20\,\mu J$$

Electric Potential due to an Electric Dipole

Consider an electric dipole consisting of two charges q and –q separated by small distance '2a'. Electric potential at any point 'P' at a distance 'r' from the centre is given by:



$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{4\pi r^2}$$

Where 'θ' is the angle between $\mathbf{p}$ and $\mathbf{r}$.

as $p\cos\theta = \mathbf{p}.\mathbf{r}$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}.\mathbf{r}}{r^2}\left(r >> a\right)$$

if $\theta = 0^0$ or π, the potential an dipole axis is given by:

$$V = \pm\frac{1}{4\pi\varepsilon_0}\frac{p}{r^2} \qquad (+\text{ sign for } \theta = 0^0,\ -\text{sign for} = 180^0)$$

If $\theta = \frac{\pi}{2}$, the potential, in the equitorial plane, is 'zero'.

Important Conclusion

The potential due to a dipole, depends not just on distance r, but also an the angle ' θ ' between $\mathbf{p}$ and $\mathbf{r}$. It, however, has axially summetry about point 'p'.

Electric Potential Energy of a System of 'n' Point Charges

The electric potential energy of a system of 'n' point charges is the work done to assemble this sytem of charges by bringing them in from an infinite distance.

$$U = \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\sum_{j=1}^{n}\sum_{i=1}^{n}\frac{q_i q_j}{r_{ij}}$$

$$i \neq j$$

If U is +ve, it indicates work is done by the external agency to assemble these charges. This work done is stored in form of potential energy. If U is –ve, it indicates work is done by the elctrostatic forces to assemble these charges. This means potential energy is released.

Potential Energy in an External Field

- Potential energy of a single point charge in external field at 'r', U = work done on q against the field.

  $$U = q\,V(r)$$

  Where V (r) is the external potential at the ponit 'r'.

- Potential energy of a system of two charges in an external field,

  U = workdone on q_1 against the field + work done on q_2 against the field + workdone in bringing q_2 in field on q_1. (Mutual interaction energy)

  $$U = q_1V(r_1) + q_2V(r_2) + \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}}$$

- Potential energy of an electric dipole havnig dipole moment 'p' and charges separated by a (small) distance '2a'

  $$U = q\,(V(+q) - V(-q)) - \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2a}$$

  $$V = -\mathbf{p}.\mathbf{E} - \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2a} \text{ where } -\mathbf{p}.\mathbf{E} = \left[q\left[V(+q) - V(-q)\right] = -qE2a\cos\theta = -pE\cos\theta\right]$$

Example–17:

Two charges $7\,\mu C$ and $-2\,\mu C$ are placed at (–9cm, 0, 0) and (9cm, 0, 0) repsectively. Suppose that system of charges is now placed in an external field, $E = A\frac{1}{r^2}, A = 9\times10^5\,cm^2$ what would be the electrostatic energy of configuration?

Solution:

Mutual interaction energy of two charges will be

$$U_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$$

$$= \frac{9\times10^9\times7\times(-2)\times10^{-12}}{0.18}$$

$$= -0.7J$$

The additional energy of interaction in external field will be

$$U_2 = q_1V(r_1) + q_2V(r_2)$$

$$= A\frac{7\mu C}{0.09m} + A\frac{-2\mu C}{0.09m} = 50\,J$$

$$U_{net} = U_1 + U_2$$

$$= (50 - 0.7) = 49.3\,J$$

Electrostatics of Conductors

In conductors, electric charges (elections) are free move throughout the volume of such bodies. When there is not net motion of free charges within the conductor, the conductor is in electrostatic equilibrium. The following are the properties of a conductor in electrostatic equilibrium.

(i) Electrostatic field is zero everywhere inside the conductor.

(ii) Net volume charge density inside a conductor is zero. Any excess charge must reside at the surface.

(iii) At the surface of a charged conductor an electrostatic field must be normal to the surface at every point.

(iv) Electrostatic potential is constant throughout the volume of conductor and has the same value (inside) as an its surafce.

(v) Electric field at the surface of a charged conductor is:

$$\mathbf{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$$

Where σ is the surface charge density and $\hat{n}$ is a unit vector normal to the surface of conductor along the outward direction.

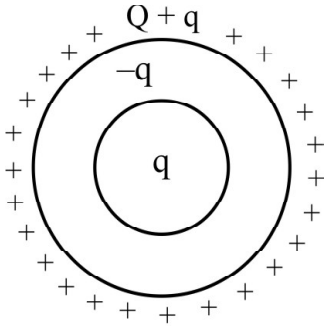
Electrostatic Shielding

If there is a cavity inside the conductor or it is hollow, the field strength inside the cavity equals zero, whatever is the field outside the conductor.

Hence whatever be the charge and field configuration outside, any cavity in a conductor remains sheilded from outside electric infulence. This is known as electrostatic sheilding.

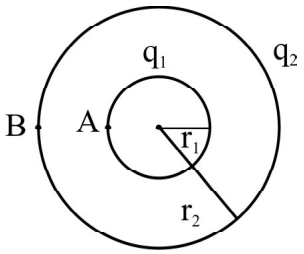
Important Points

- If a charge 'q' is kept in the cavity, then a charge (–q) will be induced on the inner surface and (+q) will be induced on the outer surface of the conductor.
- If a charge 'q' is kept inside the cavity of a conductor, and conductor is given a charge Q, then a charge (–q) will be induced on the inner surface and total charge on the outer surface will be (q+Q)



Example–18:

A small sphere, of radius r_1, having a charge q_1 is, enclosed by a shpherical shell of radius r_2 and charge q_2. Show that if q_1 is positive, charge will necessarily flow from sphere to the shell (when the two are connected by a wire) no matter what the charge q_2 on the shell is.



Solution:

Electric potential at the point 'A'

$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$$

Electric potential at the piont 'B'

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_2} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^2}$$

$$\Rightarrow V_A - V_B = \frac{1}{4\pi\varepsilon_0} q_1 \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

The p.d $(V_A - V_B)$ is independent of q_2. If q_1 is positive, then $(V_A - V_B)$ is positive.

Hence inner small sphere is always at higher potential and if connected by a wire, charge will immediately flow from the sphere to the shell.

Dielectrics

Dielectrics are (generally) non-conducting substances. Glass, plastic, mica, oil etc. are examples of dielectrics. The molecules of a dielectric, substance may be 'polar' or 'non-polar' in nature.

When a dielectric material is placed in an external field, the elctric porperties get modified. In case of non-polar molecules (zero permanent dipole moment of molecules) the positive and negative charges of the molecules experience electrostatic forces in opposite direction. Therefore, the centre of two charges get separated from each other. The molecules thus acquire an induced electric dipole moment in the direction of the field.

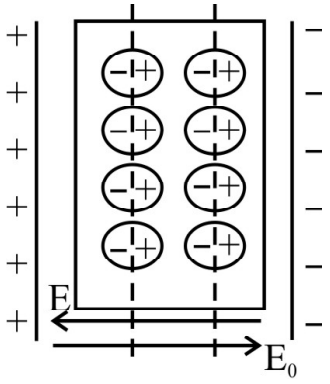
Let us now consider a dielectric whose molecules are polar (i.e. having a permanent electric dipole moments). In the absence of any external electric field, the individual dipoles (molecules) are oriented at random; hence the net dipole moment of the dielectric is zero. When such a dielectric is placed in electric field, the molecules tend to allign themselves parallel to the field due to the torque experienced by them in the field. Thus, a net dipole moment, is produced in the field.

We can thus conclude that, an electric field, induced in a dielectric, (polar or non-polar) a net dipole moment along the direction of the field. This phenomenon is known as dielectric polarisation or polarisation of matter.

Behaviour of dielectric slab is an electric field

Suppose a slab of dielectric material is placed in a uniform electric field E_0 created says, by two parallel plates. Its molecules (electric dipoles) will try to orient themselves in the direction of the field. The net effect

is the appearance of a negative charge on the face of the slab infront of the positive plate and equal positive charge on the face infront of the negative plate. The net charge in the interior is zero. These induced charges produce their own electric field E in the direction opposite to external field $\mathbf{E_0}$. Hence we conclude that when a dielectric is placed in an external field, the field in medium is reduced.



The induced dipole moment, per unit volume, is denoted by the vector $\mathbf{P}$. For a linear dielectric

$$\mathbf{P} = x_e \mathbf{E}$$

Where x_e is a constant called the electric suspectibility.

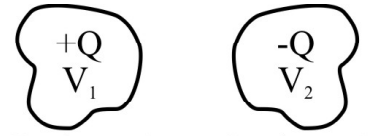
Capacitors

A capacitor is a system of two conductors separated by an insulator usually, in practice, the two conductors have charges $+Q$ and $-Q$, and a potential difference 'V' between them. [Even a single conductor can be used as capacitor by assuming the other conductor at infinity].

The conductors may be charged by connecting them to the two terminals of a battery.

Relation between Q and V

The electric field in the region between the conductors is proportional to the charge Q. That is, if the charge on the capacitor is double the electric field will also be doubled at every point.



$$p.d = V_1 - V_2 = V$$

Consequently, V is also propotional to Q.

Mathematically $Q \propto V$

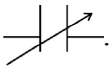
i.e. $Q = CV$, C is a constant of proportionality known as the Capacitance of the capacitor.

The capacitance C depends only on the geometrical configuration (size, shape, and separation) of the system of two conductors.

The SI unit of capacitance is $1\,\text{farad} = \dfrac{1\,\text{coulomb}}{1\,\text{volt}}$ or $1\text{F} = \dfrac{1\text{C}}{1\text{V}}$

The equation shows that for large C, V is small for a given 'Q'. This means a capacitor, with large capacitance can hold large amount of charge, at a relatively small p.d. between the conductors.

The proportionality, between Q and V, is true only untill V becomes so large that dielectric breakdown occurs. If dielectric or insulating, medium breakdown occurs the capacitor no longer behaves like a perfect one. So it is important to keep 'V' below the breakdown potential of the capacitor.

Capacitors are mainly used to store charge and electrical energy. Acapacitor with fixed capacitance is symbollically shown as —| |—, while the one with variable capacitance is shown as  .

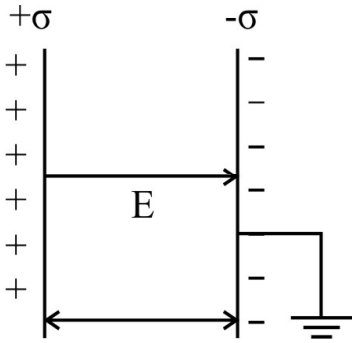
Depending upon their geometry, capacitors are categorised as:

(i) Parallel plate capacitors

(ii) Cylindrical capacitors

(iii) Spherical capacitors

Capacitance of a Parallel Plate Capacitor



A parallel plate capacitor consists of two conducting long plates parallel to each other and separated by a small distance which is small compared to linear dimensions of plates.

The electric field between the plates is given by $E = \frac{\sigma}{\varepsilon_0}$

Where σ is the surface charge density on either plate.

If A is the area of each plate and Q charge on each plate then.

$$E = \frac{Q}{\varepsilon_0 A}$$

As potential difference between the plates in given by $V = Ed = \frac{Q}{\varepsilon_0 A} d$

Capacitance of parallel plate is given by $C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$ in vaccum (or air).

If the space between the plates is filled by a medium, of dielectric constant k, the capacitance.

$$C_m = kC_0$$

Energy Stored in a Capacitor

The energy of a charged capacitor is measured by the total work done in charging the capacitor to a given potential.

$$dw = vdq$$

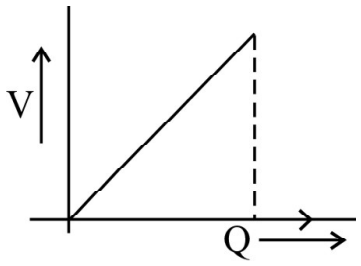
$$w = \int dw = \int_0^Q \frac{q}{c} dq$$

This work done is stored as electrostatic potential energy in the field of capactor and is given by.

$$U = \frac{1}{2}\frac{Q^2}{C}$$

$$= \frac{1}{2} CV^2$$

$$= \frac{1}{2} QV \qquad .....(1)$$



Energy density is the energy stored per unit volume. For the parallel place capacitor, we have $u = U_d Ad$. From this, we get

$$U_d = \frac{1}{2}\varepsilon_0 E^2 \qquad .....(2)$$

Combination of Capacitors

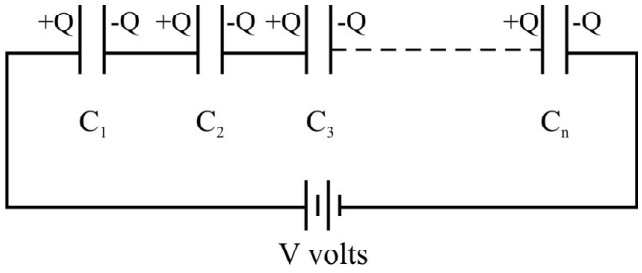
When there is a combination of capacitors in a circuit, we can replace the combination with an equivalent capacitor.

An equivalent capacitor is a single capacitor that has the same value of capacitance as that of actual combination.

(i) Series Combination : All capacitors will have same charge but different potential difference across them. As Q in constant, $V = V_1 + V_2 + V_3 ..... + V_n$. Thus, for capacitors connected Q in series; we get

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + ...\frac{1}{C_n}$$

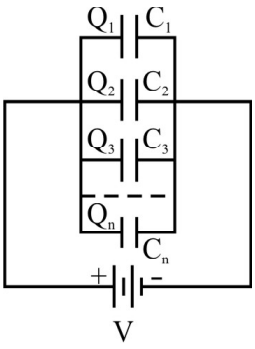
- In series combination the equivalent capacitance is always less than the least value of the capacitances, given in the electrical network.



- As charge in constant, $V \propto \frac{1}{C}$, hence in series combination the smallest capacitor has the maximum potential difference between its plates.
- Energy stored in series combination is given by

$$U_{combination} = \frac{1}{2}\frac{Q^2}{C_{eq}}$$

(ii) Parallel Combination : All the capacitors will have same potential difference but different amount of charges. As V is constant, $Q = Q_1 + Q_2 + Q_3 .... Q_n$. Thus for 'n' capacitors connected in parallel, $C_{eq} = C_1 + C_2 + ...C_n$.



- In parallel combination equivalent capacitance is always greater than the largest value of capacitance given in combination.
- As V is constant, $Q \propto C$ hence maximum change appears across the capacitors of largest value.
- Energy stored in parallel combination is given by:

$$U_{combination} = \frac{1}{2} C_{eq} V^2$$

Redistribution of Charges

Consider two insulated capacitors A and B of capacitance C_1 and C_2, carrying charges q_1 and q_2 respectively. Let V_1 and V_2 be their respetive potentials. When the two capacitors are joined by a thin metal wire, the positive changes will flow from higher potential to lower potential till both the capacitors accquire some

common potential. Thus charges are redistributed but total charge $q = q_1 + q_2$ remains constant.

$$V_{common} = \frac{Q_{net}}{C_{net}} = \frac{q_1 + q_2}{C_1 + C_2}$$

$$= \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

Before connection:

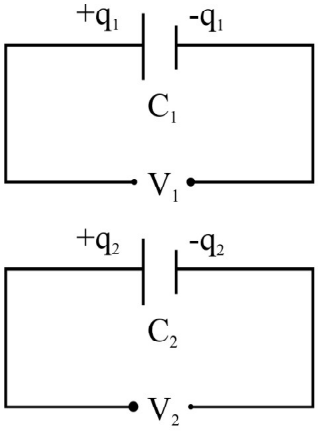
$$U_i = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_1^2$$

After connectino:

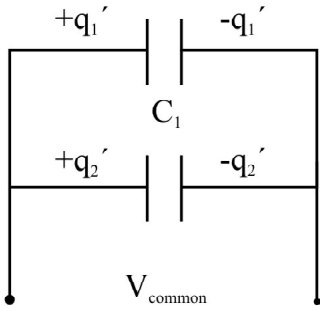
$$U_f = \frac{1}{2} (C_{net}) V_{common}^2$$

Hence there is a loss in electrostatic energy in the form of heat.

$$|U_f - U_i| = \Delta U = \frac{1}{2} \frac{C_1 C_2}{C_1 + C_2} (V_1 - V_2)^2$$



Before connection



After connection

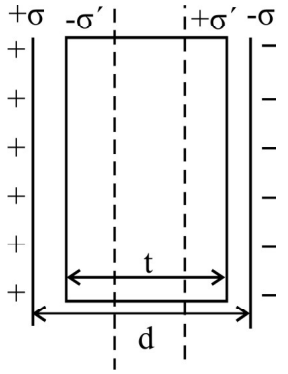
Change on C_1 after connection $q_1^1 = C_1 V_{common} = \left(\frac{C_1}{C_1 + C_2} \right) q$

Change on C_2 after connection $q_2^1 = C_2 V_{common} = \left(\frac{C_2}{C_1 + C_2} \right) q$

'Dielectric' in a Parallel Plate Capacitor

Consider a parallel plate capacitor, having two large plates, each of area A, separated by a distance 'd' in vacuum.

Situation–1: A dielectric, of constant k, is inserted between the plates of a fully charged capacitor, after the supply was disconnected.



Let the thickness of dielectric slab be t (t < d).

As capacitor is fully changed and battery disconnected, charge on each plate,

$$q = q_0 = \text{Constant}$$

$$V = E_0(d - t) + E_{mt}$$

$$\Rightarrow V = V_0\left(1 - \frac{t}{d} + \frac{t}{kd}\right)$$

The potential between the plates decreases from its value V_0 (in vacuum or air) as $C = \frac{q}{V} = \frac{q_0}{V}$ it comes out, $C = \frac{C_0}{\left(1 - \frac{t}{d} + \frac{t}{kd}\right)}$, the capacitance increases from its value C_0 (in vacuum or air)

$$U = \frac{1}{2}\frac{q_0^2}{C} = U_0\left(1 - \frac{t}{d} + \frac{t}{kd}\right)$$

The energy stored in the field of capacitor decreases from its original value U_0 (in vacuum or air)

If t = d, (the dielectric completely fills the space between the plates).

We have, $q = q_0$

and $V = \frac{V_0}{k}$

$\therefore C = C_0 k$

and $U = \frac{U_0}{k}$

Situation–2: A dielectric slab of constant k is inserted between the plates of the capacitor while the voltage supply remain connected (during charging of the capacitor).

$V = V_0 = \text{constant}$

$C = kC_0$, capacitance increases from its original value C_0 (in vacuum or air).

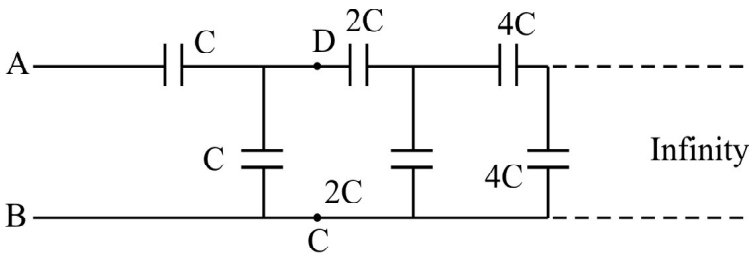
$q- = kq_0$, change storage increases from its original value q_0 (in vacuum or air).

$$U = \frac{1}{2}CV^2 = kV_0$$

Energy stored from its original value V_0 (in air or vacuum).

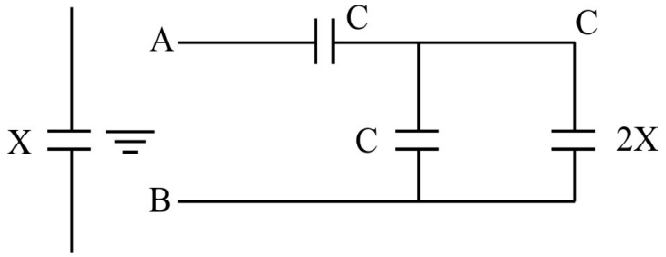
Example–19:

Find the equivalent capacitance between points A and B.



Solution:

Let the equivalent capacitance is *x*, then net capacitance after CD point will be '2*x*' because every capaciatnce becomes 2 times as compared A and B.



$$\Rightarrow x = \frac{(C + 2x)C}{C + 2x + C}$$

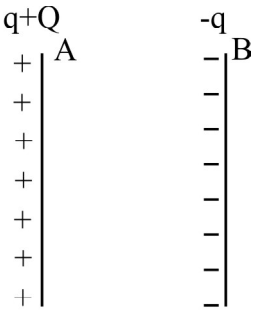
$$\Rightarrow x = \frac{C}{\sqrt{2}}$$

Example–20:

In a parallel plate capacitor of capacitance C, the plate A carries a positive charge and the plate B carries a negative charge. The potential difference between the plates is V_0. If the plate A is given an additional charge '+Q', then determine the potential difference between the plates.

Solution:

It is important to note that if the capacitance of the parallel plate capacitor is 'C', then the capacitance of each isolated plate will be 2C.



Thus, the potential of plate A will be,

$$V_A = \frac{q + Q}{2C}$$

and that of B will be, $V_B = \frac{-q}{2C}$.

The p.d. between the plates, now will be:

$$\Rightarrow V = V_A - V_B$$

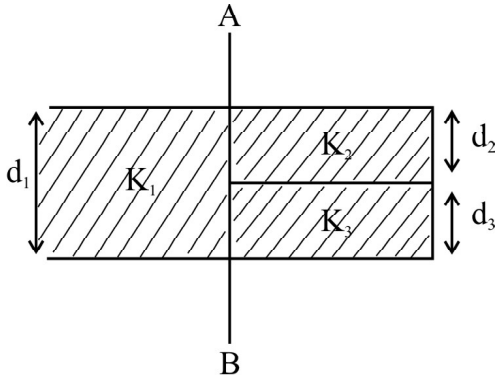
$$V = \frac{Q + q}{2C} - \frac{q}{2C}$$

$$V = \frac{q}{C} + \frac{Q}{2C}$$

Therefore $V = V_0 + \frac{Q}{2C}$ (Since $q = CV_0$ given)

Example–21:

Find out capacitance between A and B if three dielectric slabs of dielectric constant K_1, of area A_1 and thickness d_1, K_2 of area A_2 and thickness d_2 and K_3 of area A_3 and thickness d_3 are inserted between the plates of parallel plate capacitor of plate area A as shown in figure. (Given distance between the two plates $d_1 = d_2 + d_3$)



Solution:

The capacitor behaves like three individual capacitors C_1, C_2 and C_3 such that C_2 & C_3 are in series and their resultant is in parallel to C_1.

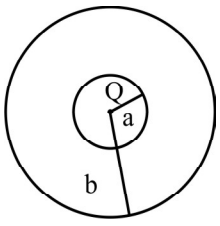
Thus, $C_{eq} = C_1 + \dfrac{C_2C_3}{C_2 + C_3}$

$$= \frac{A_1K_1\varepsilon_0}{d_2 + d_3} + \frac{\dfrac{A_2K_2\varepsilon_0}{d_2}\dfrac{A_3K_3\varepsilon_0}{d_3}}{\dfrac{A_2K_2\varepsilon_0}{d_2}\dfrac{A_3K_3\varepsilon_0}{d_3}}$$

$$\Rightarrow C_{eq} = \frac{A_1K_1\varepsilon_0}{d_2 + d_3} + \frac{A_2K_2K_2\varepsilon_0}{K_2d_3 + k_3d_2}$$

ADDITIONAL EXAMPLES

1. The region between two concentric spheres of radii 'a' and 'b', respectively (see figure), has volume charge density $\rho = \dfrac{A}{r}$, where A is a constant and r is the distance from the centre. At the centre of the sphere is a point charge Q. The value of A such that the electric field in the region between the sphere will be constant is:



(1) $\dfrac{Q}{2\pi\left(b^2 - a^2\right)}$ (2) $\dfrac{2Q}{\pi\left(b^2 - a^2\right)}$

(3) $\frac{2Q}{\pi a^2}$ (4) $\frac{Q}{2\pi a^2}$

Correct option is (4) (IIT 2016)

Solution:

Let E be the electric field at any point distance r from the centre. We than have

E = electric field, at a distance r, due to a point charge Q' where,

Q' = Q + charge in the region between r = a and r = r

$$= Q + \int_0^r \rho.\left(4\pi r^2 dr\right)$$

$$= Q + 2\pi\left(r^2 - a^2\right)A$$

$$\therefore\ E = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q + 2\pi\left(r^2 - a^2\right)A}{r^2}\right]$$

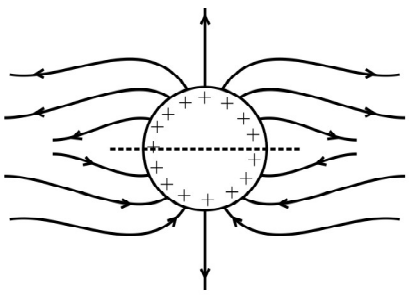
This has to be independent of r. We, therefore, have

$$\frac{Q}{r^2} = \left(\frac{2\pi a^2 A}{r^2}\right) = 0$$

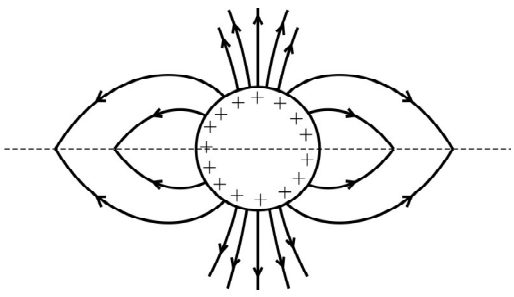
$$\therefore\ A = \frac{Q}{2\pi a^2}$$

2. A long cylinderal shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in:

(Figures are schematic and not drawn to scale)



(1)



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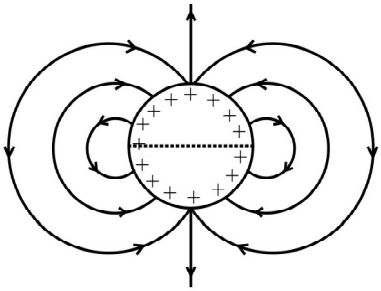
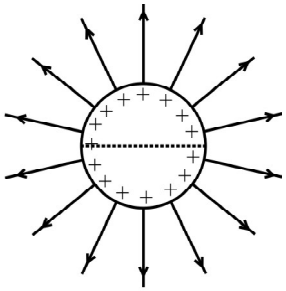


(3)



(4)

Correct option is (3) (IIT 2015)

Solution:

The correct figure is figure 3. This is because electric field lines start from positive charges and terminate on negative charges.

3. Assume that an electric field $E = 30x\,i$ exists in space. Then the potential difference $V_A - V_0$, where V_0 is the potential at the origin and V_A the potential at $x = 2$ m is:

(1) –80 V (2) 80 V

(3) 150 V (4) –150 V

Correct option is (1) (IIT 2014)

Solution:

We have, $E_x = -\dfrac{dv}{dx}$

$\therefore\ dv = -E_x dx$

$\therefore\ V_A - V_0 = -\int_0^2 30x^2 dx$

$= -30\left|\dfrac{x^3}{3}\right|_0^2 = -80\,V$

4. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potentials $\dfrac{3V_0}{2}$, $\dfrac{5V_0}{4}$, $\dfrac{3V_0}{4}$ and $\dfrac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then

(1) $R_1 = 0$ and $R_2 < (R_4 - R_3)$ (2) $2R < R_4$

(3) $R_1 = 0$ and $R_2 > (R_4 - R_3)$ (4) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$

Correct option is (1) (IIT 2016)

Solution:

Let the sphere have a charge +Q. Then Potential

(i) On the surface $V_0 = \frac{KQ}{R}$

(ii) inside $(0 < r < R)$ $V_1 = \frac{KQ}{2R^3}\left(3R^2 - r^2\right)$

$\therefore \frac{3V_0}{2} = \frac{3}{2}\frac{KQ}{R}$ (At the cenre r = 0)

Therefore, radius of equi-potential at the centre of sphere is zero. $R_1 = 0$

Let R_2 be the radius of second equi-potential surface

$\therefore \frac{5V_0}{4} = \frac{KQ}{2R_3}\left(3R^2 - R_2^2\right)$

$\therefore 2R_2^2 = R^2 \Rightarrow R_2 = \frac{2}{\sqrt{2}}$

Let R_3 be the radius of third equi-potential surface

$\therefore \frac{3V_0}{4} = \frac{KQ}{R_3}$

$\frac{3KQ}{4R} = \frac{KQ}{R_3} \Rightarrow R_3 = \frac{4R}{3}$

Let R_4 be the radius of third equi-potential surface

$\frac{V_0}{4} = \frac{KQ}{R_4}$

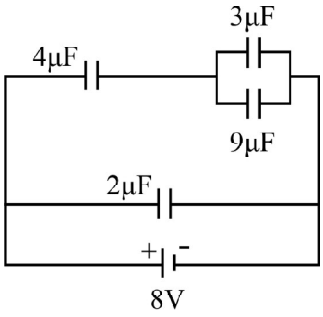
$\therefore \frac{KQ}{4R} = \frac{KQ}{R_4} \Rightarrow R_4 = 4R$

$\therefore R_4 - R_3 = 4R - \frac{4R}{3} = \frac{8R}{3} = 2.66\,R$

and $R_2 = \dfrac{R}{1.414} < R$

$\therefore\ R_2 < (R_4 - R_3)$

5. A combination of capacitor is set up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the $4\mu F$ and $9\mu F$ capacitors), at a point distant 30m from it, would equal:



(1) 360 N/C (2) 420 N/C

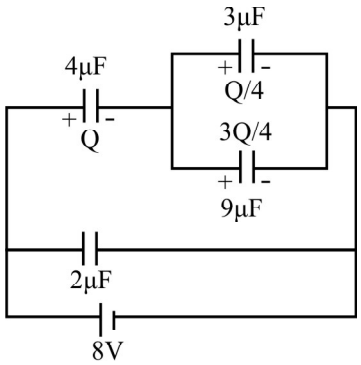
(3) 480 N/C (4) 240 N/C

Correct option is (2) (IIT 2015)

Solution:

The equivalent capacitance of the top row is $3\mu F$.

$\therefore\ Q = 3 \times 8\mu C = 24\mu C$



This charge, on the $4\mu F$ capacitor, gets divided, in the ratio 1:3 between the $3\mu F$ and $9\mu F$ capacitor.

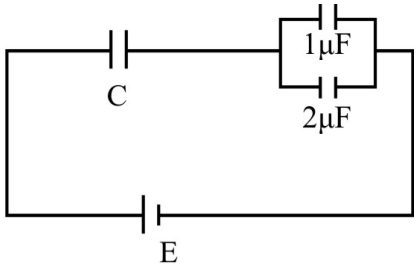
Hence $Q_4 = \dfrac{3}{4} \times 24\mu C$

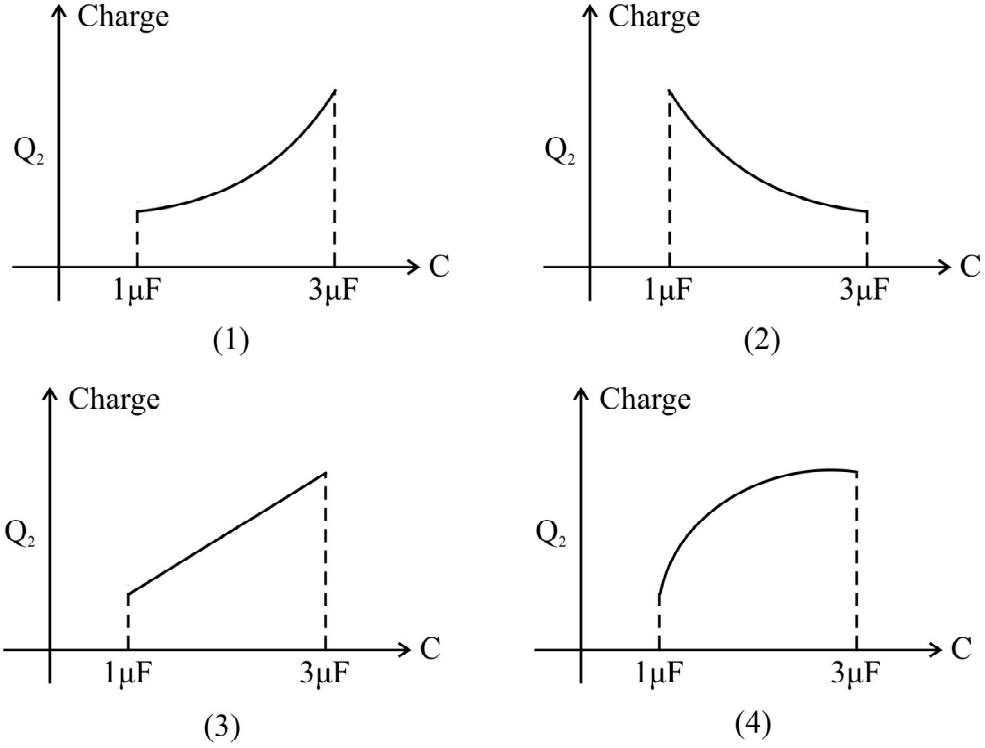
$= 18\mu C$

$\therefore$ The charges on $4\mu F$ and $9\mu F$ capacitros are charges $24\mu C$ and $18\mu C$ respectively.

Hence, $E = \dfrac{9 \times 10^9 \times (24 + 18)\mu C}{30 \times 30} = 420\ N/C$

6. In the given circuit, charge Q_2 on the $2\mu F$ capacitor changes as C is varied from $1\mu F$ to $3\mu F$. Q_2 as a function of 'C' is given properly by : (figures are drawn schematically and are not to scale).





Correct option is (1) (IIT 2015)

Solution:

For $C = 1\mu F$, equivalent capacitance $= \frac{3}{4}\mu F$

$\therefore\ Q_2 = \frac{3}{4}V \times \frac{2}{3}\mu C = \frac{V}{2}\mu C$

For $C = 2\,\mu F$, Equivalent capacitance $= \frac{5}{6}\mu F$

$\therefore\ Q_2 = \frac{5}{6}V \times \frac{2}{3}\mu C = \frac{5V}{8}\mu C$

For $C = 3\,\mu F$, Equivalent capacitance $= \frac{3}{2}\mu F$

$\therefore\ Q_2 = \frac{3}{2}V \times \frac{2}{3}\mu C = V\mu C$

$\therefore$ Q_2 iscreases (non-linearly) with increasing C.

Also change in Q_2, in changing C from 2 μ F to 3 μ F is more than that in changing C from 1 μ F to 3 μ F.

$\therefore$ Graph 1 is the correct graph.

7. A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the charge density of the positive plate will be close to:

(1) 3×10^4 C/m^2 (2) 6×10^4 C/m^2

(3) 6×10^{-7} C/m^2 (4) 3×10^{-7} C/m^2

Correct option is (3) (IIT 2014)

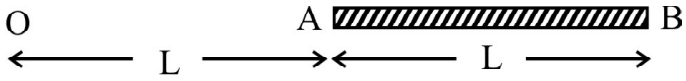
Solution:

Electric field (between the plates of the parallel plate capacitor) $E = \dfrac{\sigma}{K\varepsilon_0}$

$\therefore\ \sigma = EK\varepsilon_0 = 3\times10^4 \times 2.2 \times 8.85\times10^{-12}\,C/m^2$

$\cong 6\times10^{-7}\,C/m^2$

8. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the print O lying at a distance L from the end A is:

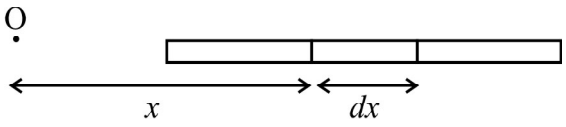


(1) $\dfrac{3Q}{4\pi\varepsilon_0 L}$ (2) $\dfrac{Q}{4\pi\varepsilon_0 L \ln 2}$

(3) $\dfrac{Q\ln 2}{4\pi\varepsilon_0 L}$ (4) $\dfrac{Q}{8\pi\varepsilon_0 L}$

Correct option is (3) (IIT 2013)

Solution:



Potential at O, due to the small element dx, is $dV = \dfrac{1}{4\pi\varepsilon_0}\left[\left(\dfrac{Q}{L}\right)dx\right].\dfrac{1}{x}$

$$\therefore V = \frac{Q}{4\pi\varepsilon_0 L}\int_{x=L}^{x=2L}\frac{dx}{x}$$

$$= \frac{Q(\ell n\, 2)}{4\pi\varepsilon_0 L}$$

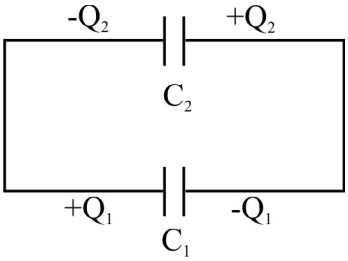
9. Two capacitors C_1 and C_2 are charged to 120 V and 200 V respectively. It is found that by connecting them together the potential on each one can be made zero. Then:

(1) $3C_1+5C_2$ (2) $3C_1+5C_2 = 0$

(3) $9C_1+4C_2$ (4) $5C_1+3C_2$

Correct option is (1) (IIT 2013)

Solution:



We need to have the charges on the two, to be equal in magnitude.

$\therefore\ 120\,C_1 = 200\,C_2$

$\therefore\ 3C_1 = 5C_2$

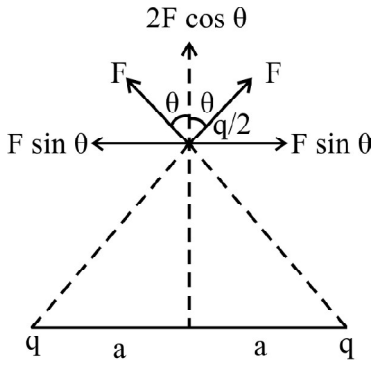
10. Two charges, each equal to q, are kept at $x = -a$ and $x = a$ on the x–axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement ($y << a$) along the y–axis, the net force acting on the particle is proportional to:

(1) $-y$ (2) $\frac{1}{y}$

(3) $-\frac{1}{y}$ (4) y

Correct option is (4) (IIT 2013)

Solution:



We see from the figure, that $F_{net} = 2F\cos\theta$

$$= 2F.\frac{y}{\sqrt{a^2+y^2}}$$

$$= 2.\frac{1}{4\pi\varepsilon_0}\frac{\left(q.\frac{q}{2}\right)}{\left(a^2+y^2\right)}\frac{y}{\sqrt{a^2+y^2}}$$

$$= \frac{q^2}{4\pi\varepsilon_0}.\frac{1}{\left(a^2+y^2\right)^{3/2}}.y$$

For $y << a$, we, therefore, have

$$F_{net} \simeq \left(\frac{q^2}{4\pi\varepsilon_0 a^3}\right)y$$

$\therefore F_{net} \alpha\, y$

PROBLEMS FOR PRACTICE

1. Two equal point charges $Q = +\sqrt{2}\mu C$ are placed at each of the two opposite corners of a square and equal point charge q at each of the other two corners. What must be the value of q so that the resultant force on Q is zero?

   Ans: $0.5\,\mu C$

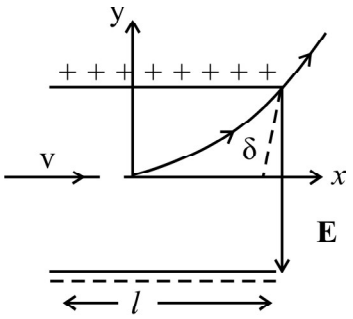
2. Two identical charged spheres are suspended in air by strings of equal lengths and make an angle of 30^0 with each other. When suspended in a liquid of density 0.8 g/cm^3 the angle remains the same. What is the dielectric constant of the liquid? (Density of the material of the spheres = 1.6 g/cm^3).

   Ans: K = 2

3. ABC is an equilateral triangle of side 10m and D is the midpoint of BC. Charges of +100, –100 and $+75\,\mu C$ are placed at B, C, and D respectively. Find the force on a $+1\,\mu C$ charge placed at A.

   Ans: $9\sqrt{2}\times10^{-3}\,N,$ at 45^0 to a line parallel to BC

4. There is a uniform electric field E between two parallel plates. An electron moving perpendicular to the field enters with velocity V. Assuming the force of gravity to be negligible, derive an expression for the vertical displacement of the electron when emerging from the plates. Charge of the electron in e and mass m.



   Ans: $\left[y = \frac{eE}{2mv^2}x^2, \delta = \frac{eE}{2mv^2}\ell^2 \right]$

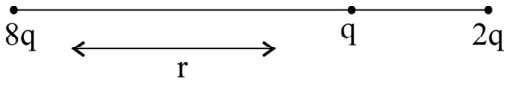
5. Two point charges of +16 μC and –9 μC are placed 8cm apart in air. Determine the position of the ponit at which the resultant electric field is zero.

   Ans: 24 cm to the right of –9 μC charge

6. Three point charges q, 2q and 8q are placed on a 9cm long straight line. Find the positions where the charges should be placed such that potential energy of this sytem is minimum. In this situation, what will be the electric field at the position of the change q due to the other two charges.

   [**Hint:** For minimum potential energy the charges of maximum magnitude should be farthest apart.

   Such that $\frac{dU}{dr} = 0$.

   U will be minimum for r = 6cm. The electric field at q within situation is E = 0.]

7. Three point charges of 1C, 2C and 3C are placed at the corners of an equilateral triangle of side 1m. Calculate the work required to move these charges to the corners of a smaller equilateral triangle of side 0.5m.

   Ans: 99×10^9 J

   [**Hint:** Work required is stored as the additional potential energy of the system.]

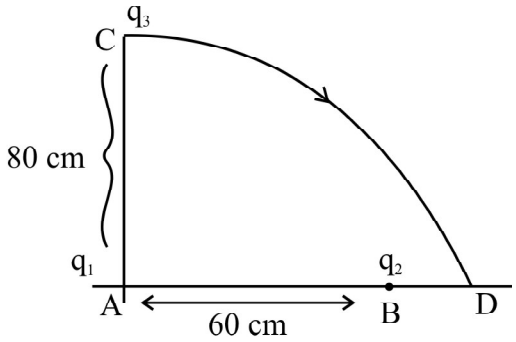
8. A change Q is distributed over two concentric hollow spheres of radii r and R (> r) such that the surface charge densities are equal. Find the potential at the common centre.

   Ans: $V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{R+r}{r^2+R^2}\right)$

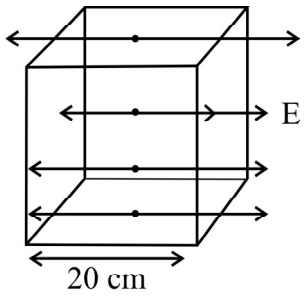
9. Two electric dipoles of moments p_1 and p_2 are in straight line show that the potential energy of each in the presence of other is $-\frac{1}{4\pi\varepsilon_0}\frac{p_1 p_2}{r_3}$, r is the distance between the dipoles (r is much greater than the length of the dipole).

10. Two charges $q_1 = +2\times10^{-8}$C and $q_2 = -0.4\times10^{-8}$C are placed 60 cm apart as shown. A third charge $q_3 = +0.2\times10^{-8}$C is moved along the arc of a circle of radius 80 cm from C to D. Find the percentage change in the potential energy of q_3.

    Ans: 76%



11. In the figure given, the electric field E = 100 N/C is shown. The side of the cube is 20 cm. Find the nature of the charge which is inside the cube and the magnitude of the charge given $\varepsilon_0 = 8.85\times10^{-12} C^2 / Nm^2$.

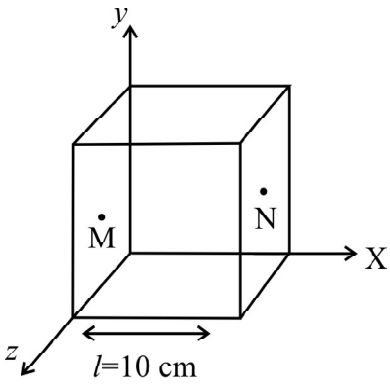


Ans: $q = +7.08\times10^{-11} C$

12. In a region of space, the electric field is given by $\mathbf{E} = 8\hat{i} + 4\hat{j} + 3\hat{k}$. Calculate the electric flux through a surface of area 100 units in *x*–y plane.

Ans: 300 Units

13. Electric field in the given figure is directed along +*x* direction and given by $E_x = 5A_x + 2B$. Where E is in N/C and *x* is in metre, A and B are constants with dimensions. Taking A = 10 N/Cm and B 5 N/C. Calculate



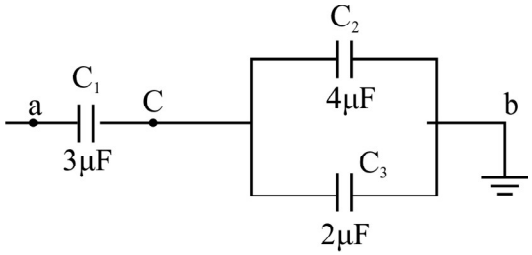
(i) The electric flux through the cube.

(ii) Net charge enlosed within the cube.

Given $\varepsilon_0 = 8.85\times10^{-12} C^2 / Nm^2$

Ans: $\phi_{net} = 0.05 N / Cm^2$, $q = 44.25\times10^{-14} C$

14. In the given circuit, if the ponit p is connected to earth and a potential of +1200 V is given to point a, calculate the charges accquired by each of the capacitors and the potential of the point C?



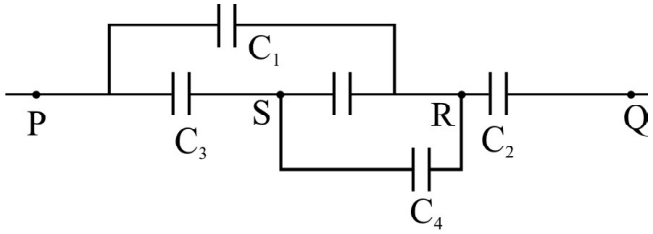
Ans: Charge on $C_1 = 2.4\times10^{-3} C$

Charge on $C_2 = 1.6\times10^{-3} C$

Charge on $C_3 = 0.8\times10^{-3} C$

Potential at C = 400 V

15. In the given network of capacitor, find the effective capacitance betwen the points P and Q. Given, $C_1 = C_2 = C_3 = 4\,\mu F$ and $C_5 = 5\,\mu F$. If a 10V battery be connected across P and Q, what will be the charges on the capacitors?



Hint: This is a wheatsone bridge arrangement. The bridge formed by capacitors is balanced R & S are at same potential. Charges on C_1, C_2, C_3 and C_4 are 20 μC and on $C_5 = 0$, C_{eq}=4 μF.

16. A 20 μF capacitor charged to 100 V is connected in parallel to a 10 μF capacitor charged to 100 V. Find the loss in energy.

    Ans: 2.70 J

17. Two capacitors 25 μF and 100 μF, connected in sereis are charged to 120V battery. The battery is then removed. The capactitors are now connected in parallel. Find the (i) p.d. across each and (ii) energy loss in the process.

    Ans: p.d. across each = 38.4V, Energy loss = 0.0518J

18. A parallel plate capacitor contains one mica sheet of thickness $d_1 = 1\times10^{-3}$m and one fibre sheet of thickness $d_2 = 0.5\times10^{-3}$m. Th dielectric constant of mica and fibre are 8 and 2.5 respectively. Fibre breaksdown in an electric field of 6.4×10^6 V/m. What maximum voltage can be applied to the capacitor.

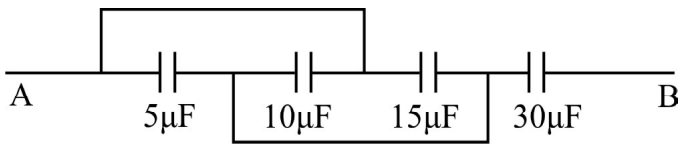
    Ans: 5200 V

19. A 2 μF parallel plate capactor with dielectric slab (K = 5) between the plates is charged to 100 V and then isolated. What will be the p.d. if the dielectric be removed? How much work will be done in removing the dielectric?

    Ans: (i) 500 V

    (ii) 0.20 J

20. Find the equivalent capacitance between the given combination.



    Ans: 30 μF

QUESTION BANK

Key Learning Points

1. There are two types of charges in nature, like charges repel and unlike charges attract.
2. Electric charges has three basic properties; quantisation, additivity and conservation.
3. Coulomb's Law: The mutual electrostatic force between two point charges (stationary) is directly propotional to the product q_1q_2 and inversely propotional to the square of the distance separeting them. $F = \frac{Kq_1q_2}{r^2}$ where K is constant of proportionality, having value in SI $9\times10^9\ Nm^2C^{-2}$ in air or vacuum. Also $K = \frac{1}{4\pi\varepsilon_0}$, ε_0 is called absolute permittivity of the free space, air or vaccum.

   $\varepsilon_0 = 8.85\times10^{-12}\ C^2N^{-1}m^{-2}$

   In vector form, coulamb's law is written as

   $$\mathbf{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}^2}\hat{r}_{21} = -\mathbf{F}_{21}$$

4. Electrostatic forces in medium decreases by a factor of dielectric constant of medium (k)

   $$\frac{F_{vacuum}}{F_{medium}} = k$$

5. We can use superposition principle for computing the net force on any charge due to any configuration of charges.

   $$\mathbf{F}_1 = \frac{1}{4\pi\varepsilon_0}q_1\sum_{i=2}^{n}\frac{q_i.q_j}{r_{ij}^2}\hat{r}_{ji}$$

6. Electric field strength $\vec{E}$ due to a source charge 'Q' is independent of test charge q_0 and is given by

   $$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

   It is radially outwards from Q, if Q is positive and radially inwords if Q is negative. Like Coulamb's law, electric field also satisfies superposition principle.

7. From the knowledge of electric field strength $\vec{E}$ at any point $\vec{r}$, we can easily calculate the magnitude and direction of force experienced by any charge q_0 held at that point i.e.

   $$\mathbf{F} = q_0\mathbf{E}$$

8. An electrostatic field line is a curve drawn in such a way that tanget at every point on the curve gives the direction of field at that point.

9. Electric field intensity at a point is equal to the number of field lines crossing normally a unit area around that point or the magnitude of field is represented by the density of field lines around that point.

10. Electric field strength of an electric dipole

    (i) On axial line of electric dipole at a distance 'r' from centre given is by:

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{2p\mathbf{r}}{\left(r^2 - a^2\right)^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{2\mathbf{p}}{r^3} \text{ for } r >> a$$

    (ii) On equitorial plane at a distance 'r' fromthe centre is:

$$\mathbf{E} = \frac{-\mathbf{p}}{4\pi\varepsilon_0}\frac{1}{\left(a^2 + r^2\right)^{3/2}}$$

$$= \frac{-\mathbf{p}}{4\pi\varepsilon_0}\frac{1}{r^3} \text{ for } r >> a$$

    Thus, the electric field due to a dipole falls off at large distances, at a much faster rate $\left(\alpha \frac{1}{r^3}\right)$ then the electric field due to a single charge $\left(\alpha \frac{1}{r^2}\right)$.

11. Electric field due to a single point charge has a spherically symmetry whereas that due to an electric dipole has cylindrical symmetry.

12. In a uniform electric field E, a dipole experiences a torque τ given by $\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$, but experiences no net force.

13. Amount of work done in rotating the dipole through a small angle is given by $w = -pE\cos\theta_f + pE\cos\theta_i$. Which is stored in the form of electrostatic potential energy in the field of electric dipole.

14. The flux $\Delta\phi$ of electric field E through a small are element Δs is given by

$$\Delta\phi = E.\Delta \mathbf{s} = E\Delta s\cos\theta$$

Where θ is the angle between $\mathbf{E}$ and $\hat{n}$ (to the area element). $\hat{n}$ is taken as outward normal by convention to calculate the total electric flux through any surface, we integrate.

$$\phi = \int_s \mathbf{E}.\mathbf{ds}$$

15. Gauss Law: The flux of electric field through any closed surface s is $\frac{1}{\varepsilon_0}$ times the total charge enclosed within the gaussian surface.

$$\phi = \frac{q_{net}}{\varepsilon_0}$$

It is used for symmetric charge configuration and is independent of size and shape of gaussian surface.

16. In the situation, when gaussain surface is so chosen that there are some charges inside and some outside, the electric field (whose flux is calculated) is due to all the charges, both inside and outside the closed surface. However for electric flux, only inside charges contribute.

17. In case of a shpere (solid / hollow), electric field intensity at an external point varies as $\frac{1}{r^2}$ where r is distance of the point from the centre of the sphere. In case of a line charge $E \alpha \frac{1}{r}$ and in case of an infinite plane sheet of charge, E does not depend upon r.

18. Electrostatic force is a conservative force work done in bringing a test charge q_0 from one point to another point is independent of path taken.

$$W_{A\to B} = q_0\left[V_B - V_A\right]$$

which is the difference of potential energy of test charge q_0 between the final and initial position.

19. Electric potential at a point is numerically equal to the total amount of work done in bringing a unit positive charge from ∞ to that point 'r'.

$$V = \frac{w}{q_0} = -\int_{\infty}^{r} Edr = \frac{1}{4\pi\varepsilon_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

For a point charge 'q', electric potential exists at every point in an electric field regardless of whether there is a charge at that point or not.

20. For a charge configuration, the potential at a point P is given by

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{1P}} + \frac{q_2}{r_{2P}} + ... \frac{q_n}{r_{nP}}\right)$$

Where r_{1P} is the distance of q from P, and so on.

21. Potential energy stored in a system of charges is the work done (by an external agency) in assembling the charges at their locations.

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}} + ...\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{q_iq_j}{r_{ij}}$$

22. Electric potential at a point due to an electric dipole with position vector 'r' is:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}.\hat{r}}{r^2}$$

On the dipole axis $\theta = 0_0$ or π

$$V = \pm\frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$$

On the equitorial plane $\theta = \frac{\pi}{2}$

$$V = 0$$

23. Magnitude of electric field is given by change in the magnitude of potential per unit displacement (potential gradient) and its direction is always towards decreasing potential.

$$E = -\frac{dV}{dr}$$

24. Electrostatic field 'E' is zero in the interior of conductor, just outside the surface of conductor, E is normal to the surface given by

$$\mathbf{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$$

Charges in a conductor can reside over the surface. Potential is constant within and on the surface of conductors. In a cavity, within a conductor, the electric field is zero. Any charge placed in the cavity, will appear over the surface of conductor.

25. For a parallel plate capacitor (with vacuum or air between the plates) its capacitance $\left(C = \frac{Q}{V}\right)$ is defined as $C = \frac{A\varepsilon_0}{d}$ where A is the area of plate and d the separation between the plates. If the medium between the plates is filled with dielectric, its capacitance increases by $C = kC_0$, where k is the dielectric constant of the insulting substance.

26. For capacitors in the series combination, the total capacitance is given by

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{Cn}$$

where 'Q' charge on each capacitor remain same. In parallel combination, the total C capacitance is given by

$$C_{eq} = C_1 + C_2 + ... + C_n$$

where p.d. across the plates of each capacitors remains same.

27. The energy 'U' stored in a capacitor of capacitance C_1, with charge Q and voltage V is:

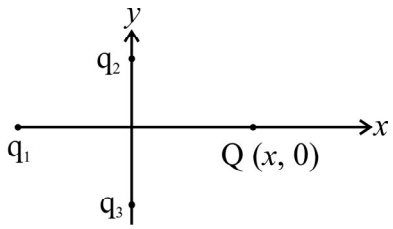
$$U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$

The energy density (energy per unit volume) in a region of $\mathbf{E}$ is $\frac{1}{2}\varepsilon_0 E^2$.

Unit–11 : Electrostatics

Average *Coulomb Force / Electrostatic Force*

1. In figure, two positive charges q_2 and q_3 fixed (symmetrically) along y–axis, exert a net electric force in the +x direction. On charge q_1 fixed along the x–axis. If a charge '+Q' is added at (x, 0), the force on q_1.



(1) shall increase along the positive x–axis (Correct)

(2) shall decrease along the positive x–axis (Incorrect)

(3) shall point along negative x–axis (Incorrect)

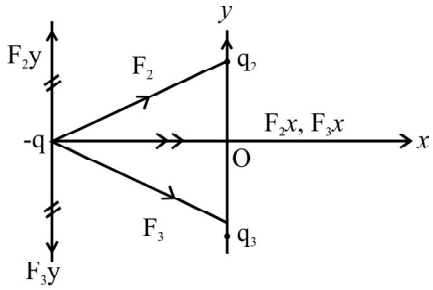
(4) shall increas but direction charnges because of the intersection of Q with q_2 and q_3 (Incorrect)

Correct answer: (1)

Solution:

As q_2; q_3 are positive charges, and net force on q_1 is along +x direction, therefore q_1 must be negative

When a positive charge 'Q' is added (x, 0). It will attract $-q_1$ along +x direction.Therefore force on q_1 will increase along positive x axis.



Correct answer: (1)

Unit–11 : Electrostatics

Easy *Coulomb Force / Electrostatic Force*

2. A small object with change q and mass m is attached to one end of a string of length 'L'. The other end is attached to a stationary support. The system is placed in a uniform horizontal electric field $\mathbf{E}$. In the presence of $\mathbf{E}$, the string makes a constant angle 'θ' with the vertical. The sign and magnitude of q is

(1) positive with magnitude $\frac{mg}{E}$ (Incorrect)

(2) positive with magnitude $\frac{mg}{E}\tan\theta$ (Correct)

(3) negative with magnitude $\frac{mg}{E}$ (Incorrect)

(4) positive with magnitude $\frac{mg}{E}\tan\theta$ (Incorrect)

Correct answer: (2)

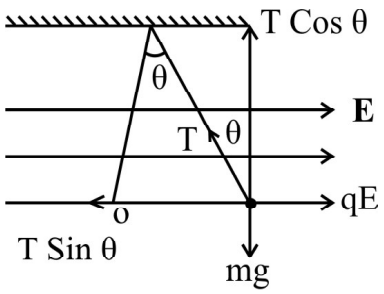
Solution:

For equilibrium $q > 0$,

$$\frac{T\sin\theta}{T\cos\theta} = \frac{qE}{mg}$$

$$\Rightarrow \tan\theta = \frac{qE}{mg}$$

or $$q = \frac{mg\tan\theta}{E}$$



Correct answer is (2)

Unit–11 : Electrostatics

Average *Coulomb Force / Electrostatic Force*

3. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field strength $\frac{81\pi}{7}\times\frac{10^5\,V}{m}$. When the field is switched off, the drop is observed to fall with terminal velocity 2×10^{-3} m/s. Given g = 9.8 m/s², viscosity of the air = 1.8×10^{-5} Nsm^{-2} and the density of oil = 9000 kg/m³; the magnitude of q is

(1) 1.6×10^{-19}C (Incorrect)

(2) 3.2×10^{-19}C (Incorrect)

(3) 4.8×10^{-19}C (Incorrect)

(4) 8.0×10^{-19}C (Correct)

Correct answer: (4)

Solution:

When drop is balanced in still air under the effect of electric field then

$$qE = \frac{4}{3}\pi r^3 \rho g$$

$$\Rightarrow q = \frac{4}{3E}\pi r^3 \rho g \qquad .....(1)$$

When electric field in switched off, drop falls with terminal velocity.

$$V = \frac{2r^2(\rho - \sigma)g}{9\eta} \quad \text{or} \quad r = \left[\frac{9\eta V}{2(\rho - \sigma)g}\right]^{1/2}$$

From (1) and (2)

$$\Rightarrow q = \frac{4}{3E}\pi\rho g\left[\frac{9\eta V}{2(\rho - \sigma)g}\right]^{3/2}$$

or $$q = \frac{4}{3}\times\frac{7}{81\pi}\times\frac{\pi}{10^5}\times 900\times 9.8\left[\frac{9\times 1.8\times 10^{-5}\times 2\times 10^{-3}}{2\times 900\times 9.8}\right]^{3/2}$$

or $$= 8.0\times 10^{-19}\,C$$

Correct answer is (4)

Unit–11 : Electrostatics

Difficult *Coulomb Force / Electrostatic Force*

4. Two identical charged sheres suspended from a common point by two massless strings of length ℓ are initially a distance d ($d << \ell$) apart because of their mutual repulsion. Now the charges begin to leak from both the spheres at a constant rate. As a result charges approach each other with a velocity 'V'. If '*x*' is the distance between them at any instant, them

(1) $V \alpha x^{-1}$ (Incorrect)

(2) $V \alpha x^{1/2}$ (Incorrect)

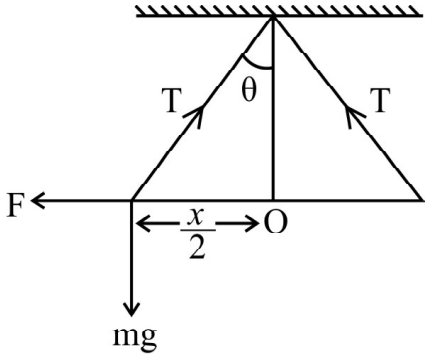
(3) $V \alpha x$ (Incorrect)

(4) $V \alpha x^{-1/2}$ (Correct)

Correct answer: (4)

Solution:

$$\tan\theta = \frac{F}{mg}$$



if $\tan\theta \simeq \theta$

$$\Rightarrow F = mg\theta \simeq \frac{mg(x/2)}{\ell}$$

$$\simeq \frac{mgx}{2\ell}$$

or $$\frac{mgx}{2\ell} = \frac{KQ^2}{x^2}$$

$$\Rightarrow Q^2 = \left(\frac{mg}{2k\ell}\right)x^3$$

or $$Q^2 \alpha x^3$$

$$Q\alpha x^{3/2}$$

$$\Rightarrow \frac{dQ}{dt} = C \times \frac{3}{2} x^{1/2} \frac{dx}{dt}$$

or $$\frac{dx}{dt} = \frac{2x^{-1/2}}{3C}\frac{dQ}{dt}$$

$$\Rightarrow \frac{dx}{dt} \alpha x^{-1/2}$$

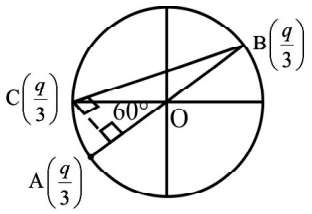
$$\Rightarrow v \alpha x^{-1/2}$$

Correct answer is (4)

Unit–11 : Electrostatics

Average *Coulomb Force / Electrostatic Force*

5. Three charges $\frac{q}{3}, \frac{q}{3}$ and $\frac{q}{3}$ are placed at the circuference of a circle of radius 'R' at points A, B, and C shown in the diagram. The magnitude of force between charges placed at C and B is



(1) $\frac{kq^2}{36R^2}$ (Incorrect)

(2) $\frac{kq^2}{9R^2}$ (Incorrect)

(3) $\frac{kq^2}{27R^2}$ (Correct)

(4) $\frac{4kq^2}{9R^2}$ (Incorrect)

Correct answer: (3)

Solution:

as $\sin 60^0 = \frac{BC}{AB}$

$\Rightarrow BC = AB \sin 60^0$

$= 2R \frac{\sqrt{3}}{2} = R\sqrt{3}$

$F_{CB} = \frac{k \frac{q}{3} \frac{q}{3}}{\left(R\sqrt{3}\right)^2} = \frac{kq^2}{27R^2}$

Correct answer is (3)

Unit–11 : Electrostatics

Easy *Electric Field*

6. Two point charges Q and –2Q are placed some distance apart. If the electric field at the location of Q is E, then the electric field at the location of –2Q will be

(1) $-E/2$ (Correct)

(2) $-E$ (Incorrect)

(3) $-3E/2$ (Incorrect)

(4) $-2E$ (Incorrect)

Correct answer: (1)

Solution:

Let 'r' be the distance between Q and –2Q. Electric field due to –2Q at the location of Q is

$$E = \frac{k(-2Q)}{r^2} \qquad .....(1)$$

Electric field due to +Q at the location of –2Q is

$$E' = \frac{k(Q)}{r^2} \qquad .....(2)$$

From (1) & (2)

$$\frac{E'}{E} = -\frac{1}{2}$$

$$E' = -\frac{1}{2}E$$

Correct answer is (1)

Unit–11 : Electrostatics

Difficult *Electric Field*

7. Let there be a spherical symmetric charge distribution with charge density varying as $\rho(x) = \rho_0\left(\frac{5}{4} - \frac{r}{R}\right)$ upto r = k and $\rho(r) = 0$ for r > R, where r is the distance from the origin. The elctric field at a distance r (r < R) from the origin is given by

(1) $\frac{\rho r}{4\varepsilon_0}\left(\frac{5}{3}-\frac{r}{R}\right)$ (Incorrect)

(2) $\frac{4\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3}-\frac{r}{R}\right)$ (Incorrect)

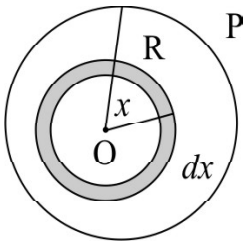
(3) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3}-\frac{r}{R}\right)$ (Incorrect)

(4) $\frac{4\pi\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3}-\frac{r}{R}\right)$ (Correct)

Correct answer: (4)

Solution:

$$q=\int \rho dv=\int_0^r \rho_0\left(\frac{5}{4}-\frac{x}{R}\right)4\pi x^2 dx$$



$$=\rho_0\left[\frac{5}{4}4\pi\frac{x^3}{3}-\frac{4\pi}{R}\frac{x^4}{4}\right]_0^r$$

$$=\rho_0\left[\frac{5}{4}\pi r^3-\frac{\pi r^4}{R}\right] \quad .....(1)$$

Electric field intensity at a point on this sheprical surface is

$$E=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}=\frac{1}{4\pi\varepsilon_0}\frac{\rho_0}{r^2}\left[\frac{5}{3}\pi r^3-\frac{\pi r^4}{R}\right] \quad [\text{From}(1)]$$

$$\Rightarrow E=\frac{\rho_0 r}{4\varepsilon_0}\left[\frac{5}{3}-\frac{r}{R}\right]$$

Correct answer is (4)

Unit–11 : Electrostatics

Difficult *Electric Field*

8. A positively charged metal ring a radius R is fixed in the *x*–y plane with its centre at origin 'O'. A negatively charged particle P is released from rest at the point $(0, 0, z_0)$ where $z_0 > 0$. Then the motion of charged particle P is

(1) periodic provided $z_0 < R$ (Incorrect)

(2) simple harmonic for all the values of z_0 (Incorrect)

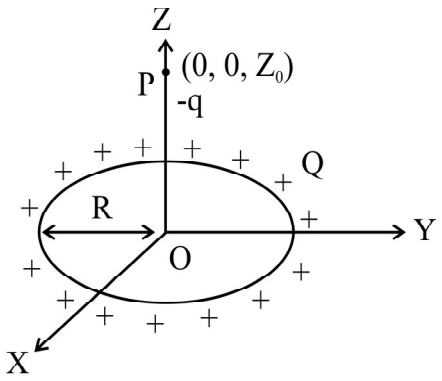
(3) approximately simple harmonic provided $z_0 << R$ (Correct)

(4) such that it crosses centre of ring 'O' and continues to move along –z axis (Incorrect)

Correct answer: (3)

Solution:

The elctric field at the point 'P' due to uniformly charged metal ring of total charge 'Q' is given by:



$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qz_0}{\left(R^2 + z_0^2\right)^{3/2}}$$

Away from the centre of the ring.

Now force on '–q' at P will be-

$$F = -qE$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{Qqz_0}{\left(R^2 + z_0^2\right)^{3/2}}$$ [Acts towards centre 'O' of the ring.]

For $z_0 << R$

$$F = -\frac{1}{4\pi\varepsilon_0}\frac{Qqz_0}{R^3}$$

or $$F = -\left(\frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^3}\right)z_0$$

or $F = -Kz_0$.

Where $K = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^3} = \text{Constant}$

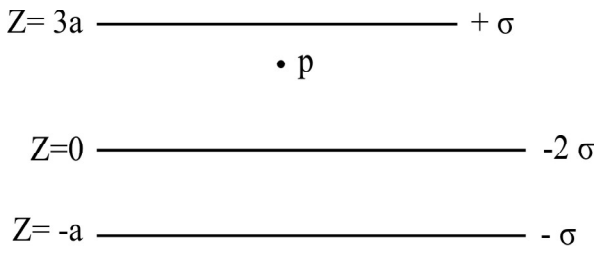
The above equation represents S.H.M. as $F \alpha z_0$ and directed towards centre.

Correct answer is (3)

Unit–11 : Electrostatics

Average *Electric Field*

9. Three infinitely charged sheets are kept parallel to *xy* plane having charge densities as shown. The electric field strength at the point 'P' is:



(1) $\frac{-4\sigma}{\varepsilon_0}\hat{k}$ (Incorrect)

(2) $\frac{4\sigma}{\varepsilon_0}\hat{k}$ (Incorrect)

(3) $\frac{-2\sigma}{\varepsilon_0}\hat{k}$ (Correct)

(4) $\frac{2\sigma}{\varepsilon_0}\hat{k}$ (Incorrect)

Correct answer: (3)

Solution:

For infinite long sheet, electric field is given by $\mathbf{E} = \frac{\sigma}{2\varepsilon_0}\hat{r}$ (Independent of r). Now electric field due to charge density $+\sigma$, at P, is

$$\mathbf{E_1} = \frac{\sigma}{2\varepsilon_0}\left(-\hat{k}\right)$$

Electric field due to charge density -2σ at P, is

$$\mathbf{E}_2 = \frac{2\sigma}{2\varepsilon_0}\left(-\hat{k}\right)$$

Electric field due to change density $-\sigma$ at point P, is

$$\mathbf{E}_3 = \frac{\sigma}{2\varepsilon_0}\left(-\hat{k}\right).$$

Hence total electric field at point 'P' is $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3$

or $$E_{net} = \frac{\sigma}{2\varepsilon_0}\left(-\hat{k}\right) + \frac{2\sigma}{2\varepsilon_0}\left(-\hat{k}\right) + \frac{\sigma}{2\varepsilon_0}\left(-\hat{k}\right)$$

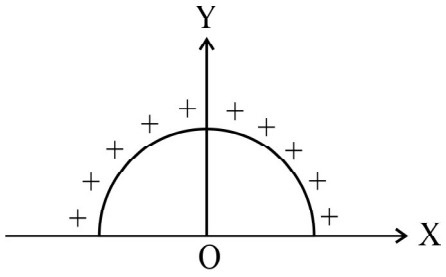
or $$E_{net} = \frac{2\sigma}{\varepsilon_0}\left(-\hat{k}\right) = -\frac{-\sigma}{\varepsilon_0}\hat{k}$$

Correct answer is (3)

Unit–11 : Electrostatics

Average *Continuous Charge Distribution*

10. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it, as shown in the figure. The net electric field at the centre 'O' is



(1) $-\frac{1}{4\pi^2\varepsilon_0}\frac{q}{r^2}\hat{j}$ (Incorrect)

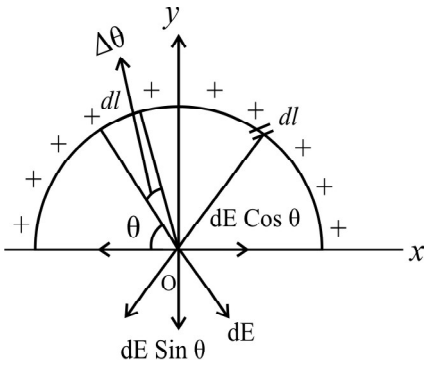
(2) $-\frac{1}{2\pi^2\varepsilon_0}\frac{q}{r^2}\hat{j}$ (Incorrect)

(3) $\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ (Correct)

(4) $\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (Incorrect)

Correct answer: (3)

Solution:



Charge on element ' $d\ell$ ', $dq = \lambda d\ell = \lambda r d\theta$

Electric field due to this element is $dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda d\theta}{r^2}$

As shown in the figure, one can realise that components along *x*–direction cancel and in pairs. Hence total electric field.

$$E = \int_0^{180^0} dE\sin\theta(-j) \qquad \left[\text{along}(-j)\right]$$

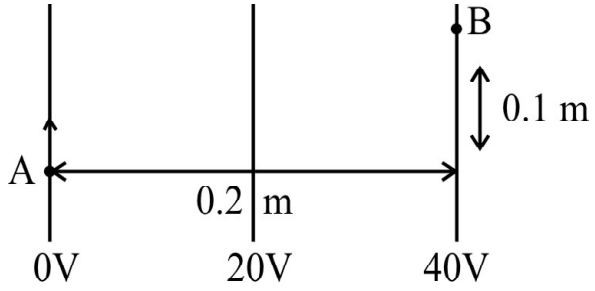
or $$E = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{r}\int_0^{180^0} dE\sin\theta(-j) = \frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$$

Correct answer is (3)

Unit–11 : Electrostatics

Average *Electric Potential*

11. In the figure given below, equipotential lines are drawn at 0, 20V and 40V.



The work done in moving a change of +3 μC from position A to position B is–

(1) 4 μJ (Incorrect)

(2) 8 μJ (Incorrect)

(3) 12 μJ (Incorrect)

(4) $120\,\mu J$ (Correct)

Correct answer: (4)

Solution:

Work done is independent of path taken. Hence

$$w_{A\to B} = \Delta q\left(V_B - V_A\right)$$

$$= 3\mu C\left(40V - 0V\right)$$

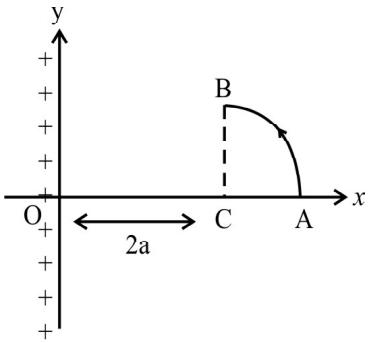
$$= 120\mu J$$

Correct answer is (4)

Unit–11 : Electrostatics

Average *Electric Potential*

12. The infinite long wire having linear change density 'λ' is lying in *x*y plane. A point charge q_0 is moved from point A to point B through the circular path AB of radius 'a' centred at C in the same plane as shown in the figure. The amount of work done is equal to:



(1) $\frac{q_0}{2\pi\varepsilon_0}\ell n\frac{2}{3}$ (Incorrect)

(2) $\frac{q_0\lambda}{2\pi\varepsilon_0}\ell n\frac{3}{2}$ (Correct)

(3) $\frac{q_0\lambda}{2\pi\varepsilon_0}\ell n\frac{2}{3}$ (Incorrect)

(4) $\frac{q_0\lambda}{\sqrt{2}\pi\varepsilon_0}$ (Incorrect)

Correct answer: (2)

Solution:

Electric field due to infinite long wire is equal to $E = \frac{\lambda}{2\pi\varepsilon_0 r}$

as $V = -\int E dr \Rightarrow V = -\frac{\lambda}{2\pi\varepsilon_0}\ell n(r)$

So potential at A, $V_A = \frac{-\lambda}{2\pi\varepsilon_0}\ell n(3a)$

and potential at B, $V_B = \frac{-\lambda}{2\pi\varepsilon_0}\ell n(2a)$

Work done in moving q_0 from A to B

$$w_{A\to B} = q_0(V_B - V_A)$$

$$= q_0\left[\left(-\frac{\lambda}{2\pi\varepsilon_0}\ell n(2a)\right)-\left(-\frac{\lambda}{2\pi\varepsilon_0}\ell n(3a)\right)\right]$$

$$= \frac{\lambda}{2\pi\varepsilon_0} q_0 \ell n\left(\frac{3}{2}\right)$$

Correct answer is (2)

Unit–11 : Electrostatics

Difficult *Electric Potential*

13. A solid conducting sphere having a charge Q is surrounded by an uncharged co-centric conducting hollow spherical shell. The p.d. between the surface of solid sphere and outer surface of hollow shell is 'V'. Now if outer shell is given a charge of –3Q, the new p.d between the same two surfaces is

(1) –2V (Incorrect)

(2) V (Correct)

(3) 2V (Incorrect)

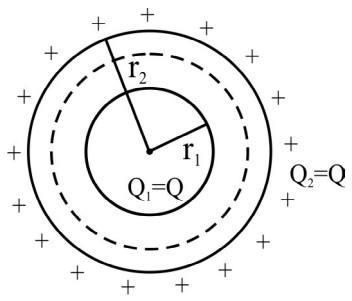
(4) 4V (Incorrect)

Correct answer: (2)

Solution:

Initially, $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_1} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_2}$

When outer shell is given a charge of '–3Q'



$$V' = \frac{1}{4\pi\varepsilon_0}\left[\left(\frac{Q}{r_1} - \frac{3Q}{r_2}\right) - \left(\frac{Q}{r_2} - \frac{3Q}{r_2}\right)\right]$$

or $$V' = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r_1} - \frac{Q}{r_2}\right] = V$$

Correct answer is (2)

Unit–11 : Electrostatics

Average *Electric Potential*

14. A conducting spherical bubble of radius a and thickness t (t << a) is charged to a potential V. Now it collapses to form a spherical droplet. The potential of the droplet is

(1) V (Incorrect)

(2) $V\left[\frac{a}{t}\right]^{1/3}$ (Incorrect)

(3) $V\left[\frac{a}{3t}\right]^{1/3}$ (Correct)

(4) $V\left[3a^2t\right]^{1/3}$ (Incorrect)

Correct answer: (3)

Solution:

As volume remains constant

$$\frac{4}{3}\pi R^3 = 4\pi a^2 t \qquad .....(1)$$

Where R is the radius of droplet formed. Using conservation of charge. [Q = CV]

$$(4\pi\varepsilon_0 R)V' = (4\pi\varepsilon_0 a)V \qquad .....(2)$$

$$\Rightarrow V' = \frac{aV}{R} = \frac{aV}{(3a^2t)^{1/3}} = V\left[\frac{a}{3t}\right]^{1/3} \qquad [\text{Using (1)}]$$

Correct answer is (3)

Unit–11 : Electrostatics

Difficult *Electrostatic Potential Energy*

15. Two point charges each of charge q are fixed at (+a, 0) and (–a, 0). Another positive charge q placed at the origin is free to move along x–axis. The charge q at origin in equilibrium will have:

(1) maximum force and minimum potential energy (Incorrect)

(2) minimum force and maximum potential energy (Incorrect)

(3) maximum force and maximum potential energy (Incorrect)

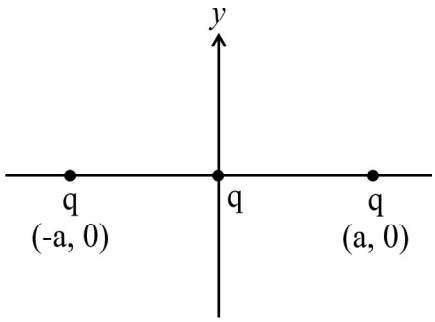
(4) minimum force and minimum potential energy (Correct)

Correct answer: (4)

Solution:

Net force on q at origin is

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\hat{i} + \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}(-\hat{i}) = 0$$



Let the charge q is displaced from origin to position (x, 0). At this point net force on the charge 'q' due to two end charges is

$$F = \frac{1}{4\pi\varepsilon_0}q^2\left[\frac{1}{(a-x)^2} + \frac{1}{(a+x)^2}\right]$$

as $|F| = \frac{dU}{dx}$ for conservative fields.

$$\Rightarrow \frac{dU}{dx} = \frac{1}{4\pi\varepsilon_0}q^2\left[\frac{1}{(a-x)^2} + \frac{1}{(a+x)^2}\right]$$

For U to be minimum $\frac{d^2U}{dx^2} > O.$

$\Rightarrow (a-x)^2 = (a+x)^2 \Rightarrow x = 0$

Hence charge at origin will have minimum minimum potential energy.

Correct answer is (4)

Unit–11 : Electrostatics

Average *Electrostatic Potential Energy*

16. A proton and an α–particle are situated at a distance 'r' apart. At very large distance apart, when released, the kinetic energy of proton will be

(1) $\frac{2Ke^2}{r}$ (Incorrect)

(2) $\frac{8Ke^2}{5r}$ (Correct)

(3) $\frac{Ke^2}{r}$ (Incorrect)

(4) $\frac{8Ke^2}{r}$ (Incorrect)

Correct answer: (2)

Solution:

According to conservation of energy–

$$\frac{K(e)(2e)}{r} = (K.E)_P + (K.E.)_\alpha$$

$$= \frac{1}{2}m_p v_p^2 + \frac{1}{2}m_\alpha v_\alpha^2 \quad .....(1)$$

According to conservation of momentum.

$$0 = m_p v_p = m_\alpha v_\alpha \quad .....(2)$$

$$m_p v_p = m_\alpha v_\alpha$$

$$1 \times v_p = 4 \times v_\alpha$$

$$v_\alpha = \frac{v_p}{4} \qquad .....(3)$$

From (1) and 3)

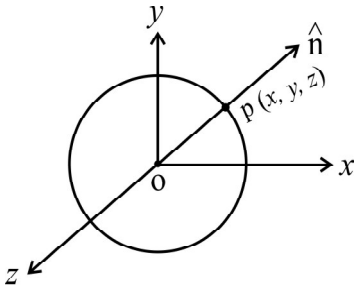
$$(K.E)P = \frac{1}{2} m_p v_p^2 = \frac{8}{5} \frac{Ke^2}{r}$$

Correct answer is (4)

Unit–11 : Electrostatics

Average *Electric Flux*

17. The electric field strength in a region is given as $\mathbf{E} = \frac{x\hat{i} + y\hat{j}}{x^2 + y^2}$. The net charge inside a sphere of radius 'a' with its centre at origin will be



(1) zero (Incorrect)

(2) $4\pi a\varepsilon_0$ (Correct)

(3) $2\pi a\varepsilon_0$ (Incorrect)

(4) $8\pi a\varepsilon_0$ (Incorrect)

Correct answer: (2)

Solution:

At a point (*x*, y, z) on the sphere a unit vector perpendicular to surface can be written as $\hat{n} = \frac{x}{a}\hat{i} + \frac{y}{a}\hat{j} + \frac{z}{a}\hat{k}$

and as given that $\mathbf{E} = \dfrac{x\hat{i} + y\hat{j}}{x^2 + y^2}$

$$\Rightarrow \mathbf{E}.\hat{n} = \left(\frac{x\hat{i} + y\hat{j}}{x^2 + y^2}\right).\left(\frac{x}{a}\hat{i} + \frac{y}{a}\hat{j} + \frac{z}{a}\hat{k}\right)$$

$= \dfrac{1}{a}$, which is independent of co-ordinate of point 'P' lying on sphere. Also $\mathbf{E}.\hat{n}$ = total flux passing through the unit area of sphere.

$\Rightarrow$ Total flux passing through sphere.

$$\phi = \left(\mathbf{E}.\hat{n}\right)4\pi a^2 = \frac{q}{\varepsilon_0}$$

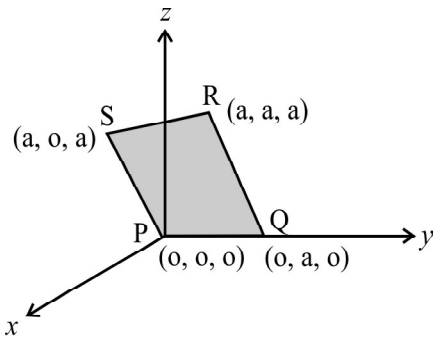
$$\Rightarrow q = \varepsilon_0 \frac{1}{a} \times 4\pi a^2 = 4\pi a\varepsilon_0$$

Correct answer is (2)

Unit–11 : Electrostatics

Average *Electric Flux*

18. Consider an electric field, $\mathbf{E} = E_0\hat{x}$, where E_0 is a constant. The flux through the shaded region (as shown in the figure) due to this field is



(1) $2E_0a^2$ (Incorrect)

(2) $\sqrt{2}E_0a^2$ (Incorrect)

(3) E_0a^2 (Correct)

(4) $\frac{E_0 a^2}{\sqrt{2}}$ (Incorrect)

Correct answer: (3)

Solution:

The shaded area $\mathbf{A} = \mathbf{P\theta} \times \mathbf{PS}$

$\therefore \ \mathbf{A} = \left(a^2\hat{i} - a^2\hat{k}\right)$

Electric flux $\phi = \mathbf{E}.\mathbf{A}$

$$\phi = \left(E_0\hat{i}\right)a^2\hat{i} - a^2\hat{k}$$

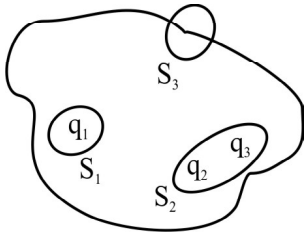
$$\phi = E_0 a^2$$

Correct answer is (3)

Unit–11 : Electrostatics

Average *Electric Flux*

19. Three charges $q_1 = 1\,\mu C$, $q_2 = 2\,\mu C$, and $q_3 = -3\,\mu C$ and four surfaces S_1, S_2, S_3 and S_4 are shown in figure. The flux emerging through surface S_2 in Nm^2/C is



(1) $36\pi \times 10^3$ (Incorrect)

(2) $-36\pi \times 10^3$ (Correct)

(3) $36\pi \times 10^9$ (Incorrect)

(4) $-36\pi \times 10^9$ (Incorrect)

Correct answer: (2)

Solution:

As the flux is due to all the charges present inside the gaussian surface. hence flux through S_2,

$$\phi_2 = \frac{\text{Total charge enclosed}}{\varepsilon_0}$$

or $\phi_2 = \frac{(2\mu C - 3\mu C)}{\varepsilon_0} = -1\times10^{-6}\times 4\pi k$

or $\phi_2 = -1\times10^{-6}\times 4\pi\times 9\times10^{9}$

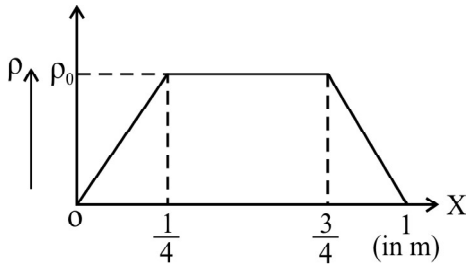
or $\phi_2 = -36\pi\times10^{3}\ Nm^2/C$

Correct answer is (2)

Unit–11 : Electrostatics

Difficult *Electric Flux*

20. The volume charge density as a function of distance *x* from one face inside a unit cube is varrying as shown in the figure. The total flux through the cube is (if $\rho_0 = 8.85\times10^{-12} C/m^3$).



(1) $\frac{1}{4}$ SI Units (Incorrect)

(2) $\frac{1}{2}$ SI Units (Incorrect)

(3) $\frac{3}{4}$ SI Units (Correct)

(4) 1 SI Units (Incorrect)

Correct answer: (3)

Solution:

Let 'q' is the total charge enclosed within the cube.

q = Area of curve (trapezium)

$$= \frac{3}{4}\rho_0$$

As flux passing throught the cube will be given by

$$\phi = \frac{q}{\varepsilon_0} = \frac{\frac{3}{4}\rho_0}{\varepsilon_0} = \frac{\frac{3}{4}\times 8.85\times 10^{-12}}{8.85\times 10^{-12}}$$

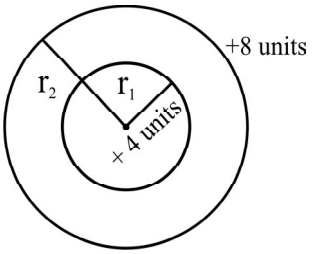
$$\phi = \frac{3}{4} \text{ SI Units}$$

Correct answer is (3)

Unit–11 : Electrostatics

Average *Conductors in Electrostatics Equilibrium*

21. A cavity is created in the conducting shell having uniform charge distribution of +8 units as shown in the figure. Now if a charge of +4 units in placed at the centre of cavity. Which set of equations correctly describe the magnitude of electric field.



	$r < r_1$	$r_1 < r < r_2$	$r < r_2$	
(1)	$E = 0$	$E = \frac{K12q}{r^2}$	$E = \frac{K8q}{r^2}$	(Incorrect)
(2)	$E = \frac{K4q}{r^2}$	$E = 0$	$E = \frac{K12q}{r^2}$	(Correct)
(3)	$E = \frac{K4q}{r^2}$	$E = \frac{K8q}{r^2}$	$E = 0$	(Incorrect)

(4) $E = \frac{K4q}{r^2}$ $E = 0$ $E = \frac{K8q}{r^2}$ (Incorrect)

Correct answer: (2)

Solution:

E_{inside} cavity will be non-zero having value $E = \frac{K4q}{r^2}$ (for $r < r_1$).

In electrostatic equilibrium, field inside conductor is zero (for $r_1 < r < r_2$).

As charge placed at cavity, will appear over the surface of conductor, hance q = 8 + 4 = 12 units.

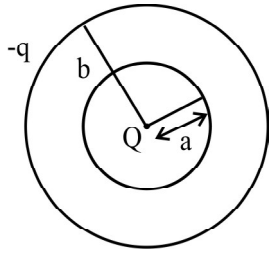
Thus, $E = \frac{K12q}{r^2}$ (for $r > r_2$)

Correct answer is (2)

Unit–11 : Electrostatics

Average *Conductors in Electrostatics Equilibrium*

22. A spherical shell with an inner surface of radius 'a' and an outer surface of radius 'b' is made of conducting material. A point charge '+Q' is placed at the centre of the spherical shell and a total charge –q is placed on the shell.



Assuming that the electrostatic potential is zero at infinite distance. The elctrostatic potential at a distance 'R' from the conducting shell where b > R > a is

(1) $\frac{KQ}{a}$ (Incorrect)

(2) $\frac{KQ}{R}$ (Incorrect)

(3) $\frac{K(Q-q)}{R}$ (Incorrect)

(4) $\frac{K(Q-q)}{b}$ (Correct)

Correct answer: (4)

Solution:

The charge 'Q' put at the centre will appear on the surface of conducting shell. So that field inside the shell remains zero in equilibrium. Also electrostatic potential in constant throughout the volume and surface of the conductor.

Correct answer is (4)

Unit–11 : Electrostatics

Average *Dielectrics*

23. A parallel plate capacitor with air as the dielectric and having the same thickness as the separation between the plates is introduced so as to fill one fourth of the capacitor as shown in the figure.

K	air
A/4	3A/4

The new capacitance will be–

(1) $(K+3)\frac{C}{4}$ (Correct)

(2) $(K+2)\frac{C}{4}$ (Incorrect)

(3) $(K+1)\frac{C}{4}$ (Incorrect)

(4) $\frac{KC}{4}$ (Incorrect)

Correct answer: (1)

Solution:

The capacitance $C = \frac{A\varepsilon_0}{d}$ (without dielectric).

Let C_1 be the capacitance of dielectric part then $C_1 = \frac{K\varepsilon_0 A/4}{d}$ and C_2 (without dielectric part) $= \frac{\varepsilon_0 3A/4}{d}$

The combination acts like parallel combination such that:

$$C_{net} = C_1 + C_2 = \frac{K\varepsilon_0 A/4}{d} + \frac{\frac{3}{4}\varepsilon_0 A}{d}$$

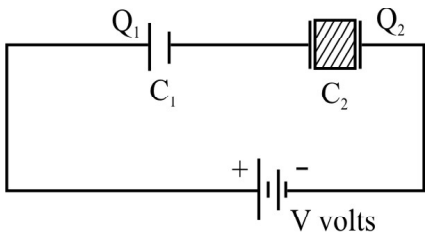
$$C_{net} = (K+3)\frac{C}{4}$$

Correct answer is (1)

Unit–11 : Electrostatics

Average *Dielectrics*

24. Two identical capacitors 1 and 2 are connected in series to a battary of 'V' volts as shown in the figure. Capacitor 2 contains a dielectric slab of dielectric constant K and Q_1, Q_2 are the charges stored in the capacitors repectively. Now the dielectric slab is removed and the corresponding charges are Q_1' and Q_2'. Then the ratio $\frac{Q_2'}{Q_2'}$ is



(1) $\frac{K+1}{K}$ (Incorrect)

(2) $\frac{K+1}{2}$ (Incorrect)

(3) $\frac{K+1}{2K}$ (Correct)

(4) $\frac{K}{2}$ (Incorrect)

Correct answer: (3)

Solution:

Let 'C_0' is the capacitance of each capacitor without dielectric.

$\Rightarrow C_1 = C_0$ and $C_2 = kC_0$

Before dielectric is removed.

$$C_{net} = \frac{C_1C_2}{C_1 + C_2} = \frac{KC_0}{K+1} \quad (\text{in series})$$

$$\Rightarrow Q_2 = Q_1 = C_{net}V = \left(\frac{KC_0}{K+1}\right)V \qquad .....(1) \qquad [\text{Charge in series remains same}]$$

When slab from C_2 is removed, now

$C_1 = C_0$ and $C_2 = C_0$

$$C_{net} = \frac{C}{2} \qquad (\text{in series})$$

As Q is same on each

$$Q_2' = Q_1' = \frac{C_0}{2}V \qquad .....(2)$$

From (1) & (2)

$$\frac{Q_2'}{Q_2} = \frac{K+1}{2K}$$

Correct answer is (3)

Unit–11 : Electrostatics

Easy *Redistribution of Charges*

25. A capacitor of 20 μF and charged to 500 V is connected in parallel with another capacitor of 10 μF charged to 200 V. The common potential of the system is:

(1) 100 V (Incorrect)

(2) 120 V (Incorrect)

(3) 350 V (Incorrect)

(4) 400 V (Correct)

Correct answer: (4)

Solution:

$$V_{common} = \frac{q_{net}}{C_{net}}$$

or $$V_{common} = \frac{C_A V_A + C_B V_B}{C_A + C_B}$$

or $$V_{common} = \frac{20\times10^{-6}\times500 + 10\times10^{-6}200}{(20+10)\times10^{-6}}$$

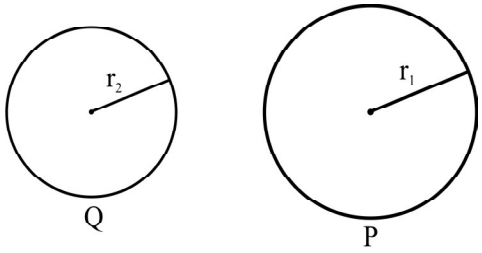
or $$V_{common} = \frac{1200\cancel{0}}{3\cancel{0}} = 400\,V$$

Correct answer is (4)

Unit–11 : Electrostatics

Easy *Redistribution of Charges*

26. Two conducting sphere P and Q of radius r_1 and r_2 are shown in the figure ($r_1 > r_2$)



Both are positively charged and isolated from surroundings. They had been connected by a connecting wire but the wire is removed after some time.

Which of the following statements are true;

(a) Sphere P is at higher potential

(b) Both spheres are at same potential

(c) Magnitude of charge on P is greater than Q

(d) Both the shperes will be having same charge after removal of wire

(1) (a) and (d) (Incorrect)

(2) (b) and (d) (Incorrect)

(3) (b) and (c) (Correct)

(4) (c) and (a) (Incorrect)

Correct answer: (3)

Solution:

When both conducting spheres are connected by a wire, there will be redistribution of charge till they accquire the same potential (statement (b) is correct).

As $V_P = V_Q$

$$\frac{1}{4\pi\varepsilon_0}\frac{Q_P}{r_1} = \frac{1}{4\pi\varepsilon_0}\frac{Q_Q}{r_2}$$

$$\Rightarrow \frac{Q_P}{Q_Q} = \frac{r_1}{r_2} \Rightarrow Q_P > Q_Q \text{ (as } r_1 > r_2\text{)} \qquad \text{(statement c)}$$

Correct answer is (3)

Unit–11 : Electrostatics

Difficult *Redistribution of Charges*

27. Two capacitor $C_1 = 1\,\mu F$ and $C_1 = 4\,\mu F$ are charged to a p.d. of 100 V and 200 V respectively. The charged capacitors are now connected to each other with the terminals of opposite sign connected together. Now the energy of the system is

(1) 0.012 J (Incorrect)

(2) 0.049 J (Correct)

(3) 0.081 J (Incorrect)

(4) 0.162 J (Incorrect)

Correct answer: (2)

Solution:

When terminals of opposite sign one connected together, the magnitude of net charges finally is equal to the difference of charges before the connection.

Initial charge on $C_1 = Q_1 = C_1V_1 = 100\,\mu C$

Initial charge on $C_2 = Q_2 = C_2V_2 = 800\,\mu C$

$Q_{net} = Q_2 – Q_1 = 700\,\mu C$

$$V_{common} = \frac{Q_{net}}{C_{net}} = \frac{700\times10^{-6}}{5\times10^{-6}} = 140\,V$$

The energy of system

$$U = \frac{1}{2} C_{net} V_{common}^2$$

or $$U = \frac{1}{2} \times 5 \times 10^{-6} \times (140)^2$$

or $U = 0.049$ J

Correct answer is (2)

Unit–11 : Electrostatics

Average *Capacitor (Parallel–Plate)*

28. The force exerted by each plate on the other plate of a parallel plate capacitor of capacitance 'C' and separation between the plates 'd' with potential difference 'V' between the plates is given by

(1) $\frac{CV^2}{2d}$ (Correct)

(2) $\frac{C^2V^2}{2d^2}$ (Incorrect)

(3) $\frac{C^2V^2}{d^2}$ (Incorrect)

(4) $\frac{dV^2}{C}$ (Incorrect)

Correct answer: (2)

Solution:

Force exerted by each plate on the other plate of a parallel plate capacitor is given by

$$F = \frac{1}{2} qE$$

where q is the magnitude of charge on either plate and E, electric field between the plates.

Also $E = \frac{V}{d}$ where V is the p.d. between the plates and 'd' plate separation.

$$\Rightarrow F = \frac{1}{2} q \frac{V}{d}$$

or $F = \frac{1}{2} \frac{CV^2}{d}$ (Putting $q = CV$)

Correct answer is (2)

Unit–11 : Electrostatics

Difficult *Combination of Capacitors*

29. Five similar capacitor plates, each of area A, are placed at a equal distance 'd' apart and are connected to a source of e.m. E as shown in the following diagram. The charge on the plate 1 and 4 will be

(1) $\frac{\varepsilon_0 A}{d}, \frac{-2\varepsilon_0 A}{d}$
(Incorrect)

(2) $\frac{\varepsilon_0 AV}{d}, \frac{-2\varepsilon_0 AV}{d}$ (Correct)

(3) $\frac{-\varepsilon_0 AV}{d}, \frac{-3\varepsilon_0 AV}{d}$ (Incorrect)

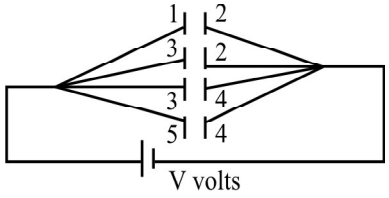
(4) $\frac{\varepsilon_0 AV}{d}, \frac{-4\varepsilon_0 AV}{d}$ (Incorrect)

Correct answer: (2)

Solution:

From equivalent circuit diagram, charge on first plate is

$$Q = CV \Rightarrow Q = \frac{\varepsilon_0 AV}{d}$$



Charge on fourth plate $Q' = (-CV) \Rightarrow Q' = \frac{-\varepsilon_0 AV}{d}$

As plate 4 is repeated twice, hence charge on 4 will be $Q'' = 2Q'$

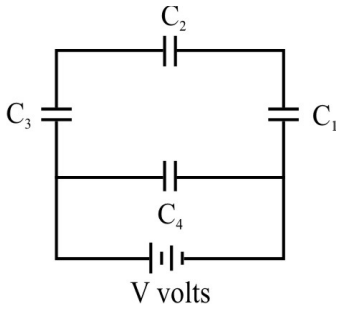
i.e. $Q'' = \dfrac{-2\varepsilon_0 AV}{d}$

Hence correct answer will be (2)

Unit–11 : Electrostatics

Average *Capacitor*

30. A network of four capacitor of capacitance equal to $C_1 = C$, $C_2 = 2C$, $C_3 = 3C$ and $C_4 = 4C$ are connected to an ideal battery of V volts as shown in figure. The ratio of charges on C_2 and C_4 is



(1) $\dfrac{4}{7}$ (Incorrect)

(2) $\dfrac{3}{22}$ (Correct)

(3) $\dfrac{7}{4}$ (Incorrect)

(4) $\dfrac{22}{3}$ (Incorrect)

Correct answer: (2)

Solution:

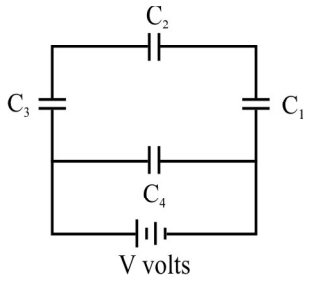
As in the given network C_1, C_2 and C_3 are in series hence, equivalent of this series combination is

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C'} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{3C}$$

$$\Rightarrow C' = \frac{6C}{11}$$

This combination C' is in parallel with C_4.



$$\Rightarrow C_{net} = C' + C_4 = \frac{6C}{11} + 4C$$

$$= \frac{50}{11}C$$

In parallel combination 'V' remains same.'

$$\Rightarrow \frac{Q'}{C'} = \frac{Q_4}{C_4}$$

or $$\frac{Q'}{Q_4} = \frac{\frac{6C}{11}}{4C} = \frac{3}{22}$$

As Q' is the charge on C' (Combination of C_1, C_2 and C_3 in series or change remains same in series. This means change on C_2 will be same as change on $C' = Q'$)

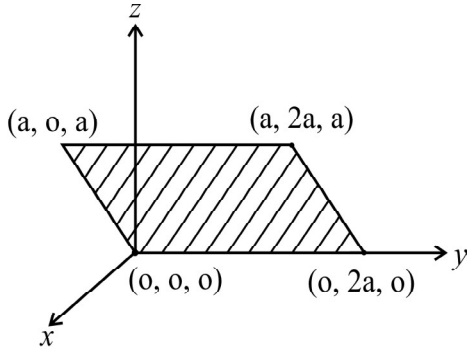
Hence, $$\frac{\text{Charge on } C_2}{\text{Charge on } C_4} = \frac{3}{22}$$

Correct answer is (2)

Unit–11 : Electrostatics

Average *Elelctric Flux*

31. Consider an electric field $\mathbf{E} = E_0\hat{i}$, **where E_0 is a constant. The flux through the shaded area as shown in the figure, due to this field E is:**



(1) E_0a^2 (Incorrect)

(2) $2E_0a^2$ (Correct)

(3) $2\sqrt{2}E_0a^2$ (Incorrect)

(4) $\sqrt{2}E_0a^2$ (Incorrect)

Correct answer: (2)

Solution:

Flux, $\phi = \mathbf{EA}$, for uniform field.

$E_0 \times (\text{Area perpendicular to } \hat{i})$

$= E_0 \times (2a \times a)$

$= 2E_0a^2$

Correct answer is $2E_0a^2$

Unit–11 : Electrostatics

Difficult *Electic Fied*

32. Two oppositely charged non-conducting solid spheres, of radii R and $\frac{3}{2}R$, having uniform volume change densitites ρ_1, and ρ_2 respectively, touch each other. The net electric field at a distance 2R from the centre of the smaller sphere, along the line joining the centres of the spheres, is zero. The ratio $\frac{\rho_1}{\rho_2}$ is equal to:

(1) -4 (Incorrect)

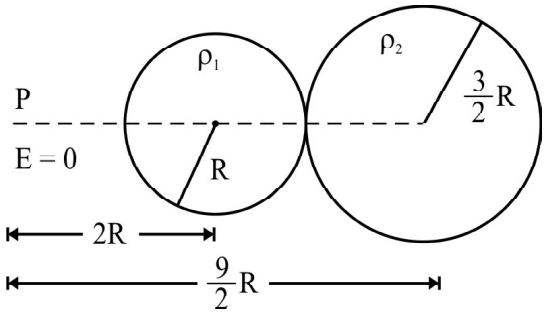
(2) $-\frac{2}{3}$ (Correct)

(3) $\frac{2}{3}$ (Incorrect)

(4) 2 (Incorrect)

Correct answer: (2)

Solution:



Since the spheres are oppositely charged, the point P, where the net field is zero, should lie on the left side of smaller sphere, as shown in the figure.

Let E_1 be the field due to smaller sphere at P and E_2 be that due to the bigger one, at P. E_1 and E_2 should be equal and opposite.

Now, $E_1 = \frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi R^3\rho_1}{(2R)^2}$ and $E_2 = \frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi\left(\frac{3}{2}R\right)^3\rho_2}{\left(\frac{9}{2}R\right)^2}$

$$\therefore \quad \frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi R^3\rho_1}{4R^2} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi\frac{3^3}{2^3}R^3\rho_2}{\frac{9^2}{2^2}R^2}$$

or $$\frac{\rho_1}{4} = \frac{3^3}{2^3}\times\frac{2^2}{3^4}\rho_2$$

or $$\frac{\rho_2}{\rho_1} = \frac{4}{6} = \frac{2}{3}$$

As one of the charge is negative,

$$\frac{\rho_1}{\rho_2} = -\frac{-2}{3}$$

Correct answer is $\frac{\rho_1}{\rho_2} = -\frac{-2}{3}$

Unit–11 : Electrostatics

Difficult *Electric Potential Energy*

33. Three piont charges q, 2q and 8q Coulomb's are to be placed on *x*–axis, such that the maximum distance between any two is 9cm and the potential energy of the system is the minimum. If q is to be placed at the origin, the locations of 2q and 8q are respectively at:

(1) 6 cm and 9 cm (Incorrect)

(2) –3 cm and 6 cm (Correct)

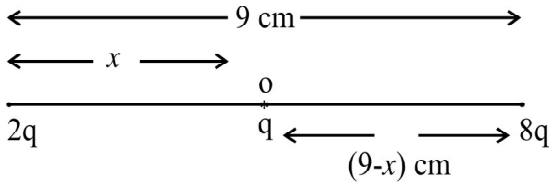
(3) 6 cm and –3 cm (Incorrect)

(4) –4 cm and 5 cm (Incorrect)

Correct answer: (2)

Solution:

For minimum potential energy, the charges of maximum energy should be at farthest position. Therefore the charges of 2q and 8q should be 9cm apart (as 9cm is the maximum possible according to the question). Let the positions of the three charges be as shown in the figure.



Here, q is at the origin.

2q at *x* to the left of origin.

8q at (9–*x*) to the right of the origin.

The potential energy of this system is:

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{2q\times q}{x} + \frac{2q\times 8q}{9} + \frac{q\times 8q}{(9-x)}\right]$$

or $$U = \frac{2q^2}{4\pi\varepsilon_0}\left[\frac{1}{x} + \frac{8}{9} + \frac{4}{(9-x)}\right]$$

But, for U to be minimum,

$$\frac{dU}{dx} = 0$$

$$\therefore \frac{2q^2}{4\pi\varepsilon_0}\left[-\frac{1}{x^2}+0+\frac{4}{(9-x)^2}\right]=0, \text{ for } U = U_{min.}$$

Hence,

$$\frac{1}{x^2}=\frac{4}{(9-x)^2}$$

or $4x^2=(9-x)^2$

or $\pm 2x = 9-x$

or $x = 3$ cm, to the left of origin

or $x = -9$ cm, or 9 cm to the right of origin, which is not possible, here.

$\therefore$ 2q is to be placed at –3 cm with respect to the origin and 8q should be placed at +6 cm from the origin, on the x–axis.

The correct answer is:

2q at –3 cm

q at origin

8q at +6 cm

Correct answer is (2)

Unit–11 : Electrostatics

Average *Electric Potential*

34. When the intensity of electric field becomes 3×10^6 Vm^{-1}, then the insulation of air is broken down. A metallic spere in air is able to store a maximum charge of 30 μC. The maximum potential the sphere can have is:

(1) 90 V (Incorrect)

(2) 2.7×10^6V (Incorrect)

(3) 1.8×10^6V (Incorrect)

(4) 0.9×10^6V (Correct)

Correct answer: (4)

Solution:

For the maximum values,

$$E = 3\times10^{6}\ Vm^{-1}$$

$$Q = 30\mu C = 30\times10^{-6} C$$

But, $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$ (R = radius of the sphere)

$$\therefore\ R^2 = \frac{1}{4\pi\varepsilon_0}\times\frac{Q}{E}$$

$$= 9\times10^{9}\times\frac{30\times10^{-6}}{3\times10^{6}}$$

$R^2 = 9\times10^{-2}\ m_2$

$\therefore\ R = 3\times10^{-1}$ m

$= 0.3$ m

Therefore, the maximum potential it can have is

$$V = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R} = E\times R$$

$= 3\times10^{6}\times0.3$

$= 0.9\times10^{6}$ V

Correct answer is (4)

Unit–11 : Electrostatics

Difficult *Electric Potential*

35. A spherical metal shell A of radius R_A and a solid metal sphere B of radius R_B ($<R_A$) are kept at a large distance and are given charges Q each. They are then connected by a thin metal wire. Which of the following statements are correct regarding the electrical conditions of A and B, after they are connected with the wire?

(i) $E_A^{inside} = E_B^{inside} = 0$

(ii) $\frac{\sigma_A}{\sigma_B} = \frac{R_A}{R_B}$

(iii) $Q_A > Q_B$

(iv) $E_A^{on\ surface} = E_B^{on\ surface}$

(1) (i), (ii) and (iii) (Incorrect)

(2) (i), (iii) and (iv) (Correct)

(3) (ii) and (iv) (Incorrect)

(4) (i), (ii), (iii) and (iv) (Incorrect)

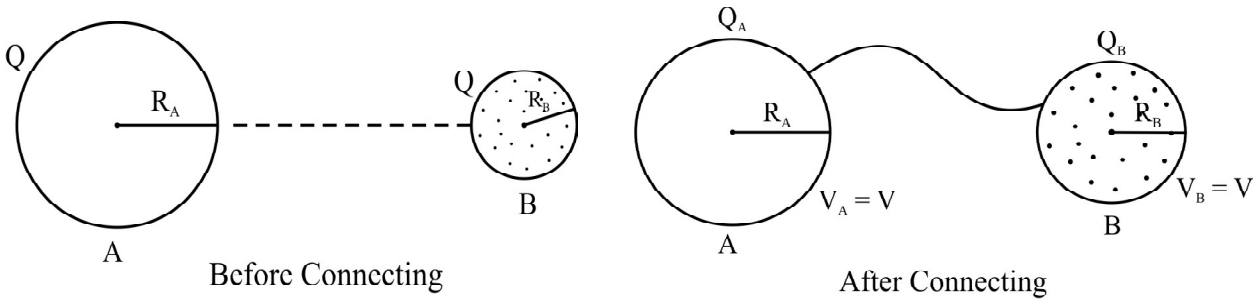
Correct answer: (2)

Solution:

For metal sphere or shell, charges reside only on the surface. As there is no charge inside, by using Gauss's theorem, we can show that $E_A^{inside} = E_B^{inside} = 0$.

So statement (i) is correct.

Let the charges on A and B, after they are connected with the wire, be Q_A and Q_B respectively.



The final potential of A and B will be equal, as charges flow from B and A, till the potential become equal.

Therefore $V_A = V_B$

So, $\frac{1}{4\pi\varepsilon_0}\frac{Q_A}{R_A} = \frac{1}{4\pi\varepsilon_0}\frac{Q_B}{R_B}$

or $\frac{Q_A}{R_B} = \frac{Q_A}{R_B}$

Since $R_A > R_B$, $Q_A > Q_B$

So, optino (3) gives a correct statement.

Now, $Q_A = 4\pi R_A^2 \times \sigma_A$

$$Q_B = 4\pi R_A^2 \times \sigma_B$$

$$\therefore \frac{Q_A}{Q_B} = \frac{R_A^2}{R_B^2}\frac{\sigma_A}{\sigma_B}$$

or $$\frac{R_A}{R_B} = \frac{R_A^2}{R_B^2}\frac{\sigma_A}{\sigma_B}$$

$$\therefore \frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$$

So, option (1) is an incorrect statement.

Electric field E, just outside the surface of A and B are:

$$E_A = \frac{\sigma_A}{\varepsilon_0} \text{ and } E_B = \frac{\sigma_B}{\varepsilon_0}$$

$$\therefore \frac{E_A}{E_B} = \frac{\sigma_A}{\sigma_B}$$

But, $\frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$

$$\therefore \frac{E_A}{E_B} = \frac{R_B}{R_A} \text{ or } E_A < E_B \qquad (\text{as } R_B < R_A)$$

Hence, option (4) is a correct statement.

Unit–11 : Electrostatics

Difficult *Electric Force*

36. A tiny spherical oil drop carrying a net charge q is balanced is still air with a vertically upward uniform electric field of strength $\frac{81\pi}{7}\times 10^5\,\text{V}-\text{m}^{-1}$. When the field is switched off, this oil drop is observed to fall with a terminal velocity 2×10^{-3} ms^{-1}. If the density of air is

neglected and taking g = 9.8 ms^{-2}, viscosity of air = 1.8×10^{-5} N-S-m^{-2}, density of oil = 900 kgm^{-3}, the number of electrons missing from the oil drop is:

(1) 2 (Incorrect)

(2) 3 (Incorrect)

(3) 4 (Incorrect)

(4) 5 (Correct)

Correct answer: (4)

Solution:

Since the drop is balanced is a vertically upward electric field, the change q on it should be positive and so few electrons will be missing from it.

$q = +ne$

$n = ?$

We find q using the following equations.

When the drop is balanced,

$qE = mg$ (Density of air is negected)

or $qE = \frac{4}{3}\pi r^3 \rho g$ .....(1)

When it falls with terminal velocity v_t, a = 0 and so net force on it is again zero.

Since field is switched off, we have,

$$6\pi\eta r v_t = \frac{4}{3}\pi r^3 \rho g \quad .....(2)$$

$$\therefore\ r^2 = \frac{9}{2}\frac{\eta v_t}{\rho g}$$

or $$r = \left(\frac{9}{2}\frac{\eta v_t}{\rho g}\right)^{1/2} \quad .....(3)$$

From (1) and (2), we have

$$qE = 6\pi\eta r v_t$$

or $q = \frac{6\pi\eta v_t}{E} \times r$

$= \frac{6\pi\eta v_t}{E} \times \left(\frac{9}{2}\frac{\eta v_t}{\rho g}\right)^{1/2}$

$= \frac{6\pi \times 1.8\times10^{-5} \times 2\times10^{-3}}{\frac{81\pi}{7}\times10^{5}} \times \left(\frac{9}{2} \times \frac{1.8\times10^{-5}\times2\times10^{-3}}{900\times9.8}\right)^{1/2}$

$= \frac{0.8\times7}{3} \times 10^{-13} \times \left(\frac{1.8}{9.8}\times10^{-10}\right)^{1/2}$

$= \frac{0.8\times7}{3} \times 10^{-13} \times \left(\frac{9}{49}\times10^{-10}\right)^{1/2}$

$= \frac{0.8\times7}{3} \times 10^{-13} \times \frac{3}{7}\times10^{-5}$

$= 0.8\times10^{-18}$

$= 8\times10^{-19}C$

But $q = ne$

$\therefore\ n = \frac{q}{e} = \frac{8\times10^{-19}C}{1.6\times10^{-19}C}$

$= 5$

Correct answer is (4)

Unit–11 : Electrostatics

Difficult *Capacitor with Dielectric*

37. The capacitance of parallel plate capacitor is 50pF and the distance between the plates is 4 mm. It is charged to 200 V, after that the chargnig battery is removed. Now, a dielectric slab of thickness 2 mm and dielectric constant 4 is placed between the plates. The exess heat produced in the slab, during this process, is;

(1) 1.0×10^{-6} J (Incorrect)

(2) 6.25×10^{-7} J (Incorrect)

(3) 3.75×10^{-7} J (Correct)

(4) 2.50×10^{-7} J (Incorrect)

Correct answer: (3)

Solution:

With air as dielectric, the capacitance of a parallel plate capacitor is;

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{4}$$

Now, with a dielectric slab of thickness t and dielectric constant K, the capacitance is:

$$C = \frac{\varepsilon_0 A}{d - t + \left(t/K\right)}$$

When C_0 is connected to the battery, then

$Q = C_0 V_0 = 50\times10^{-12}\times 200$

$= 10^{-8}$C

After charging the battery is removed. Hence the change Q will remain unchanged, there after. How, suppose the capacitance changes to C, after placing the dielectric.

Here, $C = \frac{\varepsilon_0 A}{d - t + \left(t/K\right)}$

$$= \frac{\varepsilon_0 A}{4 - 2 + \frac{2}{4}} \qquad \text{(As t = 2m, d = 3mm and K = 4)}$$

$$= \frac{\varepsilon_0 A}{10}\times 4$$

Since Q remains same,

$Q = C_0 V_0 = CV$ (V is the new potential)

$\therefore\ V = \frac{C_0 V_0}{C}$

$$= \frac{\varepsilon_0 A \times 200}{4 \times \frac{\varepsilon_0 A \times 4}{10}}$$

$$= 125 \text{ V}$$

Now, energy loss, when the slab is placed

$$\Delta U = U_i - U_f$$

$$= \frac{1}{2} QV_0 - \frac{1}{2} QV$$

$$= \frac{1}{2} Q(V_0 - V)$$

$$= \frac{1}{2} Q(200 - 125)$$

$$= \frac{1}{2} \times 10^{-8} \times 75$$

$$= 37.5 \times 10^{-8}$$

$$= 3.75 \times 10^{-7} \text{ s}$$

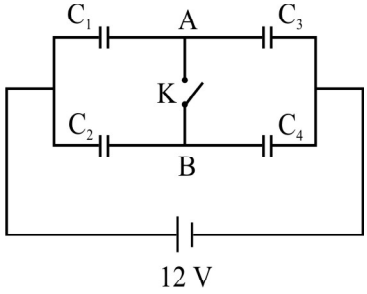
This energy loss will appear as surplus heat produced in the slab. So heat produced in the slab $= 3.75 \times 10^{-7}$ J

Correct answer is (3)

Unit–11 : Electrostatics

Difficult *Combination of Capacitors*

38. In the given network, $C_1 = 1\,\mu F$, $C_2 = 2\,\mu F$, $C_3 = 3\,\mu F$ and $C_4 = 4\,\mu F$. When the key is closed –q charge flows from A to B. The numerical value of q in μC is:



(1) 2.4 (Correct)

(2) 1.8 (Incorrect)

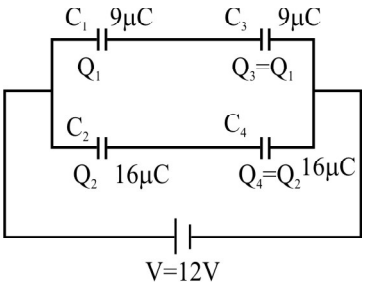
(3) 0.8 (Incorrect)

(4) 0.2 (Incorrect)

Correct answer: (1)

Solution:

When K is open, the network will be as shown in the figure. C_1 and C_3 are in series. Similarly C_2 and C_4 are in series. Both the series combination get 12 V each.



$$Q_1 = Q_3 = V\frac{C_1C_3}{C_1 + C_3} = 12 \times \frac{1\times 3}{1+3} = 9\mu C$$

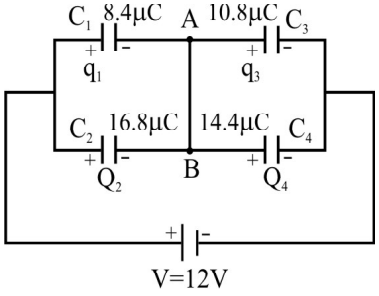
$$Q_2 = Q_4 = V\frac{C_2C_4}{C_2 + C_4} = 12 \times \left(\frac{2\times 4}{2+4}\right) = 16\mu C$$

$\therefore\ Q = 9\mu C + 16\mu C = 25\mu C$

Now, When K is closed, the network will be as shwon in the figure.

Here, C_1 and C_2 are in parallel. C_3 and C_4 are in parallel and these parallel combinations are in series.

Here, the effective capactitance C is:



$$C = \frac{(C_1 + C_2)\times(C_3 + C_4)}{C_1 + C_2 + C_3 + C_4} = \frac{(1+2)\times(3+4)}{1+2+3+4}$$

$$= \frac{3\times 7}{10} = 2.1\mu F$$

$\therefore$ Change stored in Q = CV = $2.1\times 12 = 25.2\,\mu C$.

So, chnage on $(C_1+C_2) = 3\,\mu F$ is $q_1+q_2 = 25.2\,\mu C$.

Hence, potential difference across C_1, which is equal to that across C_2 is:

$$V_1 = V_2 = \frac{25.2\mu C}{3\mu F} = 8.4V$$

Similary, $V_3 = V_4 = 3.6$ V

Now, $q_1 = C_1V_1 = 1\times 8.4 = 8.4\,\mu C$

$q_2 = C_2V_2 = 2\times8.4 = 16.8\,\mu C$

$q_3 = C_3V_3 = 3\times3.6 = 10.8\,\mu C$

$q_4 = C_4V_4 = 4\times3.6 = 14.4\,\mu C$

Initial change on the –ve plate of $C_1 = -9\,\mu C$

Final change on the –ve plate of $C_1 = -8.4\,\mu C$

Change flowing down through A, from $C_1 = -0.6\,\mu C$

Now, initial change on +ve plate of $C_3 = +9\,\mu C$

Final change on +ve plate of $C_3 = +10.8\,\mu C$

$\therefore$ Chnage flowing down through A, from $C_3 = -1.8\,\mu C$

Hence, total change flowing down through A = –0.6 + –1.8

$= -2.4\,\mu C$

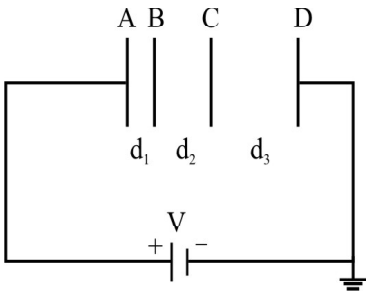
$\therefore$ Value of q in $\mu C = 2.4$

Correct answer is (1)

Unit–11 : Electrostatics

Difficult *Combination of Capacitors*

39. A, B, C, and D are four similar metallic parallel plates with separtions d_1, d_2, and d_3 respectively, such that $d_1 : d_2 : d_3 = 1 : 2 : 3$. A source of potential V is connected as shwon. The potential of A, B, C and D are respectively.



(1) $V, \frac{2}{3}V, \frac{1}{3}V$ and 0 (Incorrect)

(2) $V, \frac{2}{3}V, \frac{1}{6}V$ and 0 (Incorrect)

(3) $V, \frac{6}{6}V, \frac{1}{2}V$ and 0 (Correct)

(4) $V, \frac{1}{2}V, \frac{1}{6}V$ and 0 (Incorrect)

Correct answer: (3)

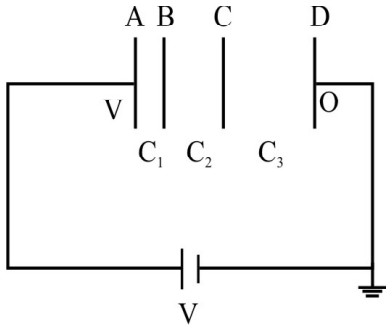
Solution:

The arrangement is equivalent to three capacitors of C_1, C_2 and C_3 connected in series. Plate area, A, is same for all and for C_1, $d_1 = d$; for C_2, $d_2 = 2d$ and for C_3, $d_3 = 3d$.

$$\therefore\ C_1 = \frac{\varepsilon_0 A_1}{d_1} = \frac{\varepsilon_0 A}{d} = C_0$$

$$C_2 = \frac{\varepsilon_0 A_2}{d_2} = \frac{\varepsilon_0 A}{2d} = \frac{C_0}{2}$$

$$C_3 = \frac{\varepsilon_0 A_3}{d_3} = \frac{\varepsilon_0 A}{3d} = \frac{C_0}{3}$$



If C is the effective capacitance, then:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

or $$\frac{1}{C} = \frac{1}{C_0} + \frac{2}{C_0} + \frac{3}{C_0}$$

or $$\frac{1}{C} = \frac{6}{C_0}$$

or $$C = \frac{C_0}{6}$$

The charge stored in system is:

$$q = CV,\ q = \frac{C_0}{6}V$$

q is same for all.

$\therefore$ Potential difference for C_1 is

$$V_1 = \frac{q}{C_1} = \frac{C_0 V}{C_0 \times 6} = \frac{V}{6}$$

Potential difference for C_2 is

$$V_2 = \frac{q}{C_2} = \frac{C_0 V}{6 \times \left(\frac{C_0}{2}\right)} = \frac{V}{3}$$

Potential difference for C_3 is

$$V_3 = \frac{q}{C_3} = \frac{C_0 V}{6 \times \left(\frac{C_0}{3}\right)} = \frac{V}{2}$$

Now, from the figure given:

$V_A = V$ and $V_D = 0$ (as D is earthed)

Now, $V_A - V_B = \frac{V}{6}$ (for C_1)

$\therefore\ V_B = V_A - \frac{V}{6} = V - \frac{V}{6} = \frac{5}{6}V$

$V_B - V_C = \frac{V}{3}$ (for C_2)

$\therefore\ V_C = V_B - \frac{V}{3}$

$$= 5\frac{V}{6} - \frac{V}{3} = \frac{3}{6}V = \frac{V}{2}$$

or $V_A = V, V_B = \frac{5}{6}V,\ V_C = \frac{V}{2}$ and $V_D = 0$

Correct answer is (3)

Unit–11 : Electrostatics

Average *Electric Field due to Spherical Body*

40. Consider a sphere of radius R with a volume distribution of charge represented by: $\rho_{(r)} = Kr$ for $r \le R$ and $\rho = 0$ for $r \ge R$. The electric field intensity at any point where r > R is given as:

(1) $E = \frac{Kr^2}{4\varepsilon_0}$ (Incorrect)

(2) $E = \frac{KR^4}{4\varepsilon_0 r^2}$ (Correct)

(3) $E = \frac{KR^2}{4\varepsilon_0}$ (Incorrect)

(4) $E = \frac{KR^4}{\varepsilon_0 r^2}$ (Incorrect)

Correct answer: (2)

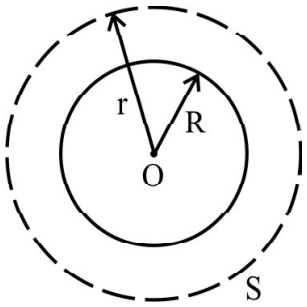
Solution:

Let S be a spherical Gaussian surface passing through the point, with r > R.

$$\oint_S \mathbf{E}.\mathbf{ds} = E \times 4\pi r^2 \qquad .....(1)$$

Now, the total charge of the sphere of radius R is;

$$q = \int_{r=0}^{R} \rho(r)\, dV$$



or $$q = \int_{r=0}^{R} Kr.4\pi r^2 dr$$

or $$q = 4\pi K \frac{R^4}{4} \qquad .....(2)$$

By Gauss's theorem,

$$\oint_S \mathbf{E}.\mathbf{ds} = \frac{1}{\varepsilon_0} \quad \text{(Charge enclosed by S)} \qquad .....(3)$$

From (1), (2) and (3), we have:

$$E \times 4\pi r^2 = \frac{1}{\varepsilon_0}.4\pi \frac{KR^2}{4}$$

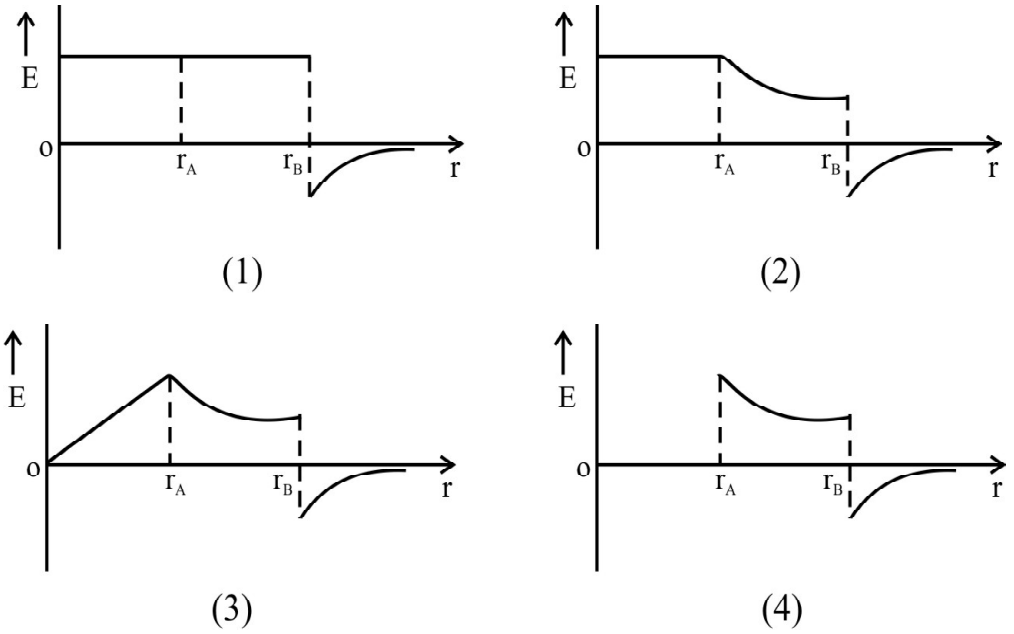
$$\therefore\ E = \frac{KR^4}{4\varepsilon_0 r^2}$$

Correct answer is (2)

Unit–11 : Electrostatics

Average *Electric Field due to a Charged Shell*

41. Two concentric conducting thin spherical shells A and B having radii r_A and r_B ($r_B > r_A$) are charged to $Q_A = +q$ and $Q_B = -2q$ respectively. The variation the electric field (E) with distance r from the common centre is repressented by the graph.



Correct answer: (4)

Solution:

As the shells are made of conductors, charges reside only on their surfaces. Therefore electric field inside the shell, due to the charge on its surface will be zero.

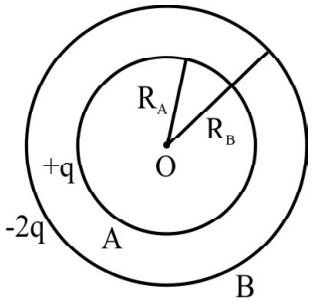
Therefor, inside shell A, there no field due to charge on A or charge on B. In the space between A and B, the field is due to charge on A only. Since charge on in $Q_A = +q$, the field in positive direction of r.

Now, outside shell B, the charges on A and B behave like point charges at the common centre.

$Q_0 = +q + -2q$

$= -q$

For $r_A > r > r_B$, field is due to +q only, as if it is at 0.



$$\therefore\ E = \frac{Kq}{r^2}$$

or $$E \alpha \frac{1}{r^2}$$

Therefore, E is positive and it varies inversely as square of r, for $r_A < r < r_B$.

For $r > r_B$, field is due to –q, as if it is at 0.

$$\therefore\ E = -\frac{Kq}{r^2}$$

Therefore, E is –ve and it also varies inversely as square of r, for $r > r_B$.

$\therefore$ E = 0, up to $r = r_A$

E is positive and $E \alpha \frac{1}{r^2}$ for $r_A < r < r_B$.

E is negative and $E \alpha \frac{1}{r^2}$ for $r > r_B$.

Correct graph is (4).

Unit–11 : Electrostatics

Easy *Electric Potential*

42. The potential of the electric field at any point **(*x*, y, z) is given by: $V = 3x^2 + 5$, where *x*, y and z are in metres and V is in volts. The intensity of electric field at (–2, 1, 0)m is:**

(1) 7 V–m^{-1} (Incorrect)

(2) –17 V–m^{-1} (Incorrect)

(3) 12 V–m^{-1} (Correct)

(4) –12 V–m^{-1} (Incorrect)

Correct answer: (3)

Solution:

$$\mathbf{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

$= -\left(6x\hat{i} + 0\hat{j} + 0\hat{k}\right)$

$= -6x\hat{i}$

$\therefore\ \mathbf{E}(-2,1,0) = -6 \times -2\hat{i}$

or $\quad E = 12\hat{i}\ V - m^{-1}$

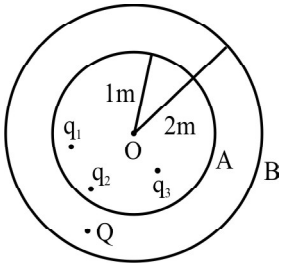
$\therefore\ E = 12\,V - m^{-1}$

Correct answer is (3)

Unit–11 : Electrostatics

Easy *Electric Flux*

43. In the figure given, A and B are two concentric spherical surfaces. Given that $q_1 = +1\,\mu C$, $q_2 = -2\,\mu C$ and $q_3 = +9.854\,\mu C$ and that the flux crossing B is four times that crossing A. The value of Q is:



(1) $8.854\,\mu C$ (Incorrect)

(2) $17.708\,\mu C$ (Incorrect)

(3) $26.562\,\mu C$ (Correct)

(4) $35.416\,\mu C$ (Incorrect)

Correct answer: (3)

Solution:

The flux crossing spherical surface A is:

$\phi_A = \frac{1}{\varepsilon_0}$ (Charge enclosed by A)

or $\quad \phi_A = \frac{1}{\varepsilon_0}(q_1 + q_2 + q_3)$

or $\phi_A = \frac{1}{\varepsilon_0}(1 + -2 + 9.854)$ (q in μC)

or $\phi_A = \frac{8.854}{\varepsilon_0}$ .....(1)

Similarly, $\phi_B = \frac{1}{\varepsilon_0}(q_1 + q_2 + q_3 + Q)$

$= \frac{1}{\varepsilon_0}(8.854 + Q)$ .....(2)

But, $Q_B = 4QA$ (given)

$\therefore \frac{1}{\varepsilon_0}(8.854 + Q) = 4 \times \frac{8.854}{\varepsilon_0}$

or $\phi = 3 \times 8.854$

$= 26.562\,\mu C$

Correct answer is (3)

Unit–11 : Electrostatics

Easy *Electric Flux*

44. Point chargers of magnitude 2Q and –Q are located at points (–a, 0, 0) and (4a, 0, 0). The ratio of electric flux crossing two spherical surfaces A and B of radii 2a and 8a respectively, centred at the origin, is:

(1) 1 : 4 (Incorrect)

(2) 1 : 2 (Incorrect)

(3) 2 : 1 (Correct)

(4) 1 : 1 (Incorrect)

Correct answer: (3)

Solution:

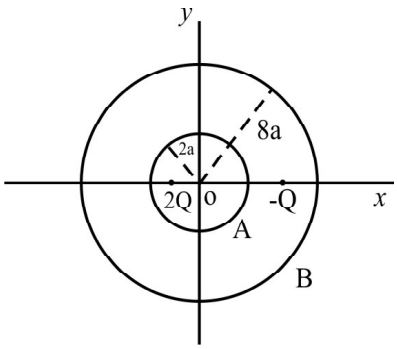
The situation is shown is the figure. Clearly, 2Q is inside sphere A and both 2Q and –Q are inside sphere B.

Now, $\phi_A = \frac{1}{\varepsilon_0}(2Q)$ .....(1)

$\phi_B = \frac{1}{\varepsilon_0}(2Q + -Q)$

$= \frac{1}{\varepsilon_0}(Q)$ .....(2)

$\therefore \quad \frac{\phi_A}{\phi_B} = 2:1$



Correct answer is (3)

Unit–11 : Electrostatics

Average *Electric Field due to Charged Sphere*

45. At a point which is 20cm from the centre of uniformly charged dielectric sphere, of radius 10cm, the elctric field intensity is 100 Vm^{-1}. The elctric field at 3cm from the centre of the sphere will be:

(1) 150 Vm^{-1} (Incorrect)

(2) 125 Vm^{-1} (Incorrect)

(3) 120 Vm^{-1} (Correct)

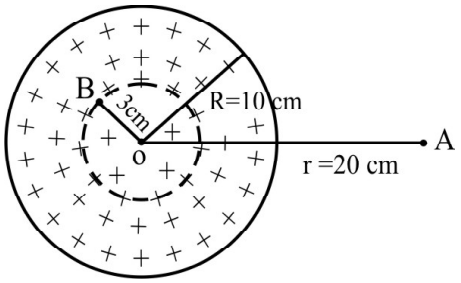
(4) zero (Incorrect)

Correct answer: (3)

Solution:

R = 10 cm

Let Q be the total charge in the dielectric sphere than



$$\rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

Now, at r = 20cm Q acts as point charge at 0.

$$\therefore \quad E_A = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

or $\frac{Q}{4\pi\varepsilon_0} = E_A r^2 = 100 \times (0.2)^2 = 4$

At B, where r = 3cm, the field is due to the charge inside the sphere of r = 3cm, which would act as point charge (q) at 0.

Now, $q = \rho \times \frac{4}{3}\pi r^3$

$$= \frac{Q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3$$

$$= Q \cdot \frac{r^3}{R^3}$$

$$= Q \cdot \frac{(0.03)^3}{(0.1)^3}$$

The field at B is:

$$E_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \times \frac{1}{(0.03)^2} \times Q \cdot \frac{(0.03)^3}{(0.1)^3} \qquad (r = 0.03m)$$

$$= \frac{Q}{4\pi\varepsilon_0} \times \frac{0.03}{0.001}$$

$$= 4 \times 30$$

$$= 120 \text{ Vm}^{-1}$$

Correct answer is (3)

Unit–11 : Electrostatics

Easy *Capacitance of a Conductor.*

46. Taking earth to be a spherical conductor of diameter 12.8×10^3 km, its electrical capacitance is

(1) 711 μF (Correct)

(2) 811 μF (Incorrect)

(3) 611 μF (Incorrect)

(4) 511 μF (Incorrect)

Correct answer: (1)

Solution:

The capacitance of a spherical conductor is $C = 4\pi\varepsilon_0 R$, where R is the radius of the spherical conductor. Here, for earth,

$$R = \frac{12.8\times10^3}{2} \text{ km}$$

$$= 6.4\times10^6 \text{ m}$$

$$4\pi\varepsilon_0 = \frac{1}{9\times10^9}$$

$$\therefore\ C = \frac{6.4\times10^6}{9\times10^9}$$

$$= 0.711\times10^3 \text{ F}$$

$$= 711\times10^{-3} \text{ F}$$

$$= 711\,\mu\text{F}$$

Correct answer is (1)

Unit–11 : Electrostatics

Average *Electric Force*

47. Four charges, equal to –Q, are placed at the four corners of a sqare and a charge q is placed at the centre of this square. If the system remains in equilibrium, the value of q, in terms of Q, is:

(1) $-\frac{Q}{4}\left(2\sqrt{2}+1\right)$ (Incorrect)

(2) $\frac{Q}{4}\left(1+2\sqrt{2}\right)$ (Correct)

(3) $-\frac{Q}{2}\left(2\sqrt{2}+1\right)$ (Incorrect)

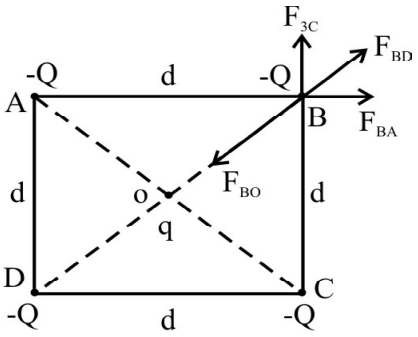
(4) $\frac{Q}{2}\left(1+2\sqrt{2}\right)$ (Incorrect)

Correct answer: (2)

Solution:

For equilibrium, the net force should be zero. Taking –Q at B, we have

$\mathbf{F}_{BO}+\mathbf{F}_{BC}+\mathbf{F}_{BA}+\mathbf{F}_{BD}=\mathbf{0}$



$\left|\mathbf{F}_{BO}\right|=\frac{kqQ}{\left(\frac{d}{\sqrt{2}}\right)^2}$

$\left|\mathbf{F}_{BC}\right|=\frac{kQ^2}{d^2};\ \left|\mathbf{F}_{BA}\right|=\frac{kQ^2}{d^2}$

$\therefore\ \left|\mathbf{F}_{BC}+\mathbf{F}_{BC}\right|=\sqrt{2}\frac{kQ^2}{d^2}$, at 45^0 with each other.

Now, $\left|\mathbf{F}_{BD}\right|=\frac{kQ^2}{\left(\sqrt{2}d\right)^2}$, at 45^0, again.

$\therefore\ \left|\left(\mathbf{F}_{BC}+\mathbf{F}_{BA}\right)+\mathbf{F}_{BD}\right|=\sqrt{2}\frac{kQ^2}{d^2}+\frac{kQ^2}{2d^2}$

For equilibrium,

$\left|\mathbf{F}_{BO}\right|=\left|\left(\mathbf{F}_{BC}+\mathbf{F}_{BA}\right)+\mathbf{F}_{BD}\right|$

or $\dfrac{kqQ}{\frac{d^2}{2}} = \sqrt{2}\dfrac{kQ^2}{d^2} + \dfrac{kQ^2}{2d^2}$

or $q = \dfrac{Q}{4}\left(2\sqrt{2}+1\right)$

Correct answer is (2)

Unit–11 : Electrostatics

Easy *Electric Potential*

48. A metal sphere A of radius 'a' is charged to some potential V. What will be its potential if sphere A is enclosed by spherical conducting shell B, of radius, 'b' and the two spheres are connected by a conducting wire?

(1) zero (Incorrect)

(2) $\dfrac{b}{a}V$ (Incorrect)

(3) $\dfrac{a}{b}V$ (Correct)

(4) V (Incorrect)

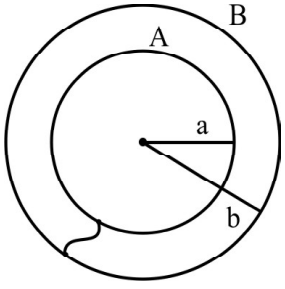
Correct answer: (3)

Solution:

Let the charge on A be 'q', intially.

Then, $V = \dfrac{q}{4\pi\varepsilon_0 a}$

or $q = \left(4\pi\varepsilon_0 a\right)V$



When A is placed inside B and connected with wire, the entire charge 'q' flows out on to the surface of B. Now, A is uncharged and it is inside the B, charged to q.

Now, potential on the surface or at any point inside a charged conductor remains the same.

$\therefore$ The potential of A = potential of B, now.

$$V_A = \frac{q}{4\pi\varepsilon_0 b}$$

or $$V_A = \frac{(4\pi\varepsilon_0 a)V}{4\pi\varepsilon_0 b} = \frac{a}{b}V$$

Correct answer is (3)

Unit–11 : Electrostatics

Difficult *Electric Potential*

49. A thin disc of radius R is uniformly charged with A charge. The potential due to this disc, at a point on its axis and distant R from its centre is

(1) $0.707\left(\frac{Q}{\pi\varepsilon_0 R}\right)$ (Correct)

(2) $\frac{Q}{2\pi\varepsilon_0 R}$ (Incorrect)

(3) $\frac{Q}{4\pi\varepsilon_0\sqrt{2R}}$ (Incorrect)

(4) $\frac{\sqrt{2}Q}{4\pi\varepsilon_0 R}$ (Incorrect)

Correct answer: (1)

Solution:

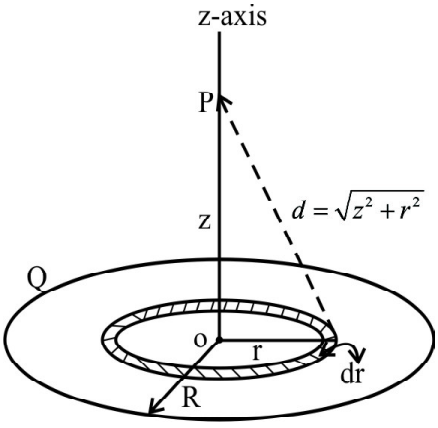
Let P be a point on the axis of the disc, at distance z from its centre.

Assuming that the disc be divided large number of elementary rings, each ring will produced a potential of dV at P. Intergration over all the dVs will give V at P.

The charge on one such elementary ring, as shown in the figure is:

$$dq = (2\pi r dr)\sigma \qquad \left(\sigma = \frac{Q}{\pi R^2}\right)$$

So, $$dv = \frac{1}{4\pi\varepsilon_0}\frac{(dq)}{d}$$



$$= \frac{1}{4\pi\varepsilon_0}\frac{2\pi r dr\sigma}{\sqrt{z^2+r^2}}$$

$\therefore$ Potential at P due to the entire disc is:

$$V = \frac{\sigma\pi}{4\pi\varepsilon_0}\int_0^R \frac{2rdr}{\sqrt{r^2+z^2}}$$

$$= \frac{2\sigma\pi}{4\pi\varepsilon_0}\left[\sqrt{r^2+z^2}\right]_0^R$$

$$= \frac{2\sigma\pi}{4\pi\varepsilon_0}\left[\sqrt{r^2+z^2}-\sqrt{0+z^2}\right]$$

$$= \frac{2\sigma\pi}{4\pi\varepsilon_0}\left[\sqrt{r^2+z^2}-z\right]$$

$$= \frac{2Q}{4\pi\varepsilon_0 R^2}\left[\sqrt{r^2+z^2}-z\right]$$

Now, at z = R,

$$V = \frac{2Q}{4\pi\varepsilon_0 R^2}\left[\sqrt{2}R-R\right]$$

$$= \frac{Q}{4\pi\varepsilon_0 R}\left[\sqrt{2}-1\right]$$

$$= 0.707\left(\frac{Q}{\pi\varepsilon_0 R}\right)$$

Correct answer is (1)

Unit–11 : Electrostatics

Difficult *Eelectric Field*

50. A simple pendulum, of length 'l', has a bob of mass 'm'. A charge 'q' is given to the bob and it is made to execute simple harmonic oscillations parallel to the field lines of a horizontal uniform electric field z. If 'g' is the acceleration due to gravity, then the time period of oscillation of the pendulum is

(1) $2\pi\sqrt{\dfrac{\ell}{\sqrt{g^2+\left(\dfrac{qE}{m}\right)^2}}}$ (Correct)

(2) $T=2\pi\sqrt{\dfrac{m\ell}{\sqrt{g^2+\left(\dfrac{qE}{m}\right)^2}}}$ (Incorrect)

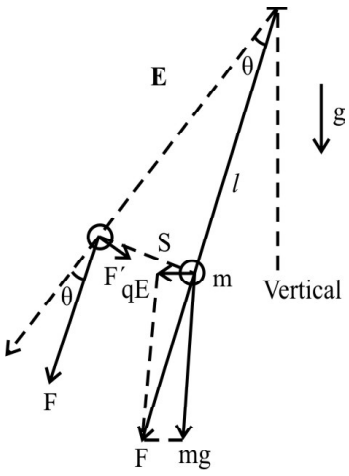
(3) $T=2\pi\sqrt{\dfrac{\ell}{\sqrt{(mg)^2+(qE)^2}}}$ (Incorrect)

(4) $T=2\pi\sqrt{\dfrac{\ell}{\sqrt{q^2+(qE)^2}}}$ (Incorrect)

Correct answer: (1)

Solution:

There are two forces on the bob, as shown in the figure. (i) mg downwords (ii) qE, horizontally.



So the resultant force on the bob is $F=\sqrt{(mg)^2+(qE)^2}$

In equilibrium position of the bob, this for F will act along the length of the pendulum. So the pendulum will rest at certain angle from the vertical.

Now, suppose the pendulum is disturbed through a small angle θ, by moving the bob in the plane of the electric field, as shown.

The restoring force developed on the bob will be:

F' = F sin θ, directed opposite to S.

F' acts on the bob. So when the bob is released, F' will produce and acceleration, 'a', such that F' = ma.

$\therefore$ ma = – Fsin θ (–ve sign as the force is opposite to S)

or $\dfrac{d^2S}{dt^2}=-\dfrac{F}{m}.\theta$

or $\dfrac{d^2S}{dt^2} = -\sqrt{\dfrac{(mg)^2 + (qE)^2}{m}} \cdot \left(\dfrac{S}{\ell}\right)$

or $\dfrac{d^2S}{dt^2} = -\left(\dfrac{\sqrt{g^2 + \left(\dfrac{qE}{m}\right)^2}}{\ell}\right) S$

This is of the form, $\dfrac{d^2y}{dt^2} = -\omega^2 y$ for S.H.M.

$\therefore\ \omega^2 = \sqrt{\dfrac{g^2 + \left(\dfrac{qE}{m}\right)^2}{\ell}}$ or $\omega = \sqrt{\dfrac{\sqrt{g^2 + \left(\dfrac{qE}{m}\right)^2}}{\ell}}$

Now, $T = \dfrac{2\pi}{\omega}$

or $T = 2\pi \sqrt{\dfrac{\ell}{\sqrt{g^2 + \left(\dfrac{qE}{m}\right)^2}}}$

Correct answer is (1)

Unit–11 : Electrostatics

Average *Motion in Electric Field*

51. A small copper ball, of density 8.6 gcm^{-3} **and diameter 4 cm, is immersed in oil (an insulator) of density 0.8 gcm^{-3}. This copper ball, of charge 'q', just remain suspented in the oil, under the effect of an electric field of intensity 2600 Vm^{-1} acting in the upward direction. The value of q is: (Take g = 10ms^{-2})**

(1) 47.54 mC (Incorrect)

(2) 43.12 mC (Correct)

(3) –47.54 μC (Incorrect)

(4) 43.12 μC (Incorrect)

Correct answer: (2)

Solution:

The ball will remain suspended in oil when the force due to electric field balances the apparent weight of the ball in the oil.

$\therefore$ qE = Apparent weight

or $qE = \frac{4}{3}\pi r^3 (\rho - \sigma) g$

or $q = \frac{\frac{4}{3}\pi r^3 (\rho - \sigma) g}{E}$

Here, Density of copper $(\rho) = 8.6\,gcm^{-3} = 8600\,kgm^{-3}$

Density of oil $(\sigma) = 0.8\,gcm^{-3} = 800\,kgm^{-3}$

$r = \frac{14\,cm}{2} = 7\,cm = 7\times10^{-2}\,m$

$E = 2600\,Vm^{-1}$

$g = 10\,ms^{-2}$

$\therefore\ q = \frac{4}{3}\times\frac{22}{7}\times\frac{\left(7\times10^{-2}\right)(8600-800)\times10}{2600}$

$= 88\times49\times10^{-5}$

$= 4312\times10^{-5}\,C$

$= 43.12\,mC$

Correct answer is (2)

Unit–11 : Electrostatics

Average *Principle of Superposition of Charges*

52. Point charges of value 4 μC and –9 μC are placed at certain distance apart in air. A third charge 'q' is placed, on the line joining these two charges, and at a suitable position, such that all the three charges remain in equilibrium. The value of 'q' is

(1) $+36\ \mu C$ (Incorrect)

(2) $+1.44\ \mu C$ (Incorrect)

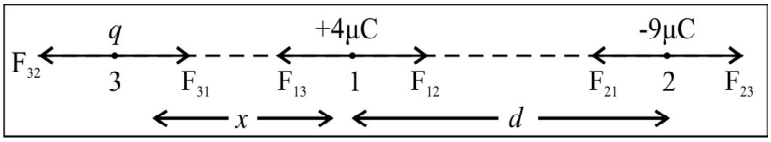
(3) $-36\ \mu C$ (Correct)

(4) $-1.44\ \mu C$ (Incorrect)

Correct answer: (3)

Solution:

For equilibrium, there should be two equal and opposite forces on each charge. This condition is statified, here, only if the third charge 'q' is placed on the line joining the first two, outside the two, and nearer to the smaller charge. This is shown diagramtically as.



And the third charge should be negative.

For charge q to be is equibrium,

$$F_{31} = F_{32}$$

or $$\frac{Kq \times 4}{x^2} = \frac{Kq \times 9}{(x+d)^2}$$

or $$\frac{2}{x} = \frac{3}{x+d}$$

$\therefore\ x = 2d$

For charge $+4\ \mu C$ to be in equilibrium,

$$F_{12} = F_{13}$$

or $$\frac{K \times 4 \times 9}{d^2} = \frac{K \times 4 \times q}{x^2}$$

or $$\frac{9}{d^2} = \frac{q}{(2d)^2}$$

$\therefore\ q = 36\mu C$ (Negative)

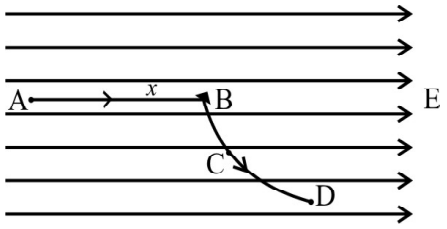
or $q = -36\mu C$

Correct answer is (3)

Unit–11 : Electrostatics

Average *Motion of Charge in Uniform Electric Field*

53. A particle of mass 'm' and carrying a charge 'q' is kept at rest at point A, is a uniform electric field of intensity E. At time instant t = 0, the particle released from A. At t = t, it hits a block at B (AB = *x*) elastically, and charges its direction of motion by 90^0, instantly. Following a parabolic path BCD, it reaches point D, as shown in the figure, at time t = 2t. The speed of the particle at D is:



(1) $\frac{1}{2}qEx^2$ (Incorrect)

(2) $\left(\frac{2Ex^2}{m}\right)^{1/2}$ (Incorrect)

(3) $2\sqrt{\frac{Exq}{m}}$ (Correct)

(4) $\left(\frac{qEx}{m}\right)^{1/2}$ (Incorrect)

Correct answer: (3)

Solution:

The electric field will produce a constant acceleration of $a = \frac{qE}{m}$ along the field direction.

The velocity attained by q at B is:

$$v^2 = 0 + 2\frac{qE}{m}x$$

or $$v = \sqrt{\frac{2qE}{m}}x$$

After collision, this *v* become perpendicular to E and hence remains constant (along $-\hat{j}$). But, the

acceleration remains the same in magnitude and direction (along $\hat{i}$). Now, along E-field, the particle again will cover *x* distance in the next t second after collision, as it reaches the point D, following the parabolic path.

Hence, its velocity, will again increases to *v* (along E-field) where $v = \sqrt{\frac{2qE}{m}}\, x$

Therefore, at D it will have $\mathbf{v}_P = v\hat{i} - v\hat{j}$

$\therefore$ Speed at 'D' will be

$$v_D = \sqrt{v^2 + v^2}$$

$$= \sqrt{2v^2}$$

$$= \sqrt{2 \times \frac{2qE}{m}\, x}$$

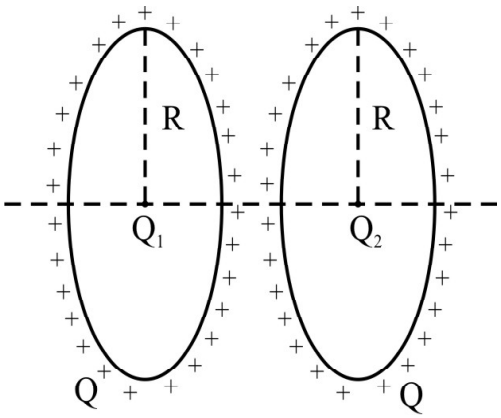
$$= 2\sqrt{\frac{Exq}{m}}$$

Correct answer is (3)

Unit–11 : Electrostatics

Difficult *Electric Field*

54. Two co-axial circular rings of radius R = 10cm each are uniformly charged with $Q = \sqrt{3}$ nC each. The distance between their centres is such that the centre of one ring, produced by the other is maximum. The magnitude of the net electric field at the centre of each ring is



(1) $600\,\mathrm{NC}^{-1}$ (Correct)

(2) $900\sqrt{3}\,\mathrm{NC}^{-1}$ (Incorrect)

(3) $\left(6+9\sqrt{3}\right)\times10^{2}\,\mathrm{NC}^{-1}$ (Incorrect)

(4) zero (Incorrect)

Correct answer: (1)

Solution:

When the field is maximum, the distance $x=\frac{R}{\sqrt{2}}$, according to the formula

$$E=\frac{KQx}{\sqrt{R^2+x^2}}$$ (x is distance along the axis)

The field due to the charge on the ring at its own centre is always zero.

$\therefore$ The net field at the centre of one is

$$E_{net}=0+\frac{KQx}{\sqrt{R^2+x^2}}, \text{ where } x=\frac{R}{\sqrt{2}}$$

Here, R = 10 cm = 10×10^{-2}m

$$\frac{R}{\sqrt{2}}=\frac{10}{\sqrt{2}}\text{ cm}=5\sqrt{2}\text{ cm}=5\sqrt{2}\times10^{-2}\text{ m}$$

$$Q=\sqrt{3}\text{ nC}=\sqrt{3}\times10^{-9}\text{C}$$

$$K=9\times10^{9}\text{ SI Units}$$

$$E_{net}=\frac{9\times10^{9}\times\sqrt{3}\times10^{-9}\times5\sqrt{2}\times10^{-2}}{\left[\left(10\times10^{-2}\right)^2+\left(5\sqrt{2}\times10^{-2}\right)^2\right]^{3/2}}$$

$$=\frac{9\times\sqrt{3}\times5\sqrt{2}\times10^{-2}}{(100+50)^{3/2}\times10^{-6}}$$

$$= \frac{9 \times \sqrt{3} \times 5\sqrt{2} \times 10^{-2}}{\left(5^2 \times 6\right)^{3/2} \times 10^{-6}}$$

$$= \frac{9 \times \sqrt{3} \times 5\sqrt{2} \times 10^{4}}{5^3 \times 6 \times \sqrt{3} \times \sqrt{2}}$$

$$= 600\ \text{NC}^{-1}$$

Correct answer is (1)

Unit–11 : Electrostatics

Easy *Continuous Charge Distribution*

55. A conducting sphere A, of radius 10 cm is charged to 10 μC. Another uncharged conducting sphere B, of radius 20 cm is brought is to contact with A and after that the spheres are separated to large distance. The surface density of charge on the spheres, now, will be in the ratio:

(1) 1 : 4 (Incorrect)

(2) 4 : 2 (Incorrect)

(3) 1 : 2 (Incorrect)

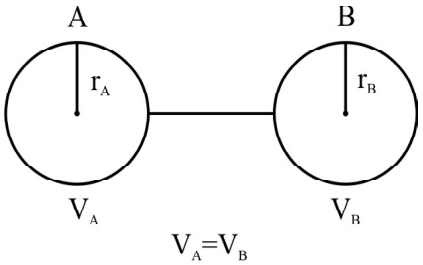
(4) 2 : 1 (Correct)

Correct answer: (4)

Solution:

When the spheres are brought to contact, they share the total charge in such a way that the potential of both are same.

i.e. $V_A = V_B$ (after they are brought in to contact).



If Q_A and Q_B are the final charges on the spheres, then

$$V_A = \frac{KQ_A}{r_A} \text{ and } V_B = \frac{KQ_B}{r_B}$$

Since $V_A = V_B$,

$$\Rightarrow \frac{Q_A}{r_A} = \frac{Q_B}{r_B}$$

or $\dfrac{4\pi r_A^2\sigma_A}{r_A} = \dfrac{4\pi r_B^2\sigma_B}{r_B}$

$\therefore \dfrac{\sigma_A}{\sigma_B} = \dfrac{r_B}{r_A} = 2:1$

Correct answer is (4)

Unit–11 : Electrostatics

Easy *Electric Dipole*

56. A dipole consisting of two point charges of –2 μ C and +2 μ C located at points A (1, 2, 2) cm and B (2, 1, 4) cm respectively, is in a uniform electric field of $\mathbf{E} = \left(3\hat{i} + \hat{j} + 2\hat{k}\right)\times 10^5\ \text{Vm}^{-1}$. The magnitude of torque experienced by the dipole is

(1) 2.772×10^{-2} Nm (Incorrect)

(2) 1.386 Nm (Incorrect)

(3) 2.772 Nm (Incorrect)

(4) 1.386×10^{-2} Nm (Correct)

Correct answer: (4)

Solution:

Let the dipole moment be $\vec{P}$

$\mathbf{P} = (2\mathbf{a})q$ ($2\mathbf{a}$ = distance vector from –q to +q)

Now, $2\mathbf{a} = \mathbf{r}_B - \mathbf{r}_A$

$= \left(2\hat{i} + \hat{j} + 4\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 4\hat{k}\right)$

$= \left(\hat{i} - \hat{j} + 2\hat{k}\right)\text{cm}$

$= \left(\hat{i} - \hat{j} + 2\hat{k}\right)\times10^{-2}\text{m}$

$q = 2\mu C = 2\times10^{-6}\,C$

$\therefore\ \mathbf{P} = \left(2\hat{i} + 2\hat{j} + 4\hat{k}\right) \times 10^{-8}$ cm

$\mathbf{E} = \left(3\hat{i} + \hat{j} + 2\hat{k}\right) \times 10^{6}\ \text{Vm}^{-1}$

$$\therefore\ \boldsymbol{\tau} = \mathbf{P} \times \mathbf{E} = 10^{-3} \times \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 4 \\ 3 & 1 & 2 \end{vmatrix} = \left[\hat{i}(-4-4) + \hat{j}(12-4) + \hat{k}(2+6)\right] \text{x} 10^{-3}$$

$$= \left[-8\hat{i} + 8\hat{j} + 8\hat{k}\right] \times 10^{-3}$$

$\therefore\ \boldsymbol{\tau} = |\boldsymbol{\tau}| = 8\sqrt{3} \times 10^{-3}$ Nm

$= 1.386 \times 10^{-2}$ Nm

Correct answer is (4)

Unit–11 : Electrostatics

Average *Principle of Superposition of Charges*

57. An infinite number of point charges, each of magnitude q = 50 μ C, are placed along the x–axis, at *x* = 1m, *x* = 2m, *x* = 4m, *x* = 8m, *x* = 16 m and so on. Consecutive charges have opposite sign, with q at *x* = 1m being positive. The net electric field at the origin 0 has a magnitude of

(1) $6.0 \times 10^{5}\ \text{NC}^{-1}$ (Incorrect)

(2) $6.0 \times 10^{9}\ \text{NC}^{-1}$ (Incorrect)

(3) $3.6 \times 10^{5}\ \text{NC}^{-1}$ (Correct)

(4) $2.7 \times 10^{5}\ \text{NC}^{-1}$ (Incorrect)

Correct answer: (3)

Solution:

$$\text{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{1^2} - \frac{q}{2^2} + \frac{q}{4^2} - \frac{q}{8^2} + \frac{q}{16^2} \ldots\ldots\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{1} - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \frac{1}{265} \ldots\ldots\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{1-\left(\frac{-1}{4}\right)}\right] \qquad \left(\because S = \frac{a}{1-r} \text{ for sum of infinite terms of a G.P.}\right)$$

$$= 9\times10^9 \times 50\times10^{-6} \times \frac{4}{5}$$

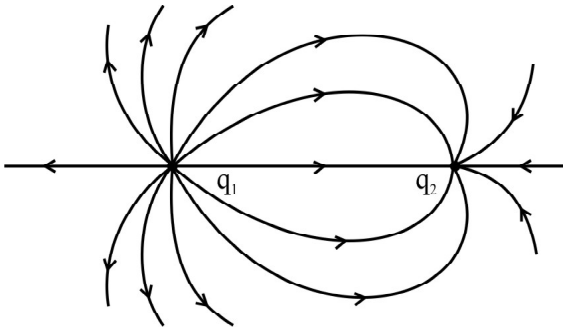
$$= 3.6\times10^5 \text{ NC}^{-1}$$

Correct answer is (3)

Unit–11 : Electrostatics

Average *Electric Field Lines*

58. The figure shows a few electric field lines for a system of two charges q_1 and q_2, fixed at two different points on the *x*–axis. These field lines suggest that



(1) $|q_1| > |q_2|$ and at a finite distance, to the right of q_2, the electric field is zero. (Correct)

(2) $|q_1| < |q_2|$ and at a finite distance, to the left of q_1, the electric field is zero. (Incorrect)

(3) $|q_1| = |q_2|$ and there no point at which the electric field is zero. (Incorrect)

(4) $|q_1| > |q_2|$ and at a finite point to the left of q_1, the electric field is zero. (Incorrect)

Correct answer: (1)

Solution:

Here, q_1 is positive and q_2 is negative as field lines originate from positive change and terminate at negative change. Also, more lines are drawn fro q_1, indicating that its magnitude is larger as compared to q_2 in the diagram.

$$\therefore \ |q_1| > |q_2|$$

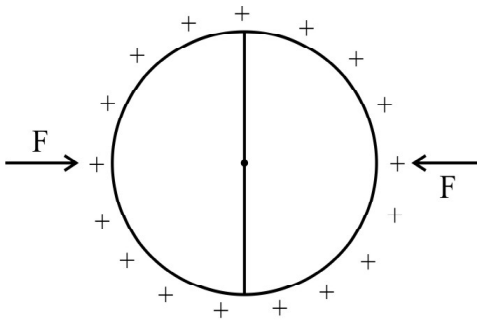
Now, for $+q_1$ and $-q_2$, the null point, where E = 0, is obtained on the line joining the two charges, outside than and nearer to the smaller charge.

Hence field is zero at some distance to the right of q_2.

Unit–11 : Electrostatics

Easy *Surface Charge Density*

59. A uniformly charged thin spherical shell, made of a co-conducting material, of radius R, carries a uniform surface charge density of σ. It is made of two hemispherical shell,s held together by pressing then with force F, as shown in the figure. F is proportional to:



(1) $\frac{1}{\varepsilon_0}\sigma^2 R^2$ (Correct)

(2) $\frac{1}{\varepsilon_0}\sigma^2 R$ (Incorrect)

(3) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ (Incorrect)

(4) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$ (Incorrect)

Correct answer: (1)

Solution:

According to the principles of homofensity of dimensions, the correct option is the one which has dimensions same as that of force i.e. $M^1L^1T^{-2}$.

Now, $\left[\frac{1}{\varepsilon_0}\right]=\left[\frac{F\times r^2}{q^2}\right]=\frac{M_1L_1T^{-2}\times L^2}{A^2T^2}=M^1L^3T^{-4}A^{-2}$

$\left[\sigma^2\right]=\left[\frac{q}{S}\right]^2=\left[\frac{A^1T^1}{L^2}\right]^2=A^2T^2L^{-4}$

$\left[R\right]=L'$

1) $\left[\frac{1}{\varepsilon_0}\sigma^2R^2\right]=M^1L^3T^{-4}A^{-2}\times A^2T^2L^{-4}\times L^2$

$=ML^1T^{-2}=\left[F\right]$

2) $\left[\frac{1}{\varepsilon_0}\sigma^2R\right]=M^1L^3T^{-4}A^{-2}\times A^2T^2L^{-4}\times L^1$

$=ML^2T^{-2}\neq\left[F\right]$

3) $\left[\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}\right]=M^1L^3T^{-4}A^{-2}\times A^2T^2L^{-4}\times L^1$

$=M^1L^{-2}T^{-2}\neq\left[F\right]$

4) $\left[\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}\right]=M^1L^3T^{-4}A^{-2}\times A^2T^2L^{-4}\times L^{-2}$

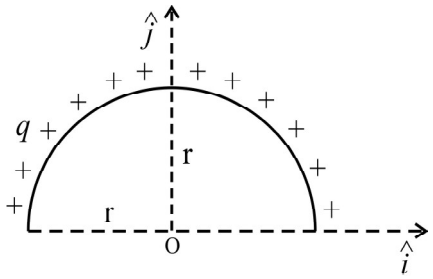
$=M^1L^{-3}T^{-2}\neq\left[F\right]$

Correct answer is (1)

Unit–11 : Electrostatics

Difficult *Electric Field*

60. A thin semicircular ring of radius 'r' has a positive charge 'q' distributed uniformly over it. The net field E at its centre is:



(1) $\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ (Incorrect)

(2) $\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (Incorrect)

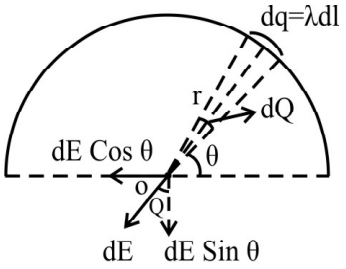
(3) $\frac{-q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (Incorrect)

(4) $\frac{-q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ (Correct)

Correct answer: (4)

Solution:

Consider an element of length $d\ell$ at angle θ.



The charge on this element on this element is $dq = \lambda d\ell$

$d\ell = rd\theta$

$\therefore\ dq = \lambda r d\theta$

Now the field dE at 0 due to this element is:

$$dE = \frac{1}{4\pi\varepsilon_0}.\frac{\lambda r d\theta}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0}.\left(\frac{\lambda}{r}\right)d\theta, \text{ as shown in the figure.}$$

Resolving dE is to two components, the cos components vanish and sin components

add-up along $\left(-\hat{j}\right)$ direction.

$\therefore\ E = \int dE \sin\theta$ (along $-\hat{j}$)

$$= \int \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{r}.d\theta \sin\theta$$

$$= \int \frac{\lambda}{4\pi\varepsilon_0 r}\int_{0^0}^{180^0} \sin\theta\, d\theta = \frac{\lambda}{4\pi\varepsilon_0 r}\left[-\cos\theta\right]_{0^0}^{180^0}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r}\left[\cos\theta\right]_{0^0}^{180^0} = \frac{\lambda}{4\pi\varepsilon_0 r}\left(1-(-1)\right)$$

$$= \frac{2\lambda}{4\pi\varepsilon_0 r} = \frac{q}{2\pi\varepsilon_0 r^2} \qquad \left(\lambda = \frac{q}{\pi r}\right)$$

or $\mathbf{E} = -\frac{q}{2\pi\varepsilon_0 r^2}\left(\hat{j}\right)$

Correct answer is (4)

Unit–11 : Electrostatics

Difficult *Continuous Charge Distribution*

61. A spherically symmetric charge distribution is set-up with the charge density varying according to the expression: $\rho_{(r)} = \rho_{(0)}\left(\frac{5}{4} - \frac{r}{R}\right)$, up to r = R and for r > R, $\rho_{(r)} = 0$. Here r is the distance from the centre O of the spherical distribution. The electric field intensity at a distance r (r < R), from O is given by:

(1) $\frac{\rho_{(0)} r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$ (Incorrect)

(2) $\frac{\rho_{(0)} 4\pi r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$ (Incorrect)

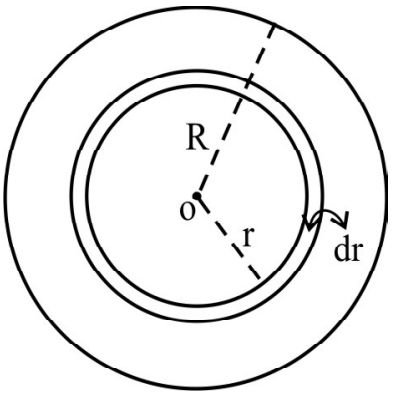
(3) $\frac{\rho_{(0)}r}{4\varepsilon_0}\left(\frac{5}{3}-\frac{r}{R}\right)$ (Correct)

(4) $\frac{4\rho_{(0)}r}{3\varepsilon_0}\left(\frac{5}{3}-\frac{r}{R}\right)$ (Incorrect)

Correct answer: (3)

Solution:

Consider a thin elementary of radius r and thickness dr. The charge of this shell is:



$$dq = \rho_{(r)} 4\pi r^2 dr$$

$$= \rho_{(0)}\left(\frac{5}{4}-\frac{r}{R}\right)4\pi r^2 dr$$

$$= \frac{5}{4}\rho_{(0)} 4\pi r^2 dr - \frac{\rho_{(0)}}{R} 4\pi r^3 dr$$

$\therefore$ Total charge in the spherical region of radius r is:

$$Q = \int_{r=0}^{r=r} dQ = \frac{5}{4}\rho_{(0)} 4\pi \frac{r^3}{3} - \frac{\rho_{(0)}}{R} 4\pi \frac{r^4}{4}$$

$$= \pi\rho_{(0)} r^3\left[\frac{5}{3}-\frac{r}{R}\right]$$

Now, $E = \frac{KQ}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\theta}{r^2}$

$$= \frac{1}{4\pi\varepsilon_0 r^2} \times \pi\rho_{(0)} r^3\left[\frac{5}{3}-\frac{r}{R}\right]$$

$$= \frac{\rho_{(0)}r}{4\varepsilon_0}\left[\frac{5}{3}-\frac{r}{R}\right]$$

Correct answer is (3)

Unit–11 : Electrostatics

Easy *Electric Field and Potential*

62. Three point charges of 2q, –q and –q are located at the vertices of an equilateral triangle. At the circumcentre of the triangle.

(1) The field is non-zero and the potential is zero. (Correct)

(2) The field is zero and the potential is non-zero. (Incorrect)

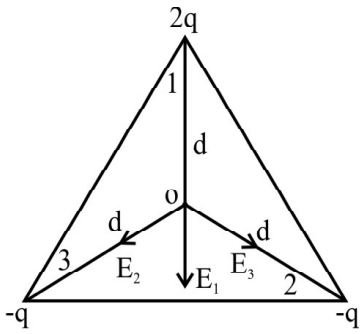
(3) Both field and potential are non-zero. (Incorrect)

(4) Both field and potential are zero. (Incorrect)

Correct answer: (1)

Solution:

As shown in the figure, the resultant of fields E_1, E_2 and E_3 can't be zero.



But the potential is:

$$V = V_1 + V_2 + V_3 = \frac{k(2q)}{d} - \frac{kq}{d} - \frac{kq}{d} = 0$$

Correct answer is (1)

Unit–11 : Electrostatics

Easy *Electrostatic Potential Energy*

63. Three point charges of 1C, 2C and 3C are placed at the corners of an equitateral triangle of side 3m. The work done is bringing these charges closer by 1m, towards each other is:

(1) 6.6×10^{10} J (Incorrect)

(2) 1.65×10^{10} J (Correct)

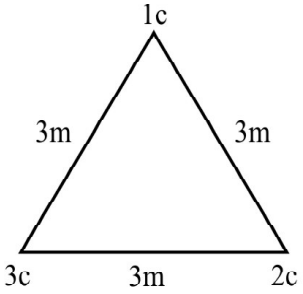
(3) 1.8×10^{10} J (Incorrect)

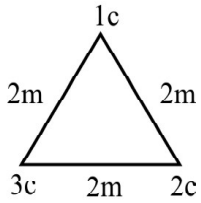
(4) 3.3×10^{10} J (Incorrect)

Correct answer: (2)

Solution:

The intial and final arrangements are as shown in the diagram.





$$U_i = \frac{kq_1q_2}{d} + \frac{kq_2q_3}{d} + \frac{kq_3q_1}{d}$$

$$= \frac{9\times10^9}{3}\left[1\times2 + 2\times3 + 3\times1\right]$$

$$= 3\times10^9\times11\,J$$

$$U_f = \frac{9\times10^9}{2}\times11 = 4.5\times10^9\times11\,J$$

$$\therefore\ w = U_f - U_i = 1.5\times10^9\times11$$

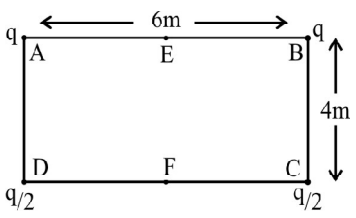
$$= 1.65\times10^{10}\ J$$

Correct answer is (2)

Unit–11 : Electrostatics

Average *Electric Potential*

64. For the rectangle ABCD in the figure, E and F are mid points of side AB and side DC respectively. If q = +10nC, then the potential difference between E and E is:



AB = CD = 6 m

CB = AD = 4 m

(1) 1.2 V (Incorrect)

(2) 12 V (Correct)

(3) 15 V (Incorrect)

(4) 18 V (Incorrect)

Correct answer: (2)

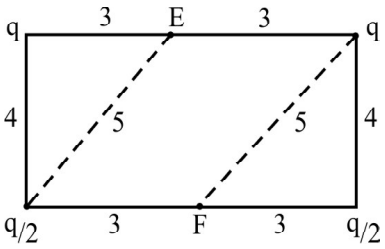
Solution:

The potential at point E is

$$V_E = k\left(\frac{q}{3}+\frac{q}{3}+\frac{q/2}{5}+\frac{q/2}{5}\right)$$

$$= kq\left(\frac{10+10+3+3}{30}\right)$$

$$= kq\left(\frac{26}{30}\right)$$



$$V_F = k\left(\frac{q}{2\times 3}+\frac{q/2}{3}+\frac{q}{5}+\frac{q}{5}\right)$$

$$= kq\left(\frac{1}{6}+\frac{1}{6}+\frac{1}{5}+\frac{1}{5}\right)$$

$$= kq\left(\frac{5+5+6+6}{30}\right)$$

$$= kq\left(\frac{22}{30}\right)$$

$$\therefore\ V_E - V_F = kq\left(\frac{26-22}{30}\right) = kq\times\frac{4}{30}$$

$$= 9\times10^{9}\times10\times10^{-9}\times\frac{4}{30}$$

$$= 12\ \text{V}$$

Correct answer is (2)

Unit–11 : Electrostatics

Difficult *Electric Potential*

65. A long straight conducting wire is uniformly charged with density of +10 mC per metre. A point charge of +2 μC is released at a perpendicular distance of 10 cm from this wire.

Kinetic energy of this charge, when it has moved through a distance of 10cm, under the field of the charged wire is:

(1) 249.57 J (Correct)

(2) 108.37 J (Incorrect)

(3) 360.03 J (Incorrect)

(4) 0.25 J (Incorrect)

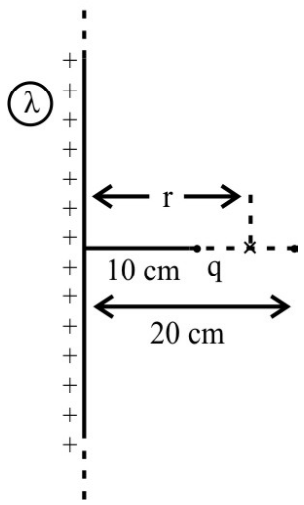
Correct answer: (1)

Solution:

The kinetic energy of the particle at 4 = 20 cm, when released at r = 10 cm is:

$K.E. = qV$

Where V is the decrease in potential between these two points.



Now, $V = \int_{r=10}^{r=20} \mathbf{E}.\mathbf{dr} = \int_{10}^{20} Edr$

$= V = \int_{10}^{20} \frac{2k\lambda}{r} dr = 2k\lambda \left[\log_e r\right]_{10}^{20}$ $\qquad E = \frac{2k\lambda}{r}$

$= 2k\lambda \times 2.303 \log_{10}\left(\frac{20}{10}\right)$

$= 2 \times 9 \times 10^9 \times 10 \times 10^{-3} \times 2.303 \times 0.3010$

$= 12.477654 \times 10^7$ volt

$\therefore$ $KE = qV$

$= 2 \times 10^{-6} \times 12.47765 \times 10^7$

$= 249.55$ J

Correct answer is (1)

Unit–11 : Electrostatics

Difficult *Principle of Superposition of Charges*

66. Ten positively charged paritcles are kept fixed on the x–axis at points $x = 10$ cm, 20 cm, 30 cm and son on up to $x = 100$ cm. The first particle has a charge of 1.0×10^{-8}C, the second 8×10^{-8}C, third 27×10^{-8}C and so on. The tenth particle has a charge of 1000×10^{-8}C. The force on 1C of charge placed at the origin due to these ten charged particles is:

(1) 4950 N (Incorrect)

(2) 495 Kilo newton (Correct)

(3) 4.95×10^7 N (Incorrect)

(4) 495 N (Incorrect)

Correct answer: (2)

Solution:

$$F_0 = F_{01} + F_{02} + F_{03} + ...F_{010}$$

$$= \frac{q_0}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1^2} + \frac{q_2}{r_2^2} + \frac{q_3}{r_3^2} + ...\frac{q_{10}}{r_{10}^2}\right]$$

$$= 9\times10^9\times10^{-6}\left[\frac{1\times10^{-8}}{(0.1)^2} + \frac{8\times10^{-8}}{(0.2)^2} + \frac{27\times10^{-8}}{(0.3)^2} + ...\frac{1000\times10^{-8}}{(1)^2}\right]$$

$$= 9\times10^9\times10^{-6}\left[1+2+3+4+.....10\right]$$

$$= 9\times10^3\times\left[55\right]$$

$$= 495\times10^3 \text{ N}$$

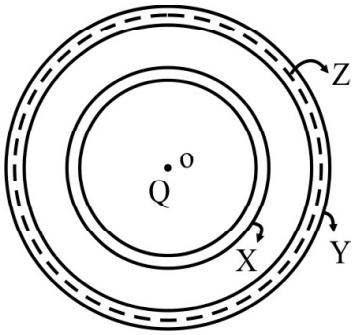
$$= 495 \text{ Kilonewton}$$

Correct answer is (2)

Unit–11 : Electrostatics

Easy *Electric Field*

67. A charge of Q is kept at common centre O of conducting shells x and y in the figure. Charge of 2Q is given to x and 3Q is given to y. The net charge enclosed by the imaginary spherical surface z (represented by dotted line) is:



(1) Q (Incorrect)

(2) 3Q (Incorrect)

(3) 6 Q (Incorrect)

(4) zero (Correct)

Correct answer: (4)

Solution:

As the spherical surface z is inside the material of the conducting shell y, E = 0 at all points on it.

Hence flux crossing z is zero.

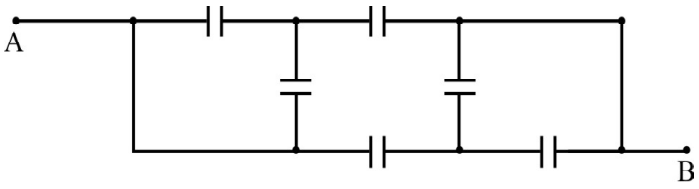
Therefore totalcharge enclosed by it also in zero.

Correct answer is (4)

Unit–11 : Electrostatics

Easy *Capacitor*

68. A network of six identical capacitors, each of value C, is made as given in the diagram.



The equivalent capacitance between A and B is:

(1) $\frac{C}{4}$ (Incorrect)

(2) $\frac{3C}{4}$ (Incorrect)

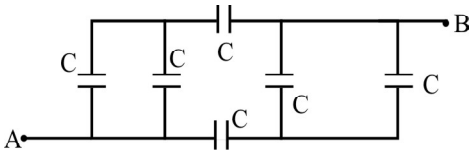
(3) $\frac{3C}{2}$ (Incorrect)

(4) $\frac{4C}{2}$ (Correct)

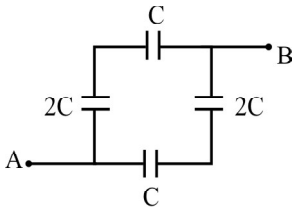
Correct answer: (4)

Solution:

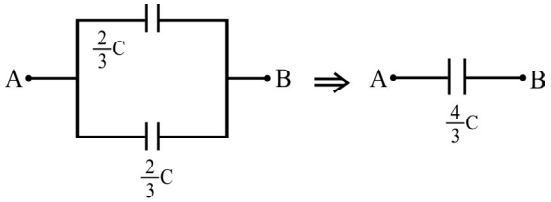
The given network can be re-drawn as below.



or



or

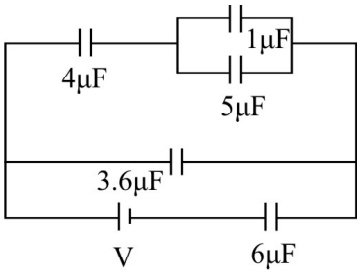


Correct answer is (4)

Unit–11 : Electrostatics

Average *Capacitor*

69. In the figure below, the charge on the 4 μ F capacitor is 24 μ C.



The potential difference (V) across the source is:

(1) 10 Volt (Incorrect)

(2) 20 Volt (Correct)

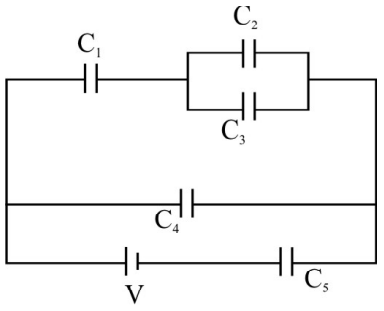
(3) 23.3 Volt (Incorrect)

(4) 46.6 Volt (Incorrect)

Correct answer: (2)

Solution:

$C_1 = 4\,\mu F$ and $Q_1 = 24\,\mu C$



$$V_1 = \frac{Q_1}{C_1} = \frac{24}{4} = 6\,\text{Volt}$$

$C_2 || C_3 = 1+5 = 6\,\mu F$

Since $6\,\mu F$ is in series with C_1, charge on $6\,\mu F$ $(1||5)$ is $24\,\mu C$.

$\therefore$ Potential difference across $6\mu F = \frac{24}{6} = 4\,\text{Volt}.$

$$C_1 \sim (C_2 || C_3) = \frac{4\times 6}{4+6} = 2.4\,\mu F$$

Charge on this $2.4\,\mu F$ is $24\,\mu C$.

$\therefore$ Potential difference across this combination is $\frac{24\,\mu C}{2.4\,\mu F} = 10\,V$

Therefore potential difference across the $3.6\,\mu F$ capacitor is also 10 V.

Now, $2.4\,\mu F$ and $3.6\,\mu F$ are in parallel.

$\therefore \left[C_1 \sim C_2 || C_3\right] || C_4 = 2.4 + 3.6 = 6\,\mu F$

With a potential difference of 10 V.

Now $C_5 = 6\,\mu F$ is in series with the above combination of $6\,\mu F$. Hence the potential difference across C_5 is also 10 V.

$\therefore$ Source supplies a potential difference of

$V = 10\ V + 10\ V$

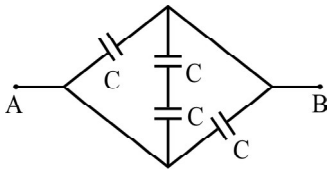
$= 20\ V$

Correct answer is (2)

Unit–11 : Electrostatics

Average *Capacitor*

70. Four identical capacitors are connected together as shown in the figure. A battery of 6 V is connected across A and B. If the energy stored in the combination is found to be 4.5 μ J, each capacitor is of capacitance.



(1) $0.1\,\mu F$ (Correct)

(2) $0.625\,\mu F$ (Incorrect)

(3) $1.0\,\mu F$ (Incorrect)

(4) $5.0\,\mu F$ (Incorrect)

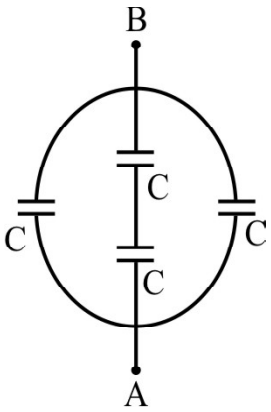
Correct answer: (1)

Solution:

$$U = \frac{1}{2} C_{eff} V^2$$

$$\therefore\ C_{eff} = \frac{2U}{V^2} = \frac{2 \times 4.5 \times 10^{-6}}{6 \times 6} = \frac{9}{36} \times 10^{-6} F$$

$$= \frac{9}{36}\ \mu F$$



Now, the above combination can be re-drawn as shown.

$$\therefore\ C_{eff} = C + \frac{C}{2} + C$$

$$= \frac{5}{2} C$$

But $C_{eff} = \frac{9}{36}\ \mu F$

$$\therefore\ \frac{9}{36} = \frac{5}{2} C$$

or $$C = \frac{18}{36 \times 5} = \frac{1}{10} = 0.1 C$$

Correct answer is (1)

Unit–11 : Electrostatics

Easy *Capacitor*

71. A 10 μF capacitor is charged to a potential difference of 1000V. The terminals of this capacitor are disconnected from the power supply and then connected to the termainals of an uncharged 6 μF capacitor. The final pontential difference across each capacitor and the the energy lost by sharing of charge, between then, are respectively.

(1) 266.6 V and 2.637 J (Incorrect)

(2) 625 V and 1.875 J (Correct)

(3) 250 V and 2.637 J (Incorrect)

(4) 750 V and 1.875 J (Incorrect)

Correct answer: (2)

Solution:

$$\text{Common potential} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

$$= \frac{10\,\mu F \times 1000V + 0}{(10\,\mu F + 6\,\mu F)}$$

$$= 625 \text{ V}$$

$$\text{Energy loss} = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$

$$= \frac{1}{2} \times \frac{10 \times 6}{10 + 6}(1000 - 0)^2\ \mu J$$

$$= \frac{15}{8} \times 10^6 \times 10^{-6} J$$

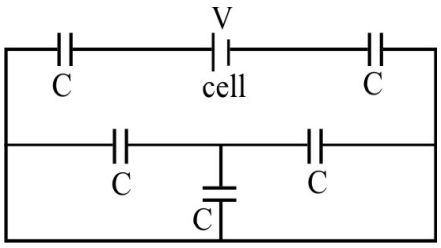
$$= 1.875 \text{ J}$$

Correct answer is (2)

Unit–11 : Electrostatics

Average *Capacitor*

72. All the capacitors in the figure have the same capacitance of C each. The cell has emf V.



The total charge stored in the combination is:

(1) $\frac{1}{4}$ CV (Incorrect)

(2) $\frac{3}{7}$ CV (Incorrect)

(3) $\frac{1}{2}$ CV (Correct)

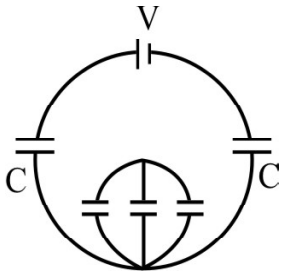
(4) 2CV (Incorrect)

Correct answer: (3)

Solution:

The combination can be re-drawn as shown. Since the three capacitors in the middle have no potential difference across, than, q = 0 for each of then.

The other two are is series with V.



$\therefore\ C_{eff} = \frac{C}{2}$

and $Q = C_{eff} \times V$

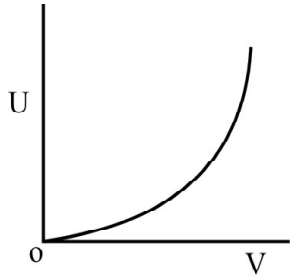
$= \frac{1}{2}$ CV

Correct answer is (3)

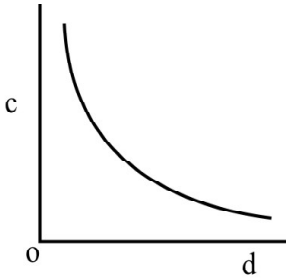
Unit–11 : Electrostatics

Average *Capacitor*

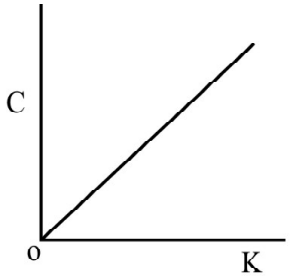
73. Keeping other relevant parameters constant, which of the follwing plots is / are incorrectly drawn, in case of a capacitor. (Notation have usual meaning).



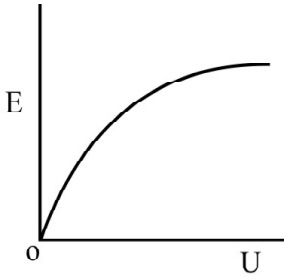
(a)



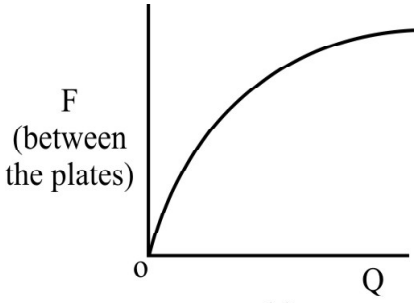
(b)



(c)



(d)



(e)

(1) (c) and (e) (Correct)

(2) (b), (c) and (d) (Incorrect)

(3) (d) and (e) (Incorrect)

(4) (b) and (e) (Incorrect)

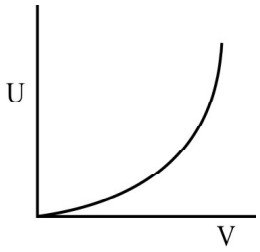
Correct answer: (1)

Solution:

(a) $U = \frac{1}{2}CV^2$

$\therefore\ U \alpha V^2$

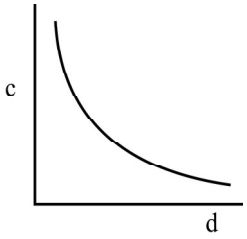
Graph of U vs V is parabola (Correct).



(b) $C = \frac{A\varepsilon_0}{d}$

$\therefore\ C \alpha \frac{1}{d}$

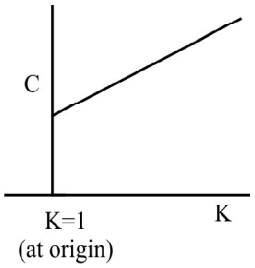
Graph is correct



(c) $C = KC_0$, where K cannot be less than 1.

So, C α K, but intercept should be C_0.

Graph is incorrect.

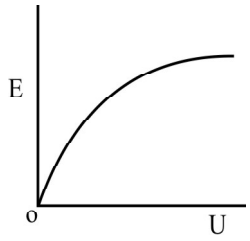


(d) $U = \overline{U} \times (Ad)$

$U = \frac{1}{2}\varepsilon_0 E^2 (Ad)$

$\therefore\ U \alpha E^2$

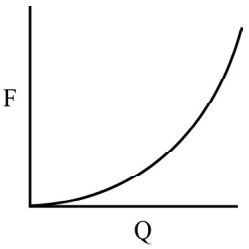
Graph is correct.



(e) $F = \frac{1}{2}\frac{Q^2}{A}$

$\therefore\ F \alpha Q^2$

So graph has parabolic bending towards F-axis is incorrect.



Correct answer is (1)

Unit–11 : Electrostatics

Average *Dielectric*

74. C, V, Q, U and E are respectively the capacitance, potential difference, charge, energy stored and inside electric field intensity of a parallel plate capacitor, which is connected to a battery. Now a material of dielectric constant 5 is filled between the plates of the capacitor, without disconnecting the battery. The new values of the above quantities are:

(1) 5C, 5V, 5Q, 5U and 5E (Incorrect)

(2) 5C, V, 5Q, 5U and E (Correct)

(3) 5C, V, 5Q, U and E/5 (Incorrect)

(4) 5C, V, 5Q, 5U and E/5 (Incorrect)

Correct answer: (2)

Solution:

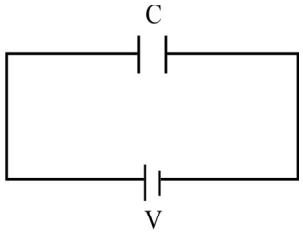
Both the cases are represented in the diagram.

(i) Capacitance increases to KC = 5C

(ii) V remains same as the potential difference of the battery. V = V

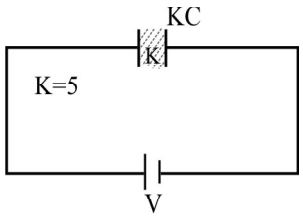
(iii) Charge = CV = Q

New charge is 5 CV = 5Q



(iv) $U_i = \frac{1}{2}CV^2 = U$

New $U_f = \frac{1}{2}5CV^2 = 5U$



(v) If E is the field inside, than

V = Ed or E = V/d

Now, V and d reamin the same.

$\therefore$ E is same in both cases.

Correct answer is (2)

Unit–11 : Electrostatics

Easy *Electrostatic Force*

75. Two positive ions, each carrying a charge 'q' are separaated by a distance 'd'. If 'F' is the force of repulsion between the ions and 'e' is the magnitude of electronic charge, the number of electrons missing from each ion is:

(1) $\frac{4\pi\varepsilon_0 F}{e^2}$ (Incorrect)

(2) $\frac{4\pi\varepsilon_0 Fd^2}{e^2}$ (Incorrect)

(3) $\left(\frac{4\pi\varepsilon_0 Fe^2}{d^2}\right)^{1/2}$ (Incorrect)

(4) $\left(\frac{4\pi\varepsilon_0 Fd^2}{e^2}\right)^{1/2}$ (Correct)

Correct answer: (4)

Solution:

$q_1 = q_2 = q = +ne$

$F = \frac{1}{4\pi\varepsilon_0}.\frac{q_1 q_2}{d^2}$

$F = \frac{1}{4\pi\varepsilon_0}.\frac{(ne)(ne)}{d^2}$

$\therefore\ n^2 = \frac{4\pi\varepsilon_0 Fd^2}{e^2}$

or $n = \sqrt{\frac{4\pi\varepsilon_0 Fd^2}{e^2}}$

Correct answer is (4)

Unit–11 : Electrostatics

Easy *Electric Dipole*

76. The dipole moment, of a deuteron and an electron, separated by a distance of 1.6 femtometre is:

(1) 4.096×10^{-34} cm (Incorrect)

(2) 5.12×10^{-34} cm (Incorrect)

(3) 8.192×10^{-34} cm (Incorrect)

(4) 2.56×10^{-34} cm (Correct)

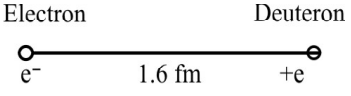
Correct answer: (4)

Solution:

$P = q\,(2a)$

Here $q = e = 1.6\times10^{-19}$C

$2a = 1.6$ femtometre



$= 1.6\times10^{-15}$ m

$\therefore\ P = 1.6\times10^{-19}\times1.6\times10^{-15}$ Cm

$= 2.56\times10^{-34}$ Cm

Correct answer is (4)

Unit–11 : Electrostatics

Average *Capacitors*

77. A series combination of n_1 capacitors, each of value C_1, is charged by a source of potential difference 4V. In another case, a parallel combination of n_2 capacitors, each of value C_2, is charged by a source of potential difference V. If both the combinations have same total energy stored, then the value of C_2 in terms of C_1 is:

(1) $\frac{16C_1}{n_1n_2}$ (Correct)

(2) $\frac{2C_1}{n_1n_2}$ (Incorrect)

(3) $\frac{16C_1n_2}{n_1}$ (Incorrect)

(4) $\frac{2C_1n_2}{n_1}$ (Incorrect)

Correct answer: (1)

Solution:

For series, $C_S = \frac{C_1}{n_1}$

and V = 4V

$\therefore\ U_S = \frac{1}{2}C_SV^2 = \frac{1}{2}\frac{C_1}{n_1}(4V)^2$

For parallel, $C_P = n_2C_2$

and V = V

$\therefore\ U_S = \frac{1}{2}C_S V = \frac{1}{2}n_2 C_2 V^2$

But, $U_S = U_P$

$\therefore\ \frac{1}{2}\frac{C_1}{n_1}(4V)^2 = \frac{1}{2}n_2 C_2 V^2$

or $\quad C_2 = \frac{16C_1}{n_1 n_2}$

Correct answer is (1)

Unit–11 : Electrostatics

Average *Electrostatic Potential Energy*

78. Three identical charges, each of $2\,\mu C$, are placed at the vertices of a triangle ABC. If AB+BC = 12 cm and AB.BC = 32 cm², the potential energy of the charge at B is:

(1) 1.53 J (Incorrect)

(2) 5.31 J (Incorrect)

(3) 3.15 J (Incorrect)

(4) 1.35 J (Correct)

Correct answer: (4)

Solution:

Given,

AB + BC = 12 cm and AB.BC = 32 cm²

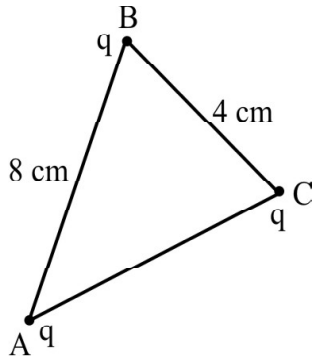
Now, $(AB - BC)^2 = (AB + BC)^2 - 4\,AB.BC$

$= 144 - 128$

$= 16\ cm^2$

$\therefore\ AB - BC = 4$ cm

Hence AB = 8 cm, BC = 4 cm



The potential V at B is:

$$V = Kq\left(\frac{1}{8\times10^{-2}} + \frac{1}{4\times10^{-2}}\right)$$

$$= \frac{9\times10^{9}\times2\times10^{-16}}{8\times10^{-2}}\times3$$

$$= \frac{27}{4}\times10^{+5}\ \text{Volt}$$

Now, $U = qV$

$$= 2\times10^{-6}\times\frac{27}{4}\times10^{+5}\ \text{J}$$

$$= \frac{27}{2}\times10^{-1}\ \text{J} = 1.35\ \text{J}$$

Correct answer is (4)

Unit–11 : Electrostatics

Easy *Electrostatic Charge*

79. Two bodies A and B carry charges of –3.00 μC and –0.44 μC respectively. How many electrons should be transferred, from which one to which, so that they acquire equal charges?

(1) 8×10^{18}, from A to B (Incorrect)

(2) 8×10^{12}, from A to B (Correct)

(3) 8×10^{13}, from A to B (Incorrect)

(4) 4×10^{12}, from A to B (Incorrect)

Correct answer: (2)

Solution:

A has higher negative charge and so electrons should be transferred from A to B.

Let n be the number of electrons transferred to make the charges equal.

$\therefore\ -3.00 + ne = -0.44 - ne$ (in μC, where $e = 1.6\times10^{-13}$ μC)

or $2ne = 2.56$ (in μC)

$$\therefore\ n = \frac{2.56}{2\times e}\ \mu\text{C}$$

$$= \frac{2.56}{2\times1.6\times10^{-13}}\ \mu\text{C}$$

$= 0.8\times10^{13}$

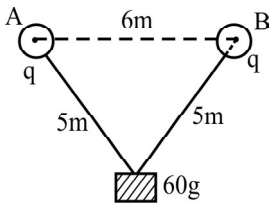
$= 8\times10^{12}$

Correct answer is (2)

Unit–11 : Electrostatics

Average *Electrostaic Force*

80. Two identical helium filled balloons A and B, charged with 'q' each and fastened to a body of 60g, by threads of length 5cm each. Float in air in equilibrium, as shown in the figure. If the seperation between the two balloons (assumed spherical) is 6m, the charge q is: (Take g = 10 ms^{-2})



(1) $30\,\mu C$ (Correct)

(2) $\sqrt{1.5}\times10^{-5}\,C$ (Incorrect)

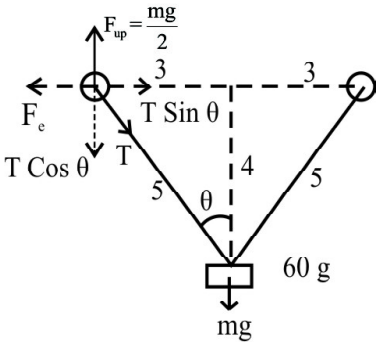
(3) $15\,\mu C$ (Incorrect)

(4) $45\,\mu C$ (Incorrect)

Correct answer: (1)

Solution:

At equilibrium



$T\sin\theta = Fe,\ T\cos\theta = \dfrac{mg}{2}$

$\therefore\ \tan\theta = \dfrac{2Fe}{mg}$

$\tan\theta = \dfrac{3}{4};\ Fe = \dfrac{kq^2}{6^2};\ mg = 60\times10^{-3}\times10$

$\therefore\ \dfrac{3}{4} = 2\times\dfrac{kq^2}{6^2}\times\dfrac{1}{60\times10^{-3}\times10} = \dfrac{2\times9\times10^9\times q^2}{6^2\times6\times10^{-1}}$

$\therefore\ q^2 = 0.9\times10^{-9} = 9\times10^{-10}\,C^2$

$q = 3\times10^{-5}\,C = 30\,\mu C$

Correct answer is (1)

Unit–11 : Electrostatics

Average *Electric Dipole*

81. A dipole of very small length and dipole moment $P = 5\times10^{-7}$ cm, lies at the origin along the positive x–axis. A point A is located at (3, 4) m. The magnitude of the electric field, of the dipole, at point A is:

(1) $36\,NC^{-1}$ (Incorrect)

(2) $36\sqrt{13}\,NC^{-1}$ (Incorrect)

(3) $14.4\sqrt{13}\,NC^{-1}$ (Correct)

(4) $14.4\sqrt{52}\,NC^{-1}$ (Incorrect)

Correct answer: (3)

Solution:

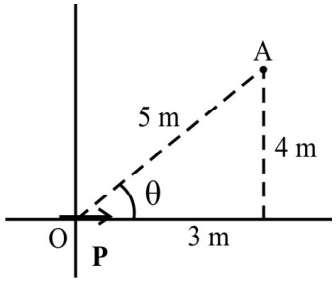
$$E = \frac{kP}{r^3}\sqrt{1+3\cos^2\theta}$$

$$= \frac{9\times10\times5\times10^{-7}}{(5)^3}\sqrt{1+3\times\left(\frac{3}{5}\right)^2}$$

$$= 7.2\times5\sqrt{\frac{25+27}{25}}$$

$$= 7.2\times2\sqrt{13}$$

$$= 14.4\sqrt{13}\,NC^{-1}$$



Unit–11 : Electrostatics

Difficult *Electrostatic Potential Energy*

82. The electrostatic energy of a nucleous can be considered as the work done against electrostatic forces in assemblying the z protons in a sphere of radius R.

Therefore the electrostatic energy assocaited with a nucleus is given by:

(1) $\frac{z^2e^2}{4\pi\varepsilon_0 R}$ (Incorrect)

(2) $\frac{ze}{4\pi\varepsilon_0 R^2}$ (Correct)

(3) $\frac{3}{5}\frac{z^2e^2}{4\pi\varepsilon_0 R}$ (Incorrect)

(4) $\frac{z^2e^2}{8\pi\varepsilon_0 R}$ (Incorrect)

Correct answer: (2)

Solution:

The uniform density of charge in a nucelus is given by:

$$\rho = \frac{ze}{\frac{4}{3}\pi R^3}$$

We regard the nucleus as a sphere having a uniform charge distribution.

The work done to increase in radius of sphere r to a sphere of radius r + dr is given by work done in bringing charge dq present in a spherical shell of thickness dr from infinity to r against the charge on sphere of radius r.

$$dq = 3\frac{ze}{R^3}r^2dr \times \frac{\frac{4}{3}\pi r^2\rho}{4\pi\varepsilon_0 r}$$

$$\therefore\ dq = \rho \times 4\pi r^2 dr$$

$$\left(\text{or } dq = 4\pi r^2 dr = \left(3\frac{ze}{R^3}\right)r^2dr\right)$$

Putting $\rho = \frac{ze}{\frac{4}{3}\pi R^3}$

$$dv = \frac{3z^2e^2}{4\pi\varepsilon_0 R^6}\pi r^4 dr$$

$$\therefore\ U = \int_0^R \frac{3z^2e^2}{4\pi\varepsilon_0 R^6} r^2 dr$$

or $$U = \frac{3}{5}\frac{z^2e^2}{4\pi\varepsilon_0 R}$$

Correct answer is (2)

Unit–11 : Electrostatics

Difficult *Electrostatic Potential Energy*

83. A conducting sphere of radius R is having a surface density σ. What is the energy stored in the surrounding space due to this charge distribution on conducting sphere? (The charge on the sphere is denoted by q)

(1) $\frac{q^2}{8\pi\varepsilon_0 R}$ (Correct)

(2) $\frac{3}{5}\frac{q^2}{8\pi\varepsilon_0 R}$ (Incorrect)

(3) $\frac{q^2}{4\pi\varepsilon_0 R}$ (Incorrect)

(4) $\frac{q^2}{8\pi\varepsilon_0 R^2}$ (Incorrect)

Correct answer: (1)

Solution:

Let us consider a point P at distance r from the centre of the sphere (r > R)

The electric field at P is give by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Let the energy density at this point P is given by

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 \frac{q^2}{16\pi^2\varepsilon_0^2}$$

The energy lying in a spherical slab formed between two concentric spheres of radii r and r + dr is $\mu\lambda 4\pi r^2 dr$ denote this energy by dU we take

$$dU = 4\pi r^2 dr \frac{q^2}{32\pi^2\varepsilon_0 r^4}$$

$$U = \int_{r=R}^{r=\alpha} du = \frac{q^2}{8\pi\varepsilon_0}\int_R^{\infty}\frac{dr}{r^2}$$

$$= \frac{q^2}{8\pi\varepsilon_0 R}$$

Correct answer is (1)

Unit–11 : Electrostatics

Easy *Dielectric*

84. A polar dielectric is placed in a external field $\mathbf{E}_e$. Which of the following statement is correct?

(1) The external field $\mathbf{E}_e$ will not effect the dielectric in any way what so ever. (Incorrect)

(2) The external field will allign all the atoms and molecules of the given dielectric in its own direction. (Incorrect)

(3) The dielectric will be have as if it is equivlent to a large number of tiny dipoles situated in vaccum. (Correct)

(4) The dielectric will behave as if it is equivalent to a large number of tiny dipoles situated in the material of dielectric. (Incorrect)

Correct answer: (3)

Solution:

A polar dielectric has a built in dipole moment. In the absence of an external field these dipoles are orineted complete by at randomly. The net dipole moment associated with any macroscoping small volume element is zero. When an electric field is applied there is is a small induced dipole moment and also a torque is exerted on all permanent dipole and the field tends to allign the dipoles in its own direction. Thermal motion tends to oppose this alignment so that at given temperature some alignment in the direction of the field is produced in equilibrium condition. Due to this aligment there is now some net dipole moment associated with each macroscoping small element of the dielectric. Hence a dielectric placed in an external field can be considered as equivalent to a large number of tiny dipole stituated in vacuum.

Correct answer is (3)

Unit–11 : Electrostatics

Easy *Dielectric*

85. In the reaction $\mathbf{P} = \varepsilon_0 x_e \mathbf{E}$

For a linear dielectric, while of the following statement is correct P.

(1) $\mathbf{E}$ stand for external field of any value in which the dielectric is placed. (Incorrect)

(2) $\mathbf{E}$ stand for external field of very large values in which the dielectric is placed. (Incorrect)

(3) $\mathbf{E}$ stands for the field produced by dipoles to which a give dielectric is equivelent to. (Incorrect)

(4) $\mathbf{E}$ stands for the vector sum of the external field and the secondary field produced by dipoles to which a given dielectric is equivalent to. (Correct)

Correct answer: (4)

Solution:

The statement (4) is established by experiment.

Unit–11 : Electrostatics

Average *Dielectric*

86. If ε_0 stand for the permitivity of vacuum, and ε the permitivity of a given dielectric, which of the follows relation is correct?

(1) $\mathbf{P} = \leftarrow \mathbf{E}$ (Incorrect)

(2) $\mathbf{P} = (\varepsilon - \varepsilon_0)\mathbf{E}$ (Correct)

(3) $\mathbf{P} = \left(\dfrac{\varepsilon - \varepsilon_0}{\varepsilon_0}\right)\mathbf{E}$ (Incorrect)

(4) $\mathbf{P} = \left(\dfrac{\varepsilon}{\varepsilon_0}\right)\mathbf{E}$ (Incorrect)

Correct answer: (2)

Solution:

We have,

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$$

or $$\mathbf{D} = \varepsilon_0 \mathbf{E} + \varepsilon_0 x_e \mathbf{E}$$

$$= \varepsilon_0 \mathbf{E}\left(1 + x_e\right)$$

Now, $\varepsilon = \varepsilon_0 \left(1 + x_e\right)$

$$\mathbf{D} = \varepsilon \mathbf{E} \qquad \text{.....(1)}$$

$$\mathbf{P} = \varepsilon_0 x_e \mathbf{E} \qquad \text{.....(2)}$$

$$\varepsilon = \varepsilon_0 \left(1 + x_e\right) \qquad \text{.....(3)}$$

From equations 1, 2 and 3 we obtain.

$$\mathbf{P} = \left(\varepsilon - \varepsilon_0\right)\mathbf{E}$$

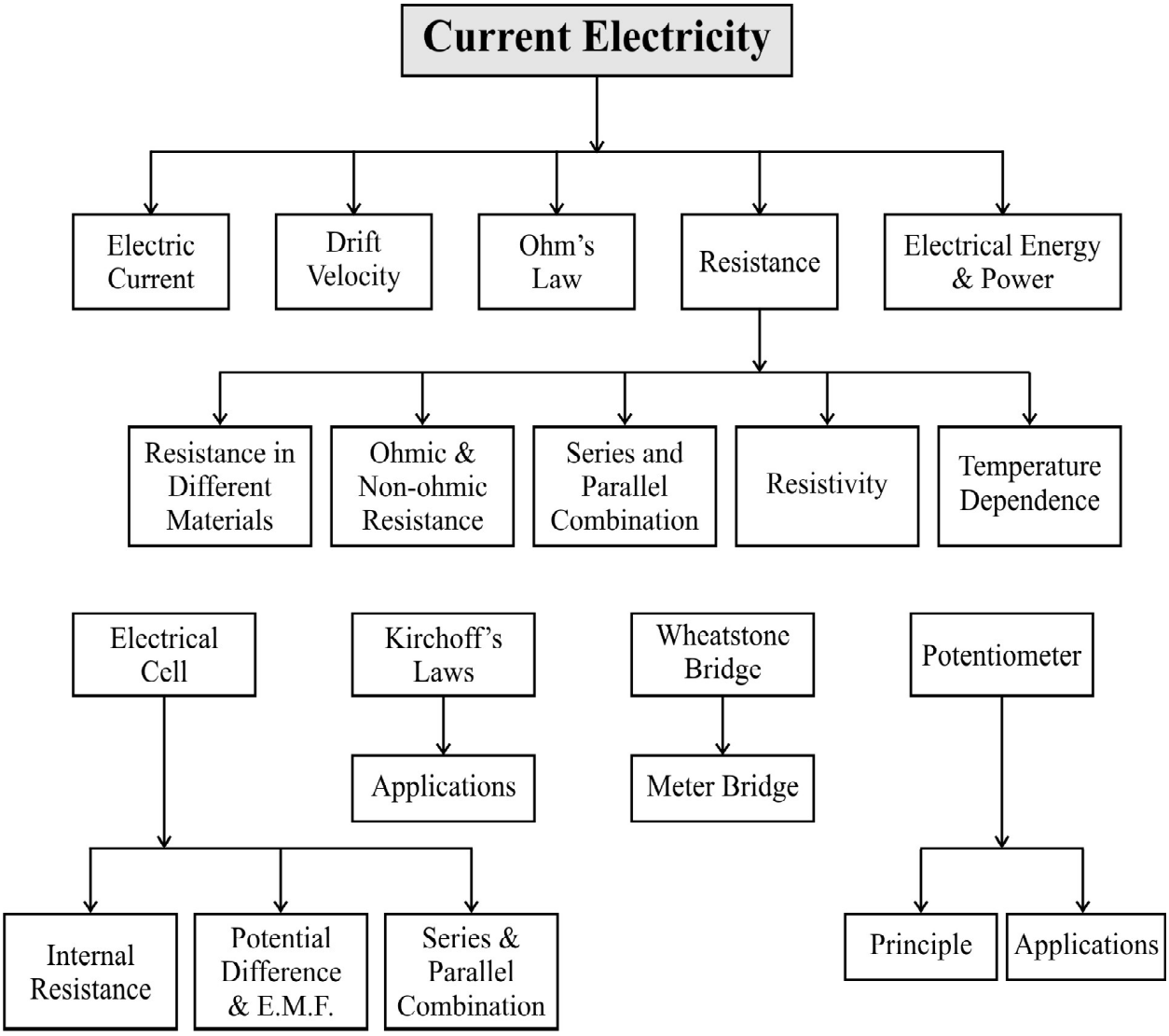
Correct answer is (2)

UNIT–12 : CURRENT ELECTRICITY

Learning Objectives

After going through this, unit you will be able to understand, appreciate and apply the following concepts:

- Electrical energy from different type of the cells.
- Meaning of emf and the essential difference between emf and terminal potential difference.
- Current and sign convention for the current.
- Drift velocity and the physics of the Ohm's law.
- Concept of resistance and its variation will temeperature.
- Combination of resistance in series and parallel.
- Joule's law and the concept of electrical power.
- Kirchoff's laws for circuit analysis.
- Wheatstone bridge: its practical form the meter bridge.
- The principle and applications of potentiometer.


Current Electricity
Electric Current
Drift Velocity
Ohm's Law
Resistance
Electrical Energy & Power
Resistance in Different Materials
Ohmic & Non-ohmic Resistance
Series and Parallel Combination
Resistivity
Temperature Dependence
Electrical Cell
Kirchoff's Laws
Applications
Wheatstone Bridge
Meter Bridge
Potentiometer
Internal Resistance
Potential Difference & E.M.F.
Series & Parallel Combination
Principle
Applications

Electric Current

It is a well known fact that the electron is an atomic particle carrying a negative charge. Electrons can be detached from the atoms of certain elements quite easily. If we connect a conductor to something which supply or absorb charge as required to maintain a applied electric field (cells, batteris etc.), this maintained field than can act on the free electrons in the conductor and will give them a resultant motion in a direction opposite to that of the field. Charges in motion constitute an electric current. If a net charge q passes through any cross section of the conductor in time t the average current is given by.

$$I = \frac{q}{t}$$

When equals quantities of charge q passes a given cross section of the conductor in successive equal interval time of t. The current I is said to be constant in magnitude or STEADY.

In general the rate of flow of charge with time is not constant and the current varies with time. It is then given by differential limit viz.

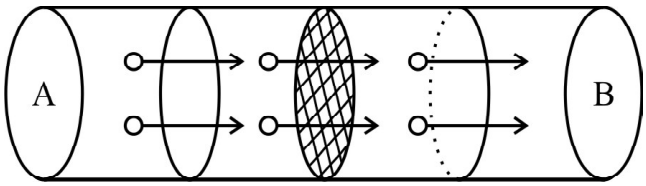
$$I = \frac{dq}{dt}$$

The current I is the same for all cross section of a conductor even though the cross sectional area may be different at different points as there can be no continued accumulation of charge at any point along the conductor.

Figure shows a portion of a wire in which there is a field towards the left and consequently the motion of the free electron towards the right. Let the electrons move with a constant drift velocity v_d (discussed later in the chapter). Then in time there dt each advances a distance $v_d t$. In this time the number of electrons crossing any place such as shaded is the number contained in a section of length $v_d t$ or volume $Av_d t$ where A is the area of cross section of the wire. Let n be the number of free force electrons per unit volume. Then the number of electrons crossing the plane in time dt is nAv_d dt and if e represents the clarge of each electron, the total clarge crossing in time dt is

$$dq = neAv_d dt$$

Hence the current $I = \frac{dq}{dt} = neAv_d$



The Direction of a Current

A logic way to define the direction of a current is the direction of motion of free electrons. However in electrolytic and gaseous conduction, the current is due to the motion of positive and negative ions. Historically the sign convention for the current was defined taking positive charge carriers as current carriers. Since these would be a incosistancy whether we define a sign convention taking positive or negative particles as charge carriers we continue to work with historical sign convention. In conductors the free electrons move in the opposite direction to the conventional current. If required we can label the current flowing in a conductor due to the motion of free electrons as *Electronic Current*.

Resistance: Proper understanding of resistance is possible only through Ohm's law. Qualitatively speaking resistance of a material is a measure of its ability to resist the flow of electricity through it.

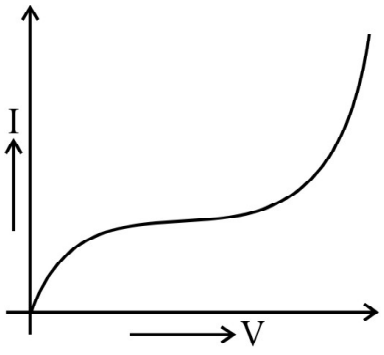
Ohm's Law

Verified experimentally Ohm's law states that the current flowing through a conductor is proportional to the voltage V across its ends when its physical conditions (temperature etc.) remains unchanged Expressed mathmatically we can write.

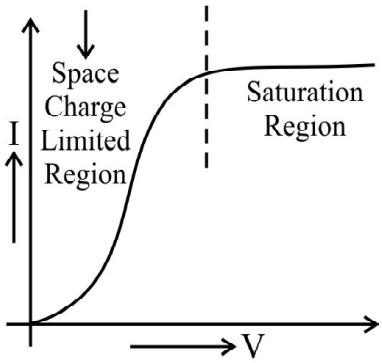
$$\frac{V}{I} = R \qquad \text{(Constant of proportionally)}$$

The constant R is known as the resistance of the conductor.

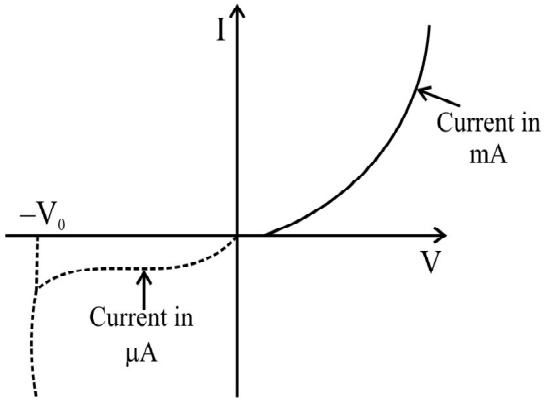
Ohm's law, a experimental law is valid only for metallic conduction. We now know that there are quite a number of substances (e.g. semi conductors) for which Ohm's law is not always valid. For such cases the current voltage graphs are not linear. We illustrate this piont by showing the plot of current voltage graph for some interesting cases:



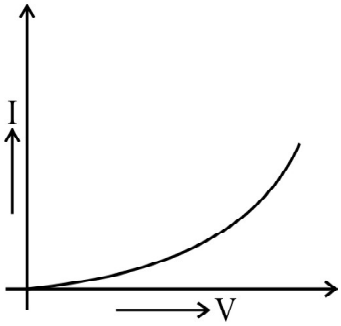
Gas Discharge Tube



Thermionic Diode



Function Diode



Thermistor

Electrical Resistivity

The resistance of a conductor depends on the nature of its material, its shape and size, Simple consideration show that the resistance of wires of a given length increases in direct proportion to their length and for wires of a given area of cross section the resistance increases in inverse proportion of their area of cross section.

We thus have,

$$R \propto L \text{ and } R \propto \frac{1}{A}$$

or $$R = \rho \frac{L}{A}$$

ρ is a constant characterstic of the material and is known as resistivity.

Colour Code for Resistances

Resistances are an integral path of practically all electrical and electronic circuits. The values of resistances needed in different circuits very over a wide range and for convenience a colour code has been developed.

The colour code works as follows:

1. We associate a colour with each digit 0, 1, 2, ...9. These colours and associated digits are given in table below:

Colour	Black	Brown	Red	Orange	Yellow	Green	Blue	Violet	Gray	White
Associated Digit	0	1	2	3	4	5	6	7	8	9

2. There are three colour bands on each resistance to indicate its value. The first two bands from one end indicate their corresponds digits while the third band colour gives the power of ten with which

the number obtained by first two colours must be multiplied to get the resistance value in ohm. For example if a resistance has three colours as yellow, orange and blue, the resistance value is $43 \times 10^6 \Omega$.

3. In addition to the above colours, two additional colours - SILVER and GOLD are also used. When used as fourth colours band they indicate the TOLERANCE or percentage realibility in the value of the resistanc obtained.

   A silver band implies a tolerance of 10% while a gold band implies a tolerance of 5%.

4. Silver band used as the third band indicate a multiplier value of 10^{-2} while a gold band used a s third band indicate a multiplies value of 10^{-1}.

Physics of the Oh'm Law

Ohm's law relate the current flowing in a conductor with the potential difference across its ends. We know that current is a flow of free elctrons under a maintained field. This could imply that free clarges in a conductor should continuously get accelerated which imply that the current should go on increasing with time. However only a stready current governed by Ohm's law flows. This implies that in a conductor, a constant electric field produces a constant velocity, a contradication of Newton's second law. Let us try to understand the reason for this observed behaviour.

We note that an electric field exerts a force n $e\mathbf{E}$ on a free electron of charge e. This force gives; the electron an acceleration 'a' in the direction of the field where

$$a = \frac{e\mathbf{E}}{m}$$

Now the electrons in a metal at any temperature are, by themselves, moving will fairly high velocities that are randomly distributed. Hence the average velocity of any free electron in the direction of the field, at the instant the field is switched on is zero. The field imports the electron an acceleration in its own direction

where $$a = \left(\frac{e\mathbf{E}}{m}\right)$$

However this acceleration lasts but for a short time as the electron on its way encounters the metal ions that are vibrating about their mean position. Since the magnitude and direction of these ions vibration at the instant of collision can have arbitrary value, they will produce a random deflection in the path of the electron when it undergoes a collision with one of these. On the average these random collisions of the free electron may occur after a time τ so that the velocity of the electron in the direction of the field just before a collision is

$$v = 0 + a\tau = \frac{e\mathbf{E}\,\tau}{m}$$

This additional velocity imparted by the field to a freely and otherwise randamly moving electron is known its DRIFT velocity. After the collision the electron may again be deflected in any random direction so that as we can assume that the (average) distribution of free electtron velocities after a time τ is the same as it was before the field was applied. Each collision causes the electron to loose the energy it gained from the field back to the atoms or ions with which it collided (cause for heating effects of current). Now the drift velocity just before and just afer a collision are $\frac{eE\tau}{m}$ and zero respectively. Hece the average drift velocity of the electron is $v_d = \frac{1}{2}\frac{eE\tau}{m} = \frac{1}{2}\frac{e}{m}\tau\frac{V}{L}$

$$v_d = \frac{1}{2}\frac{eE\tau}{m} = \frac{1}{2}\frac{e}{m}\tau\frac{v}{L}$$

If there are n free electrions per unit volume the current flowing in a conductor is given by

$$I = ne\,Av_d$$

or $$I = ne\,A\frac{1}{2}\frac{e\tau}{m}\frac{v}{L} = \frac{1}{2}\frac{ne^2\tau}{m}\frac{v}{\left(L/A\right)}$$

Putting, $\rho = \frac{2m}{ne^2\tau}$ we have $I = \frac{v}{\rho\left(L/A\right)} = \frac{v}{R}$

which is Ohm's Law

τ is known as the relexation time.

Effect of Temperature on Resistance

We have seen that the resistivity ρ is inversaly proportional to relaxtion time. The frequency of collisions us depends upon:

1. The extent of randomness in the arragement of ionic and atomic sites on the lattice of the given material.
2. The temperature of the given substance.

An increase in temperature will cause the ions and atomc to vibrate with a greater amplitude about their mean position and will increase the frequency of collisions. Hence a rise in temperature cause τ to decrease and therefore resistively ρ to increase. We thus expact a rise in resistivily with increase in temperature.

For quite a few substances, the increase in resistively with temperature is a linear one and we may write a relation of the form.

$$\rho_t = \rho_0 (1 + \alpha t)$$

where we define α as $\alpha = \frac{\rho_\tau - \rho_0}{\rho_0 t}$

α is the known as the co-efficient of resistivity and its units are ${}^0C^{-1}$.

However the variation of resistivity with temperature is not linear for all type of substances. Even for metals, the resistivity increases as a higher power of T at low temeperatures.

For insulationis and semi conductors, the resistivity increases exponentially with decreasing temperature, the variation being expressed by a relation of the form.

$$\rho(T) = \rho_0 (T) \exp\left(\frac{E_g}{kT}\right)$$

Here $\rho_0 (T)$ is a factor that varies slightly will temperature, k is the Boltzman constant and E_g is the positive energy known as GAP ENERGY.

It follows that the resistivity of insulators and semi-conductors becomes very large at low temperatures tending to infinite large values at $T \to 0$. On the other hand for metals resistivity tends to zero as $T \to 0$.

Super Conductors

The property of metals of getting currents flow through them without practically any opposition as $T \to 0$ is known as super conductivity. For metals the transtion from their normal resistivity behaviour to a zero resistance state occurs at a particular low temperature charaterstic of the metal considered. A substance in a zero resistance state is reffered as a super conductor. Recentally it has been observed that some substances acquire this super conducting behaviour at relatively higher temperature $\left(\sim 10^0 K\right)$ for Nb and $90^0 K$ for Y $Ba_2Cu_3O_7$

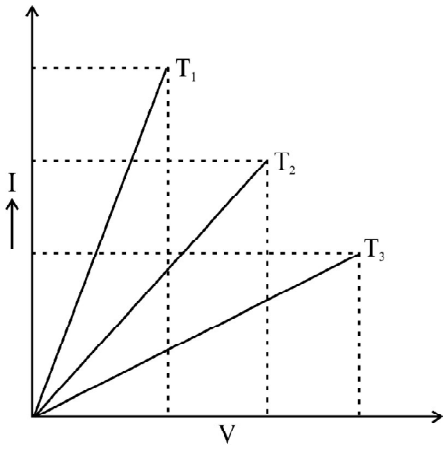
Super conductors are finding many interesting applications in technology. Some of the prominent applicatiions include.

1. Super conductor electric power transmission lines with no power loss due to heating may soon become a possiblity.
2. Super conductors may be used to product very high speed computers.
3. Powerful magnets made with superconducting coils are being used.

Example–1 :

V–I graph for a metallic wire at temperatures T_1, T_2, and T_3 are shown in the figures. Which of temperature is highest among the three?

Solution:



The slope of a I–V graph is equal to $\frac{1}{R}$.

Hence $R = \frac{1}{\text{Slope}}$

Since the slope of the graph corresponding to T_3 is having the least value the therefore the resistance of wire at temeprature T_3 is greather than T_2 or T_1. As resistance increases with temperature so temperature T_3 is greater than T_1 or T_2.

Resistance in Series and Parallel

Sereis Arrangment: When two or more resistance are joined in such a way that the second end of the first resistance is connected to the first and of the second resistance and so on as shown the resistance are said to be connected in series.

In series arrangment the current flowing through all the resistance is the same. Thus

$I = I_1 = I_2 = I_3 ..... = I_n$

Also voltage drops are added and their summation would equal to total voltage output.

$$V_T = V_1 + V_2 + ...V_n$$

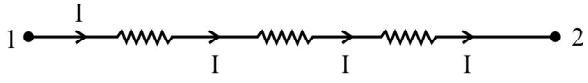
or $$V_T = I\,(R_1 + R_2 + R_3 + ... R_n)$$

$$\frac{V_T}{I} = R_T = R_1 + R_2 + R_3 + ...R_n$$

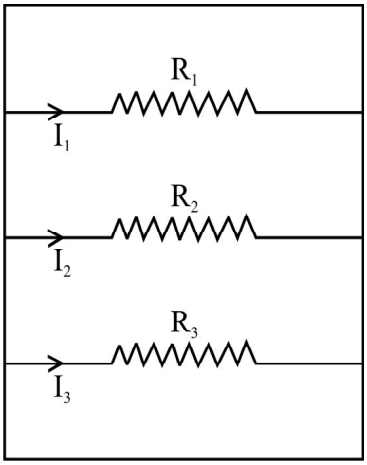
Parallel Arragnment: When two or more resistances and joined in such way that the first ends of all are connected to one common point and second ends of all connected to a second common point, the resistance are said to be joined in parallel.

In parallel arrangment the same current should leave the end 2 as enter at the end 1.

Hence $I_T = I_1 + I_2 + I_3 + ...$



The voltage that impresses itself on the side 1 of each resistor has be the same. Since the voltage must equalize itself all branches on the left side of parallel arrangement.



$$V_T = V_1 = V_2 = V_3 \ldots\ldots$$

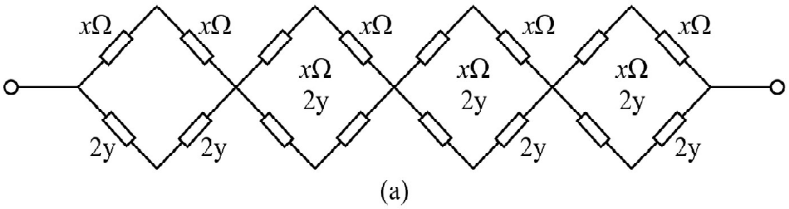
Now, $I_T = \frac{V_T}{R_T}$ $I_1 = \frac{V_T}{R_1}$ $I_2 = \frac{V_T}{R_2}$ + and so on

$$I_T = \frac{V_T}{R_T} = \frac{V_T}{R_1} + \frac{V_T}{R_2} + \ldots.\frac{V_T}{R_n}$$

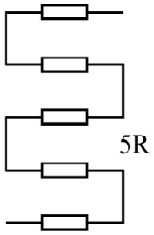
or $$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots$$

Example–2 :

Determine the equivalent resistance of the following networks.



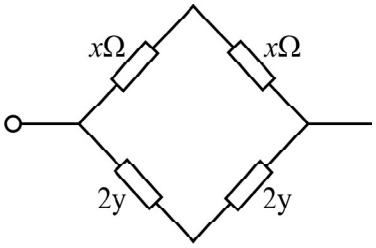
(a)



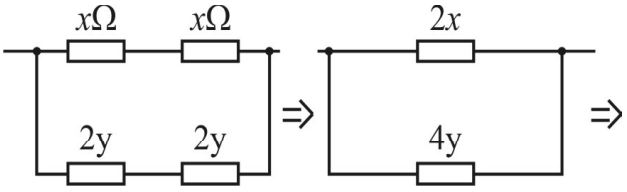
(b)

Solution :

We can regard the network of Fig. (a) as a series combination of four resistances in the form of network as shown



This can further be simplified as shown.



It is clear that the effective resistance of this networks is $\frac{1}{r_e} = \frac{1}{2x} + \frac{1}{2y} = \frac{2y + 2x}{4xy}$

$$r_e = \frac{4xy}{2(x+y)} = \frac{2xy}{x+y}$$

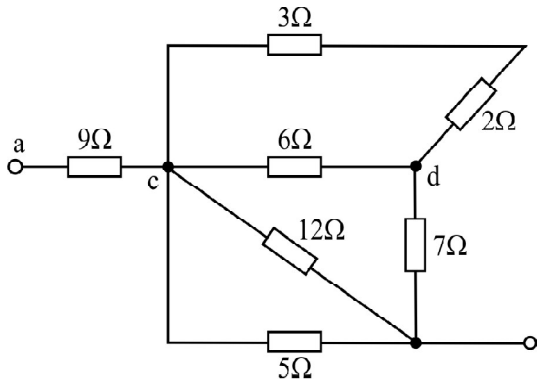
Hence the total effective resistance of the given network is given by:

$$R_{eF} = 4 \times \frac{(xy)}{x+y} = \frac{8xy}{x+y}$$

The network of figure (b) is simply a series combination of 5 resistances each of value $5\,\Omega$. Hence the effective resistance of this network is $R_{eff} = 5 \times 5 = 25\,\Omega$

Example–3 :

Find the equivalent resistance between points a and b for the combination shown in adjoining figure.

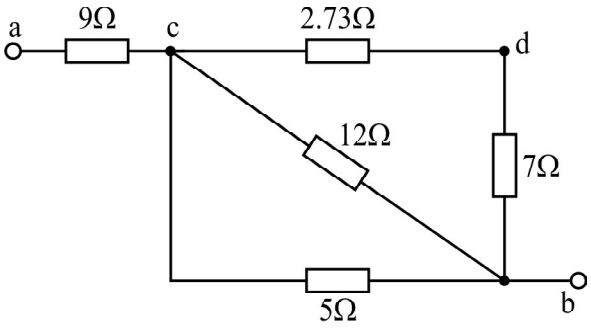


Solution :

The $3\,\Omega$ and $2\,\Omega$ resistors are in series and are equivalent to a $5\,\Omega$ resistor. This equivalent is in parallel with the $6\,\Omega$ and their equivlent is given by

$$\frac{1}{R'} = \frac{1}{5} + \frac{1}{6} = 0.20 + 0.167 = 0.367\,\Omega$$

$$\therefore\ R' = \frac{1}{0.367} = 2.730$$



The $7\,\Omega$ and $2.73\,\Omega$ resistors are in series and the their equivalent is $9.73\,\Omega$.

Now $5\ \Omega, 12\ \Omega$ and $9.73\,\Omega$ resistance are in parallel.

Let their equivalent be R". The

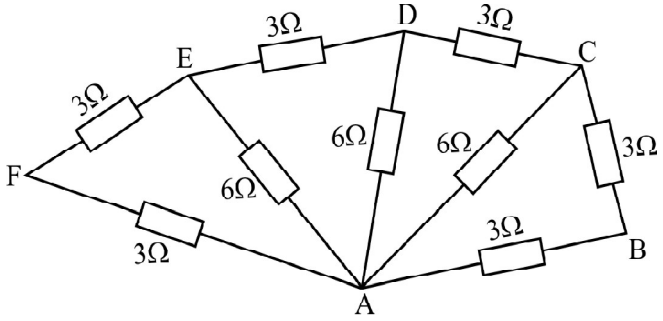
$$\frac{1}{R''} = \frac{1}{5} + \frac{1}{12} + \frac{1}{9.73} = 0.386$$

$$R'' = \frac{1}{0.386} = 2.6\,\Omega$$

Finally $2.6\,\Omega$ resistor is in series with the $9\,\Omega$ resistor. Hence the equivalent resistance of the combination is given by $9.0 + 2.6 = 11.6\,\Omega$

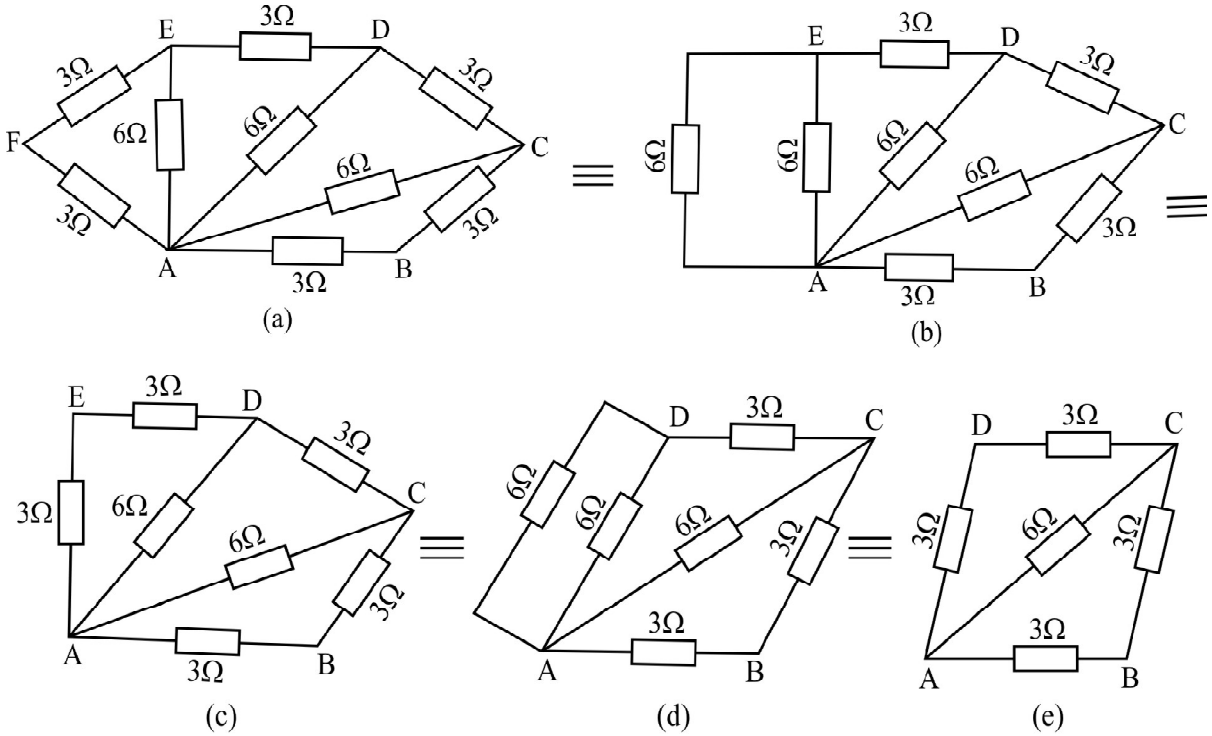
Example–4 :

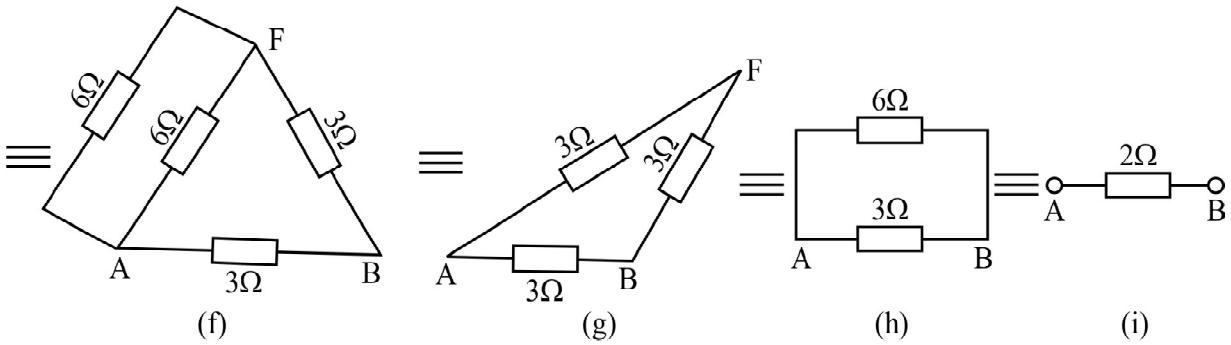
Find the effective resistance between points A and B in the circuit diagram shwon in Fig.



Solution :

Fig. shows successive reduction of the circuit.





(f) (g) (h) (i)

It is clear from Fig. that the effective resistance between points A and B is 2Ω.

Electric Cell

The simplest device to maintain a current in an electric circuit is the electrolytic cell. In this type of cell the chemical energy of a reaction is converted into electrical energy. The electrods in a simple volticcell are copper and zinc with dilute sulphuric acid as electrolyte. The action of a simple voltate cell can be understood as follows. At the zinc electrode Zn atoms are ionized and pass into solution as Zn^{++} ions leaving behind two electrons on each electrode. For each Zn^{++} ion formed two H_2^+ ions are discharged at the copper electrods taking two electrons from it. The action continue as long as electrons can pass from zinc to copper through some piece of metal.

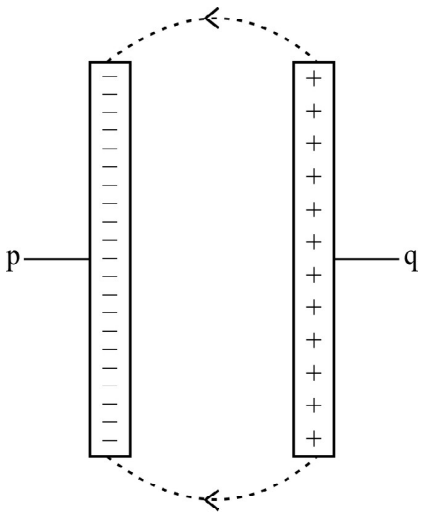
The simple voltic cell has two main defects viz (s) local action and polarization.

Daniel cell, Leclance cell and Dry cells are example of Primary cells.

Lead accumulator and alkali accumlator are exmaples of Secondary cells.

We can now directly convert the energy of light into electrical energy by using solar cells. The action of solar cells depend on proparties of semiconducting materials.

In nuclear cells the energy of β particles given out by radioactive materials in converted directly into electrical energy.



e.m.f. of a Cell

We know that electrostatic field is conservative in nature i.e.

$$\oint \mathbf{E}.\mathbf{d}\ell = 0$$

Hence electrostatic field by itself is incapable of esatablishing a current in a closed circuit. It follows that for the maintanance of current in a closed circuit a non conservative field force that can spend energy is a basic necessity.

Any source which can produce such a force is called a source of electromotive force. The electric force due to chemical ractions in a battery is an example of a non conservative field that is used for maintaining current.

Considering a battery as an example, we know that chemical energy is converted into electrical energy by chemical reactions taking place in it. The generation of energy provides the non conservative field which we assume to be directed from p to q in the battery. This is so because as a result of chemical reaction positive charges develop on plate q and equal negative charges on plate p of the battery. As a result of development of these charges an electrostatic field $\mathbf{E}_2$ which opposes the non conservative field $\mathbf{E}_1$ is established. The field $\mathbf{E}_2$ grows up a result of development of further charges. Soon the field $\mathbf{E}_2$ attains a value so that it becomes equal and opposite to E which was in first instance responsible for the development of these changes.

The battery e.m.f. is defined as

$$\varepsilon_B = \int_p^q \mathbf{E}_1 . \mathbf{d}\ell$$

Here the line integral between p and q along the straight path passing through the battary. Thus the emf of a battery is the work done when a unit charge moves from one plate to the other through the battery.

Internal resistance of a cell is defined as the resistance offered by the electrolyte and electrode of a cell when a electric current flows through it.

Terminal Potential Differnece

When a current is drawn from a cell there is a drop of potential across the internal resistance of the cell (equal to ir) where as e.m.f. of a cell is the maximum potential difference between two electrodes of a cell. when current is drawn from the cell.

Hence terminal potential difference is between two electrodes of a cell in a closed circuit.

Thus, it is clear V, the terminal potential difference is given by

$$V = \varepsilon - ir$$

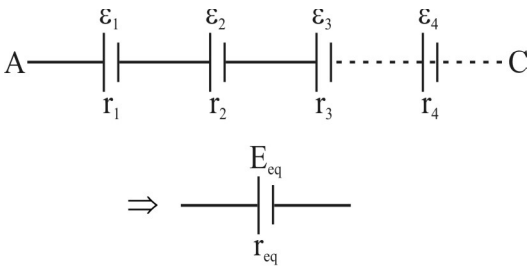
Grouping of Cells

Cells can be connected in series as shown. When n cells of emf's E_1 E_2 ..... E_n with their internal resistance as r_1 r_2, ... r_n are connected in series.

Then

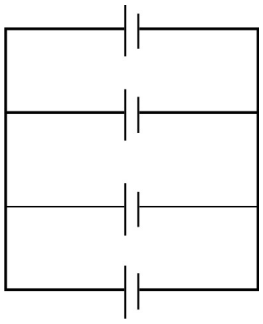
The equivalent e.m.f of a series combinations n cells is just the sum of their individual e.m.f and

The equivalent internal resistance of a series combination of n cells s just the sum of their internal resistance when n cells of e.m.f $\varepsilon_1\ \varepsilon_2 - \varepsilon_n$ and internal resistances are connected in parallel use these equivalent e.m.f and internal resistance all given by



$$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + ...\frac{\varepsilon_n}{r_n}$$

and $$\frac{1}{r_q} = \frac{1}{r_1} + \frac{1}{r_2} + ...\frac{1}{r_n}$$



We can also have a mixed grouping of cells (say N) where we connect n cells in series in row and then connect m such rows in parallel. Assuming each cell has an e.m.f. ε and internal resistance. Assuming each cell has an emf ε and internal resistance r, the equivalent e.m.f. and resistance of n cells in series in a row would be given by $n\varepsilon$ and nr.

Now there are m rows of cells in parallel therefore total internal resistance of all cells would be given by

$$\frac{1}{r_p} = \frac{1}{n_r} + \frac{1}{n_r} + ...\frac{1}{n_r}$$ up to n times.

Hence, $r_p = \frac{nr}{m}$

Since the parallel combination of rows of cells does not effect the e.m.f. of each row of cells therfore the effective e.m.f. of all cells is $n\varepsilon$. If we connect an external resistance R to such as arrangement, the current in the external resistance R is given by:

$$I = \frac{n\varepsilon}{R + \frac{nr}{m}} = \frac{m\ n\varepsilon}{mr + nr}$$

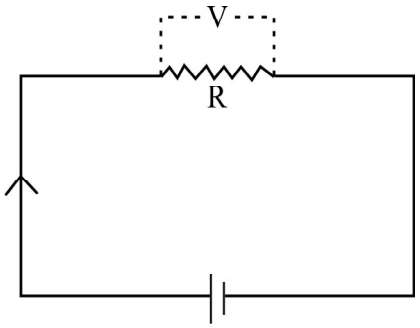
The condition for I to be maximum is given by $R = \frac{nr}{m}$

Heating Effect of Curretn (Joule Heat)

We know that in a circuit certaining resistors the flow of current electric result in the production of heat energy. We have already discussed that the metallic conduction is due to free electrons. These free electrons frequently collide with the atoms of metal in lattice. At each collision they lose some of their kinetic energy

and give it to the atoms they strike. Thus as the current flows through a wire it increases the kinetic energy of we vibrations of metal atoms and hence it generates heat in the wire, the energy lost per second by the electrons is the electrical power supplied by the battery which maintains the current. Considering a circuit as shown in figure, experimentally we observe the following facts:

1. The heat produced is proportional to the square of current.
2. The heat produced is proportional to resistance.
3. The heat produced is proportional to the time for which current flows.



Thus, we can write

$$H = i^2Rt$$

Electrical Power

The energy liberated per second in an electrical device is called electric power.

When an electric current flows through a wire or 'passive resistance' all the power which it conveys to the wire appears as heat. If I is he current R the resistance, when since

V = I R we have $P = I^2R$

as $I = \frac{V}{R}$

$$\therefore P = \frac{V^2}{R^2} \times R = \frac{V^2}{R}$$

Example–5 :

An immersion heater which is to operate on 230 V supply has to raise the temprature of 45.36 kg of water from 20^0C to 95^0C in a hour. Taking the efficiency of opection as 86%, determine the resistance of heating element in the heater.

Solution :

Mass of water = 45.36 Kg

Rise of temperature = 95–20 = 75^0C

Heat used m raising the temperature = 45.36 × 4200 × 75 J

Efficiency of operation = 86% = 0.86

Total heat produced $= \dfrac{45.36 \times 4200 \times 75}{0.86}$ J

Heat produced in one hour $= x \times 1000 \times 3600$ J

$\therefore\ x \times 1000 \times 3600 = \dfrac{45.36 \times 4200 \times 75}{0.86}$

$$x = \frac{45.36 \times 4200 \times 75}{1000 \times 3600 \times 0.86} = 4.6 \text{ kg}$$

Now wattage $= \dfrac{V^2}{R}$

$\therefore\ 4600 = \dfrac{230 \times 230}{R}$

or $\quad R = \dfrac{230 \times 230}{4600} = 13.2$ ohm

Example–6 :

A series bettery of 10 accumulation each of emf 1.5 V and internal resistance of 0.5 ohms is being charged by a D.C. source of p.d. 120V. What series resistance be connected in series in charging circuit if charging curernt is 10A. Using this resistance what is the power supplied by d.c. source and energy stored in accumulator in 2 hrs.

Solution :

Let R be the series resistance used. Then from Ohm's law

$$I = \frac{120 - 15}{R + 5} = 10$$

$\therefore\ R = 10.5 - 5 = 5.5\,\Omega$

P = power supplied by d.c. source = VI = 120 × 10 = 1200 W

Let,

P' = Power dissipicted as heat in external resistance = $I^2R = (10)^2 \times 5.5 = 550$W

P" = Power dissipicted as heat inside accumulator = $I^2 r = (10)^2 \times 5 = 500$ W

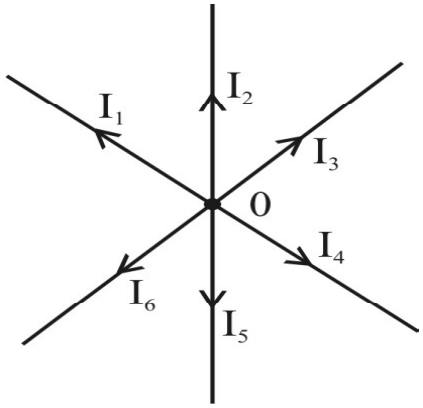
P_1 = Power stored inside accumulator P – (P'+P") = 1200–(550 + 500) = 150 W

Energy stored in 2 Hour = 150 × 2 × 60 × 60 J = 108×10^4 J

Kirchoff's Laws

Electrical circuits contains simple arrangment of resistances in series or parallel and with a known source of e.m.f. can be analysed in a straightward manner using Ohm's law. In many important cases we encounter networks where there may be junctions between two or more conductors at which current can flow along various paths and where one or more sources e.m.f. may be active. In such cases we solve the circuits by making use of Kirchoff's law. To apply these law we define two terms.

1. A branch point is a point of the circuit where three or more conductors are joined together.
2. A loop is any closed conducting path in the network.



First Law

The algebraic sum of the currents flowing toward a branch point is zero. The convention for sign is.

The current flowing toward a branch point is considered positive whereas a current flowing away from a branch point is considred negative. For example in the figure shown we have:

$$I_1 - I_2 - I_3 - I_4 + I_5 = 0$$

Second Law

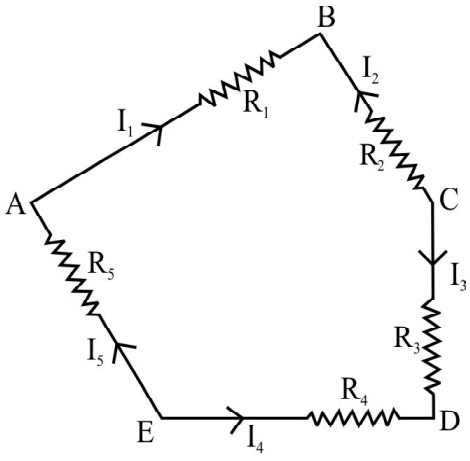
The algebraic sum of the emf's in any loop of the network is euqal to algebraic sum of IR drops in it i.e.

$$\sum \varepsilon_i = \sum I_j R_j$$

To apply the loop equation we must take some direction either clockwise or anticlockwise as the traversal direction. An emf is taken with positive sign if the emf by itself produce a positive current in the sense of traversal direction. Similarly we take an IR turn with the positive sign if the current through the resistor in question is in the direction of the traversal.

For example in the figure shown taking ABCDE as the sense of traversal we have.

$$I_1 R_1 - I_2 R_2 + I_3 R_3 - I_4 R_4 + I_5 R_5 = 0$$



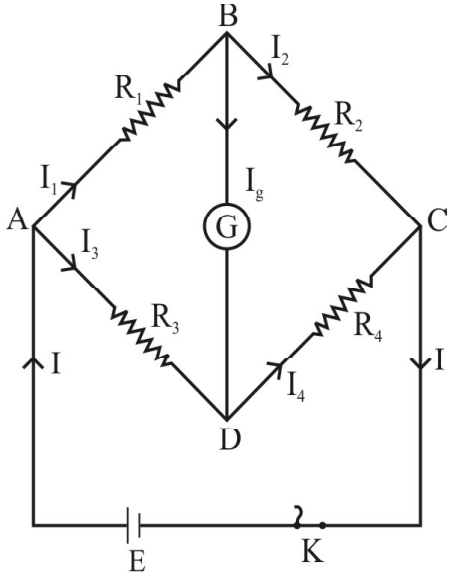
To solve the networks will the help of Kirchoff's law we proceed stepwise as follows:

(a) We assign currents with assumed directions to every branch in the network. There direction should be indicated in the circuit diagram. Then making use of Kirchoff's first law we reduce the number of unknowns as far as possible.

(b) We next write down the voltage equations using Kirchoff's second law to conveniently choosen loops. There must be

the same number of independent equation as are the unknown.

(c) Finally we solve these equations to obtain unknown quantities.



We illustrate the use of Kirchoff's law by taking the case of a Wheatstone bridge. The circuit diagram is shown in figure. Suppose we want to calculate Ig. Let the currents in all the branches be as indicated. Applying Kirchoff's fisrt law we have

at A, $I = I_1 + I_3$ or $I_3 = I - I_1$

B, $I_1 = I_2 + I_g$ or $I_2 = I_1 - I_g$

C, $I = I_2 + I_4$ or $I_4 = I - I_2 + I_g$

Applying Kirchoff's second law to mesh ABDA weget

$$I_1R_1 + I_gG_g - I_3R_3 = 0$$

or $$I_1R_1 + I_gG_g - (I - I_1) R_3 = 0$$

or $$I_1 (R_1+R_3) + I_4G_1 - IR_3 = 0 \qquad .....(1)$$

For mesh BCDB we have

$$I_2R_2 - I_4R_4 - I_gG_g = 0$$

or $$(I_1 - I_2) R_2 - (I - I_1 + I_g) R_4 - I_gGR_4 = 0$$

or $$I_1 (R_2 + R_4) - I_gG_g - I_g (R_2 + R_4) - IR_4 \qquad .....(2)$$

We eliminate I_1 from these equations by multiplying (1) by $(R_2 + R_4)$ and (2) by $(R_1 + R_3)$ and then subracting.

Solving we get,

$$I_g = I\frac{\left(R_2R_3 - R_1R_4\right)}{G_g\left(R_1 + R_2 + R_3 + R_4\right) + \left(R_1 + R_3\right)\left(R_2 + R_4\right)}$$

Condition for balance would be obtained by putting $I_g = 0$

This gives $$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

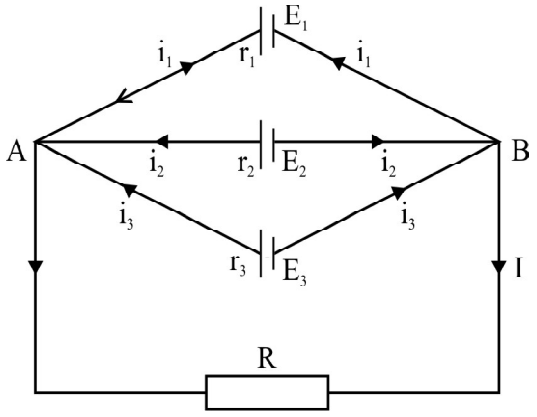
We can also obatin this condition by putting $I_g = 0$ in the equations (1_2) and obtained from Kirchoff's laws. Wheatstone bridge is a basic circuit for determination of an unknown resistance. However the balance condition

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

The above equation an IDEAL condition. This is because I_g can never be made equal to zero because a galvanometer will always draw some current, howsoever small, for its working. Hence in practice we obatin a value R_3' instead of R_3 in the balancing condition. $\left(R_3' - R_3\right)$ is known as WANT OF BALANCE. For a Wheatstone bridge to be sensitive the galvanometer should show as large a deflection as possible for a given want of balance. Broadly speaking we can say that a sensitive cap wheatstonte brdige is reasonable senstive if the resisance in all the arms are of the same order of magnitude.

Example–7 :

Three cells of emf's E_1, E_2 and E_3 and internal resist are r_1, r_2, r_3 and connected in parallel through an external resistance R. Calculate the current flowing through each cell.



Solution :

Suppose i_1, i_2 and i_3 be the currents in given three cells and r be this current through external resistance R. Applying Kirchoff's first law to the point, we have A

$$i = i_1 + i_2 + i_3 \qquad .....(1)$$

Applying second law to the mesh E_1 A R B E_1, we have

$$i_1 r_1 + iR = E_1 \qquad .....(2)$$

Similarly, $i_2 r_2 + iR = E_2$ (Mesh E_2 A R B E_2) .....(3)

and $i_3 r_3 + iR = E_3$ (Mesh E_3 A R B E_3) .....(4)

Divides equation (2) by r_1, equation (3) by r_2 and equation (4) by r_3 and adding we get

$$\left(i_1 + i_2 + i_3\right) + iR\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right) = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3}$$

or $$i + iR\left(\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}\right)=\frac{E_1}{r_1}+\frac{E_2}{r_2}+\frac{E_3}{r_3}$$

$$i\left[1+R\left(\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}\right)\right]=\frac{E_1}{r_1}+\frac{E_2}{r_2}+\frac{E_3}{r_3}$$

$$i\left\{\frac{1}{r_1r_3r_3}\left[r_1r_2r_3+R\left(r_2r_3+r_3r_1+r_2r_3\right)\right]\right\}=\frac{1}{r_1r_3r_3}\left(E_1r_2r_3+E_2r_3r_1+E_3r_1r_2\right)$$

or $$i=\frac{E_1r_2r_3+E_2r_3r_1+E_3r_1r_2}{r_1r_3r_3+R\left(r_2r_3+r_3r_1+r_2r_3\right)}$$

$$i_1=\frac{E_1-iR}{1}$$

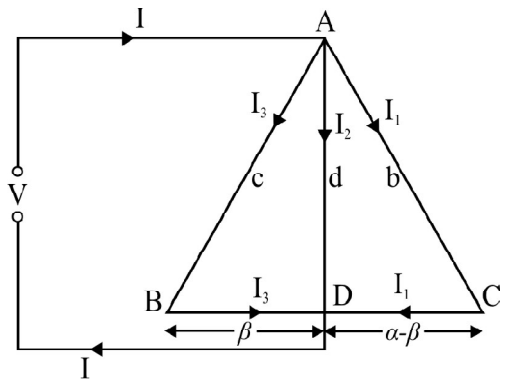
$$i_1+\frac{1}{r_1}\left[E-R\left\{\frac{\left(E_1r_2r_3+E_2r_3r_1+E_3r_1r_2\right)}{r_1r_2r_3\left[R\left(r_2r_3+r_3r_1+r_1r_2\right)\right\}}\right\}\right]$$

Similar expression for i_2 and i_3 can be written from the symmetry.

Example–8 :

The resistance of three wires BC, CA and AB of the same material and uniform cross section are a, b, c resectively. Another wire from A of constant resistance can make a contact with BC. If a current enters at A and leaves at the point of contant with BC. Show that the maximum resistance of the network is:

$$\frac{(a+b+c)d}{a+b+c+4d}$$



Solution :

Let the sliding were make contact with wire BC at point D. Let the resistance of part BD of wire be β and of DC $(\alpha-\beta)$. Let I be the current staring at A and leaving at D. From Kirchoff's second law.

We have for network ADVA

$$I_2d=V \quad \text{or} \quad I_2=\frac{V}{d} \qquad .....(1)$$

For network ABDA

$$I_3 c + \beta I_3 - I_2 d = 0$$

or $$I_3 (c + \beta) = I_2 d = V$$

$$\therefore\ I_3 = \frac{V}{c + \beta} \qquad .....(2)$$

Similarly for network ACDA

$$I_1 b + I_1 (a - \beta) - I_2 d = 0$$

$$I_1 (a + b - \beta) = I_2 d = V$$

$$I_1 = \frac{V}{a + b - \beta} \qquad .....(3)$$

$$I = I_1 + I_2 + I_3 = \frac{V}{a + b - \beta} + \frac{V}{c + \beta} + \frac{V}{d}$$

or $$I = V\left[\frac{1}{a + b - \beta} + \frac{1}{d} + \frac{1}{c + \beta}\right]$$

If R is the resistance of equivalnet network then $I = \frac{V}{R}$

Hence combining we get,

$$\frac{1}{R} = \frac{1}{a + b - \beta} + \frac{1}{d} + \frac{1}{c + \beta}$$

For maximum value of R, $\frac{dR}{d\beta} = 0$

or $$-\frac{1}{R^2}\frac{dR}{d\beta} = \frac{1}{(a + b - \beta)^2} - \frac{1}{(c + \beta)^2}$$

or $$-\frac{1}{R^2}\frac{dR}{d\beta} = \frac{(c + \beta)^2 - (a + b - \beta)^2}{(a + b - \beta)^2 (c + \beta)^2}$$

For $\frac{dR}{d\beta} = 0$ we must have

$$a + b - \beta = c + \beta$$

or $$\beta = \frac{a + b - c}{2}$$

Substituting the value of β we get

$$\frac{1}{R_{max}} = \frac{1}{a + b - \frac{a + b - c}{2}} + \frac{1}{d} + \frac{1}{c + \frac{a + b - c}{2}}$$

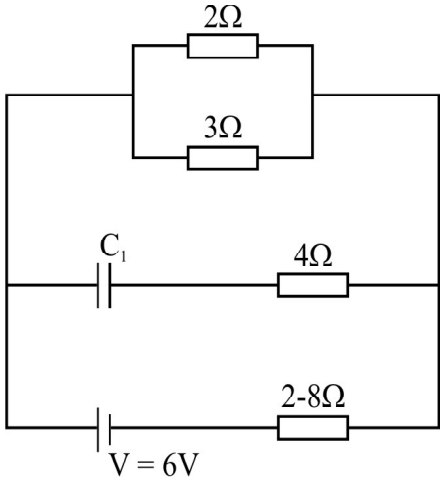
$$= \frac{2}{a + b + c} + \frac{1}{d} + \frac{2}{a + b + c}$$

$$= \frac{2d + a + b + c + 2d}{(a + b + c)d} = \frac{a + b + c + 4d}{(a + b + c)d}$$

Hence, $R_{max} = \frac{(a + b + c)d}{a + b + c + 4d}$

Exmaple–9 :

Calculate the steady stage current in the 2Ω resistor shown in the circuit. The internal resistance of battery is neglible and the capacitance of the capacitor is 0.2 microfarad.



Solution :

The current distribution in the various arms of the network is shown in Fig. It is important to note the capacitor C_1 has infinite resistance for d.c. current. At branch point B, therefore no current passes through the arm BC_1F

From loop ABCD H EFGA

$$2i_1 + 2.8i = 0 \qquad .....(1)$$

From loop DHEJD

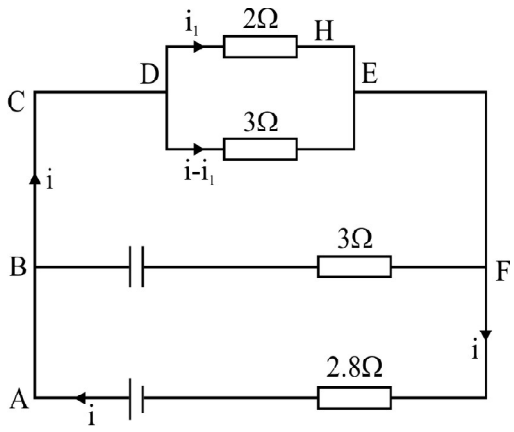
$$2i_1 - 3\,(i - i_1) = 0$$

or $$5i_1 - 3i = 0 \qquad .....(2)$$

From (1) and (2) we have,

$$2i_1 + 2.8\,[(5/3)\,i_1] = 0$$

$$i_1 = 0.9\ A$$



Metre Bridge

A meter bridge is a pratical application of a Wheatstone bridge. It consists of a straight and uniform wire one meter long. The internal wire has a high resistivity and low temperature co-efficient of resistance (Manganin is the material used).

The wire is stretched along a meter scale with its ends soldered to two copper strips of neglible resistance between the copper strips and third one fixed along the panel have two gaps AB and CD.

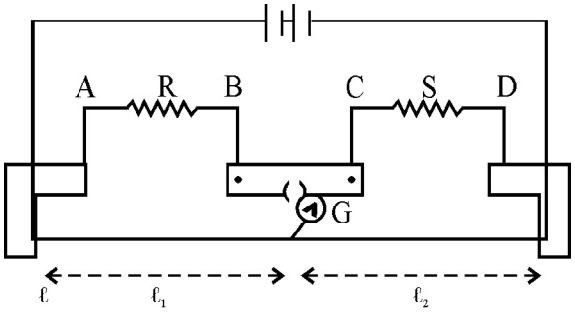
The unknown resistance is connected in the gap AB and a suitable resistance is connected in the gap. There is a jockey which series to make contact will the stretched wire and a pointer attached it to indicates we position of the point of contact with scale.

The position of jockey is adjusted until the galvanometer shows no deflection. Since the wire is of uniform thickness, the resistance is proportional to length.

Let $xz = \ell_1$ and $zy = 100 - \ell_1 = \ell_2$

Then we have,

$$\frac{R}{S} = \frac{xz}{zy} = \frac{\ell_1}{\ell_2}$$



The determination here may be inaccurate due to the following errors:

(a) Non uniformaty of the wire. To eliminate this error the metar bridge wire must be calibrated in an auxillary experiment.

(b) The position of the point where the jockey makes contact with the wire may not be shown accurately by the pointer attached to the scale.

This error may be eliminated by interchanging R and S and repeating the experiment. The mean of two gives the accurate balancing length.

(c) The ends of the wire may not coincide will the ends of the scale and there may be a small resistance where the wire is connected to the copper strip. This resistance is known as END correction. To determine we take R and S known and obtain a balance point a length L_1 of the melor bridge wire. If α and β are end corrections at the two ends we have,

$$\frac{R}{S} = \frac{L_1 + \alpha}{100 - L_1 + \beta}$$

Now interchanging the positions of R and S we obtain a balance at a length L_2 of the meter bridge wire.

$$\frac{S}{R} = \frac{L_2 + \alpha}{100 - L_2 + \beta}$$

Solving these two equations for α and β we get

$$\alpha = \frac{SL_1 + RL_2}{R - S}$$

$$\beta = \frac{RL_1 + SL_2}{R - S}$$

(d) If a current is passed for a longer direction of time the point may flicker. To avoid this the current in the wire may be kept as small as possible and the key should be pressed for as short a time as possible while taking a reading.

Example–10 :

Two parallel resistance 4Ω and 8Ω are connected in the right gap of a meler bridge and a resistance of 6Ω is connected in the left gap. Where would be the balance point on the meter bridge wire?

Solution :

S is the combination of parallel combination of 4Ω and 8Ω resistance.

$$\frac{1}{S} = \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$$

or $S = \frac{8}{3}\ \Omega$

Let the balance point be at a distance of x from the left end of the wire.

$$\frac{R}{S} = \frac{x}{100 - x}$$

$$\frac{6 \times 3}{8} = \frac{x}{100 - x}$$

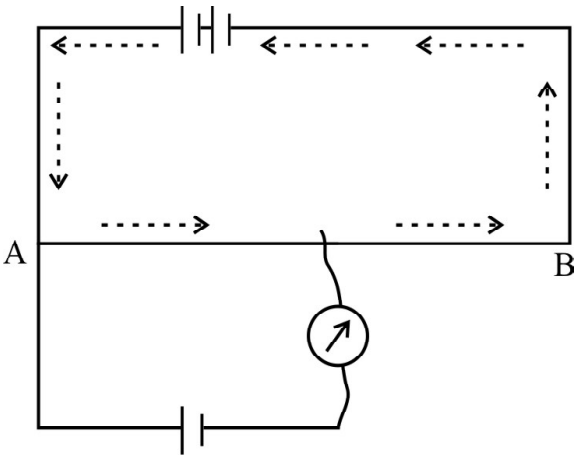
Solving for x we get,

$x = 69.2$ cm

Potentiometer

The potentiometer is a very useful instrument which can be used to measure a unknown e.m.f.

The principle of a potentiometer can be understodd by referring to figure.



Here AB is a long uniform wire connected to a battery E_1. The source of unkonwn e.m.f. E_2 is connected across AB through a sliding contact J. The sliding contact is moved along AB till there is no deflection in the galvanometer. When this happens the emf E_2 is equal in magnitude of the potential difference between A and J (For obatining a balance E_1 must be greater than E_2).

Let us cnosider the wire AB is uniform and the fall of potential across unit length (potential gradient) be ρ. If the balancing length for an unknown e.m.f. is ℓ than,

$$e = \rho\ell$$

Given a choice, we would prefer to use a ten wire potentlometer over a four wire potentiometer for a better accuracy of measurement (ρ would be smaller for a ten wire potentiometer).

Usually one find various faults in potentiometer circuit. We discuss below various possibilties:

(a) Both defiections are in the same direction the one with the contact maker at the zero end of the scale being the smaller of two. This error arises because the p.d. under test is connected in the wrong way round and the connections are to be reveresed.

(b) Both deflections are in the same direction the one with the contact maker at the zero of the scale being the larger of two.

The reason for this fault is that the accumlator that drives the current through the potentiometer produces too small a p.d. across the wire. To eliminate this error we must either recharge the accumulator or if there is a resistance in series with the wire this must be reduced.

(c) The same deflection is obatined for all position of the contact maker. This happen when there is a broken connection somewhere in the part of the circuit in series with the potentionmeter wire and accumulator.

(d) No deflection is obatined for all positions of the contact maker. This happens when there is a broken connection in the galvanometer circuit.

(e) The deflecion of the galvonometer are in opposite directions for the two positions of the contact maker but the pointer flickers unsteady.

There is a loose connection some where in the circuit.

A potentiometer is a very verstile instrument and can be used for:

(i) Comparison of emf's of two cells.

(ii) Determination of internal resistance of a cell.

(iii) Determination of a low resistance.

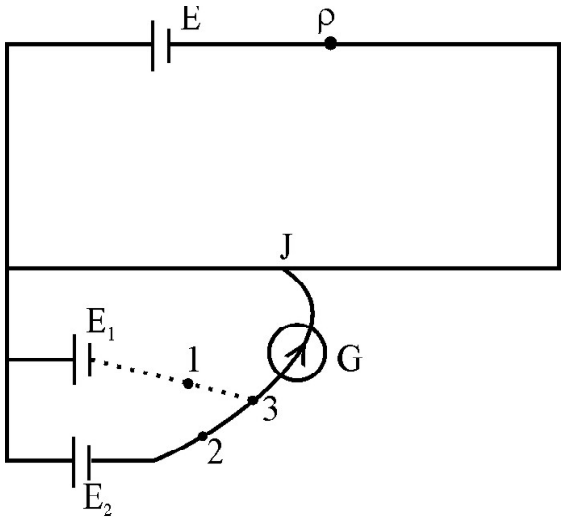
(iv) Measurement of current.

(v) Measurement of high emf.

(vi) Measurement a very low e.m.f. (thermocouple)

Comparision of e.m.f. of Two Cells

Connect point 1 of the two way key and obtain a balance point at a length L_1 of the potentiometer wire. Than $E_1 = \rho L_1$



Where ρ is the potential gradient of the potentiometer wire.

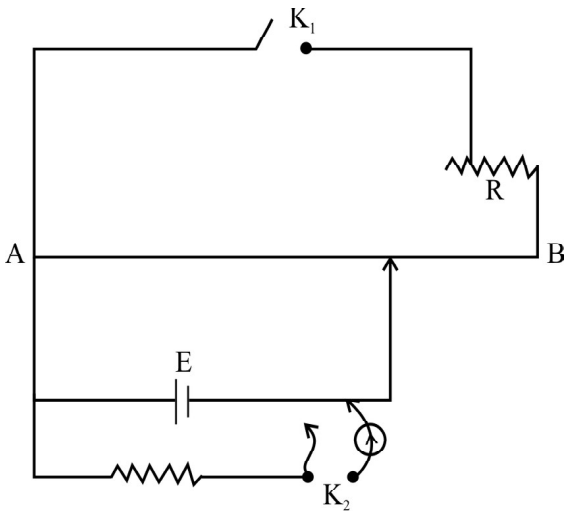
Now connect contact 2 of the two way key and obtain a balance point at a length L_2 of the potentiometer wire. Than

$$E_2 = \rho L_2$$

Hence, $\frac{E_1}{E_2} = \frac{L_1}{L_2}$

Determination of Internal Resistance of a Cell

The circuit arrangment is shown in the figure. E is the cell whose internal resistance ir is to be measured A resistance box R is connected across the cell through a key K_2.



By adjusting the rheostat R_1 and closing key K_1 a balancing point is obtained is on the potentiometer wire at a point L_1 with K_2 open.

Then $E_1 = \rho L_1$

Where ρ is the potential gradient of the potentiometer wire.

Now key K is closed with a known resistance R out of resistance box. Once again without disturbing rheostat R_1 the balancing length L_2 is measured. The we have,

$$V = L_2 \rho$$

$$\therefore \frac{E}{V} = \frac{L_1}{L_2}$$

Let I be the current drawn throught the cell by resistance R. Then the potential difference of the cell is

$$V = IR$$

Also, $I = \frac{ER}{R+r}$

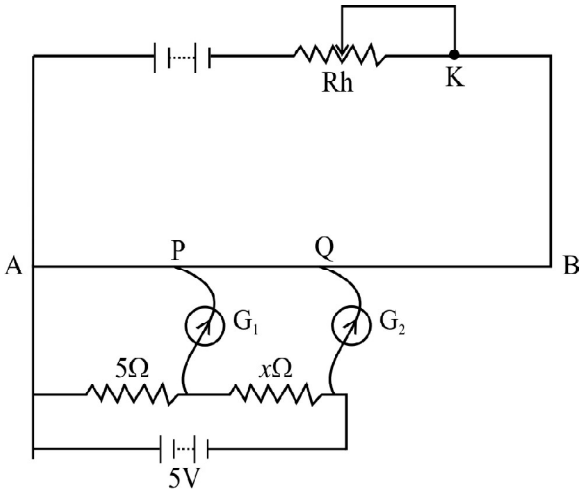
$$\therefore \frac{E}{V} = \frac{R+r}{R}$$

Sovling we get,

$$r = R\left(\frac{L_1 - L_2}{L_2}\right)$$

Example–11 :

The circuit shown in figure is designed to measure an unknown. Resistance using a potentiometer. When there is no deflection m the galvanometer, the balance point is obtained on P of the potentiometer in the galvonometer G_1. Another balance point is located Q of the potentiometer wire in the galvanometer G_2. It is found that $AP = \frac{AQ}{3}$. What is the value of *x*?



Solution :

We have,

$$\frac{5}{5+x} \times 5 = \rho(AP) = \rho\left(\frac{AQ}{3}\right)$$

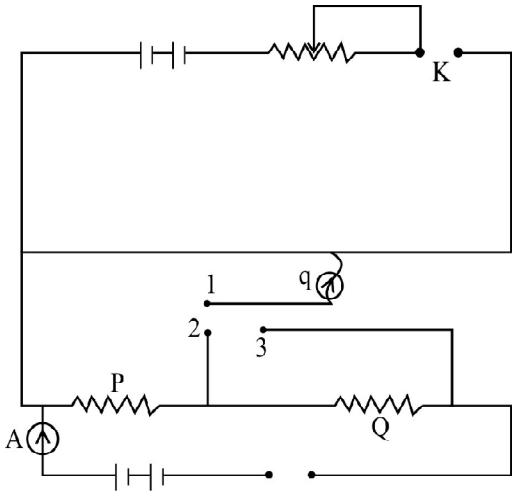
and $$\frac{5}{5+x}(5+x) = \rho\ AQ$$

Hence, $\frac{5}{5+x} = \frac{1}{3}$

$$x = 10\,\Omega$$

Example–12 :

A potentiometer (with potential gradient ρ) is set up as shown. The ammeter reads 1.0 when the two way key is switched off. When the points 1, 2 of the key are connected the balance is obatined at *x* cm and when the points 1 and 3 are connected the balance is obatined at y cm of the potentiometer wire. Then the magnitude of the resistors R and Q are equals when respectively to $\rho(y - x)$ and ρx.



Solution :

When the key between 1 and 2 is plugged in P is in circuit.

$\therefore \quad V_1 = IP = 1P = \rho x$

When the key between 1 and 3 is plugged in coil P and Q are in the circuit.

$V_2 = I(P+Q) = 1(P+Q)y$

$\therefore \quad V_2 - V_1 = \rho + Q - \rho = k(y - x)$

PROBLEMS FOR PRACTICE

1. The resistivity of the material of a wire is 4×10^{-5} ohm-cm. If the resistance of the wire is 13 ohm and its diameter is 1 mm. Calculate the length of the wire.

   Ans: 27.5 m

2. A carbon resistor of $74\,\Omega$ to be marked with rings of different colour for its identification. Write the sequence of colours.

   Ans: Violet, yellow and orange

3. Two wires of equal length one of aluminum and the other of copper have the same resistance. Which of the two is lighter?

   $\rho_{Al} = 2.63\times10^{-8}\,\Omega m,\ \rho_{Cu} = 1.72\times10^{-8}\,\Omega m$

   Relative density of Al and Cu are 2.7 and 8.9

   Ans: Aluminium

4. A wire with a resistance of $6.0\,\Omega$ is drawn out so that its new length is three times its original length. Find the resistance of longer wire assuming that its resistivity and density of meterial are not changing during the process.

   Ans: $5\,\Omega$

5. At what temeperature would the resistance of a copper conductor be double its resistance at 0^0C.

   Ans: 260^0C

6. An iron wire 2 km long and a copper wire 3 km long are connected in parallel across a source of voltage 200 V. Both wires carry the same current. The diameter of copper wire is 1 mm and its temeperature is 100^0C. Obtain the diameter of iron wire. The temperature coefficient of copper and iron at 20^0C is 3.9×10^{-3} and $5\times10^{-8}\,(^0C)^{-1}$ respectively. Resistivity of copper and iron is 1.7×10^{-8} and $10^{-7}\,\Omega m$. Neglect the thermal conductivity of the wires.

   Ans: 2.046 mm

7. A heater connected to a 230 V supply initially draws a current of 3A which becomes 2.3 A after 5 second. What is the steady temeperature of the heating element if the initial temperature is 27^0 C. Temperature cofficient is $0.00017C^{-1}$

   Ans: 1817^0C

8. A transmission line 1 km long is prepared from 17 kg of magnin 20^0C. What is the maximum and minimum temperature within which it can operate if its resistance is to be within $0.76\,\Omega m$ of its calibrated value. Given

   Coefficient of linear expansion $1.8\times10^{-3}\,(^0C)^{-1}$

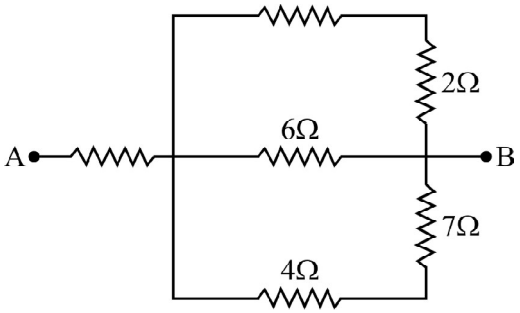
Temperature coefficient = $10^{-5}\ (^0C)^{-1}$

Density at $20^0C = 8.5\times10^3\ m^{-3}$

Resistively at $20^0C = 4.4\times10^{-7}\ \Omega m$

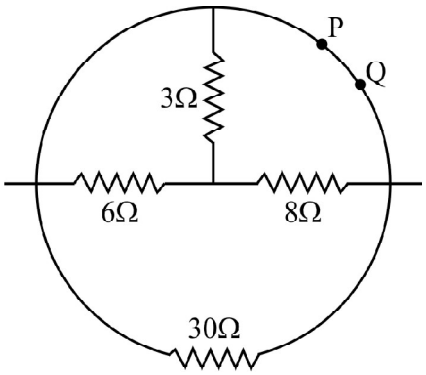
Ans: –80 or 120

9. Calculate the equivalent resistance between A and B for the combination shown in figure.



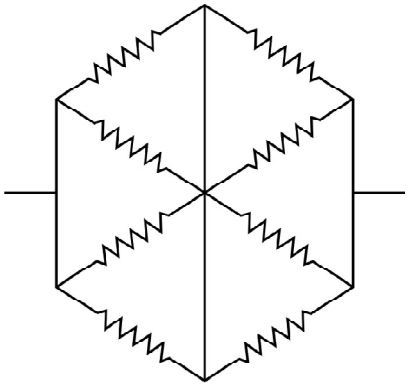
Ans: 3.18 Ω

10. Find the equivalent resistance between P and Q of the figure.



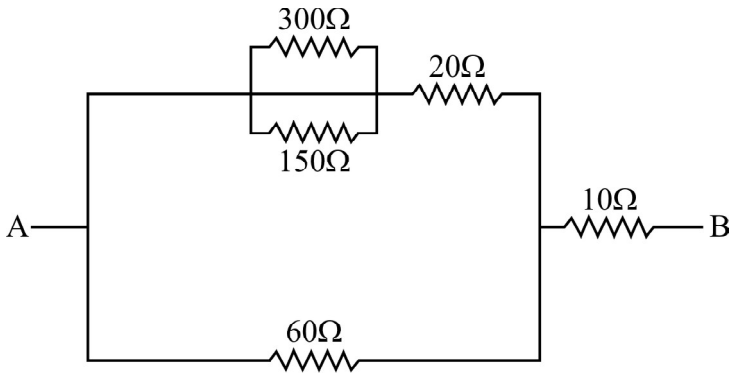
Ans:

11. Find the effective resistance between points *x* and y of the hexagonal circuit shown in fig. Each resistor has a value of 10 Ω.



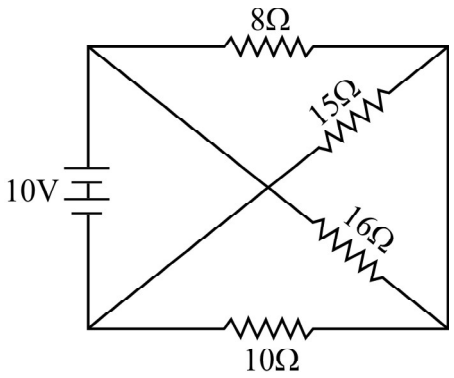
Ans: 5 Ω

12. Calculate the equivalent resistance between point A and B of the network shown.



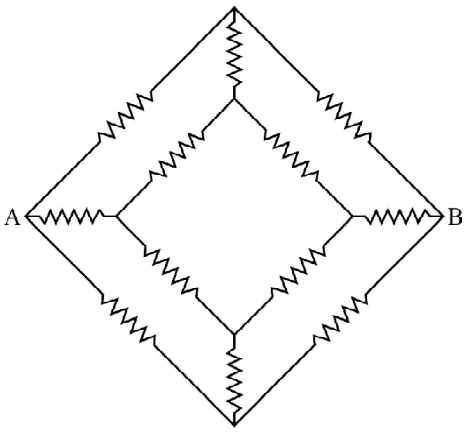
Ans: 50 Ω

13. Find the equivalent resistance between A and B of the network shown in figure. Neglect the internal resistance of the battery.



Ans: 11.33 Ω

14. Twelve resistance each of value 4Ω are connected as shown in the figure. Find the effective resistance between A and C.

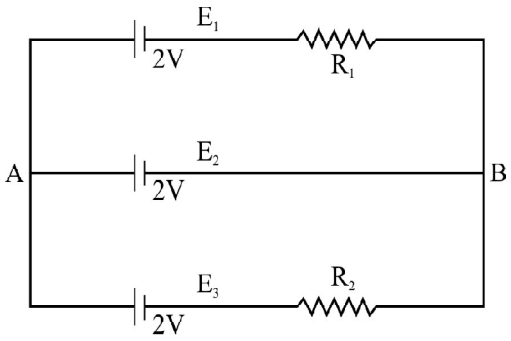


Ans: 3Ω

15. Four identical cells each of e.m.f. E are connected in parallel providing supply of current to external circuit consisting of two resistances of resistance R in parallel. The terminal voltage of the cells as read by an ideal voltmeter is V. Calculate the internal resistance of each cell.

Ans: $\left(\frac{E-V}{V}\right)\times 2R$

16. In the circuit shown, calculate the current flowing between A and B through E_2.



Ans: 2A from B to A

17. In an electric furnace 100 kg of tin are to be melted in an hour from an initial temperature of 15^0C. Find the power required for this operation.

Given,

Specific heat of tin = 0.056

Melting point of tin = 235^0C

Latent heat of fusion of tin = 13.3 kilocalorie per kg.

Thermal efficienty = 70%

J = 0.24 Calorie per joule

Ans: 4.2377 Kwh

18. What is the wattage and voltage ratings of a bulb which will produce maximum brightness when connected across a battery of e.m.f. 4.5 V and internal resistance 20.

Ans: 2.53 W and 2.25 V

19. Two bulbs rated 100 W, 200 V, 200 W, 200 V are connected in (1) series (2) in parallel and joined to a source of p.d 200 V. In each case which bulb glow more brightly. What is total power consumed in each case.

Ans: In series: 100 W bulb glows more brightly then 200 W bulb and power consumed = 66.6 W.

In parallel: 200 W bulb glows more brightly than 100 W bulb and power consumed is 300 W.

20. A lead fuse in a circuit consisting of a copper wire with cross sectional area 5 mm^2 metls if the wire heat up by 25 K. The initial temperature of the lead fuse is 293 K. What is the cross sectional area of the fuse lead wire.

    Ans: 2.5 mm^2

21. A high speed lift weighing 15.7 KN goes up at 1.0 m/s. The supply line is 220 V, the motor efficienty is 92%. What is the power of the electric motor driving the lift.

    Ans: 17 Kw

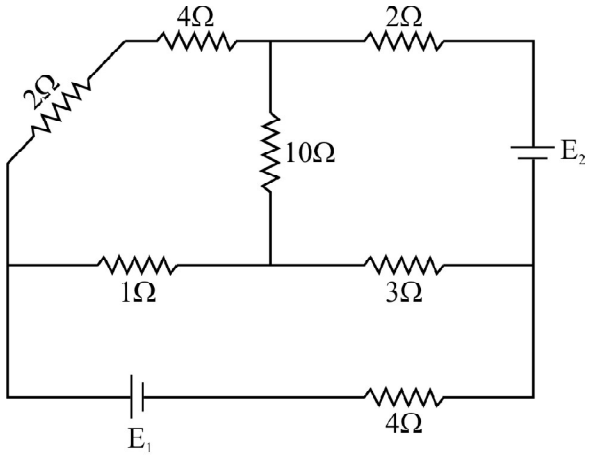
22. ABCD is a quadrileteral of which arms have resistance AB = 1Ω, BC = 2Ω, CD = 3Ω and DA = 4Ω. A galvanometer of resistance 352 is placed across BD. If a current of 1A is passed at A and leaves at C calculate the current in the galvonometer.

    Ans: 0.00667 A

23. An electric current of 5A is divided into three branches, the lenghts of the wires in three branches are proportional to 1, 2 and 3.Find the current in each branch. The wires are of the same material and cross section.

    Ans: 0.3075A, 2A, 0.9225A

24. In the circuit shown in figure, what is the current through 4Ω resistance.



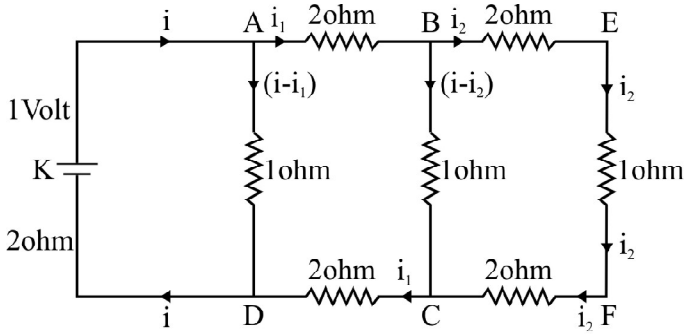
    Ans:

25. Two squares ABCD and AEFC have the side BC in common. The sides made of conducting wires have resistance as follows.

    AB, BE, FC, CD each 2Ω

    AD, BC EF each 1Ω

A cell of e.m.f. 2 volt and internal resistance 2Ω is joined across AD. Find the current in various parts of the circuit.

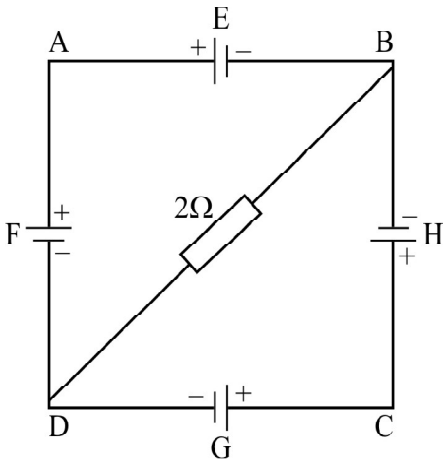


Ans: $I_2 = \frac{2}{99}A$

$I = \frac{70}{99}A$

$I_1 = \frac{12}{99}A$

26. In the circuit shown in figure E, F, G and H are cells of emf 2, 1, 3 and 1 volt respectively and their internal resistance are 2, 1, 3 and 1Ω respectively. Calcualte:

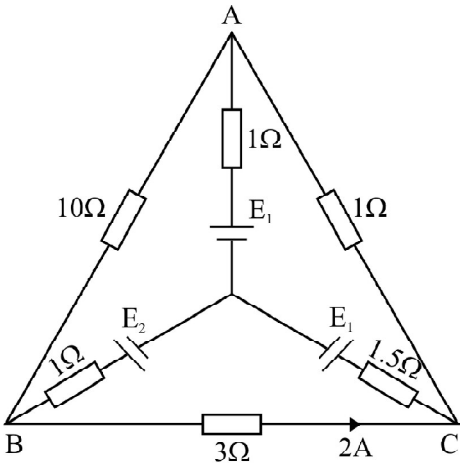


(i) The potential difference between B and D.

(ii) The potential difference across the terminals of the cells G and H.

Ans: $V_{DB} = \frac{2}{13}V$, $V_G = \frac{21}{13}V$, $V_H = \frac{19}{13}V$

27. In the circuit shown in figure, the current through 3Ω resistance. 2A, $E_1 = 14V$,



What is the value of E? Internal resistance of each battery is 1Ω.

Ans: 7Ω

28. Twelve equal wires each of resistance r are joined up to form a skelton cube. A current enters at one corner and leaves at the opposite corner. What resistance does the cube offer in this situation.

Ans: $\frac{5}{6}$r

29. A letter is constructed of a uniform wire of resistance $1\Omega\,cm^{-1}$. The side of the letter are 20 cm long and the across piece in the middle is 10 cm long while the apex angle is 60^0. Find the current flowing through the different pairs when a battery of emf 15V and internal resistance $3.33\ \Omega$ is connected across the two ends of the legs.

Ans: 0.167A; 0.33A

30. In a meter bridge, the balance point is found to be at 40 cm from the left end of the meter bridge wire when the resistance in the left gap is 15Ω. Find the balance point when the resistance in left and right gaps are interchanged.

Ans: 60 cm

31. The resistance of a meter bridge wire is 2Ω. The galvonometer used in the circuit has a resistance of 100Ω and the battery used has an emf of 2V. If the resistance in the left and right gaps are 1Ω and 3Ω, where is the balance point obatined.

Ans: 25 cm

32. A resistance of 2Ω is connected in the left gap of a meter bridge wire which has a length of 1m. An unknown resistance X (X < *x*) is connected in the right gap. The balance point is found at L cm from the positive end of the battery. On interchanging the resistances in two gaps it is found that the balance point further shifts by 20 cm away. Neglecting the end corrections calculate the value of unknown resistance *x* used.

Ans: 3Ω

33. *x* is an unknown resistance to be determined by a meter bridge by using a 10Ω resistance in the right hand side gap of the meter bridge wire. The balance point is obtained at 52 cm mark. The end corrections are 1 cm and 2 cm respectively for the left and right ends. What is the value of *x*.

Ans: 10.6 cm

34. The resistance of a potentiometer wire is 8Ω and its length is 8 meler. A high reisistance box and a 92 volt accumlator are placed in series will it. What should be the value of the resistance box if it is desired to have a potential drop of one microvolt per millimeter?

Ans: 912Ω

35. A potentiometer wire of length 100 cm has a resistance of 10Ω. It is connnected in series with a resistance and a cell of emf 2 volt and neglible internal resistance source of emf 10 milli volt is balanced by a length of 40 cm of the potentiometer wire. What is the value of the external resistance.

    Ans: 790Ω

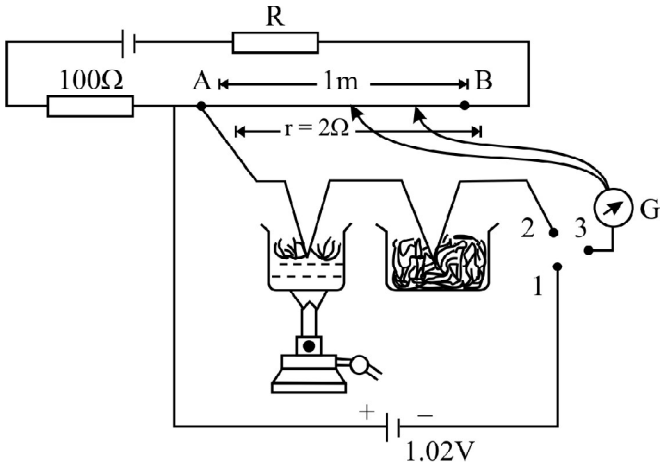
36. In an experiemnt with a potentiometer to measure the internal resistance of a cell when the secondary circuit is shunted by 5Ω, the null point is at 2.20 m. When the cell is shunted by 20Ω the null point is at 3.00 m. Calculate the internal resistance of cell.

    Ans: 2.76Ω

37. A 1.5 V poteniometer is used for finding the internal resistance of 1.2 V cell. The balance point of the cell comes at 65 cm. If a resistor of 10Ω is used in the external circuit of the cell, the balance point changes to 55 cm. Calculate the internal resistance of the cell.

    Ans: 1.82Ω

38. A potentiometer circuit using a standard cell of emf 1.02 volt is setup as shown in fig. If the balancing length with 1, 3 connection is 90 cm and with the 2, 3 connection is 45 cm. What is the e.m.f. of thermocouple? Also find the value of R.



    Ans: 0.51 V and 1.004Ω

QUESTION BANK

Key Learning Points

1. The free electrons in a conducter in a maintained electric field constitute a current.

2. The electric field that acts on the electrons in a conductor does not produce a net acceleration because the electrons keep on colliding with the ctoons leat make up the conductor and thereby acquire a constant drift velocity v_d in a direction opposite to the field.

$$v_d = \frac{1}{2}\frac{e}{m}\tau\frac{v}{L}$$

Where τ is known as relaxtion time and other symbols have their usual meaning.

3. The general expression for the current is $i = nev_dA$

where n is the number of free electrons per unit volume and A is the area of cross section of the conductor.

4. The same current flows through the every cross section of the conductor even of its is not uniform.

5. According to ohm's law the current is directly proportional to the potational difference provided the physical conditions of the conductor remains unchanged.

i.e. $V = IR$

Where R is the resistance offered by the conductor.

6. The resistance of any conductor can be expressed in terms of the length ℓ and area of cross section A as $R = \rho\frac{\ell}{A}$

Where ρ is a constant of the material and is called its resistivity. The receprocal of ρ is known as conductivity any is denoted by σ.

7. The equivalnce resistance of resistance $R_1, R_2, R_3, \ldots\ldots R_n$, connected.

(i) In series, is $R_{eq} = R_1 + R_2 + \ldots\ldots R_n$

(ii) In parallel, is $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots.\frac{1}{R_n}$

8. The variation of resistance with temperature is given by

$$R_\tau = R_0(1+\alpha t)$$

where α is the temperature co-efficient of material.

9. The resistance of metals increases with temperature. However for semi conductor and insulators resistivity increases exponnentally with decreasing temperature.

10. Super conductors (metals like Nb and other meterial likes $Ba_2Cu_2O_7$) have zero resistance.

11. For solving complicated network use is made of Kirchoff's laws.

12. Wheatstone bridge is a special type of resistance network for measuring the resistance (It cannot be used for measurement of very low and very high resistances). The condition of balance is:

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

13. A Wheatstone bridge has optimum sensitivity when the resistance in its all branches are of the same order of magnitude.

14. A cell is a device in which by means of chemical reactions, chemical energy is converted into electrical energy. A cell provides the necessary potential difference for maintaining a current in an electric circuit.

15. Emf a cell is the potential difference between its terminal when the cell does not draw any current (open circuit) whereas the terminal potential difference is defined as the potential difference between its terminal in a closed circuit.

    The two are related as:

    $E = V - ir$

    where r is the internal resistance of the cell.

16. When a number of cells of emf's $E_1, E_2 ..... E_n$ are connected

    (i) In series, then, $E_{eq} = E_1 + E_2 + ... E_n$

    (ii) In parallel, then, $E_{eq} = \frac{E_1}{r_1} + \frac{E_2}{r_2} + ... \frac{E_n}{r_n}$

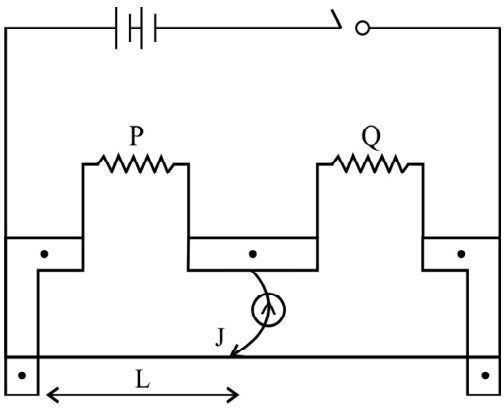
    and $\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + ... \frac{1}{r_n}$

17. Meter bridge is based on the Wheatstone bridge and is used to measure resistance.

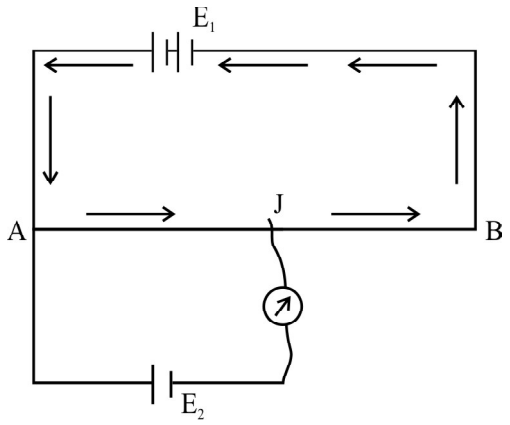
    Referring to figure we have,

    $$P = Q\frac{L}{100 - L}$$

    For better accuracy we have to take into account end corrections.



18. Potentiometer is a verstile instrument. Here we balance an unknown e.m.f. E_2 against a length L of the potentiometer wire. We can use potentiometer as:



(i) To compare two emf's $\left(\frac{E_1}{E_2} = \frac{\ell_1}{\ell_2}\right)$ (Reference Text)

(ii) To determine the internal resistance of a cell $\left(r = \left(\frac{L_1 - L_2}{L_2}\right)R\right)$

(iii) Measurements of current.

(iv) Measurement of high emf.

(v) Measurement of a very small emf.

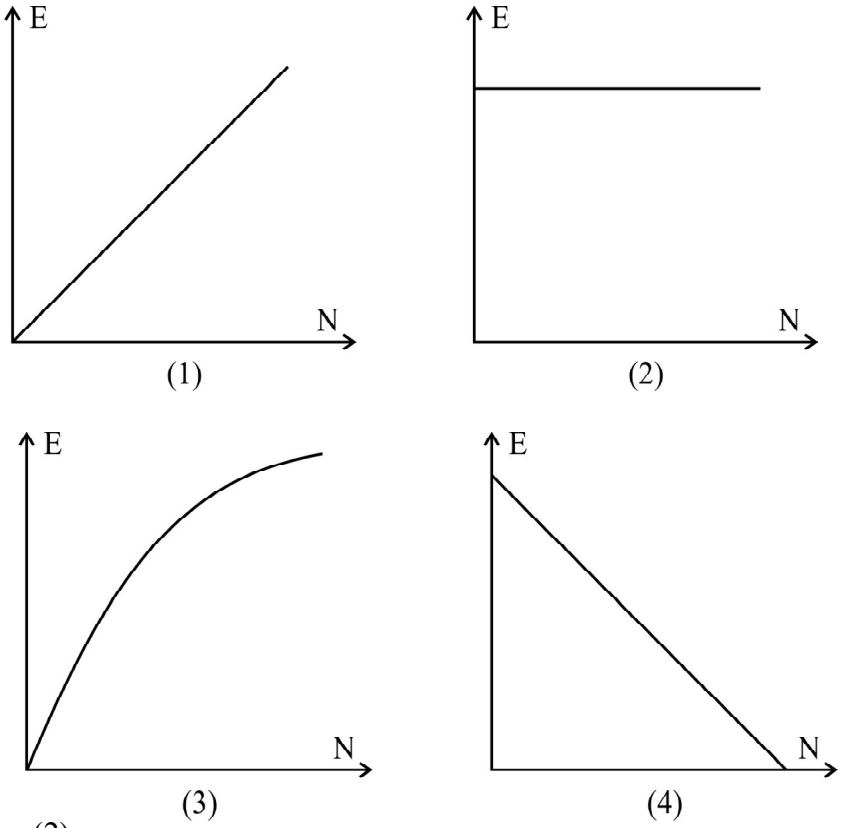
19. If a current of I ampere traverses a conductor between the ends of which there is a potential difference equal to V, the work done during a time t is V I t. If the whole of energy liberated is converted into heat, the heat generated is given by

$H = I^2 R t$

Unit–12 : Current Electricity

Average *Combination of Cells*

1. N identical cells, each of emf E, and internal resistance r, are connected in parallel. The variation of the net emf, E of the combination, with the number of cells N, is expressed corretly by the graph labelled as:



Correct answer: (2)

Solution:

If n cells, of e.m.f. $E_1, E_2 \ldots\ldots E_n$ and internal resistance $r_1, r_2 \ldots\ldots r_n$, are connected in parallel, then we have,

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \ldots\ldots \frac{1}{r_n}$$

and $$\frac{E_{eq}}{r_{eq}} = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \ldots\ldots \frac{E_n}{r_n}$$

From these equations we have,

$$\frac{1}{r_{eq}} = \frac{1}{r} + \frac{1}{r} + ...\frac{1}{r} = \frac{N}{r}$$

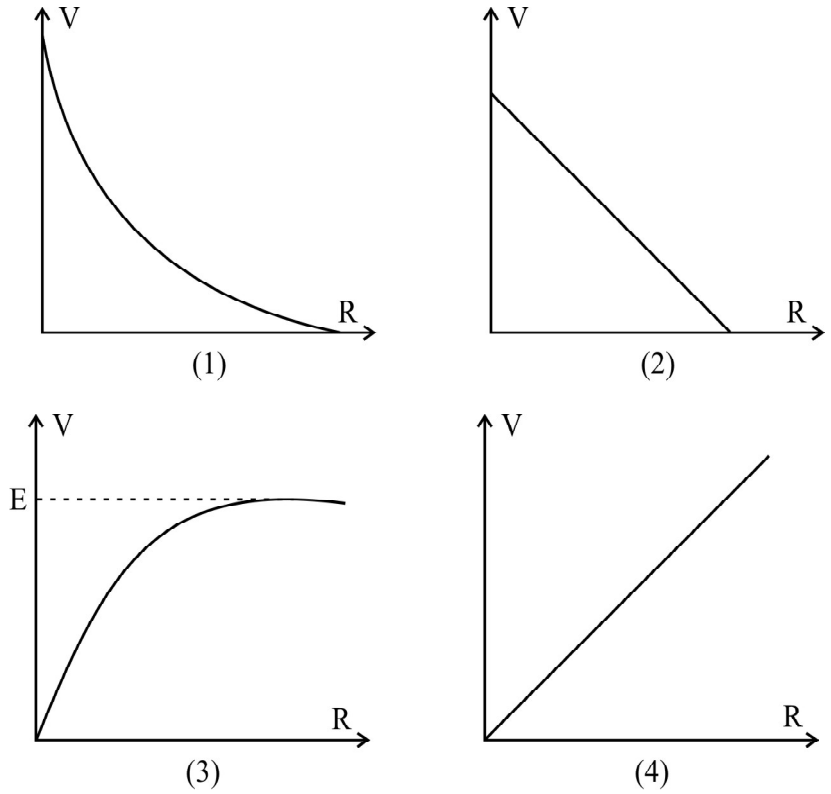
and $$\frac{E_{eq}}{r_{eq}} = \frac{E}{r} + \frac{E}{r} + ...\frac{E}{r} = \frac{\frac{nE}{r}}{\frac{n}{r}} = E$$

Hence option (2) is correct.

Unit–12 : Current Electricity

Average *Terminal p.d*

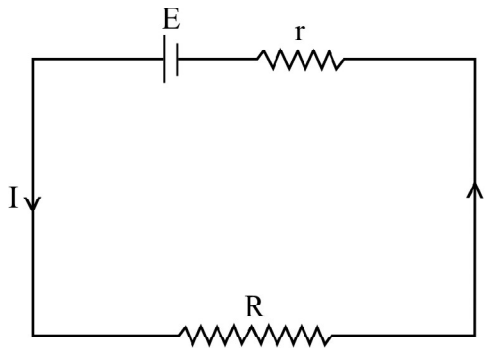
2. A cell having an emf E and internal resistance r is connected across a variable resistance R. When the resistance R is increasing and the potential difference V across R is measured, the graph between increases value of R and the corresponding value of V is represented by the graph, labelled as graph.



Correct answer: (3)

Solution:

The circuit is a shown.



The current in this circuit is $i = \frac{E}{R + r}$

Potential V across R is

$$V = iR = \left(\frac{E}{R + r}\right)R = \frac{E}{1 + \frac{r}{R}}$$

It is easy to see that

When $R \to 0$ $V = 0$ and

When $R \to \infty$ $V \to E$

Hence option (3) is correct.

Unit–12 : Current Electricity

Average *Cells in Series*

3. N identical cells, each of emf E and internal resistance r, are joined in series. Out of these n cells are incorrectly connected, i.e., these terminals are connected in reverse of that required for series connection $\left(n < \frac{N}{3}\right)$. E_0 and r_0 represent the resulting emf and internal resistance of the combination.

Then E_0 and r_0 are given by:

(1) $(N - 2n)E,\ r_0 = Nr$ (Correct)

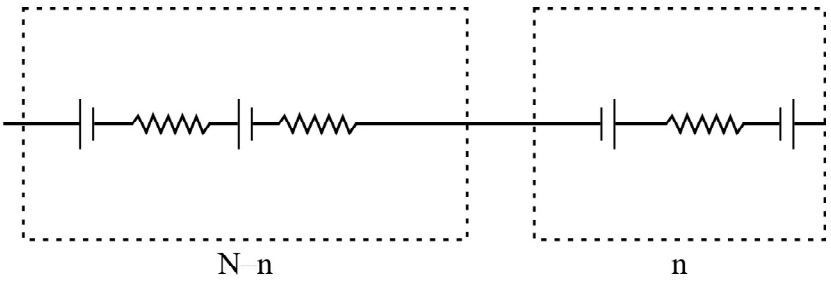
(2) $\left(\frac{N-n}{2}\right)E,\ r_0 = \left(\frac{N-n}{2}\right)r$ (Incorrect)

(3) $(n - n)E,\ r_0 = \left(\frac{N}{n}\right)r$ (Incorrect)

(4) $(2N - n)E,\ r_0 = (N - 2n)r$ (Incorrect)

Correct answer: (1)

Solution:



$F_0 = (N - n)\,E - nE = (N - 2n)E$

$r_n = r + r + \ldots\ldots$ n times $= Nr$

Hence the option (1) is correct.

Unit–12 : Current Electricity

Average *Cell Combinations*

4. A battary of 24 cells each of emf 1.5 V and internal resisatnce 2Ω is connected so as to send maximum current through a 125 resistance. In order to achieve this we must connect.

(1) 6 cells in series and connect 4 such rows in parallel (Incorrect)

(2) all the 24 cells in series (Incorrect)

(3) all the 24 cells in parallel (Incorrect)

(4) 12 cells in series and connect 2 such rows in parallel (Correct)

Correct answer: (4)

Solution:

Let us connect cells in mixed grouping where n cells are connected in series in one row and m such rows are connected in parallel. Let $N = Nr = 24$

Then for maximum current in external resistance.

$$r = \sqrt{\frac{NR}{r}} = \sqrt{\frac{24 \times 12}{2}} = 12$$

$$m = \frac{N}{n} = 2$$

Hence option (4) is correct.

Unit–12 : Current Electricity

Difficult *Cell Combinations*

5. (MN) identical cells, of e.m.f. E and internal resistance r, connected in N parallel rows where each row contains M cells in series. This combination of cells is connected across an external resistance R. What should be arrangment of the cells for getting maximum current through R if the total number of cells is fixed?

(1) Number of cells in a row = $\frac{RN}{r}$ and number of rows = $\frac{Nr}{R}$ (Incorrect)

(2) Number of cells in a row = $\sqrt{\frac{RN}{r}}$ and number of rows = $\sqrt{\frac{Nr}{R}}$ (Correct)

(3) Number of cells in a row = $\sqrt{\frac{MR}{r}}$ and number of rows = $\sqrt{\frac{MR}{R}}$ (Incorrect)

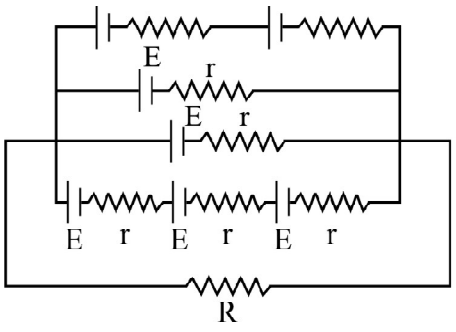
(4) Number of cells in a row = $\sqrt{\frac{Nr}{R}}$ and number of rows = $\sqrt{\frac{nR}{r}}$ (Incorrect)

Correct answer: (2)

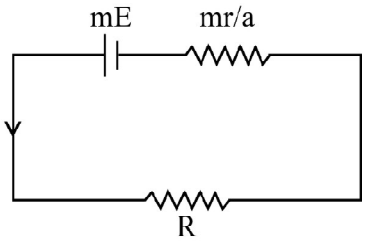
Solution:

The simplified version of the given circuit is shown in diagram (2) we have,

$$i = \frac{mE}{R + \frac{mr}{n}} = \frac{mnE}{mr + nR}$$



(1)



(2)

If the total number of cell is fixed, than m is fixed. Then we need to find the value of the m and n so that the current is maximum. We can write

$$i = \frac{m\ n\ E}{\left(\sqrt{m\,r} - \sqrt{n\,r}\right)^2 + 2\sqrt{m\,n\,R}}$$

As mn is fixed. I is maximum when denominator is minimum

$\therefore \quad \left(\sqrt{m\,r} - \sqrt{n\,r}\right)^2$

or $\quad \frac{m}{n} = \frac{R}{r}$

If N is the total number of cells then we can get $mn = N$ and $\frac{m}{n} = R/r$

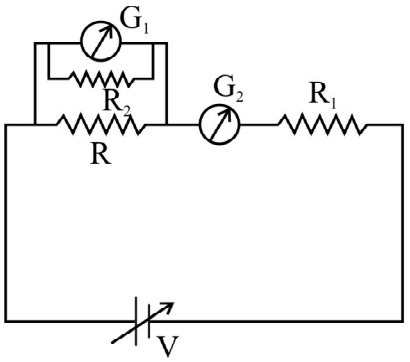
Hence for the maximum current. Number of cell in a row = $\sqrt{\frac{RN}{r}}$

and number of rows = $\sqrt{\frac{Nr}{R}}$

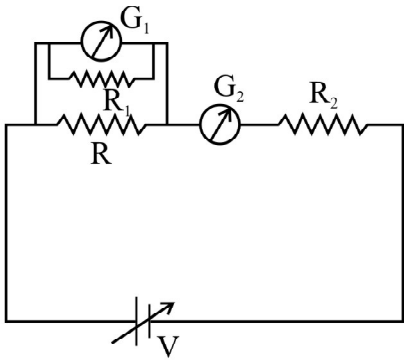
Unit–12 : Current Electricity

Average *Circuit Analysis*

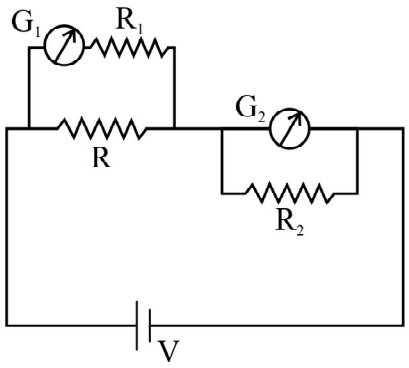
6. You are provided with a test resistance R, a high resistance R_1 and a small resistance R_2, two identical galvanometer G_1 and G_2 and a variable voltage source V. You are required to set up a circuit for the verification of ohm's law. Which of the following would be a correct circuit?



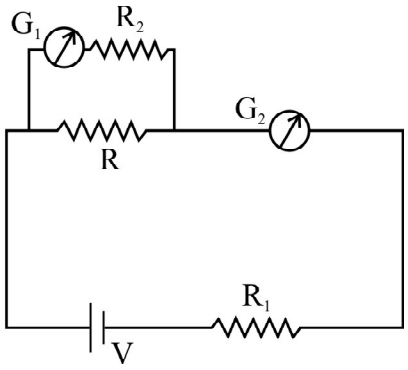
(1)



(2)



(3)



(4)

Correct answer: (3)

Solution:

Here we have to use galvanometer G_1, with a high resistance in series, in place of the voltmeter. Similarly galvanometer G_2, with a parallel small resist once,takes the place of the ammeter. Hence the circuit diagram (3) is the correct circuit for the verification of ohm's law.

Unit–12 : Current Electricity

Average *Drift Velocity*

7. A conductor contains 8×10^{-28} free electrons per cubic meter. The conductor carries a current of 1A and has a length of 10 Cm. The area of cross section of the conductor is $7.81\times10^{-6}m^2$. The time taken by an electron to move from one end of the conductor to the other end is:

(1) 10^{-3}s (Incorrect)

(2) 10^{-4}s (Incorrect)

(3) 10^{5}s (Correct)

(4) 10^{6}s (Incorrect)

Correct answer: (3)

Solution:

We known that drift velocity v_d is given by the relation.

$$v_d = \frac{I}{n\,e\,A}$$

where the symbols have their usual meanings.

Now it is easy to see that v_d can be written as $v_d = \frac{L}{t}$

where L is the distance between one end of a conductor to the other end and t is the time taken by the free electrons to travel from one one to the other end of the conductor.

Hence, $\frac{L}{t} = \frac{I}{neA}$

or $\frac{t}{L} = \frac{neA}{I}$

or $t = \frac{neA}{I} L$

Substituting we given values we get, $t = 10^{-3}$ s

Hence option (1) is correct.

Unit–12 : Current Electricity

Easy *Mean Free Path*

8. The mean free path of electron in a metal is 4×10^{-8} m. The electric field which can give on average 2eV energy to an electron in the metal will be

(1) $10\times10^{7}\ \frac{V}{m}$ (Incorrect)

(2) $5\times10^{-7}\ \frac{V}{m}$ (Incorrect)

(3) $5\times10^{7}\ \frac{V}{m}$ (Correct)

(4) $2.5\times10^{7}\ \frac{V}{m}$ (Incorrect)

Correct answer: (3)

Solution:

It is easy to see that the work done is (qE) (λ).

Hence,

$2eV = qE \times 4 \times 10^{-8}$

or $E = \frac{1}{4 \times 10^{-8}} \frac{V}{m} = 5 \times 10^{7} \frac{V}{m}$

Unit–12 : Current Electricity

Easy *Resistivity*

9. The length of a wire is doubled and current through it is increased by a factor of $\frac{3}{2}$ without any charge of temperature. The resistivity of the material of the wire

(1) remains same (Correct)

(2) increase by a factor of 3 (Incorrect)

(3) decrease by a factor of 3 (Incorrect)

(4) increase by a factor of 2 (Incorrect)

Correct answer: (1)

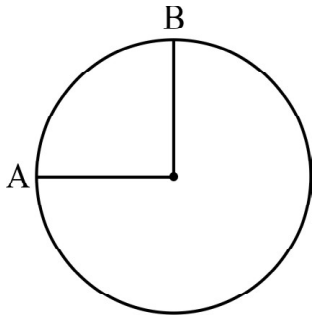
Solution:

The resistivity of a wire depends only on the nature of wire. Changing factors like length, area of cross section does not charge the resistivity of the material of the wire.

Unit–12 : Current Electricity

Average *Combination of Resistance*

10. A wire of uniform resistance z r m^{-1} is sent into a circle of radius r. The same wire is connected between point A and B as shown. The equivalent resistance between A and B is:



(1) $\frac{zr}{(3\pi+16)}$ (Incorrect)

(2) $\frac{6\pi(z/r)}{(3\pi+16)}$ (Incorrect)

(3) $\frac{6\pi zr}{(3\pi+16)}$ (Correct)

(4) $\left(\frac{z}{r}\right)(3\pi+16)$ (Incorrect)

Correct answer: (3)

Solution:

Let R_1 be the resistance of one quadrant and R_2 be the resistancr of three quadrants of circle.

$$R_1 = \left(\frac{2\pi r}{4}\right)z = \frac{\pi zr}{2}$$

$$R_2 = \frac{3}{2}\pi zr$$

If the resistance of straight wires AO and OB taken together is R_3. Then

$R_3 = 2rz$

R_1, R_2 and R_3 are connected in parallel

$$\frac{1}{R} = \frac{2}{\pi zr} + \frac{2}{3\pi zr} + \frac{1}{2zr} = \frac{3\pi+16}{6\pi zr}$$

$$R = \frac{6\pi zR}{3\pi+16}$$

Hence optino (3) is correct.

Unit–12 : Current Electricity

Easy *Non Ohmic Devices*

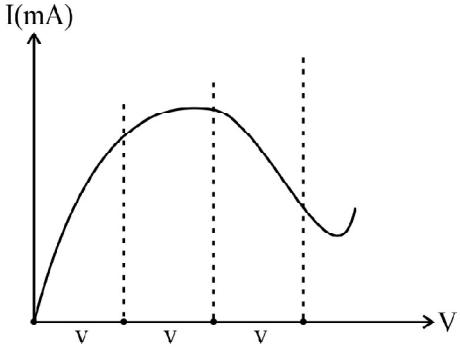
11. The relation between V and I for a device not obeying Ohm's is shown in the figure where there is more than one value for current for a given interval of V. This device can be

(1) a junction diode (Incorrect)

(2) a super conductor (Incorrect)

(3) a light emitting diode (Correct)

(4) a transmiter (Incorrect)



Correct answer: (3)

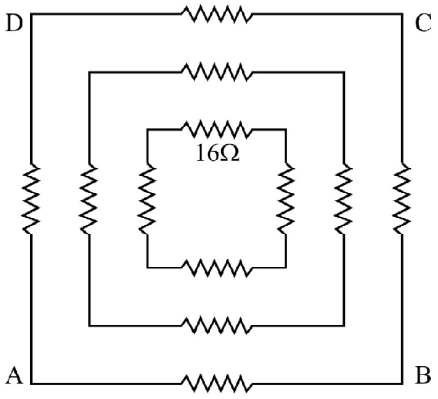
Solution:

Informative (The material of the light emitting diode is GaAs).

Unit–12 : Current Electricity

Average *Combination of Resistances*

12. Sixteen resistance, each of 16Ω are connected as shown in figure. The net resistance, between A and B, is.



(1) 4Ω (Incorrect)

(2) 6Ω (Incorrect)

(3) 8Ω (Incorrect)

(4) 3Ω (Correct)

Correct answer: (4)

Solution:

In each of the four arms of network of resistance four resistance are converted in parallel. The effective resistance of each arm of network is

$$\frac{R}{4}=\frac{16}{4}=4\Omega$$

The total resistance is then given by:

$$R_T=\frac{(4+4+4)\times 4}{(4+4+4)+4}=3\Omega$$

Hence option (4) is correct.

Unit–12 : Current Electricity

Easy *Resistance of Wire*

13. A copper wire is stretched to make it 0.z% longer. Then the percentage charge in its resistance is

(1) $0.\left(\frac{1}{2}\right)\times z\ \%$ (Incorrect)

(2) No charge (Incorrect)

(3) $0.4\,(z\ \%)$ (Incorrect)

(4) $2\times(0.z\ \%)$ (Correct)

Correct answer: (4)

Solution:

If A is the area of cross sectino of the wire of length L and density d its mass is given by:

$$m = ALd \qquad \text{or} \qquad A = \frac{m}{Ld}$$

The resistance, R of the wire of, resistively ρ is given by

$$R = \frac{\rho L}{A} = \frac{\rho L^2 d}{m}$$

Differentating we get,

$$\frac{dR}{R} = \frac{2dL}{L}$$

Hence option (4) is correct.

Unit–12 : Current Electricity

Difficult *Resistance Combinations*

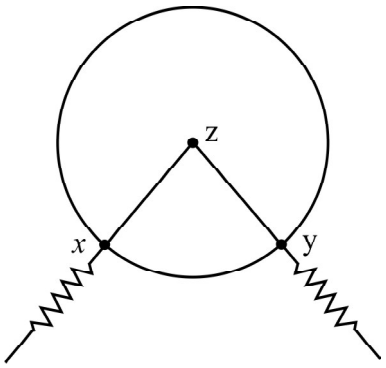
14. *x* and y are two points on a uniform ring of resistance R. The angle xzy = θ where z is the centre of ring. Then the equivalent resistance between *x* and y is:

(1) $\frac{R}{4\pi^2}(2\pi-\theta)\theta$ (Correct)

(2) $\frac{R}{4\pi^2}\theta$ (Incorrect)

(3) $\frac{R}{2\pi}\theta(2\pi-\theta)$ (Incorrect)

(4) $\frac{R}{(2\pi-\theta)}\times 4\pi^2\theta$ (Incorrect)



Correct answer: (1)

Solution:

$$R_{xy}=\frac{\rho\ell_{xy}}{A}$$

Also, $R=\frac{\rho(2\pi a)}{A}$ and $\ell_{xy}=\theta A$

$$R_{xy}\left(\text{Smaller portion}\right)=\frac{R}{2\pi a}(\theta a)=\frac{R\theta}{2\pi}$$

Similarly, $R_{xy}\left(\text{Larger portion}\right)=\frac{R}{2\pi}=(2\pi-\theta)$

$$R_{eq}=\frac{\frac{R\theta}{2\pi}\times\frac{R}{2\pi}(2\pi-\theta)}{\frac{R\theta}{2\pi}+\frac{R}{2\pi}(2\pi-\theta)}$$

$$=\frac{R\theta(2\pi-\theta)}{4\pi^2}$$

Hence the option (1) is correct.

Unit–12 : Current Electricity

Difficult *Resistance Combinations*

15. A cylinder of radius a, lengths L is made of a material of resistivity ρ_1. It is electroplated with a material, of resistivity ρ_2, till it radius increases to b. The resistance of the

elctroplated cylinder will be given by the relation.

(1) $\left[\dfrac{1}{\dfrac{a^2}{\rho_1}+\left(\dfrac{b^2-a^2}{\rho_2}\right)}\right]$ (Incorrect)

(2) $\dfrac{1}{\pi}\left[\dfrac{1}{\dfrac{a^2}{\rho_1}+\left(\dfrac{b^2-a^2}{\rho_2}\right)}\right]$ (Correct)

(3) $\dfrac{a^2}{\rho_2}+\left(\dfrac{b^2-a^2}{\rho_1}\right)$ (Incorrect)

(4) $\dfrac{1}{2\pi}\left[\dfrac{a_1\rho_1+b\rho_2}{\dfrac{a^2}{\rho_1}+\dfrac{b^2}{\rho_2}}\right]$ (Incorrect)

Correct answer: (2)

Solution:

Let R_1 be resistance of inner cylinder of radius a and resistivity ρ. Then $R_1=\rho_1\dfrac{L}{\pi a^2}$

The area of cross section of electroplated part is $\pi\left(b^2-a^2\right)$. The resistance R_2 of electroplated part of cylinder is

$$R_2=\rho_2\frac{L}{\pi\left(b^2-a^2\right)}$$

Resistance R_1 and R_2 are connected in parallel. The equivalent resistance R is

$$R=\frac{R_1R_2}{R_1+R_2}=\frac{\rho_1\rho_2L^2\Big/\left[\pi^2a^2\left(b^2-a^2\right)\right]}{\dfrac{1}{\pi}\left[\dfrac{\rho_1}{a^2}+\dfrac{\rho_2}{\left(b^2-a^2\right)}\right]}$$

$$= \frac{\rho\rho_2 L}{\pi\left[\rho_1\left(b^2-a^2\right)+\rho_2 a^2\right]} = \frac{1}{\pi}\left[\frac{1}{a^2/\rho_1 + \frac{\left(b^2-a^2\right)}{\rho^2}}\right]$$

Hence option (2) is correct.

Unit–12 : Current Electricity

Difficult *Continuous Variation of Resistance*

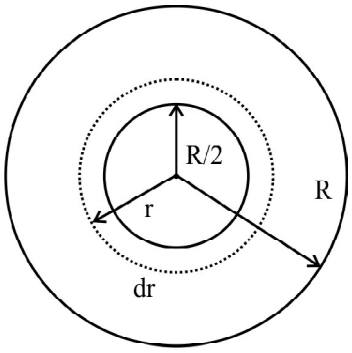
16. A cylinderical wire of radius R and is of uniform area of cross section. The current density varies with radial distance r as $J = \alpha r^2$. Then, the current, through the outer portion of the wire, between radial distances, $\frac{R}{2}$ and R, is given by

(1) $\pi a^2 R^4$ (Incorrect)

(2) $\frac{15}{32}\pi a R^4$ (Correct)

(3) $\frac{15}{32}\pi a R^3$ (Incorrect)

(4) $\frac{15}{32}(\pi a R)^3$ (Incorrect)



Correct answer: (2)

Solution:

We consider an element of the cylinderical wire of radius r and thikness dr as shown. Then the area of cross section of the element is $dA = 2\pi r\, dr$.

It can be seen that the current is along the wire and the elementary area dA is perpendicular to the cross section of the wire and both have the same direction.

Hence the current through the element of the wire is

$$dI = dA = \alpha r^2 \times 2\pi r\, dr = 2\pi\alpha\, r^3 dr$$

Hence the current through the outer portion of the wire is

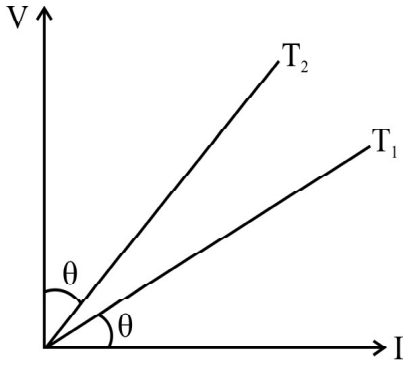
$$I = \int_{R/2}^{R} 2\pi\alpha R^3 dr = 2\pi\alpha\left[\frac{r^4}{4}\right]_{R/2}^{R} = \frac{15}{32}\pi\alpha R^4$$

Hence option (2) is correct.

Unit–12 : Current Electricity

Average *V–I Graphs*

17. The voltage current graph for a platinum wire conductor at temperature T_1 and T_2 is as shown in the figure.



(Given temperature coefficient of platinum is α and R_0 to the resistance of platinum wire at 0^0C). The value of $(T_2 - T_1)$ is

(1) $\frac{2}{\alpha R_0}\tan\theta$ (Incorrect)

(2) $\frac{R_0\alpha}{2}\cos 2\theta$ (Incorrect)

(3) $\frac{2}{\alpha R_0}\sin 2\theta$ (Incorrect)

(4) $\frac{2}{\alpha R_0}\cot 2\theta$ (Correct)

Correct answer: (4)

Solution:

$$R_1 = \tan\theta = R_0(1 + \alpha T_1)$$

$$R_2 = \cot\theta = R_0\left(1+\alpha T_2\right)$$

$$\cot\theta - \tan\theta = R_0\left(1+\alpha T_2\right) - R_0\left(1+\alpha T_1\right)$$

or $$T_2 - T_1 = \frac{1}{\alpha R_0}\left(\cot\theta - \tan\theta\right)$$

$$T_2 - T_1 = \frac{1}{\alpha R_0}\left(\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}\right)$$

$$= \frac{2\cos 2\theta}{\alpha R_0 \sin 2\theta} = \frac{2}{\alpha R_0}\cot 2\theta$$

Unit–12 : Current Electricity

Easy *Dependence of Resistivity on Temperature*

18. Which of the following statement is correct?

(1) For metals, resistivity increases linearly with temperature, at all temperatures. (Incorrect)

(2) For insulators resistivity decreases expontentially with decreasing temperatures. (Incorrect)

(3) For semi conductors resistivity becomes very large at low temperatures tending to infinitely large values as . $T \to 0$ (Correct)

(4) Resistivity increases with temperatures for metals, insulators as well as semi conductors. (Incorrect)

Correct answer: (3)

Solution:

For semi conductor the resistivity increases exponentially with decreasing temperature, the variation being expressed by the relation of the form

$$\rho(T) = \rho_0(T)\ \exp\left(\frac{E_g}{kT}\right)$$

Here $\rho_0(T)$ is a factor that varies slightly will temperature, k is the Boletzman's constant and E_g is the positive energy known as gap energy.

Because of expontatial dependance the resistance of semi conductors becomes very large at low temperature tending to infinite large values as $T \to 0$.

Hence option (3) is correct.

Unit–12 : Current Electricity

Average *Dependance of Resistance on Temperature*

19. Which of the following statements is incorrect?

(1) The resistance of a thermistor charges very rapidly with charge in temperature. (Incorrect)

(2) The temperature co-efficient of resistivity can be both positive and negative. (Incorrect)

(3) A thermistor can easily measure a charge in temperature of 10^{-3}C. (Incorrect)

(4) The temperature co-efficient of resistivily of a thermistor is normally very low. (Correct)

Correct answer: (4)

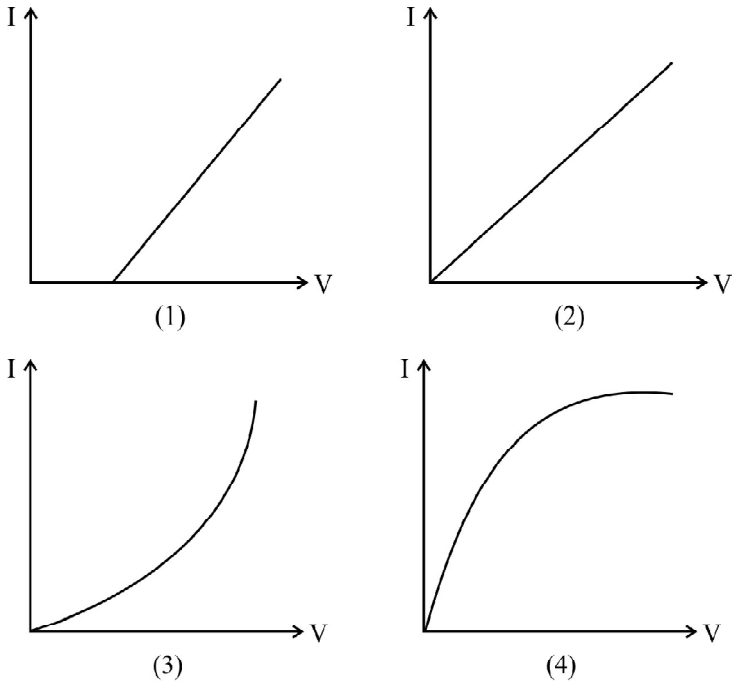
Solution:

The thermistor is a semi conducting device. The resistivity of a semi conductor increases expontentially with decrease in temperature. The resistivity of a semi conductor increases very large with decrease in temperature tending to inifinity as $T \to 0$. The option 1, 2 and 3 are known characterstics of a thermistor. Hence the option (4) is not correct.

Unit–12 : Current Electricity

Easy *Non-Ohmic Devices*

20. Which of the following graphs represent current voltage relationship for a water voltamtere?



Correct answer: (1)

Solution:

For certain electrolytes like platinum and water, we find that the current flowing through is extremely small until the potential difference increase to a value V'. After that the current increases linearly, with V, as shown in option (1).

Hence option (1) is correct.

Unit–12 : Current Electricity

Average *Circuit Analysis*

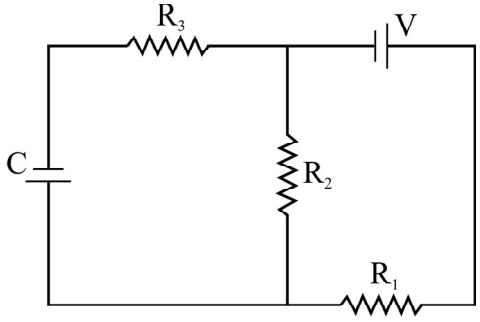
21. Find the steady state voltage drop across the capacitor C in the diagram given below.

(1) V (Incorrect)

(2) $\dfrac{VR_1}{\left[R_2+\left(\dfrac{R_1R_3}{R_1+R_3}\right)\right]}$ (Incorrect)

(3) $\dfrac{VR_2}{R_1+R_2}$ (Correct)

(4) $\dfrac{VR_2}{R_1+R_3}$ (Incorrect)



Correct answer: (3)

Solution:

Since no d.c. current flows through a capacitor the steady state voltage drop across capacitor is same as across resistance R_2. The current is given by

$$I=\frac{V}{R_1+R_2}$$

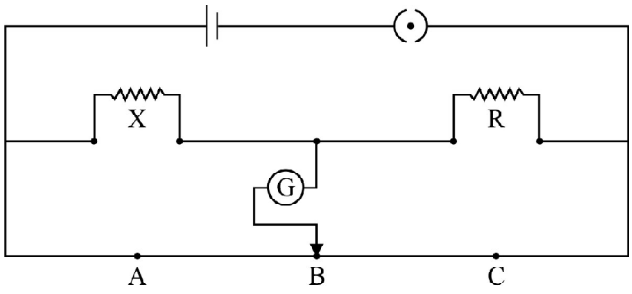
Here the voltage drop across $R_2 = IR_2 = \dfrac{VR_2}{R_1+R_2}$

Hence option (3) is correct.

Unit–12 : Current Electricity

Easy *Meter Bridge*

22. Figure shows a meter bridge circuit for the determination of an unknown resistance. When we use different values of R, (say R_1, R_2 and R_3, respectively) we obtain 'balance points' at A, B and C, respectively. For which resistor, the value of ***x*** **will be determined more accurately?**



(1) R_1 (Incorrect)

(2) R_2 (Correct)

(3) R_3 (Incorrect)

(4) The accuracy of determination will be practically the same whether we use R_1, or R_2, or R_3. (Incorrect)

Correct answer: (2)

Solution:

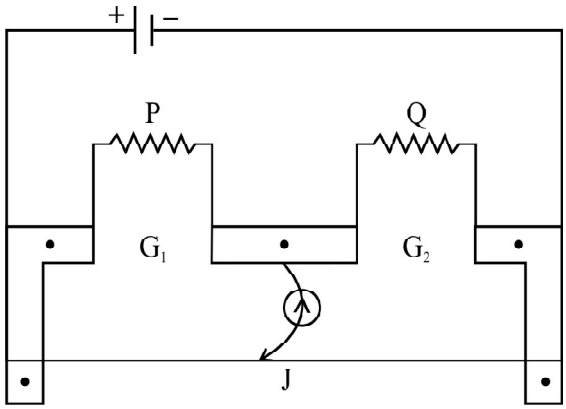
Meter bridge is based on the principle of Wheatstone bridge. The balance condition $\frac{R_1}{R_2}=\frac{R_3}{R_4}$ in a Wheatstone bridge is an ideal one since it is not achieveable in practice. This is because current in the galvanoemeter can never be made equal to zero. Hence the value of variable resistance (R_5) in a Wheatstone bridge will always be a little different from ideal value R_3 for balance. $\left(R_3^{'}-R_3\right)$ is known as want of balance. It can be shown that, for optimum sensitivity of a wheatstone bridge for a given want of balance is obtained when the resistance in its all four arms are of the same order of magnitude. The balance point B, using resistance R_2, is close to the middle of the wire. This satisfies the condition for optimum sansitivity. Hence option (2) is correct.

Unit–12 : Current Electricity

Average *Meter Bridge*

23. Figure shows a meter bridge circuit for measurements of resistance. However the meter bridge wire has end corrections α and β (in terms of the length of meter bridge wire).

Taking known P and Q, we find a balance point is obtained at a length ℓ_1 of meter bridge wire. If we inter change, the position of P and Q, the balance point is obtained at a length ℓ_2 of meter bridge wire; then the value of α and β are



(1) $\alpha = \dfrac{Q\ell_2 - P\ell_1}{P - Q},\ \beta = \dfrac{Q\ell_1 - P\ell_2}{Q - P}$ (Incorrect)

(2) $\alpha = \dfrac{P\ell_2 - Q\ell_1}{Q - P},\ \beta = \dfrac{P\ell_1 - Q\ell_2}{P - Q}$ (Incorrect)

(3) $\alpha = \dfrac{\ell_1 - \ell_2}{P},\ \beta = \dfrac{\ell_2 - \ell_1}{Q}$ (Incorrect)

(4) $\alpha = \dfrac{Q\ell_1 - P\ell_2}{P - Q},\ \beta = \dfrac{P\ell_1 - Q\ell_2}{P - Q}$ (Correct)

Correct answer: (4)

Solution:

We can write

$$\frac{P}{Q} = \frac{\ell_1 + \alpha}{100 - \ell_1 + \beta}$$

and $$\frac{Q}{P} = \frac{\ell_2 + \alpha}{(100 - \ell_2 + \beta)}$$

Solving these two equations for α and β we get the values of α and β as given in option (4).

Unit–12 : Current Electricity

Average *Wheatstone Bridge*

24. To obtain the optimum sensitivity of a Wheatstone bridge, for a given 'want of balance', and with a freedom of choice of arrangment, we should.

(1) Connect the battery so that resistance in series with the resistance to be measured is greater than the resistance in parallel will it. (Correct)

(2) Connect the battary so that the resistance in series with the resistance to be measured is less than the resistance in parallel will it. (Incorrect)

(3) Interchange the position of the battery and the galvanometer. (Incorrect)

(4) Use a galvanometer with a reasonable high resistance. (Incorrect)

Correct answer: (1)

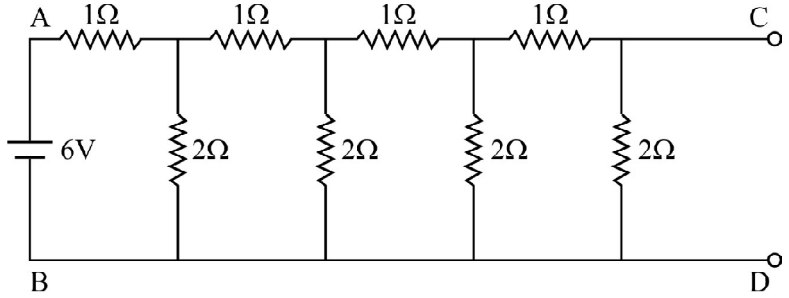
Solution:

We can obtain an ideal relation for the sensitivity of a Wheatstone bridge for a given want of balance. However this ideal relation cannot be satisfied in actual working conditions. Hence we look for a rule for optimum senstivity of a wheatstone bridge. The option (1) is a working rule given by Callender for optimum senstivity of a Wheatstone bridge.

Unit–12 : Current Electricity

Average *Combination of Resistance*

25. An infinite ladder network of resistance is constructed with 1Ω and 2Ω resistances as shown. The 6V battery has negligible resistance. Then the effective resistance between A and B is



(1) 1Ω (Incorrect)

(2) 2Ω (Correct)

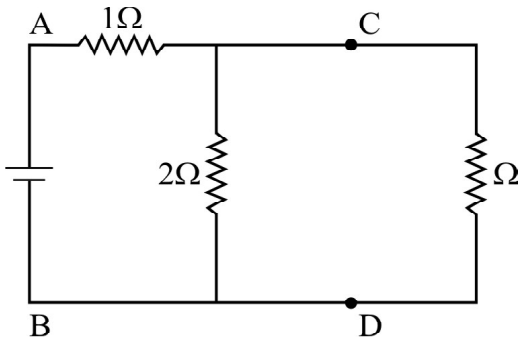
(3) 3Ω (Incorrect)

(4) Infinite (Incorrect)

Correct answer: (2)

Solution:

Let x be the effective resistance between points A and B. Since the chain is an infininte one this may also be taken as the effective resistance of the chain beyond the points C and D. Hence the may replace the given chain by the equivalent diagram as shown:



We this have,

$$x = 1 + \frac{1}{\left(\frac{1}{2} + \frac{1}{x}\right)}, \quad x = 1 + \frac{2x}{2+x}$$

or $(x - 1)(2 + x) = 2x$

Simplifying we have, $x^2 - x - 2 = 0$

or $\quad x = 1 \pm \sqrt{\frac{1+8}{2}} = 2 \text{ or } -1$

Since the equivalent resistance cannot be negative we have,

$$x = 2\,\Omega$$

Hence option (2) is correct.

Unit–12 : Current Electricity

Average *Combination of Resistance*

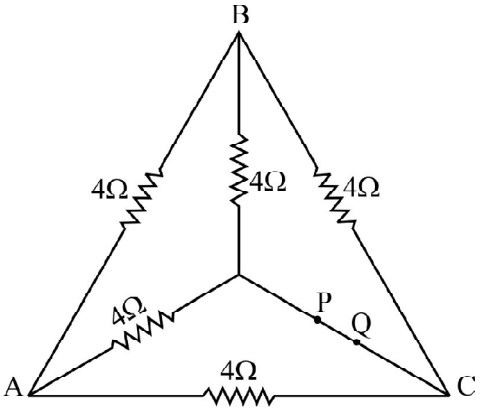
26. Figure shows a network of 5 resistances each of value $4\,\Omega$ connected as shown. The value of the resistnace between terminals P and Q is

(1) $2\,\Omega$ (Incorrect)

(2) $4\,\Omega$ (Correct)

(3) $8\,\Omega$ (Incorrect)

(4) $0.4\,\Omega$ (Incorrect)



Correct answer: (2)

Solution:

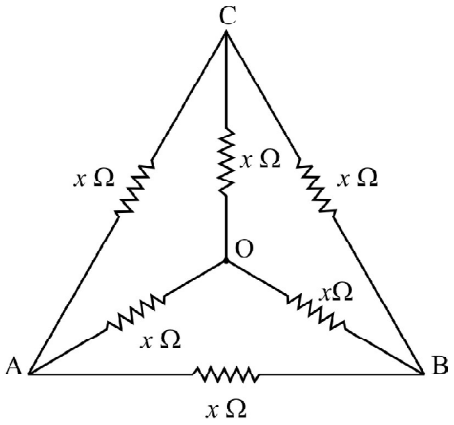
The given network forms a balanced wheatstone bridge. Hence the resistance between AB can be removed. All the resistance have value of 4Ω. Hence the resistance between points P and Q is also 4Ω.

Hence option (2) is correct.

Unit–12 : Current Electricity

Average *Combination of Resistances*

27. Six equal resistance each of value x ohm are connected to form a network as shown in figure. Then the resistance between A and B is



(1) $2x$ (Incorrect)

(2) $6x$ (Incorrect)

(3) $x/2$ (Correct)

(4) $x/6$ (Incorrect)

Correct answer: (3)

Solution:

The upper five resistance between A and B are forming a balanced Wheatstone bridge. Hence there will be no current in arm CO. Hence CO are can be removed. Also resistance of arm ACB, AOB and AB are all in pararllel.

The effective resistance R is given by

$$\frac{1}{R} = \frac{1}{(x+x)} + \frac{1}{(x+x)} + \frac{1}{x}$$

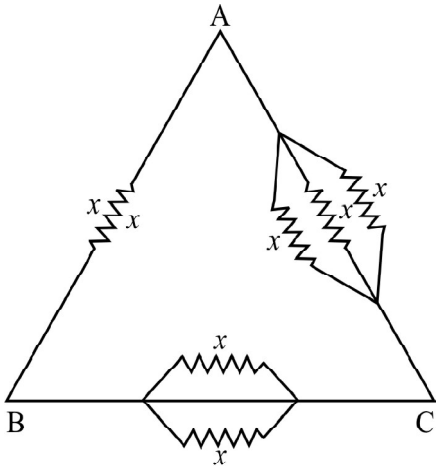
or $$R = \frac{x}{2}$$

Hence option (3) is correct.

Unit–12 : Current Electricity

Difficult *Combination of Resistances*

28. Figure shows the connections of six resistances each of value *x* between the points A, B and C. Then the net resistance will be maximum between.



(1) A and B (Correct)

(2) B and C (Incorrect)

(3) A and C (Incorrect)

(4) The net value of the resistance between A and B; B and C and A and C will be the same. (Incorrect)

Correct answer: (1)

Solution:

In the branch AC the effective resistance is $\frac{x}{3}$. In the branch BC the effective resistance is $\frac{x}{2}$

$\therefore$ Resistance in the branch $\text{ACB} = \frac{x}{3} + \frac{x}{2} = \frac{5x}{6}$

The resistance of branch ACB is in parallel to resistance of branch AB. This effective resistance between A and B $= \frac{x \times 5x/6}{x + 5x/6} = \frac{5}{11}x$.

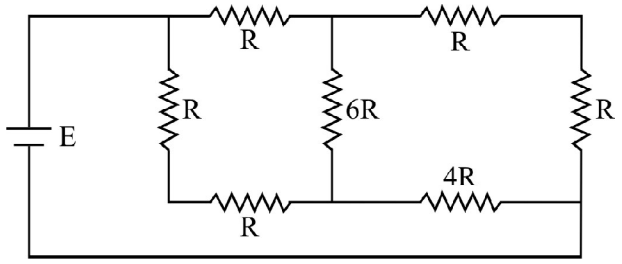
Similarly the resistance between B and C is $\frac{4}{11}x$ and the resistance between A and C is $\frac{3}{11}x$.

Hence option (1) is correct.

Unit–12 : Current Electricity

Difficult *Circuit Analysis*

29. A battery, of internal resistance 4Ω, is connected to the network of resistance as shown in the following figure.



To deliver the maximum power to the network the value of, R, should be

(1) 2 (Correct)

(2) $\frac{4}{9}$ (Incorrect)

(3) $\frac{2}{9}$ (Incorrect)

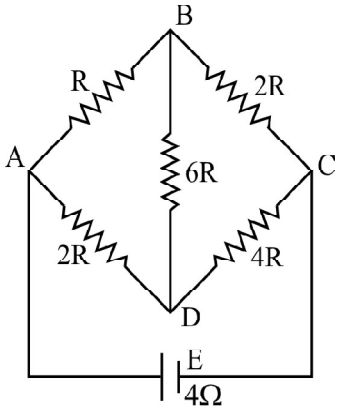
(4) $\frac{2}{3}$ (Incorrect)

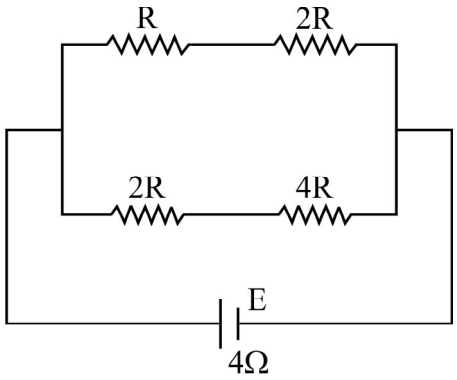
Correct answer: (1)

Solution:

The given network can be reduced to a balanced Wheatstone bridge as shown. Hence no current flows through side BD of 6R resistance.

The given circuit forther reduces to.





To obtain maximum power, the net external resistance must be equal to total internal resistance.

$\therefore$ $2R = 4$

or $R = 2\Omega$

Hence option (1) is correct.

Unit–12 : Current Electricity

Average *Potentiometer*

30. A potentiometer circuit is shown in figure. *x* and y being the terminal of an unknown e.m.f. It is observed that when the jockey J is at A there is a small deflection in the galvanometer. But when the jockey is moved to B the deflection remains in the same direction but increases. This happens because

(1) The main battery is connected in an incorrect manner. (Correct)

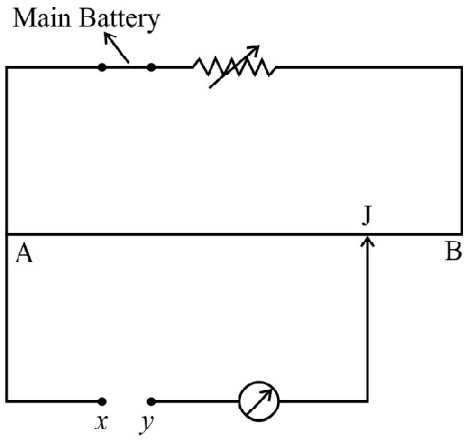
(2) The e.m.f. of the main battery is less than unknown e.m.f. (Incorrect)

(3) The unknown e.m.f. is less than that of the main battery. (Incorrect)

(4) That is a broken connection in the galvanomter circuit. (Incorrect)

Correct answer: (1)

Solution:



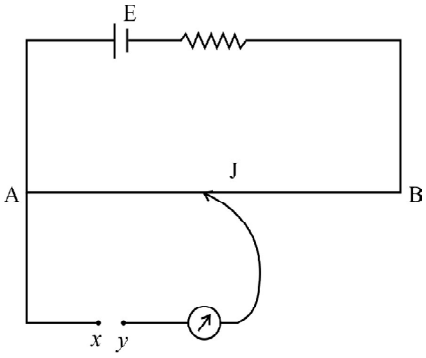
In a potentiometer we balance an unknown e.m.f. against potential drop a portion of length of the potentiometer wire. We find that the deflaction is in the same direction and increases as we move from one end A to another and B of the potentiometer wire it is clear that external e.m.f. is superimposing on drop of potential across AJ. This indicates the external e.m.f. between points *x*y is wrongly connected.

Hence option (1) is correct.

Unit–12 : Current Electricity

Average *Potentiometer*

31. A potentiometer circuit is shown in figure *x* and y being the tarminals of an unknown e.m.f. It is observed that when the jockey J is at A, there is a small deflection in the galvanometer and the deflection remains m the same direction but decreases when the jockey is moved to B. This is because



(1) The unknown e.m.f. *x*y is connected incorrectly. (Incorrect)

(2) The unknown e.m.f. *x*y is just less than E. (Incorrect)

(3) The unknown e.m.f. is much less than E. (Incorrect)

(4) The total drop of potential e.m.f. across the pontentiometer wire is less than the unknown. (Correct)

Correct answer: (4)

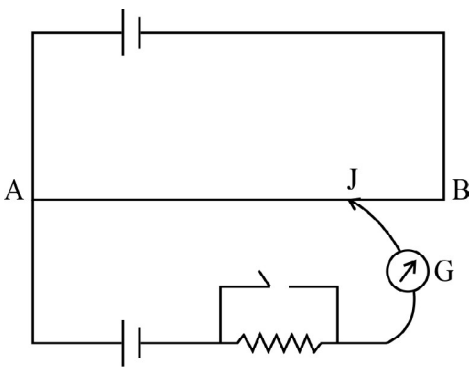
Solution:

When we find that deflation decreases when we move form end A to the other end B, the unknown emf *x*y is correctly connected. We all not able to obtain a balance point since the total drop of potential across the total length of potential meter is less than unknown e.m.f. Hence to obtain the balance we must use source battery E of higher value of emf.

Unit–12 : Current Electricity

Difficult *Potentiometer*

32. Figuqre shows a potentiometer with a cell of 20 V and internal resistance of $0.04\,\Omega$ maintaining a potential drop across the resistor wire AB. A standard cell which maintains a constant emf 1.02 V gives a balance point at 67.3 length of the wire. To ensure very low currents drawn from the standard cell, a very high resistance of $600\,k\Omega$ is put in series with it which is shorted close to the balance point. The standard cell is replaced by a cell of unknown emf E and the balance point found



similarly turn out to be 82.3 cm length of the wire. The value of E from the above given data is found to be 1.25 volt. We wish to determine the emf of, say, a themocouple (or the order of a few millivolt.

(1) To aclieve this measurement we should replace the high resistance of $600\,\Omega$ with an infinite resistance. (Incorrect)

(2) Increase the potential gradient of the potentiometer wire by a factor of at least 100. (Incorrect)

(3) Decrease the potential gradient of the potentiometer wire by at least a factor of 100 by connecting suitable low resistance in series with the potentiometer wire. (Incorrect)

(4) Decrease in potential gradient of potentiometer wire by a factor of more than 100 by connecting a suitable high resistance in series with the potentiometer wire. (Correct)

Correct answer: (4)

Solution:

The potential gradient of given wire is

$$\rho = \frac{2R}{(R+0.5)L} \quad \frac{V}{cm}$$

Usually R is of the order of a few ohm while L is of the order of 400 cm to 1000 cm.

Hence $\rho = \frac{2.0 \times 10}{10.5 \times 10^3}$

It follows that the balance length for a small e.m.f. would be only a few cm. We would need to reduce the value of s by a factor of at least hundred or so. This can be achieved by adding a suitable high resistance in series with the potentiometer wire.

Hence option (4) is correct.

Unit–12 : Current Electricity

Potentiometer

33. Which of the following statements is correct?

(1) A potentiometer can only be used for comparing two e.m.f.'s (Incorrect)

(2) A potentiometer cannot be used to measure a high e.m.f. (Incorrect)

(3) A potentiometer cannot be used for measuring thermo e.m.f. (Incorrect)

(4) A potentiometer can be used for measuring high emf's as well as very small e.m.f.'s (Correct)

Correct answer: (4)

Solution:

A potentiometer can be used for measuring a high e.m.f. The circuit diagram is shown in figures. Here the large e.m.f. to be measured is connected with two resistances R_1 and R_2

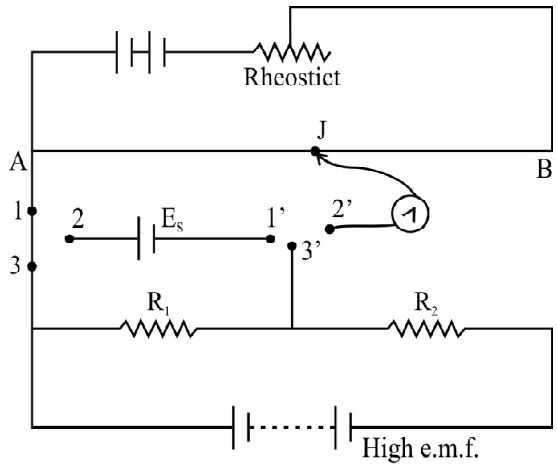
$(R_1 \sim 100\Omega \;\; R_2 \sim 9900\Omega)$

We first connect 1 and 3 and 2' and 3'. The potential drop produced across R_1 due to H. T is balanced across a length L_1 of the potentiometer wire. Then potential drop across $R_1 \propto L_1$. Since $R_1+R_2 = 10000\ \Omega$ we can assume that the total potential drop across R_1 and R_2 is equal to e.m.f. of high e.m.f. using ohm's law, we can write.

Potential drop across $R_1 = \dfrac{E}{R_1+R_2}R_1$

$$\frac{100}{10000}E = \frac{E}{100}$$

or $\dfrac{E}{100} \propto L_1$



Drop across potentiometer wire is $E_s\left(\dfrac{r}{R_2}\right)$

The potential gradient therefore is $E_s \dfrac{r}{R}\cdot\dfrac{1}{L}$

The small e.m.f. to be measured is now brought into circuit by making contact between 2 and K. If L' is the balance length, the e = small emf = $\left(E_s \dfrac{r}{R_2}\dfrac{1}{L}\right)L'$

The circuit shown is used for the measurement of low resistance. Here we set the total potential drop across the potentioneles small.

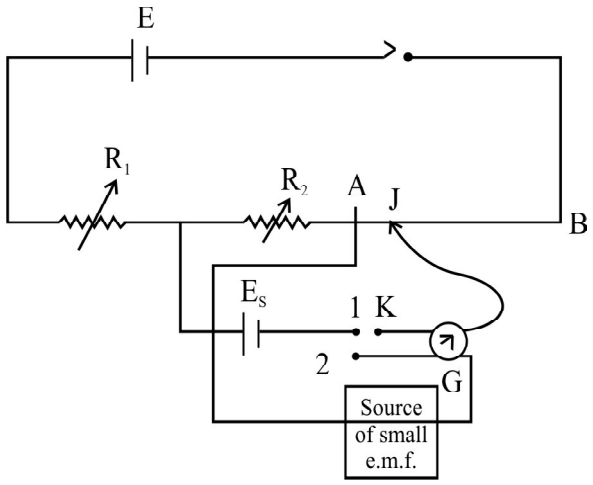
The small potential drop cross the potentiometer is measured accurately using a standard cell let r be the resistence per unit length of potentiometer wire and L is its total length then the current is given by

$$I = \frac{E}{(R_1+R_2+r)}$$

Total potential drop across potentiometer wire

$$I^r = \frac{Er}{R_1+R_2+r}$$

$$\therefore \text{ Potential Gradient } \rho = \frac{Er}{(R_1+R_2+r)L}$$



We adjust the value of $(R + R_2)$ so that ρ is of the order of magnitude required to determine ρ very accurately use is made of standard cell E_s. With the help of two way key contact is made between 1 and R. The values of R_1 and R_2 are altered keeping their sum constant till the balance point is obtained of the end of potentiometer wire. Now the potential drop across R_2 is exactly equal to e.m.f. of standard cell. The current I therefore is E_S/R_2. The potential drop across potentiometer wire is $E_s\left(r/R_2\right)$. The potential gradient therefore is $E_S \frac{r}{R} \cdot \frac{1}{L}$.

The small emf to be measured is now brought into circuit by making connection between 2 and k. If L′ is the balancing length. Then

$$e = \text{small e.m.f.} = \left(E_s \, r/R_2 \cdot \frac{1}{L}\right) L'$$

Hence option (4) is correct.

Unit–12 : Current Electricity

Average *Heating Effect of Current*

34. An electric tea kettle has two heating coils. When one of coils is switched on, the kettle begins to boil in 6 minutes. When the other is switched on the kettle begin to boil in 8 minutes. It boil the coils are switched on simultaneously and connected in parallel then the kettle will begin to boil in.

(1) 14 minutes (Incorrect)

(2) 205. 7 seconds (Correct)

(3) 7/24 minutes (Incorrect)

(4) 48 minutes (Incorrect)

Correct answer: (2)

Solution:

Let t be the time the kettle take to begin boiling when they are connected in parallel.

Now, $H = \frac{V^2}{R_1} t_1 = \frac{V^2}{R_2} t = \frac{V^2}{R_1 R_2} (R_1 + R_2) t$

Thus, $R_1 = \frac{V^2 t_1}{H}$ and $R_2 = \frac{V^2 t_2}{H}$

$H = \frac{V^2 (R_1 + R_2)}{R_1 R_2} t$, $H = V^2 t \left(\frac{H}{V^2 t_1} + \frac{H}{V^2 t_2} \right)$

or $t = \frac{t_1 t_2}{t_1 + t_2} = \frac{6 \times 8}{6 + 8} = \frac{24}{7}$ minute

$= 205.7$ s

Hence option (2) is correct.

Unit–12 : Current Electricity

Easy *Heating Effect of Current*

35. Lamps A and B have filaments of carbon and tangsten respectively. At room temperature the resistance of two bulbs is same. Both are connected in parallel to a 22 V supply. Which of the following statements is correct?

(1) Both bulbs have same intensily (Correct)

(2) Intensity of bulb A is more than that of B (Incorrect)

(3) Intensity of bulb A is less than thab B (Incorrect)

(4) Initially both have same intensity but, after a sufficiently long time, intensity of bulb A is less than that of B (Incorrect)

Correct answer: (1)

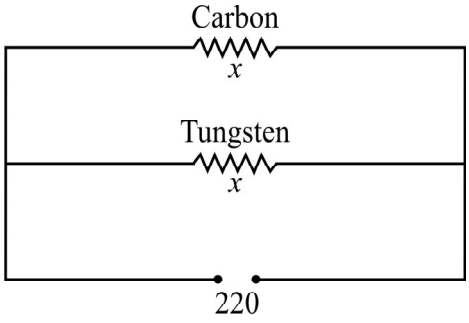
Solution:

Since both carbon and tengsten have same resistance (say x) and are connected in parallel the potential drop across each of then is 220 V.

Q_1 = heat produced in carbon filament bulb = $\frac{(220)^2}{x}$

Q_2 = heat produced in tungsten filament bulb = $\frac{(220)^2}{x}$

$\therefore \quad \frac{Q_1}{Q_2} = 1$



Hence both bulbs have the same intensity.

Hence option (1) is correct.

Unit–12 : Current Electricity

Average *Power Consumption in a Circuit*

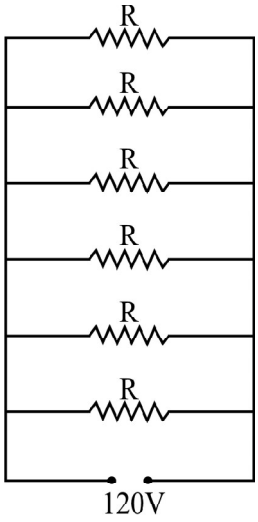
36. Six Diwali tree lights are arranged in a parallel circuit as shown in figure. Each bulb dissipates 10 W when operated at 120 V. The total power consumption, of the array, would be (nearly)

(1) 600 Watt (Incorrect)

(2) 0.6 Watt (Incorrect)

(3) 120 Watt (Incorrect)

(4) 60 Watt (Correct)



Correct answer: (4)

Solution:

Resistance of the each buls = $R = \frac{V^2}{P} = \frac{120 \times 120}{10} = 1440\,\Omega$

The given 6 resistance are connected in parallel, therefore equivalent resistance is given by:

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{6}{R}$$

$$R_{eq} = \frac{R}{6} = \frac{1440}{6} = 240$$

Hence the power consumed by the given arrangment is given by

$$P = \frac{V^2}{R} = \frac{120 \times 120}{240} = 60\,W$$

Hence option (4) is correct.

Unit–12 : Current Electricity

Average *Heating Effect of Currents*

37. A copper wire, having cross sectional area of 0.5 mm^2 and a length of 0.1 m, is initially at 25^{0}C and thermally insulted from the surroundings a current of 1.0 A is set up in this wire.

Neglecting the change of resistance with the temperature, the time in which wire will start melting is given by :

(1) (555) s (Incorrect)

(2) (55.5) s (Correct)

(3) (5.55) s (Incorrect)

(4) (0.55) s (Incorrect)

Melting point of copper = 1050^{0}C

Spefic resistance of copper = $1.6 \times 10^{-8}\,\Omega$

Density of copper = 9×10^3 Kg/m^3

Specific heat capacity of copper = 9×10^{-2} Cal/kg ^{0}C, 1 Cal = 4.2 J

Correct answer: (2)

Solution:

We can easily see that $I^2Rt = ms(\Delta\theta)$

or $I^2\left(\rho_1 \frac{\ell}{A}\right)t = A\ell d(s\Delta\theta)$

In above relation ρ_1 is the specific resistance and d the density of the copper.

$$\therefore\ t = \frac{A^2 d(s\Delta\theta)}{I^2\rho_1}$$

Substituting the values we get

$$t = \frac{\left(0.5\times10^{-6}\right)^2\left(9\times10^{3}\right)\left(9\times10^{-2}\times4.18\times1050\right)}{(10)^2\left(1.6\times10^{-8}\right)}$$

$$= 55.5\,S$$

Hence option (2) is correct.

Unit–12 : Current Electricity

Average *Heating Effect of Currents*

38. Two wires, made of tinned copper, having identical cross section and different lengths, are used as fuses. Then

(1) Two wires fuses will melt at different values of the current. (Incorrect)

(2) Two wire fuses will melt at the same value of current (Correct)

(3) Two wire fuses will only melt at the same value of the current only if they we of equal lenghts (Incorrect)

(4) Two wires fuses will only melt at the same value of current if they have same resistivity (Incorrect)

Correct answer: (2)

Solution:

If a current I flows into a fuse wire, the rate of loss of heat from the fuse wire is given by the relation.

$$I^2 = 2\pi^2\frac{Qr^3}{\rho}$$

or $$Q = \frac{I^2\rho}{2\pi^2 r^3}$$

or $Q \propto \frac{\rho}{r^3}$ for the same value of the current.

Since the area of cross section of bulb wires is identical, so the radius of both the wires will be same. Since Q is independent of ℓ, both the wires of different lengths will melt at the same value of the current.

Hence option (2) is correct.

Unit–12 : Current Electricity

Average *Heating Effect of Currents*

39. A fuse, with a radius of 0.1 mm blows at 10A. What should be the radius of fuse wire made of the same material which melts at 20A?

(1) 0.5 mm (Incorrect)

(2) 0.05 mm (Incorrect)

(3) 0.159 mm (Correct)

(4) 0.318 mm (Incorrect)

Correct answer: (3)

Solution:

The relation between the heat produced and current in a fuse wire of radius r and having same material is given by

$$I^2 = \frac{2\pi^2 Hr^3}{\rho}$$

Hence we have,

$$(10)^2 = \frac{2\pi^2 H(0.1)^3}{\rho}$$

$$(20)^2 = \frac{2\pi^2 Hx^3}{\rho}$$

$$\frac{(20)^2}{(10)} = \frac{x^3}{(0.1)^3}$$

Solving for x we get

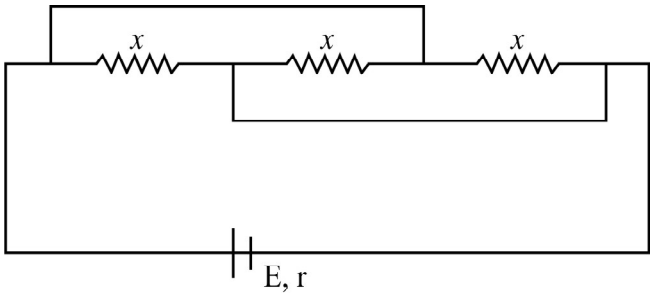
$$x = 0.159\,\text{mm}$$

Hence option (3) is correct.

Unit–12 : Current Electricity

Average *Heating Effect of Currents*

40. Three equal resistance each of value *x* ohm are connected as shown in figure. A battery of e.m.f. E and internal resistance r is connected across the circuit. To generate maximum heat in the circuit, the value of *x* should be



(1) $3R_e$ (R_e being the equivalent resistance of resistances *x*) (Incorrect)

(2) 3r (Correct)

(3) $\frac{3}{x}r^2$ (Incorrect)

(4) $\frac{3x^2}{r}$ (Incorrect)

Correct answer: (2)

Solution:

If H is the heat produced, then

$H = I^2R_e t$ where I is the current in crcuit and R_e is the equivalent resistance.

Now, $I = \frac{E}{(R_e + r)}$ where r is the internal resistance of the cell.

Also, $\frac{1}{R_e} = \frac{1}{x} + \frac{1}{x} + \frac{1}{x} = \frac{3}{x}$

$\therefore R_e = {}^{x}\!/_{x}$

Hence, $H = \frac{E^2}{(R_e + r)}(R_e t)$

For maximum heat to be produced $\frac{dH}{dx}$ should be zero.

$$\therefore \frac{E^2 t}{(r+R_e)^2} - \frac{3E^2 R_e t}{(r+R_e)^3} = 0$$

or $\quad I = \frac{2R_e}{r+R_e} \quad$ or $\quad r = R_e$

But, $R_e = (x/3)$

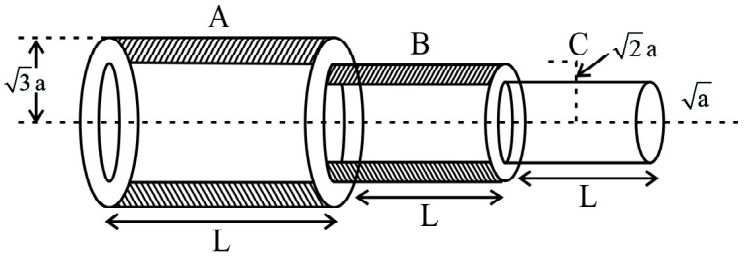
$\therefore (x/3) \quad$ or $\quad x = 3r$

Hence option (2) is correct.

Unit–12 : Current Electricity

Average *Resistance Variation with Length, Area & Nature of Material*

41. Following figure showns cross section through three long conductor of same length and material, with circular cross sections as shown in figure. Conductor B will fit exactly within conductor A, and conductor C will fit exactly withing conductor B, relationship between their end to end resistance is



(1) $R_A = R_B = R_C$ (Correct)

(2) $R_A > R_B > R_C$ (Incorrect)

(3) $R_A < R_B < R_C$ (Incorrect)

(4) Given data not sufficient (Incorrect)

Correct answer: (1)

Solution:

As the length of all conductors is equal and 'ρ' reistivity is also same as they are made up of same material.

Now, area of cross section of conductor A $= \pi\left[\left(\sqrt{3a}\right)^2 - \left(\sqrt{2a}\right)^2\right] = \pi a^2$

and of conductor B $= \pi\left[\left(\sqrt{2a}\right)^2 - \left(\sqrt{a}\right)^2\right] = \pi a^2$

Area of cross section of conductor C = πa^2

$\therefore$ Areas of cross section of the conductors A, B and C are also equal.

As resistance of a conductor is given by $R = \rho\frac{\ell}{A}$

$\therefore$ $R_A = R_B = R_C$.

Hence answer (1) is correct.

Unit–12 : Current Electricity

Difficult *Grouping of Resistances*

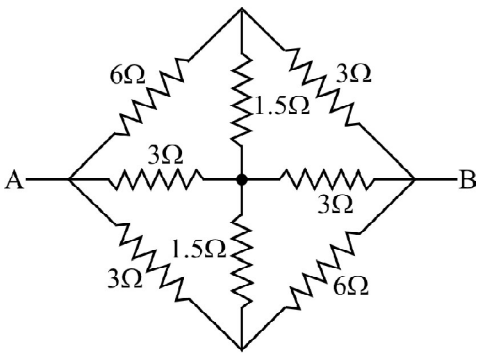
42. The equivalent resistance of the network between points A and B in the given figure.

(1) $\frac{14}{9}\ \Omega$ (Correct)

(2) $\frac{21}{5}\ \Omega$ (Incorrect)

(3) $\frac{18}{7}\ \Omega$ (Incorrect)

(4) $\frac{11}{2}\ \Omega$ (Incorrect)



Correct answer: (1)

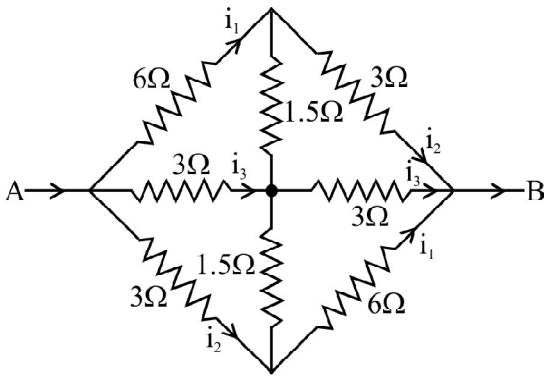
Solution:

Current distribution at point A and B will be similar

Therefore, circuit can be redrawn like as shown

Applying KCl at junection Q.

$i_1 + i' - i_2 = 0 \quad i' = i_2 - i_1$



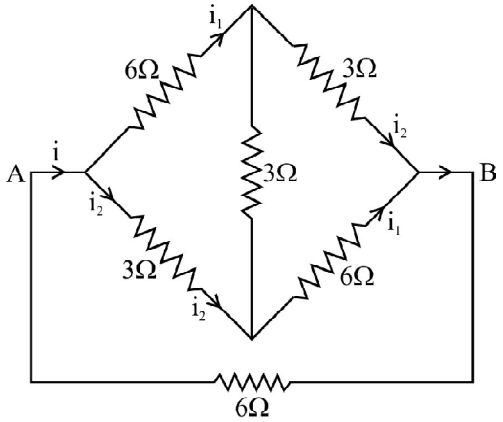
$V_P - V_R = 6i_1 + 3i_2$

$V_P - V_R = 6i_1 - 3(i_2 - i_1) + 6i_1 = 15i_1 - 3i_2$

$\Rightarrow 6i_1 + 3i_2 = 15i_1 - 3i_2$

$6i_2 = 9i_1 \Rightarrow i_2 = \left(\frac{3}{2}\right) i_1$

But, $i = i_1 + i_2 = i_1 + \frac{3}{2} i_1 = \left(\frac{5}{2}\right) i_1$



$i_1 = \frac{2}{5} i \;\; \& \;\; i_2 = \frac{3}{2} \times \frac{2}{5} i = \frac{3}{5} i$

$V_P - V_Q = i \times R'$

$6i_1 + 3i_2 = iR'$

$6 \times \frac{2}{5} i + 3 \times \frac{3}{5} i = iR'$

$\frac{12i}{5} + \frac{9}{5} i = iR'$ (Where R' is the equivalent resistance of Wheatstone bridge)

$\frac{21}{5} i = iR'$

$R' = \frac{21}{5} \Omega$

Hence effective resistance between ponits A & B

$$\frac{1}{R_{eq}} = \frac{1}{R'} + \frac{1}{6} = \frac{1}{R_{eq}} = \frac{5}{21} + \frac{1}{6} = \frac{30+21}{21 \times 6} = \frac{51}{21 \times 6} = \frac{27}{21 \times 2} = \frac{9}{14} \Omega$$

$$R_{eq} = \frac{14}{9} \Omega$$

Alternative (Short cut)

R' = equivalent of Wheatstone bridge

$$R' = \frac{(P+Q)RS+(R+S)PQ+G(P+Q)(R+S)}{G(P+Q+R+S)+G(P+R)(Q+S)}$$

Here, $P = 6\Omega, Q = 3\Omega, R = 3\Omega, S = 6\Omega, G = 3\Omega$

$$R' = \frac{18\times 9+9\times 18+3\times 9\times 9}{3(18)+9\times 9} = \frac{12}{5}\ \Omega$$

Hence effective resistance between point A & B

$$\frac{1}{R_{eq}} = \frac{5}{12}+\frac{i}{6} = \frac{14}{9}\ \Omega$$

Hence option (1) is correct

Unit–12 : Current Electricity

Difficult *Ohm's Law and Current Density*

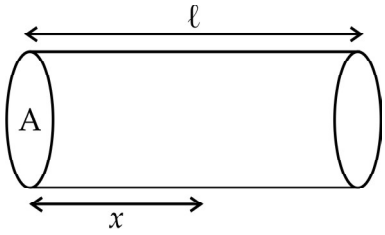
43. The given figure shows a conductor of length 'ℓ' and cross sectional area 'A', its conductively varies as $\sigma(x) = \frac{\sigma_0 \ell}{\sqrt{x}}$ where '*x*' is the distance along the axis of the cylinder from one end of it. The electric field at each point in the cylinder will be; when is the potential difference across its ends is V_0.

(1) $\frac{2\sqrt{\ell}\ V_0}{3A\sigma_0}$ (Incorrect)

(2) $\frac{3V_0\sqrt{x}}{2\ell^{3/2}}$ (Correct)

(3) $\frac{2}{3}\frac{V_0\sqrt{x}}{\ell^{3/2}}$ (Incorrect)

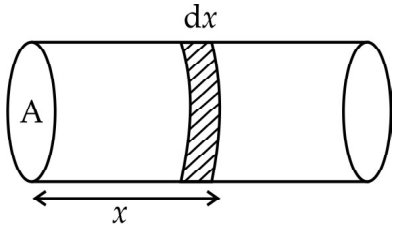
(4) $\frac{3}{2}V_0\sqrt{\frac{x}{\ell}}$ (Incorrect)



Correct answer: (2)

Solution:

$$dR = \frac{1}{\sigma(x)}\frac{dx}{A} = \frac{\sqrt{x}\,dx}{\sigma_0 \ell A}$$



$$R = \int_0^{\ell} \frac{\sqrt{x}\,dx}{\sigma_0 \ell A} = \frac{1}{A\sigma_0 \ell}.\frac{2}{3}(\ell)^{3/2} = \frac{2\sqrt{\ell}}{3A\sigma_0}$$

Current flowing $i = \frac{V_0}{R} = \frac{3A\sigma_0 V_0}{2\sqrt{\ell}}$

Current density $j = \frac{i}{A} = \frac{3\sigma_0 V_0}{2\sqrt{\ell}}$

Electric field $(E) = \frac{j}{\sigma} = \frac{2V_0\sqrt{x}}{2\ell^{3/2}}$

Hence option (2) is correct.

Unit–12 : Current Electricity

Average *Discharging of Cell*

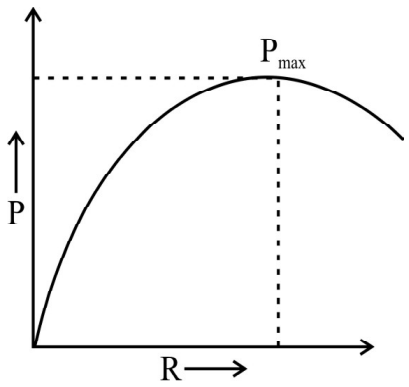
44. Given graph shows the variation of power dissipated in external resistnace (Load) R, delivered by a cell of emf 'E', internal resistance 'r', when supplying current 'i' to the load. The value of load corresponding to point A in the given graph is

(1) r (Correct)

(2) 2r (Incorrect)

(3) $\frac{r}{2}$ (Incorrect)

(4) $\sqrt{r}$ (Incorrect)



Correct answer: (1)

Solution:

Current given by the cell of the resistance $r = \frac{E}{R + r}$

Power dissipated in external resistance (load) $P = i^2R = \frac{E^2}{(R+r)^2}R$

Power will be maximum when $\frac{dP}{dR} = 0$

$$\frac{dP}{dR} = \frac{d}{dR}\left[E^2R(R+r)^{-2}\right]$$

$$= E^2\left[R\times(R+r)^{-3}\times(-2)+(R+r)^{-2}\times 1\right] = E^2\left[\frac{-2R}{(R+r)^3}+\frac{1}{(R+r)^2}\right]$$

For $\frac{dP}{dR} = 0$,

$$\frac{2R}{(R+r)^3} = \frac{1}{(R+r)^2} \Rightarrow R = r$$

Hence option (1) is correct

Unit–12 : Current Electricity

Difficult *Potentiometer*

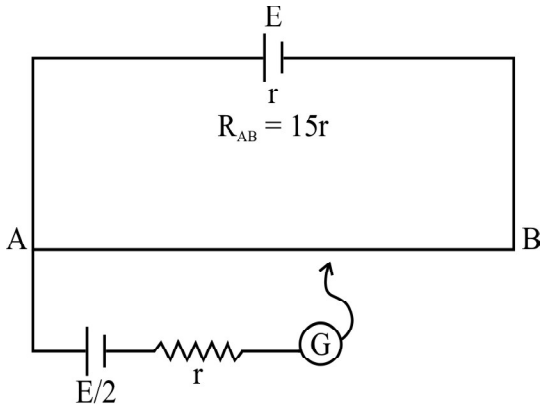
45. In the given figure AB is a potentiometer wire of length 600 cm, the current flowing through the galvanometer, if jockey is touched at 560 cm lengths of wire from end A.

(1) $\frac{4E}{22r}$ (Incorrect)

(2) $\frac{3E}{22r}$ (Correct)

(3) $\frac{7E}{22}r$ (Incorrect)

(4) $\frac{E}{22}r$ (Incorrect)



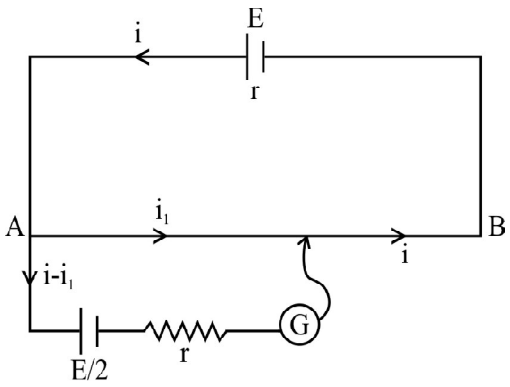
Correct answer: (2)

Solution:

Resistance of potentiometer wire = 15r

Resistance of the length 560 cm

$$= \frac{15r}{600} \times 560 = 14\,r$$



Applying KVL in loop APMA

$$-14r\,i_1 + (i - i_1)\,r + \frac{E}{2} = 0$$

$$-15r\,i_1 + ir + \frac{E}{2} = 0$$

$$\Rightarrow 15 i_1 r - ir = \frac{E}{2} \qquad .....(1)$$

Applying KVL to loop AEBA

$$-E + ir + ir + i_1 14r = 0$$

$$14 i_1 r + 2ir = E$$

$$7 i_1 r + ir = \frac{E}{2} \qquad .....(2)$$

Solving (1) & (2)

$$i_1 = \frac{E}{22r}$$

Putting the value of i_1 in equations (2)

$$\frac{7E}{22} + ir = \frac{E}{2}$$

$$ir = \frac{E}{2} - \frac{7E}{22} = \frac{4E}{22}$$

$$i = \frac{4E}{22} r$$

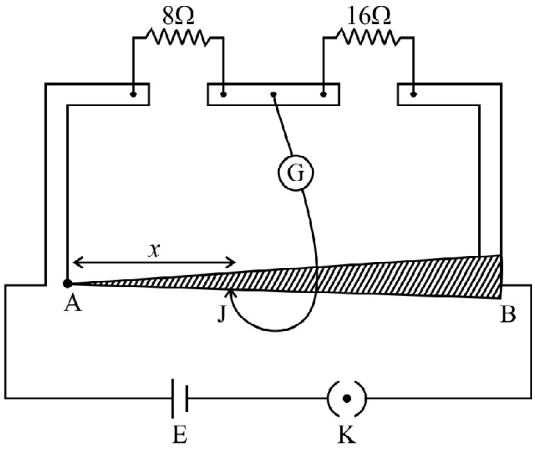
Hence current through galvanometer $i - i_1 = \frac{4E}{22r} - \frac{E}{22r} = \frac{3E}{22r}$

Hence option (2) is correct.

Unit–12 : Current Electricity

Difficult *Meter Bridge*

46. In the given experimental setup of meter bridge the length of the wire is one meter but its radius is increasing linearly from end A to end B which becomes double at end B with respect to end A. The null point will be obtained at?



(1) 66.67 cm (Incorrect)

(2) 20 cm (Correct)

(3) 33.33 cm (Incorrect)

(4) 80 cm (Incorrect)

Correct answer: (2)

Solution:

Let 'dx' be a small element of length at distance x from end A, and radius of the wire at that position will be

$= \left(r + \frac{r}{1}x\right) = r(1+x)$ where r is the radius at end A.

'dR' = Resistance of this element $dR = \frac{\rho dx}{\pi r^2 (1+x)^2}$

$$R_1 = \int_0^x dR = \int_0^x \frac{\rho dx}{\pi r^2 (1+x)^2} = \frac{\rho}{\pi r^2}\int_0^x (1+x)^{-2}\, dx$$

$$R_1 = \frac{\rho}{\pi r^2}\left(1 - \frac{1}{1+x}\right) = \frac{\rho x}{\pi r^2 (1+x)}$$

Similarly resistance of the remaining wire i.e. from J to B.

$$R_2 = \frac{\rho}{\pi r^2}\int_x^1 (1+x)^{-2}\,dx = \frac{\rho}{\pi r^2}\left(-\frac{1}{2}+\frac{1}{1+x}\right)$$

$$R_2 = \frac{\rho(1-x)}{2\pi r^2(1+x)}$$

As the bridge is balanced at distance x from end A. Hence,

$$\frac{R_1}{R_2} = \frac{8}{16} \text{ or } \frac{2x}{(1-x)} = \frac{1}{2}$$

$$\Rightarrow 4x = 1-x \text{ or } 5x = 1$$

$$x = \frac{1}{5}\text{ m} = 20\text{ cm}$$

Hence option (2) is correct.

Unit–12 : Current Electricity

Difficult *Grouping of Resistors*

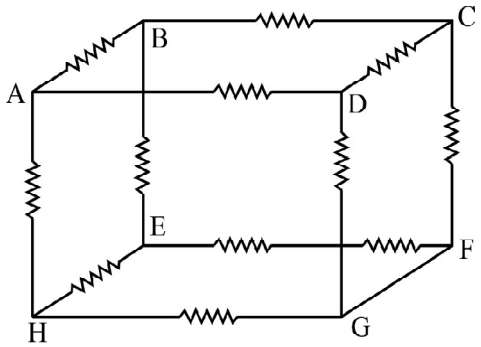
47. In the given circuit, each resistance is of $12\,\Omega$, the effective resistance between points A and B will be

(1) $7\,\Omega$ (Correct)

(2) $9\,\Omega$ (Incorrect)

(3) $12\,\Omega$ (Incorrect)

(4) $10\,\Omega$ (Incorrect)



Correct answer: (1)

Solution:

As the paths AD and AH are equivalent, hence they carry equal currents i.e. I_1, therefor current through AB is $I-2I_1$

The potential difference V between points A and B may be written from the path AB, AHEB, AHGFCB.

$V = (I-2I_1)\,12$

$V = 12I_1 + 12I_2 + 12I_1 = (2I_1 + I_2)12$

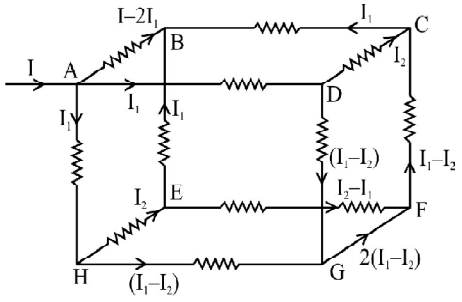
$V = 12I_1 + 12\,(I_1 - I_2) + 24\,(I_1 - I_2) + 12\,(I_1 - I_2) + 12\,I_1$

$= (6I_1 - 4I_2)\,12$

Hence on elimination of I_1 and I_2

$V = 7\,\Omega$

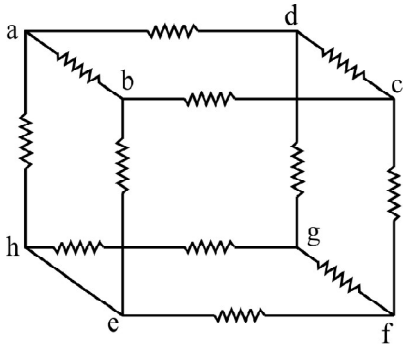
Hence option (1) is correct.



Unit–12 : Current Electricity

Difficult *Combination of Resistances*

48. The equivalent resistance between points a and c in the given figure will be when each resistance is of $8\,\Omega$



(1) $\frac{20}{3}\Omega$ (Incorrect)

(2) $8\,\Omega$ (Incorrect)

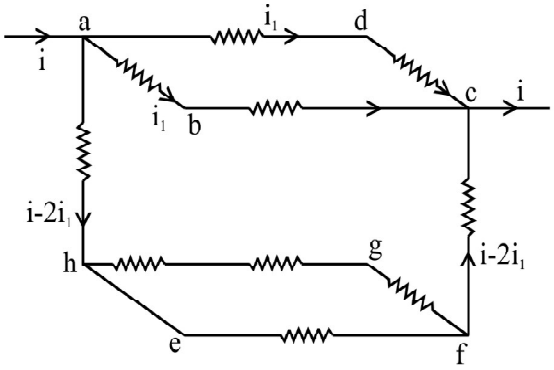
(3) $6\,\Omega$ (Correct)

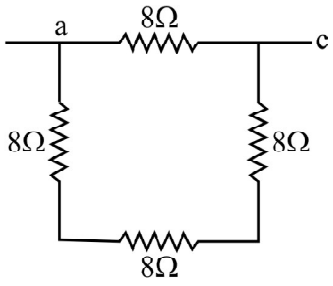
(4) $32\,\Omega$ (Incorrect)

Correct answer: (3)

Solution:

Current distribution in the given figure will be as follows, (due to symmetry at points a and c).





$$\frac{1}{R_{ac}} = \frac{1}{8} + \frac{1}{24} = \frac{3+1}{24} = \frac{4}{24} \Rightarrow R_{ac} = 6\,\Omega$$

Hence option (3) is correct.

Unit–12 : Current Electricity

Difficult *Power Delivered to a Resistor (Max. Power Transfer Theorem)*

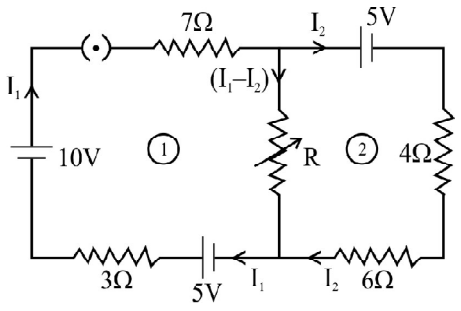
49. In the given figure, R is a variable resistor and its value is 1Ω at the time t = 0, i.e. when switch is closed. It is increasing continuously by an automatic regulator at the rate of 4.0×10^{-2} r/s and regulator is switched of when maximum power is delivered (dissipated) in the resistor (R), the time for which regulator was on

(1) 500 second (Incorrect)

(2) 400 second (Incorrect)

(3) 100 second (Correct)

(4) 150 second (Incorrect)



Correct answer: (3)

Solution:

In loop 1: $-7I_1 - R(I_1 - I_2) + 5 - 3I_1 + 10 = 0$

$$I_1(R+10) = 15 + RI_2 \quad \text{.....(i)}$$

In loop 2: $-5 - 4I_2 - 6I_2 + R(I_1 - I_2) = 0$

$$I_2(10+R) = I_1R - 5$$

$$\Rightarrow I_1 = \frac{(10+R)I_2 + 5}{R} \quad \text{.....(ii)}$$

$$I_2 = \frac{I_1R - 5}{10+R}$$

Putting the value of I_2 in equation (i)

$$I_1(R+10) = 15 + R\left(\frac{I_1R-5}{10+R}\right) \Rightarrow I_1 = \frac{R+15}{2R+10}$$

$$\therefore I_2 = \frac{\left[\frac{R+15}{2R+10}\right].R - 5}{10+R} = \frac{R-5}{2R+10}$$

$\therefore$ Current flowing through variable resistor R, $= I_1 - I_2$

$$= \frac{R+15}{2R+10} - \frac{(R-5)}{(2R+10)} = \frac{10}{R+5}$$

Hence power dissipated $P = (I_1 - I_2)^2 R = \frac{100R}{(R+5)^2}$

For power to be maximum $\frac{dP}{dR} = 0$

$$= 100\left[-2R \times (R+5)^{-3} + (R+5)^{-2} \times 1\right]$$

$$= 100\left[-\frac{2R}{(R+5)^3} + \frac{1}{(R+5)^2}\right]$$

$$\Rightarrow \frac{2R}{(R+5)^3} = \frac{1}{(R+5)^2} \Rightarrow 2R = R+5 \Rightarrow R = 5\Omega$$

Since rate of increase of resistance is $4.0 \times 10^{-2}\ \Omega / s$.

$\therefore$ Time required to attain value (5Ω) from $1(\Omega)$ will be 100 seconds.

i.e. $R_t = R_{initial} + \alpha(\text{time})$

$$5 = 1 + 4 \times 10^{-2} \times t$$

$$\frac{4.0}{4.0 \times 10^{-2}} = t \Rightarrow 100\,\text{second}$$

Hence optino (3) is correct.

Unit–12 : Current Electricity

Difficult *Heating Effect*

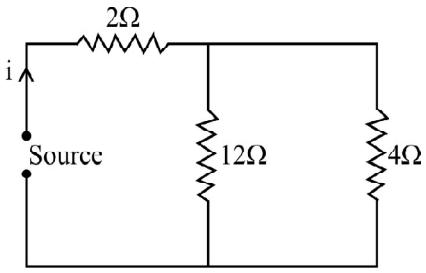
50. In the given figure, the charge supplied by the source varies as a function of time as $Q = 3t - 5t^2$, total heat produced in a resistance of 4Ω will be

(1) 2.025 J (Correct)

(2) 1.350 J (Incorrect)

(3) 4.050 J (Incorrect)

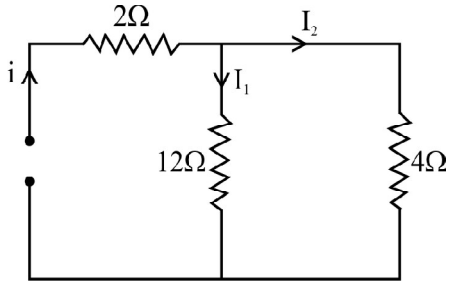
(4) 0.750 J (Incorrect)



Correct answer: (1)

Solution:

Current $i = \frac{dQ}{dt} = 3 - 10t$



Hence current in 4Ω

$$i_2 = \frac{i \times 12}{12+4} = \frac{12i}{16} = \frac{3i}{4}$$

$$i_2 = \frac{3}{4}(3-10t)$$

Charge flowing from the source will be zero at $t = 0$ and will be maximum when $\frac{dQ}{dt} = 0$ i.e. at $t = \frac{3}{10}$.

Hence total heat produced in 4Ω resistor.

$$H = \int I^2 R dt = \int_0^{3/10} \left[\frac{3}{4}(3-10t)\right]^2 4\,dt$$

$$H = \frac{9}{4}\left[\int_0^{3/10} 9\,dt + \int_0^3 100t\,dt - \int_0^3 60 + dt\right] = \frac{9}{4}\left[9(t)_0^{3/10} + 100\left(\frac{t^3}{3}\right)_0^{3/10} - 60\left(\frac{t^2}{2}\right)_0^{3/10}\right]$$

$$= \frac{9}{4}\left[9\left(\frac{3}{10}-0\right) + \frac{100}{3}\left(\frac{27}{1000}-0\right) - \frac{60}{2}\left(\frac{9}{100}-0\right)\right]$$

$$= \frac{9}{4}\left[\frac{27}{10} + \frac{9}{10} - \frac{27}{10}\right] = \frac{81}{40}\text{ J} = 2.025\text{ J}$$

Hence option (1) is correct.

Unit–12 : Current Electricity

Difficult *Current Density*

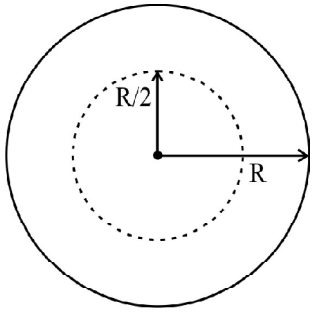
51. The current density in a cylindrical wire of radius R = 3cm varies with radial distance 'r' as mr^3, where 'm' is a constant having value 2.0×10^7 amp/m^5 and distance 'r' is in meter, the current flowing through the outer portion of the wire between R/2 and R is

(1) 0.591 A (Correct)

(2) 0.047 A (Incorrect)

(3) 1.52 A (Incorrect)

(4) 1.47 A (Incorrect)



Correct answer: (1)

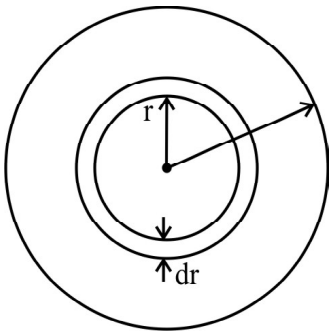
Solution:

Area $dA = 2\pi r\, dr$

$I\,(\text{current}) = \oint \mathbf{j}.\mathbf{dA}$

$\oint \mathbf{j}.\mathbf{dA} = jdA\cos\theta = jdA$ (as current flows normal to area)

Hence, $I = \oint jdA = \int_{R/2}^{R} mr^3 2\pi dr$



$$= m\times 2\pi\int_{R/2}^{R} r^4 dr = 2\pi m\left[\frac{r^5}{5}\right]_{R/2}^{R}$$

$$= \frac{2\pi m}{5}\left[R^5 - \frac{R^5}{32}\right] = \frac{2\pi m}{5}\times\frac{31R^5}{32}$$

$$= \frac{2\times3.14\times2.0\times10^7}{5}\times\frac{31}{32}\times\left(3\times10^{-2}\right)^5 = 591.34\times10^{-3} = 0.591\,\text{A}$$

Hence option (1) is correct.

Unit–12 : Current Electricity

Average *Current*

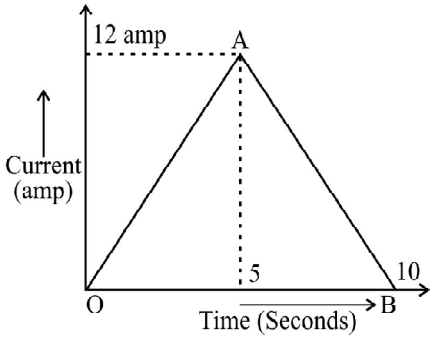
52. The given graph shows the variation of current with time in a circuit for 0 second to 10 second. It increases upto 12A linearly in 5 second. Then decreases to zero in next five second. The charge flowing in the circuit between 2 second and 6 seconds will be

(1) 60 Coulomb (Incorrect)

(2) 30 Coulomb (Incorrect)

(3) 24 Coulomb (Incorrect)

(4) 36 Coulomb (Correct)



Correct answer: (4)

Solution:

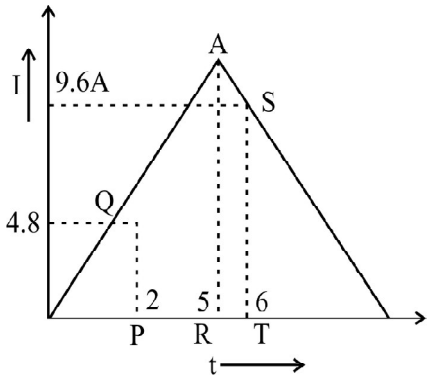
The rate of growth of current upto 5 seconds.

$$= \frac{12-0}{5} = 2.4\,\text{amp/s}$$

$\therefore$ Current at the end of 2 second $= 2.4 \times 2 = 4.8$ amp

The rate of decay of current from A to B = –2.4 amp/s

$\therefore$ Current at sixth second = 12–2.4 = 9.8 amp.



Hence chrage between 2 second and 6 second

= area of trapezium PQAR + Area of trapezium RAST

$$= \frac{1}{2}(4.8+12)\times 3 + \frac{1}{2}(12+9.6)\times 1$$

= (25.2 + 10.8) = 36 Coulomb

Hence option (4) is correct.

Unit–12 : Current Electricity

Difficult *Unbalanced Wheatstone Bridge*

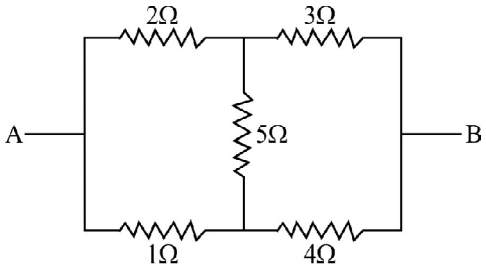
53. The equivalent resistance between points A and B will be

(1) $\frac{45}{14}\ \Omega$ (Incorrect)

(2) $\frac{115}{14}\ \Omega$ (Incorrect)

(3) $\frac{175}{71}\ \Omega$ (Correct)

(4) 2.5 Ω (Incorrect)



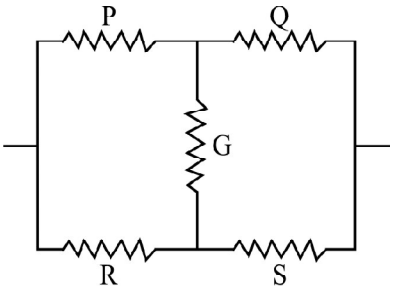
Correct answer: (3)

Solution:

$$R_{eq} = \frac{PQ(R+S)+(P+Q)RS+G(P+Q)(R+S)}{G(P+Q+R+S)+(P+R)(Q+S)}$$

$$\therefore\ R_{eq} = \frac{2\times3(1+4)+(2+3)(1\times4)+5(2+3)(1+4)}{5(2+3+1+4)+(2+1)(3+4)}$$

$$= \frac{6\times5+5\times4+5\times5\times5}{5\times10+3\times7} = \frac{30+20+125}{50+21} = \frac{175}{71}\Omega$$



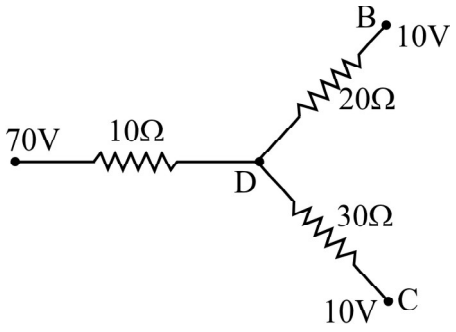
Hence option (3) is correct.

Alternative : can be determined using Kirchoff's laws.

Unit–12 : Current Electricity

Average *Current Distribution*

54. In the network points AB and C are at potentials of 70V, 0V and 10V respectively. Then



(1) Potential at point D is 40V (Correct)

(2) The currents in the section AD, DB and DC are in the ratio 1:2:3 (Incorrect)

(3) The network draws a totoal power of 360 W. (Incorrect)

(4) All the above (Incorrect)

Correct answer: (1)

Solution:

$V_A - V_D = 70 - V_D = 10i$

$V_D - V_B = V_D - 0 = 20i_1$

$V_D - V_C = V_D - 10 = 30(i - i_1)$

$\Rightarrow 70 - V_D = 10i$ or $70 - 20i_1 = 10i_i$

$70 = 10(2i_1 + i)$ or $70 = 20(i_1 + 10i)$ .....(1)

$20i_1 - 10 = 30(i - i_1)$

$10 = -30i + 50i_1$ .....(2)

From (1) & (2)

$(70 = 20i_1 + 10i) \times 3$, or $10 = 50i_1 - 30i$

$220 = 110i_1$

$i_1 = \frac{220}{110} = 2A$

Hence $V_D = 20i_1 = 20 \times 2 = 40\,V$

Hence option (1) is correct

Unit–12 : Current Electricity

Aeverage *Combination of Resistances*

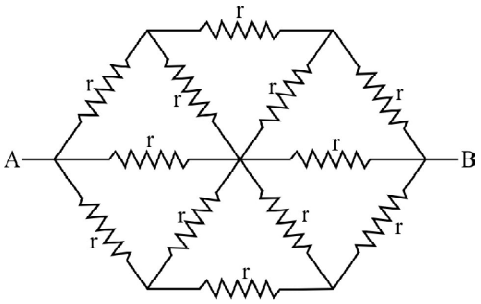
55. The equivalent resistance between point A & B in the given circuit will be

(1) $\frac{8}{7}$ r (Incorrect)

(2) $\frac{10}{3}$ r (Incorrect)

(3) $\frac{4}{5}$ r (Correct)

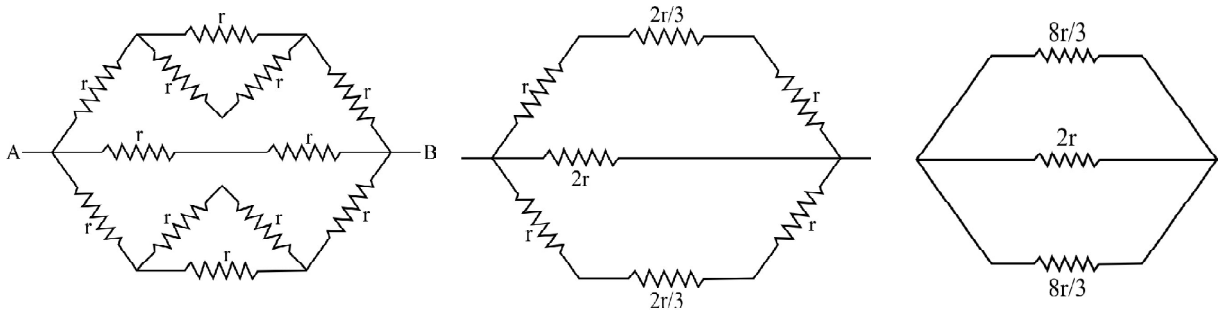
(4) $\frac{22}{3}$ r (Incorrect)



Correct answer: (3)

Solution:

Due tosymmetry of the figure. It can be reduced.



$$\frac{1}{R_{eq}} = \frac{3}{8r} + \frac{1}{2r} + \frac{3}{8r}$$

$$\frac{1}{R_{eq}} = \frac{3+4+3}{8r} = \frac{10}{8r}$$

$$\Rightarrow R_{eq} = \frac{8r}{10} = \frac{4r}{5}$$

Hence option (3) is correct

Unit–12 : Current Electricity

Average *Calculation of Resistance*

56. A material of resistivity 'ρ' is filed in space between two concentric spheres of radius a and 'b' uniformly (a < b), the resistance of this material will be:

(1) $\frac{\rho}{4\pi(b+a)}$ (Incorrect)

(2) $\frac{\rho}{4\pi}\left(\frac{1}{b^2}-\frac{1}{a^2}\right)$ (Incorrect)

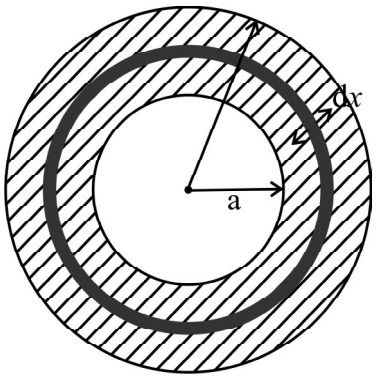
(3) $\frac{\rho}{4\pi}\left(\frac{1}{b}-\frac{1}{a}\right)$ (Incorrect)

(4) $\frac{\rho}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$ (Correct)

Correct answer: (4)

Solution:

Let the resistance of the small element dx of radius x is dR



$$dR = \frac{\rho dx}{4\pi x^2}$$

$$\therefore\ R = \frac{\rho}{4\pi}\int_a^b \frac{dx}{x^2} = \frac{\rho}{4\pi}\left[-\frac{1}{x}\right]_a^b$$

$$= \frac{\rho}{4\pi}\left[\frac{1}{a}-\frac{1}{b}\right]$$

Hence option (4) is correct.

Unit–12 : Current Electricity

Average *Potentiometer*

57. In an arrangement, for the determination of internal resistance of a battery using potentiometer, the null point is obatined at 300 cm of the wire when the experimental cell is shunted with resistance of 5Ω, when another resistance of 20Ω is connected in parallel to this shunt, galvanometer shows no deflection at 270 cm of wire, the internal resistance of the cell and the balancing length for the cell without any shunt will be

(1) 4.0Ω and 540 cm (Correct)

(2) 0.55Ω and 300 cm (Incorrect)

(3) 0.45Ω and 270 cm (Incorrect)

(4) $7.78\,\Omega$ and $435\,cm$ (Incorrect)

Correct answer: (1)

Solution:

$r = \left(\frac{\ell}{300} - 1\right)5$, where ℓ being balancing length without shunting.

When a resistance of $20\,\Omega$ is joined in parallel to $5\,\Omega$ the effective shunt $= \frac{5 \times 20}{5 + 25} = 4\,\Omega$

$\therefore\ r = \left(\frac{\ell}{270} - 1\right)4$

$\Rightarrow \left(\frac{\ell}{300} - 1\right)5 = \left(\frac{\ell}{270} - 1\right)4$

$\Rightarrow \left(\frac{\ell - 300}{6}\right) = \left(\frac{\ell - 270}{27}\right)4 \ = 9(\ell - 300) = 8(\ell - 270)\ \ \ell = 2700 - 2160$ or $\ell = 540\,cm$

Hence internal resistance $r = \left(\frac{540}{300} - 1\right)5 \ = \frac{240}{300} \times 5$

$r = 4\,\Omega$

Hence option (1) is correct.

Unit–12 : Current Electricity

Average *Heating Effect of Current*

58. A nichrome Filament heater consumes 100W power at 27^0C, when it is used its temperature rises to 3027^0C, the power consumed by it at the moment when switch is, put on, voltage remaining same. Given the temperature coefficient of resistively of nichrome is $4\times10^{-4}\ K^{-1}$.

(1) 90.9 W (Incorrect)

(2) 220 W (Correct)

(3) 1300 W (Incorrect)

(4) 7.96 W (Incorrect)

Correct answer: (2)

Solution:

$$P_0 = \frac{V^2}{R_0}, \Rightarrow R_0 = \frac{V^2}{P_0}$$

$$P_t = \frac{V^2}{R_t} \Rightarrow R_t = \frac{V^2}{P_t}$$

As, $R_t = R_0(1+\alpha\Delta t)$

$$\frac{V^2}{P_t} = \frac{V^2}{P_0}(1+\alpha\Delta t)$$

$$\Rightarrow P_0 = P_t(1+\alpha\Delta t)$$

$$= 100\left[1+(3027-27)\times 4\times 10^{-4}\right]$$

$$= 100\left[1+3000\times 4\times 10^{-4}\right] = 100\times 2.2 = 200\ \text{W}$$

Hence correct option is (2).

Unit–12 : Current Electricity

Average *Grouping of Cells*

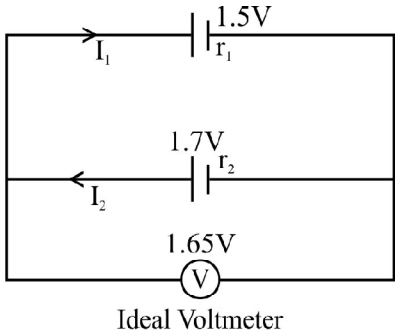
59. Two cells A and B of emf 1.5 V and 1.7 V, internal resistances r_1 and r_2 respecting are arranged as shown in figure, the voltmeter reads 1.6V (the voltmeter is ideal) the relationship between r_1 and r_2 will be.

(1) $r_1 = 3r_2$ (Correct)

(2) $r_1 = -3r_2$ (Incorrect)

(3) $r_2 = 2r_2$ (Incorrect)

(4) $r_2 = 2r_1$ (Incorrect)



Ideal Voltmeter

Correct answer: (1)

Solution:

$1.65 = 1.5 + I_1r_1 \Rightarrow I_1r_1 = 1.65–1.5 = 0.15$

Also, $1.65 = 1.7–I_1r_2 \Rightarrow I_1r_2 = 1.70–1.65 = .05$

Dividing $\frac{r_1}{r_2} = \frac{0.15}{.05} = 3$

$\Rightarrow r_1 = 3r_1$

Hence option (1) is correct.

Unit–12 : Current Electricity

Average *Potentiometer*

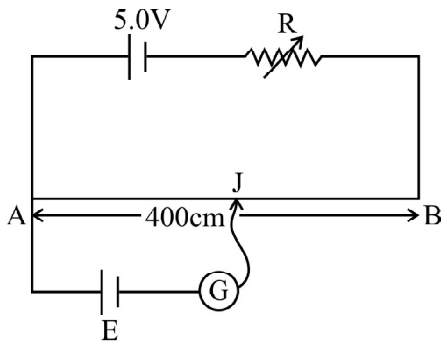
60. The given figure shows a potentiometer wire of length 400 cm resistance 'r' and uniform area of cross section, A driver cell of 5.0 V along with a variable resistance 'R' is connected across it, to supply the current. During an experiment the null point is obtained at the length 160 cm, when R = 0 and at 320 cm for certain value of R from end A, the emf of the experimental cell (E) and the value of variable resistance R will be

(1) 2V and 1.25r (Correct)

(2) 1.0V and 3r (Incorrect)

(3) 2.0V and 3.5r (Incorrect)

(4) 1.0V and 2.5r (Incorrect)



Correct answer: (1)

Solution:

From figure, when the value of resistance R = 0

then current in the potentiometer wire $i_1 = \frac{5.0}{r}A$

Hence potential drop across the wire = 5.0 V

$\therefore$ Potential gradient $k = \frac{5.0}{4.0} = \frac{5}{4}$ volt / m

Since cell of emf 'E' is balanced at 160 cm length of the wire, therefore

$$E = k\ell = \frac{5}{4} \times 1.6 = 2.0\,V$$

Now current becomes $i_2 = \frac{5.0}{R + r}$ when resistance R has certain value.

$\therefore$ Potential drop across the whole length of the wire $= i_2 r = \dfrac{5r}{R+r}$

Now potential gradient $k' = \dfrac{5r}{4(R+r)}$ and null point is obatined at 360 cm

$\therefore\ E = k'\ell' \Rightarrow 2.0 = \dfrac{5r}{4(R+r)} \times 3.6$

$$2.0 = \frac{4.5r}{R+r}$$

$$2.0 \text{ or } + 2.0r = 4.5r$$

$$2.5r = 2.0\ R$$

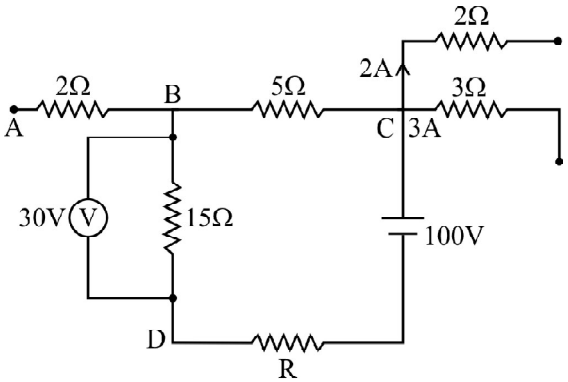
$$R = \frac{2.5r}{2.0} = 1.25\,r$$

Hence option (1) is correct.

Unit–12 : Current Electricity

Average *Current Distribution*

61. In the given figure, ideal voltmeter reads 30V when connected across 15Ω resistance. Then potential difference across B and C and the resistance R is given by



(1) 15V and 42.5Ω respectively (Correct)

(2) 10V and 40Ω respectively (Incorrect)

(3) -15V and 42.5Ω respectively (Incorrect)

(4) $-10\,V$ and $70\,\Omega$ respectively (Incorrect)

Correct answer: (1)

Solution:

The current flowing in branch BD is $\frac{30}{15} = 2A$

$\therefore$ Current in branch BC = 3A

Hence potential difference between points B & C $= 3 \times 5 = 15\,V$

Applying KVL to loop BDCB

$-15 \times 2 - IR + 100 + 5 \times 3 = 0$

$IR = -30 + 15 + 100$

$2 \times R = 115 - 30 = 85\ V$

$R = \frac{85}{2} = 42.5\,\Omega$

Hence option (1) is correct.

Unit–12 : Current Electricity

Average *Combination of Resistances*

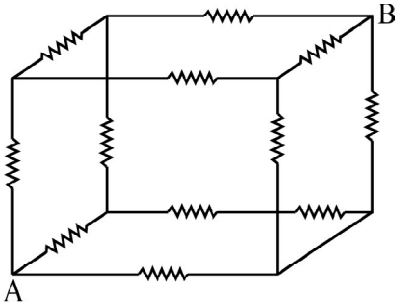
62. In the given figure, each resistance is of $6\,\Omega$, the effective resistance between points A and B will be

(1) $3.5\,\Omega$ (Incorrect)

(2) $5\,\Omega$ (Correct)

(3) $7.2\,\Omega$ (Incorrect)

(4) $4.5\,\Omega$ (Incorrect)

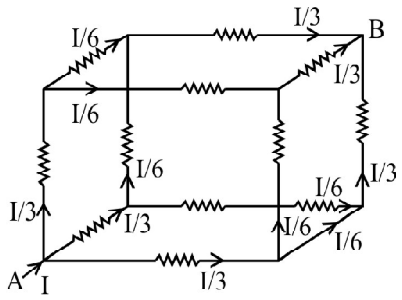


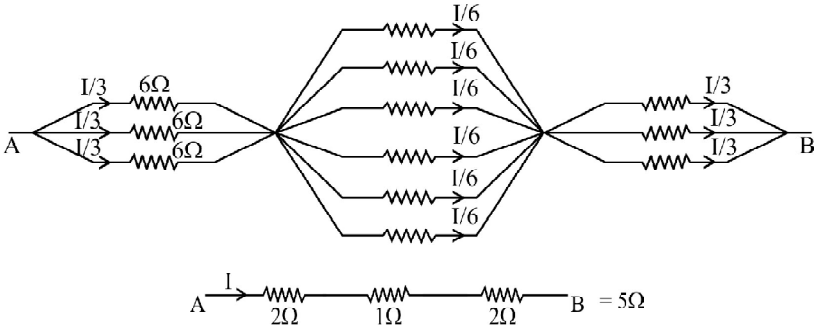
Correct answer: (2)

Solution:

Let current I enters at point A, and divide into three branches equally i.e. $\frac{I}{3}$. Due to symmetry same current will leave at end B.

Hence $R_{AB} = 5\Omega$





Option (2) is correct.

Unit–12 : Current Electricity

Average *Resistance of conductors*

63. Two conductos, one solid and other hollow are made up of same material, the length of solid conductor is twice the length of hollow conductor and diameter is D, while the outer diameter of hollow conductor is 2D and inner is D, the ratio of resistance between the ends of two conductor will be

(1) 2 : 3 (Incorrect)

(2) 1 : 6 (Incorrect)

(3) 3 : 2 (Correct)

(4) 3 : 1 (Incorrect)

Correct answer: (3)

Solution:

$$R_1 = \rho\frac{2\ell}{\pi\frac{D^2}{4}} = \frac{8\rho\ell}{\pi D^2}$$

$$R_2 = \rho\frac{\ell}{\left(\frac{\pi D^2}{4} - \frac{\pi D^2}{16}\right)} = \frac{16\rho\ell}{3\pi D^2}$$

$$\frac{R_1}{R_2}=\frac{\frac{8\rho\ell}{\pi D^2}}{\frac{16\rho\ell}{3\pi D^2}}=\frac{8\times 3}{16}=3:2$$

Hence option (3) is correct.

Unit–12 : Current Electricity

Average *Potentiometer*

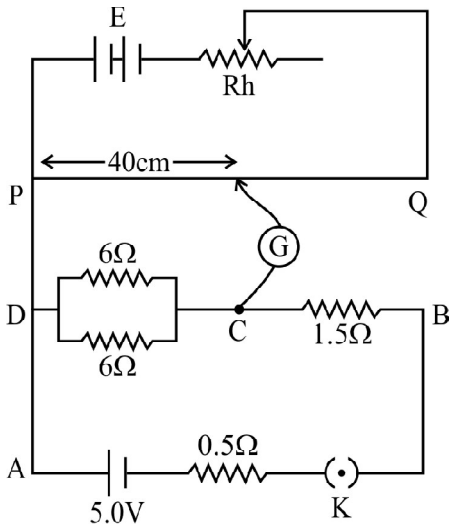
64. In the following circuit, the potential difference between the points C and D is balanced against 40 cm length of the potentiometer wire, in order to balance the potential difference between the points B and C, where should jockey on the wire be pressed?

(1) 10 cm (Incorrect)

(2) 40 cm (Incorrect)

(3) 60 cm (Incorrect)

(4) 20 cm (Incorrect)



Correct answer: (4)

Solution:

Current in loop AKBCDA

$$-5.0+I\times 0.5+I\times 1.5+3I=0$$

$$5I=5 \qquad I=1A$$

Hence potential difference between points D and C $=3\times 1=3V$

and between points C and B $=1.5\times 1=1.5\,V$

According to given figure

$$V_{DC}=K\times 40\Rightarrow 3=K\times 40\Rightarrow K=\frac{3}{40}\text{ volt / cm}$$

Now, $V_{CB}=K\times\ell\Rightarrow\ell=\frac{1.5\times 40}{3}=20\text{cm}$

Hence option (4) is correct.

Unit–12 : Current Electricity

Average *Current Distribution*

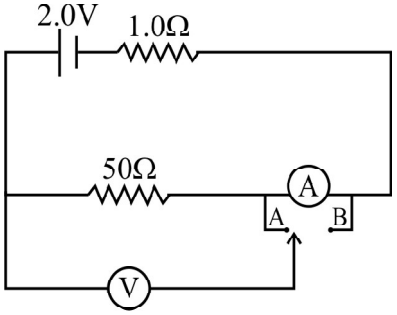
65. In the given circuit, the battery of emf 2.0 V and internal resistance 1.0Ω is connected to an external resistance 50Ω, the resistance of voltmeter and ammeter are 200Ω and 2.0Ω respectively. The readings of voltmeter and ammeter will be, when switch 'S' is connected to point A.

(1) 3.72 V & 0.047 A (Incorrect)

(2) 1.86 V & 0.093 A (Incorrect)

(3) 3.72 V & 0.093 A (Incorrect)

(4) 1.86 V & 0.047 A (Correct)



Correct answer: (4)

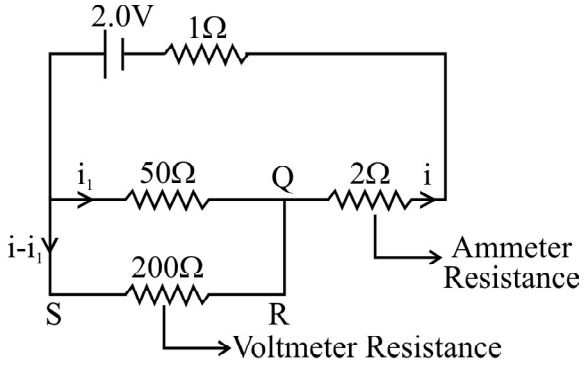
Solution:

Net resistance of the combination.

$$R = \left(\frac{50\times 200}{200+50}\right) + 2\Omega + 1\Omega = 43\Omega$$

Hence current i drawn from the cell.

$$i = \frac{2}{43}A$$



In loop PQRS

$$-50i_1 + 200(i - i_1) = 0$$

$$200i = 250i_1 \text{ or } 4i = 5i_1$$

$$\Rightarrow i_1 = \frac{4}{5}\times i = \frac{4}{5}\times\frac{2}{43}A = \frac{8}{215}A$$

Hence current through voltmeter $= i - i_1 = \frac{2}{43} - \frac{4}{5}\times\frac{2}{43} = \frac{1}{5}\times\frac{4}{43} = \frac{4}{215}A$

Hence voltmeter reading $= 200\times\frac{4}{215} = 3.72$

Ammeter readind $= \frac{2}{43} = 0.047A$

Unit–12 : Current Electricity

Easy *Calculation of Current*

66. The electron in an hydrogen atom moves in a circular orbit of radius 5×10^{-11} m with a speed of $0.6\pi\times10^{6}$ m/s, then

(1) The frequency of the electron is 3×10^{15} revolutions (Incorrect)

(2) The electron carries 3.2×10^{-19}C charge around the loop (Incorrect)

(3) The current in the orbit is 0.96 mA (Correct)

(4) The current flows in the same direction in which electrons move (Incorrect)

Correct answer: (3)

Solution:

$$I = \frac{q}{T} = \frac{q}{\left(\frac{2\pi}{\omega}\right)} = \frac{q\omega}{2\pi}$$

Also, $\omega = \frac{v}{r}$

$$\Rightarrow I = \frac{qv}{2\pi r} = \frac{1.6\times10^{-19}\times0.6\times1_0^6\pi}{2\times5\times10^{-11}} = 0.96\,\text{mA}$$

Hence option (3) is correct.

Unit–12 : Current Electricity

Average *Potential Differnece*

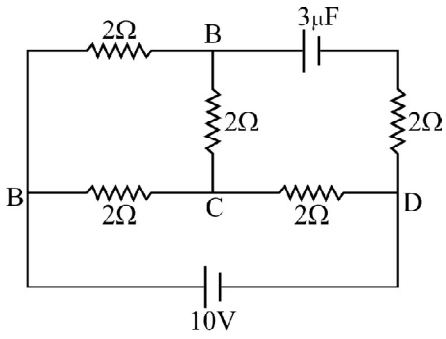
67. In the circuit shown, the cell is ideal, with emf 10V, each resistance is of 2Ω, the potential difference across the capacitor is

(1) 12V (Incorrect)

(2) 10V (Incorrect)

(3) 8V (Correct)

(4) zero (Incorrect)



Correct answer: (3)

Solution:

No current will flow through the branch containing capacitor.

$$\therefore\ R_{eff} = \left(\frac{4\times 2}{4+2}\right) + 2 = \frac{10}{3}\ \Omega$$

Hence current drawn from the cell $I = \frac{V}{R_{eff}} = \frac{10}{10/3} = 3A$

$\therefore$ Current in branch ABC $= 1A$

Hence p.d. between points B & D

$= (V_B - V_C) + (V_C - V_D)$

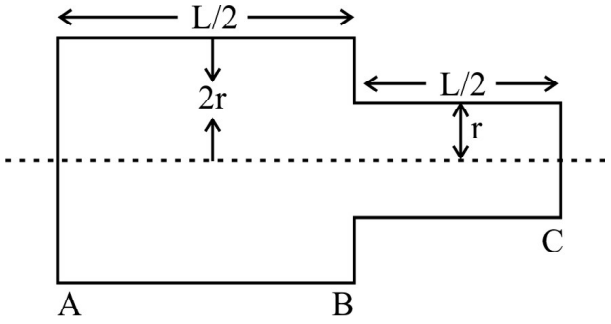
$= 2 \times 1 + 2 \times 3 = 2 + 6 = 8V$

Hence option (3) is correct.

Unit–12 : Current Electricity

Easy (Knowledge Based) *Current Density*

68. Two bars of radius r and 2r are kept in contact as shown in figure, an electric current i is passed through bars, which one of the following statement is cuuret;



(1) Heat produced in bar BC is 4 time the heat produced in bar AB (Correct)

(2) Electric field in both bars have the same value (Incorrect)

(3) Current density across AB is double that of across BC (Incorrect)

(4) Potential difference across AB is 4 time that of across BC (Incorrect)

Correct answer: (1)

Solution:

As the resistance of Bar BC will be four time of resistance of Bar AB.

(Area) $A_{BC} = \frac{1}{4} A_{AB}$

$R_{BC} = 4$ times of R_{AB}

Heat produced $H = i^2Rt$

For same current heat produced will be four times.

Hence option (1) is correct.

Unit–12 : Current Electricity

Average *Effect on Resistance on Stretching*

69. A wire of resistance 16Ω is cut into four equal parts, each part is stretched to the original length of the wire, then each streteched part first connected in series than in parallel, the ratio of resistance will be.

(1) 16 : 1 (Correct)

(2) 1 : 16 (Incorrect)

(3) 4 :1 (Incorrect)

(4) 1 : 4 (Incorrect)

Correct answer: (1)

Solution:

Resistance of each part on cutting = 4Ω

After stretching resistance of each part will be 16 times = 64Ω

Effective resistance in series = $64 \times 4 = 256\Omega$

Effective resistance in parallel $\frac{64}{4} = 16\Omega$

$\therefore$ Ratio of resistances in series to parallel = 256 : 16

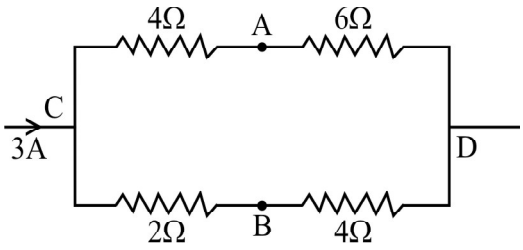
= 16 : 1

Hence option (1) is correct.

Unit–12 : Current Electricity

Average *Combination of Resistance and Division of Current in Parallel Combination*

70. The current of 3A flows in the combination of resistance as shown in figure, the potential difference between points A and B. Will be



(1) +0.75 V (Incorrect)

(2) zero (Incorrect)

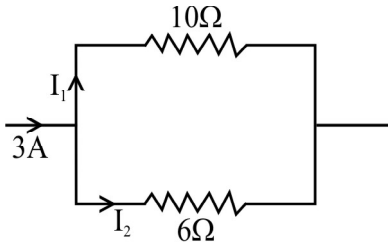
(3) 8.25 V (Incorrect)

(4) –0.75 V (Correct)

Correct answer: (4)

Solution:

The given circuit can be reduced to the circuit as shown here



Current in $10\,\Omega$ resistance

$$I_1 = \frac{3\times 6}{10+6} = \frac{18}{16} = \frac{9}{8}\text{A}$$

Current in $6\,\Omega$ resistance $\frac{3\times 10}{10+6} = \frac{30}{16} = \frac{15}{8}\text{A}$

From given figure $V_C - V_A = \frac{9}{8}\times 4 = \frac{9}{2}\text{V} = 4.5\text{V}$

$$V_C - V_B = \frac{15}{8}\times 2 = \frac{15}{4}\text{V} = 3.75\text{V}$$

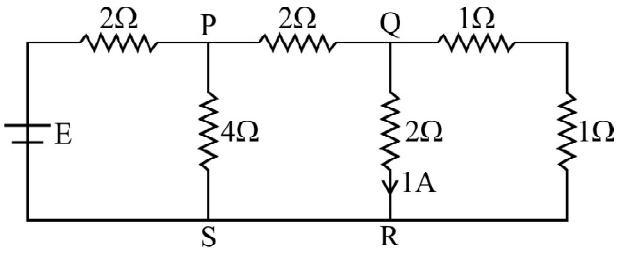
Hence, $(V_C - V_B) - (V_C - V_A) = 3.75 - 4.5 = -0.75\text{V}$

Hence option (4) is correct

Unit–12 : Current Electricity

Average *Cell and Distribution of Current*

71. In the given figure, the emf of the battery will be



(1) 12V (Incorrect)

(2) 13V (Correct)

(3) 16V (Incorrect)

(4) 18V (Incorrect)

Correct answer: (2)

Solution:

Current entering at point Q is 2A as it will divide equally.

Hence current through PQ = 2A

and current in branch PQRS = 2A

$\therefore$ Current in branch PS $= \frac{6}{4}\text{A} = 1.5\text{ A}$

$\therefore$ Therefore current drawn from cell = (2.0 + 1.5)A = 3.5A

Total resistance in the circuit $= 2\Omega + \frac{12}{7}\Omega = \frac{26}{7}\Omega$

Hence emf E = IR $= 3.5 \times \frac{26}{7} = 13\text{ V}$

Hence option (2) is correct.

Unit–12 : Current Electricity

Average *Distribution of Current*

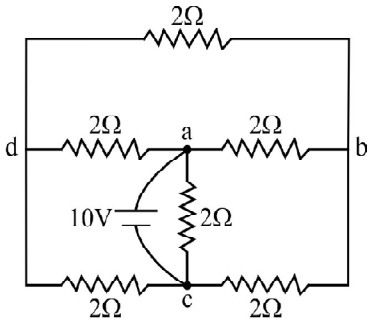
72. In the given circuit diagram, the current flowing through branch ab and bd will be:

(1) 2.5A and 10A (Incorrect)

(2) 0A and 10A (Incorrect)

(3) 0A and 2.5A (Incorrect)

(4) 2.5A and 0A (Correct)



Correct answer: (4)

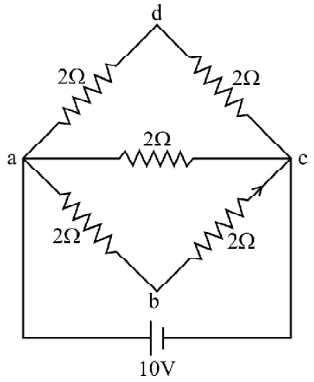
Solution:

The resistance between the points b and d becomes ineffective in the circuit, because points b and d are equal potential points.

hence current in branch bd = 0A

Current in branch ab = $\frac{10}{4} = 2.5$ A

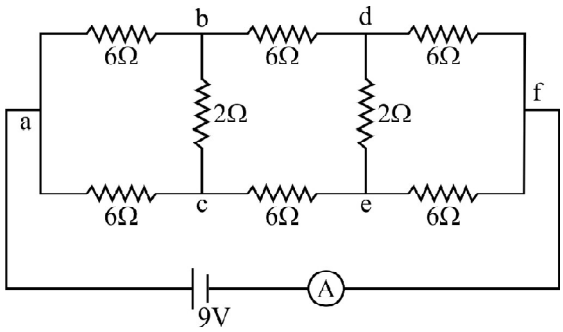
Hence correct option is (4).



Unit–12 : Current Electricity

Average *Distribution of Current*

73. In the given circuit, current through branch de and the ammeter reading will be



(1) 0A and $\frac{9}{13}$A (Incorrect)

(2) 0A and $\frac{9}{40}$A (Incorrect)

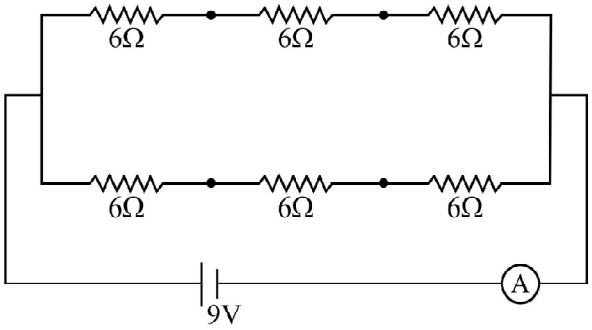
(3) 0A and 1A (Correct)

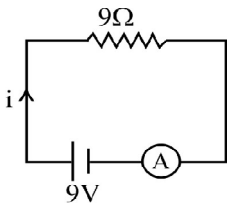
(4) 0.5A and 0.5A (Incorrect)

Correct answer: (3)

Solution:

The effective resistance between points a and f.





Hence current drawn from battery $i = \frac{9}{9} = 1A$

Therefore current through branch de = 0 (d and e being equal potential points, and ammeter reading 1A).

Hence option (3) is correct.

Unit–12 : Current Electricity

Easy *Combination of Resistances*

74. Two resistances R_1 and R_2 provides series to parallel equivalents as $\left(\frac{n}{1}\right)$, the correct relationship between R_1, R_2 and is

(1) $\left(\frac{R_1}{R_2}\right)^2 + \left(\frac{R_2}{R_1}\right)^2 = n^2$ (Incorrect)

(2) $\left(\frac{R_1}{R_2}\right)^{3/2} + \left(\frac{R_2}{R_1}\right)^{3/2} = n^{3/2}$ (Incorrect)

(3) $\frac{R_1}{R_2} + \frac{R_2}{R_1} = n$ (Incorrect)

(4) $\left(\frac{R_1}{R_2}\right)^{1/2} + \left(\frac{R_2}{R_1}\right)^{1/2} = \sqrt{n}$ (Correct)

Correct answer: (4)

Solution:

In series $R_S = R_1 + R_2$

In parallel $R_P = \frac{R_1 R_2}{R_1 + R_2}$

Given, $\frac{R_S}{R_P} = \frac{n}{1}$

$$\Rightarrow \frac{(R_1 + R_2)^2}{R_1 R_2} = \frac{n}{1} \Rightarrow \frac{R_1 + R_2}{\sqrt{R_1 \times R_2}} = \sqrt{n}$$

$$\Rightarrow \frac{\sqrt{R_1^2}}{\sqrt{R_1R_2}}+\frac{\sqrt{R_2^2}}{\sqrt{R_1R_2}}=\sqrt{n}$$

$$\Rightarrow \frac{\sqrt{R_1}}{\sqrt{R_2}}+\frac{\sqrt{R_2}}{\sqrt{R_1}}=\sqrt{n}$$

$$\Rightarrow \left(\frac{R_1}{R_2}\right)^{1/2}+\left(\frac{R_2}{R_1}\right)^{1/2}=\sqrt{n}$$

Hence alternative (4) is correct.

Unit–12 : Current Electricity

Average *Resistance*

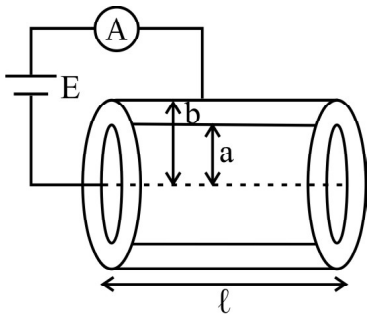
75. A hollow cylinder of length 'ℓ' and radii a and b is filled with a material of resistivity 'ρ' and is connected to a battery of emf 'E' through an ammeter, the reading of ammeter will be

(1) $\frac{E\,2\pi\ell}{\rho\log_e\left(b/a\right)}$ (Correct)

(2) $\frac{E\,2\pi\ell}{\rho\log_e\left(a/b\right)}$ (Incorrect)

(3) infinite (Incorrect)

(4) zero (Incorrect)



Correct answer: (1)

Solution:

Resistance of an imaginary cylinder of radius x, thickness dx, $dR=\frac{\rho dx}{2\pi x\ell}$

So the resistance of given cylinder $(R)=\int_a^b\frac{\rho dx}{2\pi x\ell}=\frac{\rho}{2\pi\ell}\int_a^b\frac{1}{x}dx=\frac{\rho}{2\pi\ell}\log_e\left(b/a\right)$

Hence reading of ammeter $I=\frac{E}{R}=\frac{2\pi\ell E}{\rho\log_e\left(b/a\right)}$

Hence option (1) is correct.

Unit–12 : Current Electricity

Easy *Temperature Dependence of Resistance*

76. A piece of silver and another of germanium cooled from room temperature to 85K. Then the resistance of

(1) Each of then increases (Incorrect)

(2) Each of then decreases (Incorrect)

(3) That of silver increases and germanium decreases (Incorrect)

(4) That of silver decreases and that of germanium decreases (Correct)

Correct answer: (4)

Solution:

Silver is a metal and germanium is a semi conductor. Resistance of metal decrease and that of a semi conductor increases with decreases in temperature.

Hence option (4) is correct

Unit–12 : Current Electricity

Average *Using Ohm's Law*

77. In a discharge tube the number of hydrogen ions drifting across a cross section per second is n_1 while the number of electrons drifting in the opposite direction across another cross section is n_2. If the supply voltage is V, the effective resistance, of the tube is

(1) $\frac{V}{(n_1 - n_2)e}$ (Incorrect)

(2) $\frac{V}{(n_1 + n_2)e}$ (Correct)

(3) $\frac{eV}{n_1 - n_2}$ (Incorrect)

(4) $\frac{eV}{n_1 + n_2}$ (Incorrect)

Correct answer: (2)

Solution:

The effective current in the tube will be obtained by adding the current due to flow of protons in one direction and the flow of electrons in opposite direction. Hence the total current through the tube is $(n_1+n_e)|e|$.

Now from Ohm's law, the resistance of the tube is given by

$$R = \frac{V}{(n_1 + n_2)e}$$

Unit–12 : Current Electricity

Average *Driff Velocity*

78. A silver wire has a radius of 0.1 cm and carries a current of 2A. The drift velocity of the electron in the silver wire is:

We are given:

Auogaidro's number = 6.023×10^{23}

Atomic weight of silver = 108

Density of silver 10.5×10^3 Kg/m^3

Charge on the electron = 1.6×10^{-19}C

(1) 7×10^{-4} m/s (Correct)

(2) 7×10^{-6} m/s (Incorrect)

(3) 7×10^{-3} m/s (Incorrect)

(4) 0.7 (Incorrect)

Correct answer: (1)

Solution:

Since the valence of silver is one so each atom of silver can be assumed to contribute one electron.

By Avogadro's law, we have 1g mole of substance = 6.023×10^{23} atoms

Hence number of electrons per unit volume will be

$$n = \frac{6.023\times10^{23}}{10^8\times10^{-3}}\times10.5\times10^3 = 5.86\times10^{28}\ /\ m^3$$

Cross sectional area $= \pi r^2 = 3.14\times10^{-6}\ m^2$

Now we know I = nevA where v is the drift velocity.

$$\therefore\ v = \frac{I}{neA} = \frac{2}{5.86\times10^{28}\times1.6\times10^{-19}\times3.14\times10^{-6}}$$

or $v = 7\times10^{-4}\ m/s$

Hence option (1) is correct.

Unit–12 : Current Electricity

Average *Factors Affecting Resistance*

79. A one meter long metallic wire is broken into two unequal parts P and Q. P part of the wire is uniformly extended into another wire R. Length of R is twice the length of P and resistance of R is equal to that of Q. The ratio of the resistance of P and R, is

(1) $\frac{1}{2}$ (Incorrect)

(2) $\frac{1}{4}$ (Correct)

(3) $\frac{1}{8}$ (Incorrect)

(4) $\frac{1}{16}$ (Incorrect)

Correct answer: (2)

Solution:

Let ℓ be the length of part P of the wire.

The length of wire $R = 2\ell$

Length of wire $Q = 1-\ell$

Mass of P = Mass of R

$\pi r_P^2 \ell d = \pi r_R^2 2\ell d$

$r_P^2 = 2r_R^2$ or $r_P = \sqrt{2}\, r_R$

Than, $R_P = \frac{\rho \ell}{\pi r_P^2}$, $R_Q = \frac{\rho(1-\ell)}{\pi r_Q^2}$

and $R_R = \rho \frac{2\ell}{\pi r_R^2}$

Hence, $\frac{R_P}{R_R} = \frac{r_R^2}{2r_P^2} = \frac{r_R^2}{2\left(\sqrt{2}\ r_R\right)^2} = \frac{1}{4}$

Hence option (2) is correct.

Unit–12 : Current Electricity

Average *Heating Effects of Currents*

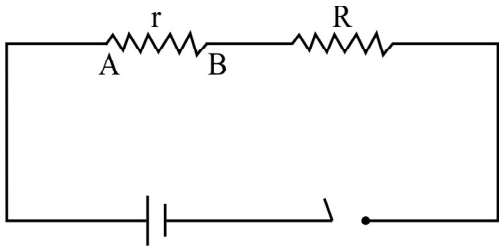
80. A wire, of resistance r, connects points A and B; two points in a circuit; the resistance of the remainder of which is R. Without any other charge being made A and B are also connected by (n-1) other wires, the resistance of each of which is r. Then the heat produced in the n wires null be greater than that produced in the first wire; if

(1) $r > \left(\sqrt{n}\right)R$ (Incorrect)

(2) $r = \left(\sqrt{n}\right)R$ (Incorrect)

(3) $r < \sqrt{n}\,R$ (Correct)

(4) $r > \left(\sqrt{n-1}\right)R$ (Incorrect)



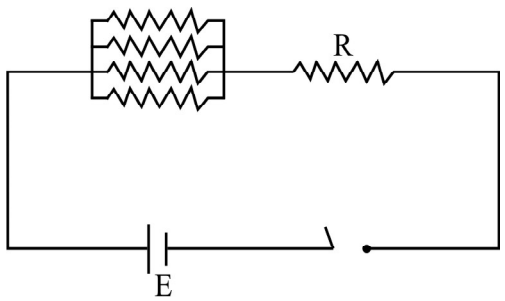
Correct answer: (3)

Solution:

In the first case have,

H_1 = heat produced per second in the wire $r = i_1^2 r$

or $H_1 = \frac{E^2}{(R+r)^2} r$



In the second case the n wire are are connected in parallel having are equivalent resistance $\frac{r}{n}$

$$\therefore\ H_2 = i_2^2 \frac{r}{n} = \frac{n^2E^2}{(nR+r)^2}\frac{r}{n} \qquad \left[\because\ i_2 = \frac{E}{R+\frac{r}{n}} = \frac{nE}{nR+r}\right]$$

or $$H_2 = \frac{nE^2}{(nR+r)^2} r$$

$$= \frac{H_2}{H_1} = \frac{n(R+r)^2}{(nR+r)^2}$$

If $H_2 > H_1$ then we must have,

$$n(R+r)^2 > (nR+r)^2$$

or $$n\left[R^2 + 2Rr + r^2\right] > n^2R^2 + 2nRr + r^2$$

or $$nR^2 - n^2R^2 > r^2 - nr^2$$

$$nR^2(1-n^2) > r^2(1-n)$$

$$nR > r^2$$

$$R > \frac{r}{\sqrt{n}}$$

or $$r < (\sqrt{n})R$$

Hence option (3) is correct.

81. The supply voltage to a room is 120 V. The resistance of the lead wires is 6Ω. A 60 W bulb is already switched on. What is the decrease of voltage across the bulb, when a 240 W heater is switched on in parallel to the bulb?

(1) 2.9 Volt

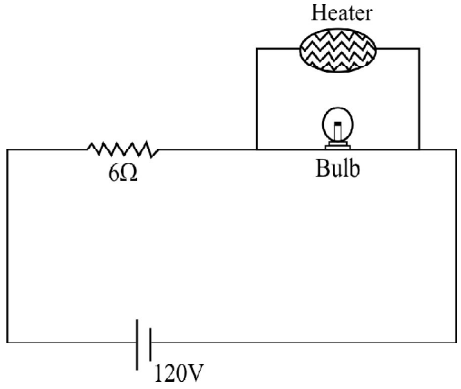
(2) 13.3 Volt

(3) 10.04 Volt

(4) zero Volt

Correct answer: (3) (IIT 2013)

Solution:



'R' for bulb $= \frac{120 \times 120}{60}\Omega = 240\ \Omega$

'R' for Heater $= \frac{120 \times 120}{240}\Omega = 60\ \Omega$

Voltage, across the bulb,

(i) Before heater is switched on $V_1 = \frac{120}{246} \times 240\ V$

(ii) After heat is switched on $V_2 = \frac{120}{54} \times 48\ V$

$\therefore$ Decrease in voltage $= V_1 - V_2 = 120\left[\frac{240}{240} - \frac{48}{54}\right]V$

$\simeq 10.04\,V$

Hence option (3) is correct.

82. In a large building, there are 15 bulbs of 40 W, 5 bulbs of 100 W, 5 fans of 80 W and 1 heater of 1 kW. The voltage of the electric mains is 220 V. The minimum capacity of the main fuse of the building will be:

(1) 12 A

(2) 14 A

(3) 8 A

(4) 10 A

Correct answer: (1) (IIT 2014)

Solution:

Total power of all the devices

$= [15 \times 40 \times 5 \times 100 + 5 \times 80 + 1 \times 1000]\ W = 2500\ W$

$\therefore$ Current $= \frac{2500\,W}{200\,V} = 11.36\,A \approx 12\,A$

$\therefore$ Minimum capacity of the main fuse should be 12 A.

Hence option (1) is correct.

83. When 5V potential difference is applied across a wire of length 0.1m, the drift speed of electrons 2.5×10^{-4} ms^{-1}. If the electron density of the wire is 8×10^{28} m^{-3}, the resistivity of the materials is close to

(1) $1.6\times10^{-6}\ \Omega m$

(2) $1.6\times10^{-5}\ \Omega m$

(3) $1.6\times10^{-8}\ \Omega m$

(4) $1.6\times10^{-7}\ \Omega m$

Correct answer: (2) (IIT 2015)

Solution:

$$R=\frac{\rho L}{A}=\frac{V}{I}=\frac{V}{neAv_d}$$

$$\therefore\ \rho=\frac{V}{neV_d}.\frac{1}{L}$$

$$\simeq 1.6\times10^{-5}\ \Omega-m$$

Hence option (2) is correct.

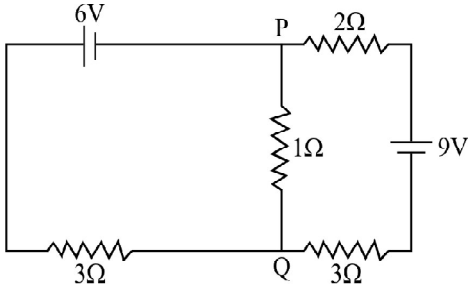
84. In the circuit shown, the current in the 1Ω resistor is:

(1) 0.13 A, from Q to P

(2) 0.13 A, from P to Q

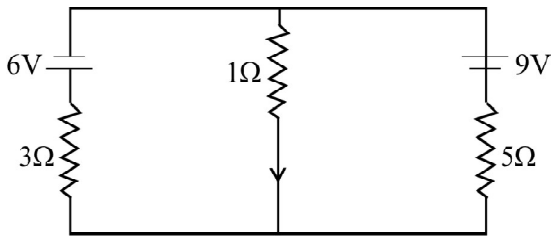
(3) 1.3 A, from P to Q

(4) 0A



Correct answer: (1) (IIT 2015)

Solution:



By using Kirchoff's laws, we find the current in the 1Ω resistor, as per option (1).

85. The temperature dependence of resisatnces of Cu and undoped Si in the temperature range 300-400K, is best described by:

(1) Linear increase for Cu, exponential increase for Si

(2) Linear increase for Cu, exponential decrease for Si

(3) Linear decrease for Cu, linear decrease for Si

(4) Linear increase for Cu, linear increase for Si

Correct answer: (2) (IIT 2016)

Solution:

For conductors

Resistance increases linearly (for not to large increase in temperature) with temperature.

For semi-conductors

Resistance decreases non-linearly is an (almost) exponential way.

Hence option (2) is correct.

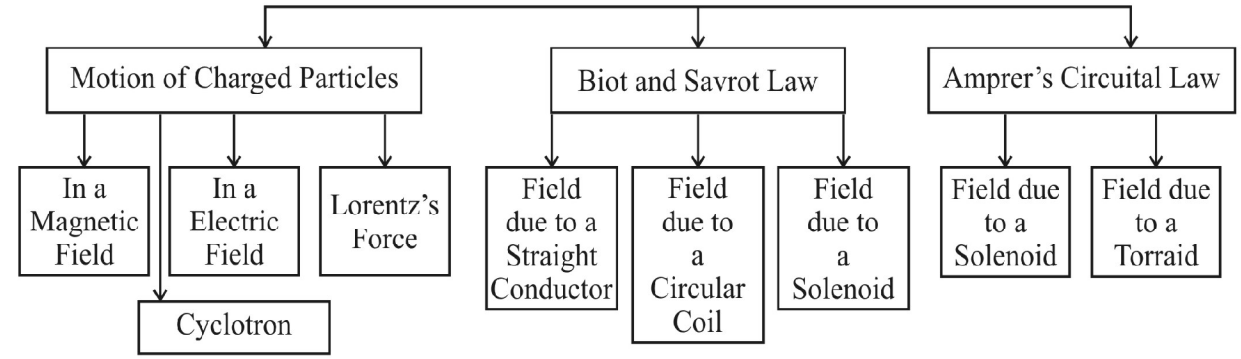
UNIT–13 : MAGNETIC EFFECTS OF CURRENT AND MAGNETISM

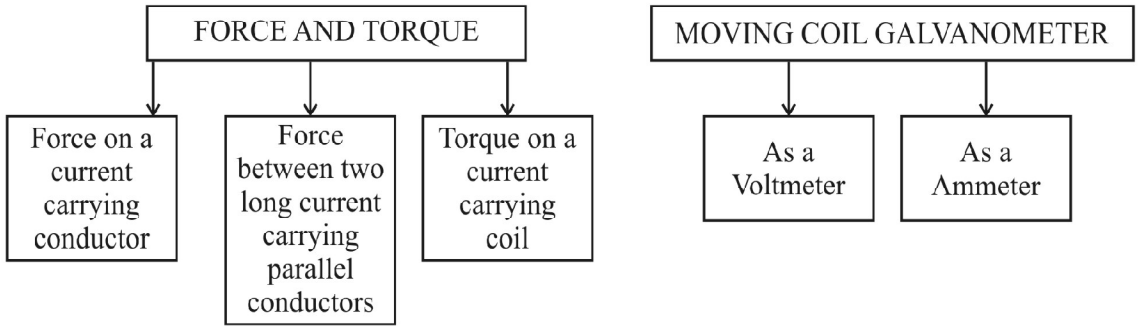
Learning Objectives

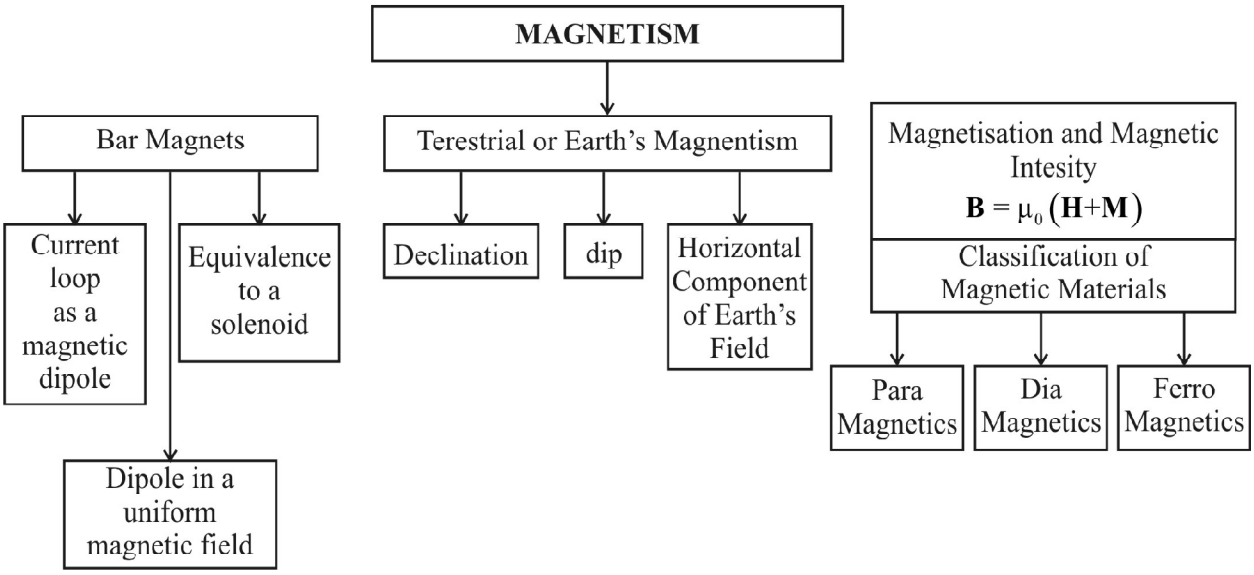
After going through this unit you will be able to understand, appreciate and apply the following concept as:

- Basic ideas related to Biot Savart law.
- Application of Biot Savart law to a current carrying circular coil.
- Ampere's (circulated) law.
- Applying Ampere's law to a (i) long straight current carrying wire and (ii) solenoid.
- Force on a moving charge in uniform electric and magnetic fields – concept of Lorentz force.
- Cyclotron – its principle of working.
- Force on a current carrying conductor in a uniform magnetic field.
- Definition of the (SI) unit of current – the ampere.
- Torque expereinced by a current loop in a uniform magnetic field.
- Moving coil galvanometer – its principle and working.
- Current sensitivity of a moving coil galvanometer – factors affecting it.
- Conversion of a given moving coil galvanometer into (i) an ammeter (ii) a voltmeter.
- Equivalent magnetic dipole moment of a current carrying loop.
- Equivalence between a bar magnet and a current carrying solenoid.
- Magnetic field lines.
- Magnetic field of the earth.
- The (magnetic) elements of earth's magnetic field.
- Classification of different materials into para, dia and ferromagnetic materials.
- Magnetic susceptibility, permeability, hysteresis.
- Differences, similarities and uses of electromagnets and permanent magnets.

MAGNETIC EFFECTS OF CURRENT


Motion of Charged Particles
Biot and Savrot Law
Amprer's Circuital Law
In a Magnetic Field
In a Electric Field
Lorentz's Force
Cyclotron
Field due to a Straight Conductor
Field due to a Circular Coil
Field due to a Solenoid
Field due to a Solenoid
Field due to a Torraid


FORCE AND TORQUE
MOVING COIL GALVANOMETER
Force on a current carrying conductor
Force between two long current carrying parallel conductors
Torque on a current carrying coil
As a Voltmeter
As a Ammeter

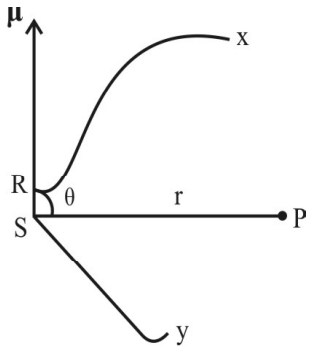


Magnetic Effects of Current

In 1802, an Italian jurist Romagnosi discovered that a current carrying wire affects a nearby magnetic needle in much the same way as a magnet affects it. In 1820 Oersted made a similar observation. It was Ampere who made a significant contribution in further understanding of the magnetic effects, associated with currents. Biot and Savart then experimentally established the factors, on which the magnetic field produced by a current carrying conductor, depends.

Biot and Savart Law

Let us consider a current carrying conductor xy and let P be a point where the magnetic field is to be calculated. Let RS be a current element of length $d\ell$. It was found that magnitude of d**B**, is



(i) directly proportional to current, i, through the conductor.

(ii) inversely proportional to square of distance r, of the point P from the current element and (*iii*) directly proportional to $d\ell \sin\theta$ where θ is the angle bewteen the current element $d\ell$ and the radius vector **r**.

Combning these we get

$$|d\mathbf{B}| \propto \frac{id\ell \sin\theta}{r^2}$$

or $$|d\mathbf{B}| = K\frac{id\ell \sin\theta}{r^2}$$

where K is a constant of proportionality

The constant K is analogus to constant of proportionality appearing in Coulomb's law. It depends upon the system of units. In SI units K is choosen, experimentally, (keeping in mind that the unit of the current is the 'ampere') to have the value.

$$K = \frac{\mu_0}{4\pi}$$

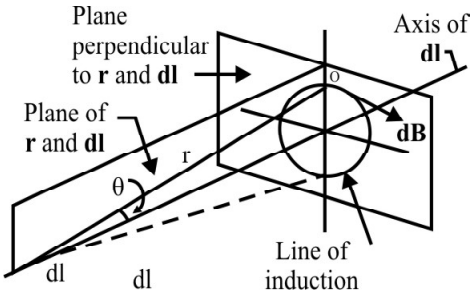
Here μ_0 is a constanst, called the PERMEABILITY of free space, and its value is

$$4\times10^{-7}\ \frac{\text{weber}}{\text{ampere meter}}$$

Hence $$|d\mathbf{B}| = \frac{\mu_0}{4\pi}\frac{id\ell \sin\theta}{r^2}$$

To incorporate the direction, we have to write

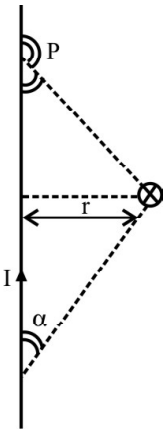
$$|d\mathbf{B}| = \frac{\mu_0}{4\pi}\left(\frac{i d\ell \times \hat{r}}{r^2}\right)$$

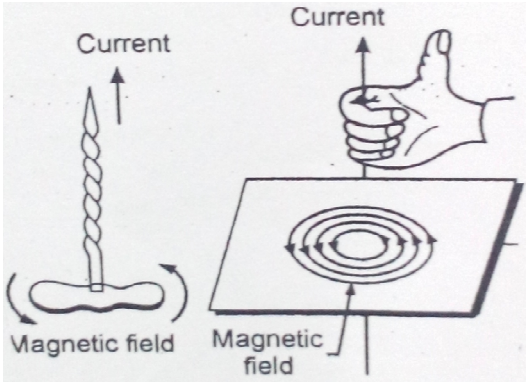


Thus d**B** is always perpendicular to the plane formed by **r** and $d\ell$ as shown. This can be further understood by imagining a right hand screw to be placed at P such that it is normal to the plane formed by $d\ell$ and **r**. Rotate the screw in the dirction that makes $d\ell$ coincide with **r**. The direction of advancement of this screw than gives the direction of d**B**.

Using Biot and Savart law, we can form a general procedure for carrying out mathematical calculations. We can show that for straight wire, carrying a current i,

$$|\mathbf{B}| = \frac{\mu_0 i}{4\pi r}[\cos\alpha + \cos\gamma]$$





For an infinitely long straight wire, we have, in this limiting case, both α and γ tending to zero. Thus, for an infinite straight wire,

$$|\mathbf{B}| = \frac{\mu_0 i}{2\pi r}$$

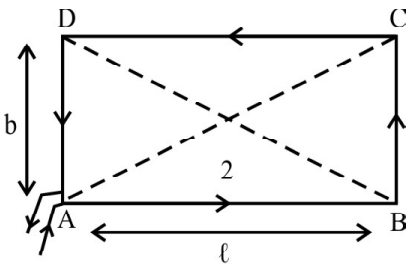
The pattern of field lines of this magnetic field is shown in figure. We have the so called right hand thumb rule to find the direction of this field.

According to this rule – "If we grasp the straight conductor in the palm of our right hand so that the thumb points in the direction of flow of current, the sense in which fingers curl, gives the direction of the magnetic field lines".

Example–1 :

Show that the magnetic field at the centre of a rectangular coil, of sides ℓ and b, carrying a current I, is given by

$$|\mathbf{B}| = \frac{2\mu_0 I\sqrt{\ell^2 + b^2}}{(\pi \ell b)}$$



Solution :

Denoting by ℓ_1 and ℓ_2 the distances of two ends of the wire, from the foot of the perpendicular, drawn from the field point, on a wire of finite length, we can write.

$$|\mathbf{B}| = \frac{\mu_0 I}{4\pi a}\left[\frac{\ell_1}{\sqrt{\ell_1^2 + a^2}} + \frac{\ell_2}{\sqrt{\ell_2^2 + a^2}}\right]$$

Now the direction of field due to all the wires, AB & BC, CD and DA are the same; perpendicular to the plane of the rectangle and directed outward.

The field at O, due to the wires AB and CD are equal and is given by

$$|\mathbf{B}_1| = \frac{\mu_0 I}{4\pi\frac{b}{2}}\left[\frac{\frac{\ell}{2}}{\sqrt{\left(\frac{\ell^2}{2}\right)^2 + \left(b^2/2\right)^2}} + \frac{\frac{\ell}{2}}{\sqrt{\left(\frac{\ell^2}{2}\right)^2 + \left(b^2/2\right)^2}}\right]$$

Similarly the fields, due to wires BC and DA, at O, are equal and is given by

$$|\mathbf{B}_2| = \frac{\mu_0 I}{4\pi\left(\frac{\ell}{2}\right)}\left[\frac{\frac{b}{2}}{\left(b/2\right)^2 + \left(\ell/2\right)^2} + \frac{\frac{b}{2}}{\left(\frac{b}{2}\right)^2 + \left(\frac{\ell}{2}\right)^2}\right]$$

Hence the net magnetic field, at O, is

$$|\mathbf{B}| = 2|\mathbf{B}_1| + 2|\mathbf{B}_2|$$

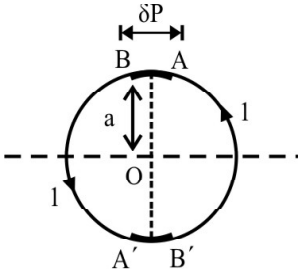
Substituting the values and simplifying we get

$$|\mathbf{B}| = \frac{2\mu_0 I\sqrt{\ell^2 + b^2}}{(\pi b\ell)}$$

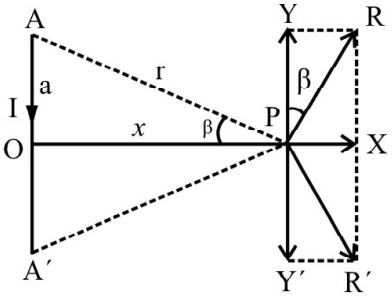
Field due to a Circular Coil (i) at a Point on the Axis of the Coil and (ii) at the Centre of the Circular Coil

To calculated the field due to a circular coil at point P on it axis (see figure) we take a element AB of the circular coil. The field due to this element is

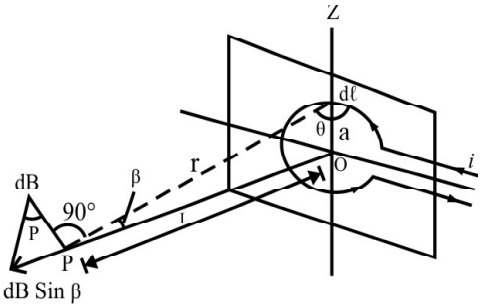
$$dB = \frac{\mu_0}{4\pi}\frac{id\ell}{r^2} \qquad (\theta = 90^0)$$



We resolve this field into perpendicular component $(dB\cos\beta)$ and parallel component $(dB\sin\beta)$. Now if we take a symmetrically situated element A'B' on the opposite end of the diameter and similarly resolve into parallel and perpendicular components and add the field component's due to these two elements we find that the perpendicular components cancels out. Whatever is true for one pair of elements is also trace for every other pair. Hence we conclude that the field due to a circular coil at a point other on its axis is along the axis of the coil and is given by



$$B = \left[\frac{\mu_0 i}{4\pi}\right]\frac{d\ell}{r^2}\sin\beta$$



Now $\sin\beta = \frac{a}{r}$ and $r^2 = a^2 + x^2$

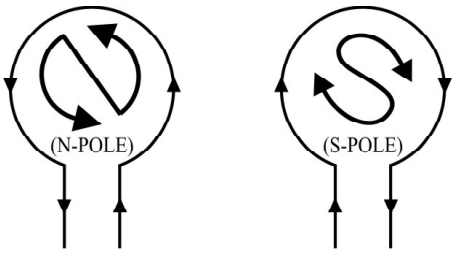
Putting these values we obtain, after integration.

$$|\mathbf{B}| = \frac{\mu_0}{2}\frac{ia^2}{(a^2+x^2)^{3/2}}$$

At the centre of the coil, $x = 0$ and we have

$$B = \frac{\mu_0 i}{2a}$$

There is a simple rule for finding the polarity of the two faces of a current carrying circular coil. According to the rule when an observer looking at the circular coil, finds the current to be flowing in the anticlockwise sense that face of the coil behaves like the N–pole of the equivalent magnet. On the other hand, if the current is seem to flow in the clockwise sense that face of the coil behaves like the S–pole of the equivalent magnet.



(a) Anticlockwise current (b) Clockwise current

'Far Off' Field of a Circular Coil

In the expression

$$|\mathbf{B}| = \frac{\mu_0 i a^2}{2\left(a^2 + x^2\right)^{3/2}}$$

if we have $x >> a^2$ than the field due to a circular coil, at a far off point on its axis, becomes equal to

$$|\mathbf{B}| \simeq \frac{\mu_0 i a^2}{2x^3} = \frac{\mu_0 A i}{2\pi x^3} \qquad \left(A = \pi x^2\right)$$

We will discuss later that a current carrying circular loop is equivalent to a magnetic dipole with dipole magnetic moment $|\mathbf{m}|$ equal to (iA) where A is the area a of the loop and i the current flowing in the loop.

Hence we can will be

$$|\mathbf{B}| \simeq \frac{\mu_0 m}{2\pi x^3}$$

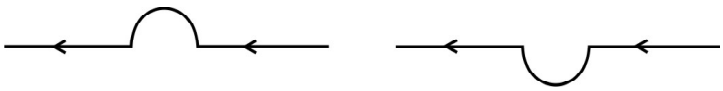
which can be shown to be the field due to a (small) magnetic dipole at a far off point.

Hence a current carrying circular loop behaves like a magnetic dipole with magnetic moment.

$$|\mathbf{m}| = iA$$

Example–2 :

A straight wire, carrying a current of 12A is bent into a semicircular arc of radius 2cm, as shown. What is the direction and magnitude of B at the centre of the arc? What will happen if the wires were bent into a semicircular arc of the same radius but in the opposite sense as shown?



Solution :

The magnetic field at the centre of a circular coil of radius r is given by

$$B = \frac{\mu_0 i}{2r}$$

Hence the magnetic field, at the centre of a semicircular arc, of radius, r, would be given by

$$|\mathbf{B}| = \frac{1}{2}\left(\frac{\mu_0 i}{2r}\right) = \frac{\mu_0 i}{4r}$$

Hence $B = \dfrac{4\pi \times 10^{-7} \times 12}{4 \times 0.02} T = 1.9 \times 10^{-4} T$

The field, would be normal to the plane of paper, going into it.

In the second case, the field would be normal to the plane of paper but going out of it.

Example–3 :

If the field, normal to the plane of a wire of n turns and radius r, carrying a current i, is measured on the axis of coil, at a small distance h, from the centre of the coil, show that it is smaller than the field at the centre, by the fraction $\dfrac{3h^2}{2r^2}$.

Solution :

The magnetic field B, at a point on the axis distant h from the centre, is given by

$$B = \frac{\mu_0 N i r^2}{2\left(r^2 + h^2\right)^{3/2}}$$

The field B_C at the centre of the coil is $B_C = \dfrac{\mu_0 N i}{2r}$

$$\therefore \quad B_C - B = \frac{\mu_0 N i}{2r} - \mu_0 N i \frac{r^2}{\left(r^2 + h^2\right)^{3/2}}$$

or $$\therefore \quad B_C - B = \frac{\mu_0 N i}{2}\left[\frac{1}{r} - \frac{r^2}{r^3\left(1 + \dfrac{h^2}{r^2}\right)^{-3/2}}\right]$$

$$= \frac{\mu_0 N i}{2}\left[\frac{1}{r} - \frac{1}{r}\left(1 + \frac{h^2}{r^2}\right)^{-3/2}\right]$$

$$= \frac{\mu_0 N i}{2}\left[\frac{1}{r} - \frac{1}{r}\left(1 - \frac{3}{2}\frac{h^2}{r^2} + ...\right)\right] = \frac{\mu_0 N i}{2}\left(\frac{3h^2}{2r^3}\right)$$

The fractional decrease in the field is given by

$$\frac{(B_C - B)}{B_C} = \frac{\left(\frac{\mu_0 i}{2}\right)\left(\frac{3h^2}{2r^3}\right)}{\left(\frac{\mu_0 Ni}{2r}\right)} = \frac{3}{2}\left(\frac{h^2}{r^2}\right)$$

Example–4 :

A straight wire, of length $\left(\frac{\pi}{2}\right)$ m , is bent into a circular shape. If the wire were to carry a current of 5A, calculate the magnetic field due to it, before bending, at a point distance 0.01 times the radius of the circle formed from it. Also calculate the magnetic field at the centre of the circular loop formed for the same value of current.

Solution :

The radius, r, of the circular coil, is given by $2\pi r = \frac{\pi}{2} \;\therefore\; r = \frac{1}{4} m = 0.25\, m$

When the wire is straight, the distance of the field point is $r' = 0.01\ r = (0.01 \times 0.25)$ m

It is obvious that this distance is very small as compared to the length of the wire. ($\therefore$ wire can be taken of infinite length).

$\therefore$ Magnetic field due to the straight wire is

$$B = \frac{\mu_0 i}{2\pi r} = \frac{2\times10^{-7}\times5}{0.01\times0.25}\ T = 4\times10^{-7}\,T$$

The magnetic field, at the centre of the circle, carrying a current of 5A, is given by

$$B_{loop} = \frac{\mu_0 i}{2r} = \frac{10^{-7}\times5}{2\times0.25}\ T$$

$$\simeq 1.25\times10^{-5}\ T$$

Example–5 :

In the Bohr model of the hydrogen atom the electron circulaters around nucleus in an orbit of radius 5.1×10^{-11} m at a frequency of 6.8×10^{15} Hz. Find the value of B set up at the centre of the orbit.

Solution :

Since $i = \frac{\text{charge}}{\text{time}}$ and $\text{time period} = \frac{1}{\text{frequency}}$

We have, $i = e\upsilon$

$\therefore \quad i = \left(1.6\times10^{-19}\times6.8\times10^{15}\right)A = 1.1\times10^{-3}\,A$

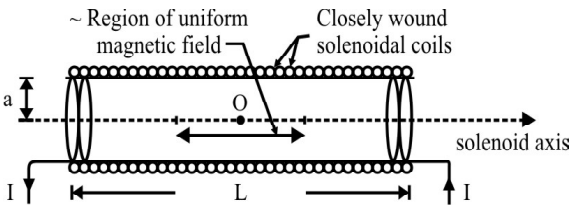
Hence the magnetic field, at the centre of the circular orbit, is

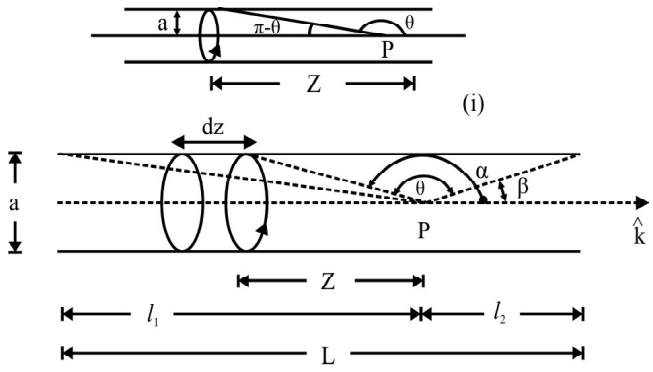
$$B = \frac{\mu_0 i}{2r} = \frac{4\pi\times10^{-7}\times1.1\times10^{-3}}{2\times5.1\times10^{-11}}\ T \simeq 13.55\,T$$

(It is quite a large magnetic field indeed!)

Field due to a Solenoid

A solenoid consists of a large number of turns of (insulated) wire wound very closely on an appropriate cylindrical core. Let us calculate the field due to a solenoid at a point P on its axis. Let the solenoid be wound with N turns of wire and let a and L be its radius and length. We than have $n = \frac{N}{L}$, where n is the number of turns per unit length.





Let P be any point on the axis of the solenoid

The magnetic field at this point due to any one turn of the solenoid whose centre is at a distance z from this point is

$$dB' = \frac{\mu_0 i a^2}{2\left(a^2+z^2\right)^{3/2}}$$

If we consider a length dz of the solenoid around this turn, this length would contain ndz circular coils each of which can be very nearly regarded as having its centre at the same distance z from the point P. Hence the total magnetic field at P due to this length dz of the solenoid is given by

$$d\mathbf{B} = (ndz)\,\mathbf{d}B' = \frac{\mu_0\, i\, n\, a^2}{2\left(a^2+z^2\right)^{3/2}}dz$$

Hence $|\mathbf{B}|$, the field due to solenoid can be written as

$$|\mathbf{B}| = \frac{\mu_0 i\, na^2}{2}\int_{z=\ell_2}^{z=\ell_1}\frac{dz}{\left(a^2+z^2\right)^{3/2}}$$

where ℓ_1 and ℓ_2 are the distance of the two ends of the solenoid from P.

We put $z = a\cos\theta$

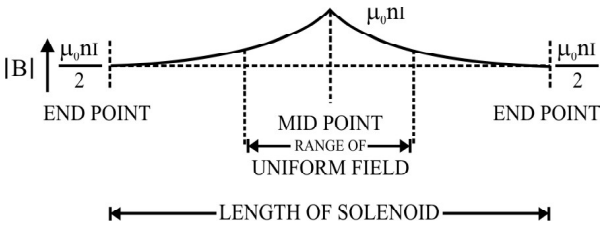
On integrations, we get

$$\mathbf{B} = \frac{\mu_0 i\, na^2}{2}\frac{1}{a^2}(\cos\beta - \cos\alpha)$$

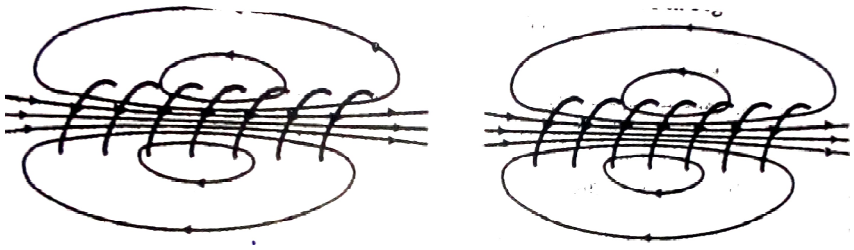
For an infinte solenoid we then have $\mathbf{B} = \left|\mu_0 i\left(N/L\right)\right| = \mu_0\, in$

$|\mathbf{B}|$ (at the end points), is $\frac{\mu_0\, in}{2}$

The axial field of a long solenoid thus varies as shown in the figure. We can thus conclude the axial field of a long solenoid is very nearly uniform over a small range around its midpoint.



We show below the field lines of this magnetic field. This pattern is very similar to the magnetic field lines of magnetic field of a bar magnet. Hence a solenoid is often viewed as the equivalent of a bar magnet.



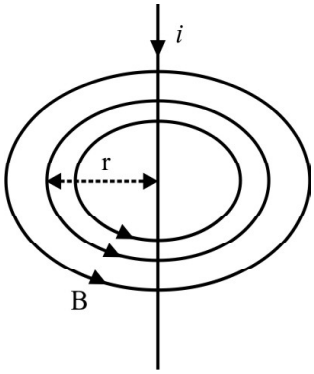
(i) (ii)

Field lines of the magnetic field of a (i) (long) selenoid (ii) bar magnet

Ampere's Circuital Law

According to this law, the line intergral of the magnetic field, around a closed curve, is equal to $\mu_0 i$ where i is the current enclosed by the path of integration.

Thus, $\oint \mathbf{B}.\mathbf{d}\ell = \mu_0 i$



Since $\oint \mathbf{B}.\mathrm{d}\ell$ is not equal to zero, it is important to realize that the magnetic field is not a *Conservative Field*.

We can easily check the validity of Ampere's Circuital Law. We have seen that the magnetic field due to a long straight wire is given by

$$B = \frac{\mu_0 i}{2\pi r}$$

This field has its field lines as shown in the figure. Let us calculate the line integral of this field around any circle of radius r.

Along the circle considered, all the linear line elements $\mathrm{d}\ell$ are directed along **B**, so that we can write

$$\mathbf{B}.\mathbf{d}\ell = \mathbf{B}\mathrm{d}\ell$$

Hence $\oint \mathbf{B}.\mathbf{d}\ell = \oint \mathbf{B}.\mathrm{d}\ell = \mathbf{B}\oint \mathrm{d}\ell$

$$\therefore \quad \oint \mathbf{B}.\mathbf{d}\ell = \left(\frac{\mu_0 i}{2\pi r}\right) \times (2\pi r) = \mu_0 i$$

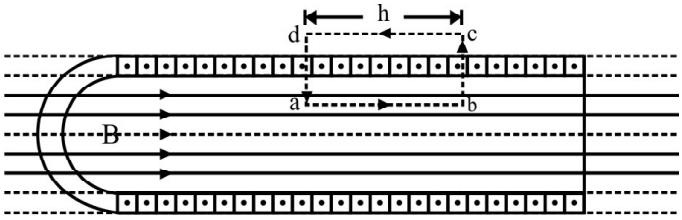
Thus $\oint \mathbf{B}.\mathbf{d}\ell = \mu_0 i$, as per Ampere's circuital law.

Applications of Ampere's Circuital Law

(i) We apply Ampere's law to calculate the field due to long solenoid. For a long solenoid the field is uniform near the centre and parallel to axis of the solenoid. We calculate its line integral around the path abcd, as shown.

We can write,

$$\oint \mathbf{B}.\mathbf{d}\ell = \int_a^b \mathbf{B}.\mathbf{d}\ell + \int_b^c \mathbf{B}.\mathbf{d}\ell + \int_c^d \mathbf{B}.\mathbf{d}\ell + \int_d^a \mathbf{B}.\mathbf{d}\ell$$



The second and fourth integrals are zero each because at every point of these paths $\theta = 90^0$. The integral $\int_a^b \mathbf{B}.\mathbf{d}\ell = Bh$ $\quad (\because \quad \mathbf{B}.\mathbf{d}\ell = Bd\ell)$

Also for chosen path, the side cd can be taken very far off from the solenoid, so that B at every point, along it zero. Hence $\int_c^d \mathbf{B}.\mathbf{d}\ell = 0$

Hence $\oint \mathbf{B}.\mathbf{d}\ell = Bh = \mu_0 i_{enclosed}$

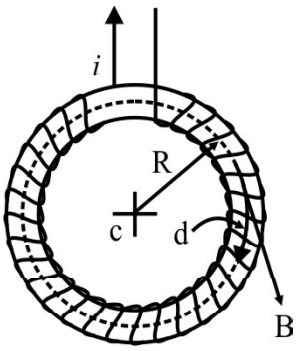
Now $i_{enclosed} = \mu_0 i\left(\frac{N}{L}\right)h$ (i = current through each turn)

$\therefore \quad B = \mu_0 i\left(\frac{N}{L}\right) = \mu_0 i\, n$

where $n\left(=\frac{N}{L}\right)$ is the number of turns per unit length of the solenoid.

(ii) Magnetic Field of A toroidal Solenoid

A toroidal solenoid can be thought of a straight solenoid bent into the shape of a circle so that its ends join together. A toroidal winding is shown in the figure.



To calculate $|\mathbf{B}|$ at a point P, we consider a closed path of integration shown by the dotted lines; a circle of radius r. From symeetrary $|\mathbf{B}|$ will have the same magnitude at every point along the path of integration. We have seen that, the magnetic field due to a long straight solenoid, is along its axis. Since there are no ends of a toroidal solenoid the same conditions apply. Also the axis of the solenoid has become a circle. Hence

$\oint \mathbf{B}.\mathbf{d}\ell = B\oint \mathbf{d}\ell = 2\pi rB$

$$\therefore \quad \oint \mathbf{B}.\mathbf{d}\ell = 2\pi rB = \mu_0 I$$

$$\therefore \quad B = \frac{\mu_0 I}{2\pi r}$$

Now the considered path of integration, passes through, all the N turns of solenoid, each carry a current i, therefore $I = N i$

$$B = \left(\frac{\mu_0 iN}{2\pi r}\right)$$

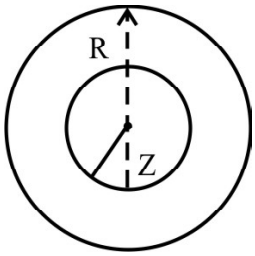
Example–6 :

A long straight solid cylinderical wire, of radius R, carries a current i uniformly distributed all over its circular cross section. FInd the magnetic field at a distance z from the axis of wire.

Solution :

A section of solid metal wire of radius R prependicular to its length is represented in the figure.

Let us consider a point P, lying inside the wire at a perpendicular distance r, from the axis of wire. To apply Ampere's Circuital law, we considers a circular path of radius z around the axis of wire. Over this path, because of symmetry consideractions, the magnetic field produced due to the current in the wire, will be equal at all points on the circular path and will have a tangential direction.



By Ampere's circuital law, we then have

$$\oint \mathbf{B}.\mathbf{d}\ell = \mu\left(\frac{iz^2}{R^2}\right) \text{ (for } z < R)$$

$\because$ Current inclosed by the circular path is $\frac{i}{\pi R^2} \times \pi z^2 = \frac{iz^2}{R^2}$

$$\therefore \quad B2\pi z = \frac{\mu i z^2}{Rz}$$

However, the current enclosed by the circular path would be i when P lies outside the wire.

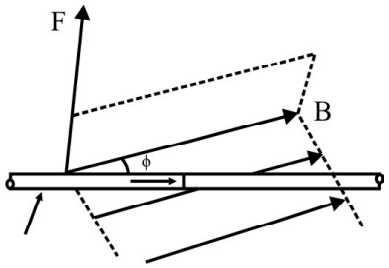
Hence we have, for $z > R$,

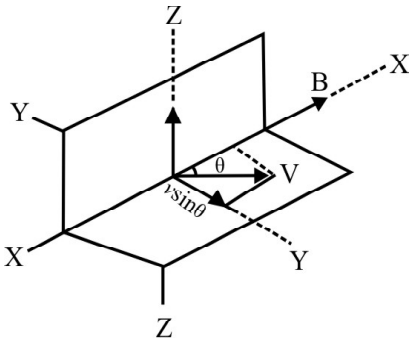
$$B\ 2\pi z = \mu_0 i$$

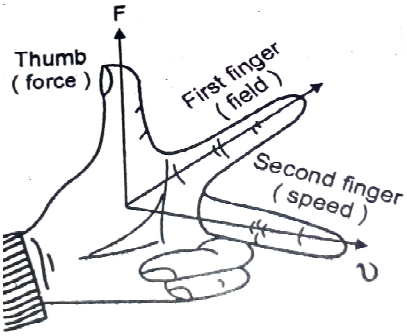
$$B = \frac{\mu_0 i}{2\pi z}$$

Motion of Changed Particles in Magnetic and Electrid Fields

Let us consider a region in space where the magnetic field is uniform and is along x–axis. Let a positive change q move along the y axis with a consted speed v. Experiments show that the changed particle does not experience any force when it is moving parallel to the direction of the magnetic field. Also it is observed that (in other cases) the charged particle experiences a force in a direction perpendicular to both **v** and **B**, the magnitude of the force experienced by charged particle when it is moving perpendicular to **B**, is given by $|\mathbf{F}| = qvB$







For a positive charge the direction of the force according to Fleming left hand rule, is explained in the adjoining figure.

For a negatively charged particle the direction of force is opposite to that of a positive charge.

However when the charged particle is moving with its velocity **v**, making an angle θ with the direction of **B**, the parallel component $v\cos\theta$ does not experience any force. It is only the perpendicular component, $v\sin\theta$, that experience a force given by

$$|\mathbf{F}| = qBv\sin\theta$$

We can, therefore, write

$$\mathbf{F} = q(\mathbf{v}\times\mathbf{B})$$

We define **B**, in magnetism, with the help of this equations.

It is important to note that a magnetic field cannot change the kinetic energy of a moving charge.

A moving changed particle in a uniform transverse magnetic field, always moves in a plane perpendicular to **B**. Hence its trajectory of motion is a circle of radius R_B such that

$$R_B = \frac{mv}{qB}$$

If a charged particle having a charge q moves in a uiform electric field, the force acting on the charged particle is given by

$$\mathbf{F} = q\mathbf{E}$$

Under the influence of an electric field, directed perpendicular to its direction of motion, the trajectory of the charged particle is a parabola. Usually, however, the deflection of the moving particle in the field is so small that we can approximate the trajectory of a moving charged particle in the electric field also by a circle of radius R_E, for which we can write.

$$eE = \frac{mv^2}{R_E}$$

Lorentz Force

A particle moving simultaneously in an electric and a magnetic field, will experience a net force, given by

$$\mathbf{F} = q\mathbf{E} + q\,(v \times \mathbf{B})$$

F is known as the Lorentz force.

This equation for **F** is valid for constant as well as for varying electric and magnetic fields.

Example–7 :

An electron, emitted by a heated cathode and accelerated through a potential difference of 2.0 KV, enters a region of a uniform magnetic filed of 0.15 T. The field is transverse to the initial velocity of the election. Find the radius of the trajectory of the electron.

Solution :

A charged particle, subjected to a uniform magnetic field, transverse to its initial velocity **v**, moves along an arc of a circle. The radius, R, of the circular arc, is obtained from the relation.

$$Bev = \frac{mv^2}{R}$$

$$\therefore \quad R = \frac{mv}{eB}$$

Now when a charged particle is accelerated through a potential V, it gains an energy eV which must be equal to its kinetic energy.

$$\therefore \quad eV = \frac{1}{2}mv^2$$

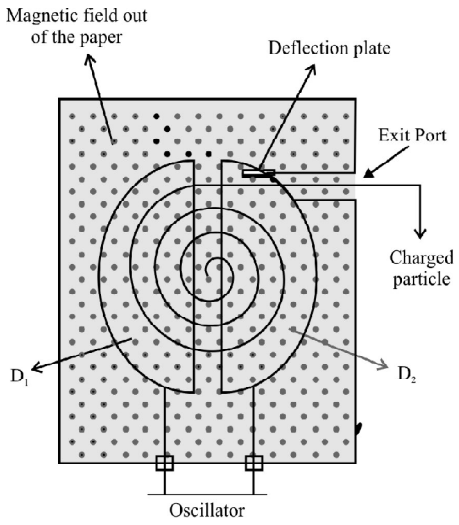
or $$v = \sqrt{\frac{eV}{m}}$$

$$\therefore \quad v = \left(\frac{2 \times 1.6 \times 10^{-19} \times 2 \times 10^{3}}{9.1 \times 10^{-31}} \right)^{1/2} m/s = 2.65 \times 10^{7} \text{ m/s}$$

Hence $$R = \left[\frac{9.1 \times 10^{-31} \times 2.65 \times 10^{7}}{0.5 \times 1.6 \times 10^{-19}} \right] m \simeq 1 mm$$

Cyclotron

Designed by Lawrence and Livingston, a cyclotron is based on principle of giving repeated accelarating pushes to a beam of charged particle while the particles are moving round in a circular path.



The particles are made to move in a circular path by applying a transverse magnetic field B.

We have the relation $$Bev = \frac{mv^2}{r}$$

$$\because \quad r = \frac{mv}{Be}$$

The time period, of one revolution of the particle, will be

$$T = \frac{2\pi r}{v} = \frac{2\pi mv}{Bev} = \frac{2\pi m}{Be}$$

Thus particles orbit around in their circular path, at a frequency, f, given by,

$$f = \frac{I}{T} = \frac{Be}{2\pi m}$$

The charged particles move in a circular chamber made up of two parts called the D's (dees). The particles, to be accelerated, are introduced into the arrangement near the centre of dees and a magnetic field transverse to their plane is applied. The dees are connected to an oscillator of frequency $\left(Be/2\pi m \right)$ and this maintains the alternating potential and ions in phase. Thus every time the particles return to the gap, between the dees, they experience an accelerating push and are consequently accelerated to very high energies. Every time the particle get accelerated its velocity increases and, in order to satisfy the equilibrium condition, it

must move in a circle of ever increasing radius. Thus the particle travel in a SPIRAL path. When the velocity is increased to a desired value, the particles are led outwords for their appropriate USE.

The 'final energy', to which the cyclotron can accelerate the charged particles is limited by the relativistic increase in mass with velocity.

Example–8 :

A cyclotron, in which flux density is 14000 gauss, is employed to accelerate the protons. How rapidly should the electric field between the dees be reversed? Given ($m_p = 1.67\times10^{-27}$ kg and $e = 1.6\times16^{-19}$C).

Solution :

We have

$$f = \frac{eB}{2\pi m}$$

$$\therefore \quad f = \frac{1.6\times10^{-19}\times14\times10^{3}\times10^{-4}}{2\times3.146\times1.67\times10^{-27}}\ \text{Hz} \qquad \left(\because\ 1G = 10^{-4}T\right)$$

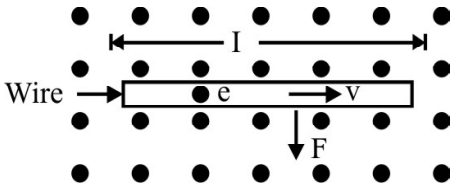
$$= 21.34\ \text{MHz}$$

Force on a Current Carrying Conductor in a Magnetic Field

Since a magnetic field exerts a force on a moving change, it is clear that a magnetic field will also exerts force on a conductor carrying a current.

Consider a straight wire, carrying a current i, placed in uniform magnetic field B. The wire is kept at right angles to the field. Let n be the number of free electrons per unit volume and let us suppose that they have a average drift velocity v in a direction perpendicular to **B**. The magnitude of the average force on each electron is given by

$$|\mathbf{F}'| = ev|\mathbf{B}|$$



Also the current density $|\mathbf{J}| = nev$

Therefore we have $v = \frac{|\mathbf{J}|}{ne}$

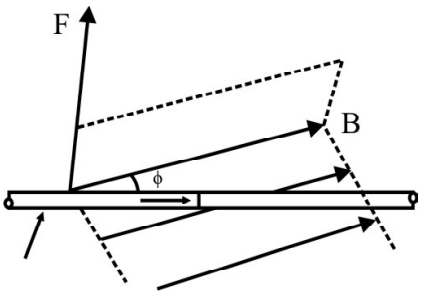
Hence $|\mathbf{F}'| = e|\mathbf{B}|\frac{j}{ne} = \frac{jB}{n}$

Total number of electron in a conductor of length L is AL × n

Since all these electrons are moving in the same direction, in a uniform magnetic field the force is in the same direction for all the free electrons.

Thus the force $|\mathbf{F}| =$ on the whole conductor is $(nAL)\times|\mathbf{F}'| = nAL\left(\frac{jB}{n}\right) = (Aj)LB$

$= iL|\mathbf{B}|$ $\quad [\because AJ = i]$

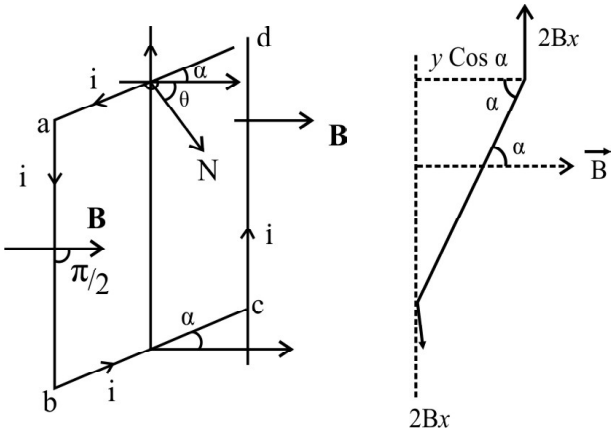


However, if the current is flowing at an angle θ, with respect to **B**, this result can be generalized as

$$\mathbf{F} = i(\mathbf{L}\times\mathbf{B})$$

Torque on a Current Carrying Coil in a Magnetic Field

As shown in the figure let us consider a coil abcd suspended in a uniform magnetic field **B**. Let the sides ab and cd, of the coil, have a length *x* and let sides bc and da have a length y. We denote by α, the angle which the plane of the coil, makes with the direction of the field. Current, flowing in the coil abcd, is denoted by i.



The force on the arm da is given by $iy\,B\sin(\pi-\alpha) = iy\,B\sin\alpha$. The direction of this force is in the plane of the coil and directed upwards.

Similarly the force on arm bd is $(iy\,B\sin\alpha)$ and is in the plane of the coil and directed down wards.

These two forces do not produce any effect on the coil as they are in the same plane and oppositely directed.

Similarly the magnitude of forces acting on the sides ab and cd is given by (i*x* B)

The direction of the force on side ab and cd is perpendicular to the plane of paper directed, respectively, outwards, and into, the plane of the coil.

Since these two forces are equal parallel and act in opposite directions, and have different line of action, they consitute a torque. This torque is given by

$\tau = i\,x\,B \times y\cos\alpha$ [$y\cos\alpha$ is the arm of the couple]

or $\tau = i\,AB\cos\alpha$ [A, the area of the coil, equals (xy)]

Le the angle, which the normal to the plane of the coil, makes with the direction of the field be θ. We than have

$$\alpha = \left(\frac{\pi}{2} - \theta\right) \text{ and } \cos\alpha = \sin\theta$$

Hence torque τ is given by $\tau = i\,AB\sin\theta$

If the coil has n turns than the torque becomes $\tau = ni\,AB\sin\theta$

Taking into consideration, the direction of the torque, we can write.

$$\boldsymbol{\tau} = ni\left(\mathbf{A} \times \mathbf{B}\right)$$

Denoting $ni\left(\mathbf{A}\right) = \mathbf{M}$, the equivalent magnetic dipole moment of the (rectangular) coil, we can write.

$$\boldsymbol{\tau} = \left(\mathbf{M} \times \mathbf{B}\right)$$

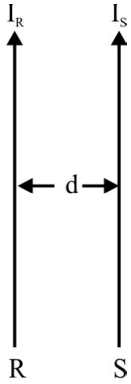
The tendency of this torque is to rotate the coil about its own axis and to bring it to its 'equilibrium position', the position in which **M** is parallel to **B**.

[The above expression is analogous to the expression for the couple on an electric dipole in an electrostatic field i.e. $\boldsymbol{\tau} = \left[\mathbf{P} \times \mathbf{E}\right]$

Force between two Parallel Long Current Carrying Wires

When two parallel current carrying wires are kept near to each other, each exerts a force on other. We can think of either one of the wirs as being situated in the magnetic field caused by the other current carrying wire. Hence there results a force between these current carrying conductors. To calculate this force we consider two long straight conductors R and S, carrying currents I_R and I_S, respectively, and seperated by a distance d. The magnetic field B_R, at the location of the conductor S, due to current I_R, flowing in conductor R, is given by.

$$B_R = \mu_0 \frac{I_R}{2\pi d}$$



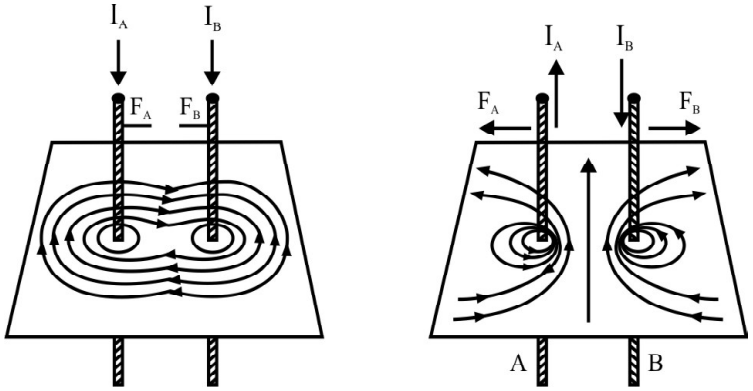
Due to this field the force on a length L of the (other) conductor, carrying a current I_S, is given by

$$F = B_R I_S L$$

Hence $F = \frac{\mu_0 IR}{2\pi d} \times I_S \times L = \frac{\mu_0 I_R I_S L}{2\pi d}$

The force, per unit length, therefore, equals $\left(\frac{\mu_0 I_R I_S}{2\pi d}\right)$

We represent below the field lines and direction of force, between two parallel curernt carrying conductors, carrying currents that are (i) LIKE and (ii) UNLIKE, currents.



Example–9 :

A uniform magnetic field, of 3000G, is established along the positive z axis. A rectangular loop, of sides 10 cm and 5 cm, carries a current of 12A. Find the torque on the loop, in the different cases shown in figure. Which cases corresponds to the position of equilibrium?

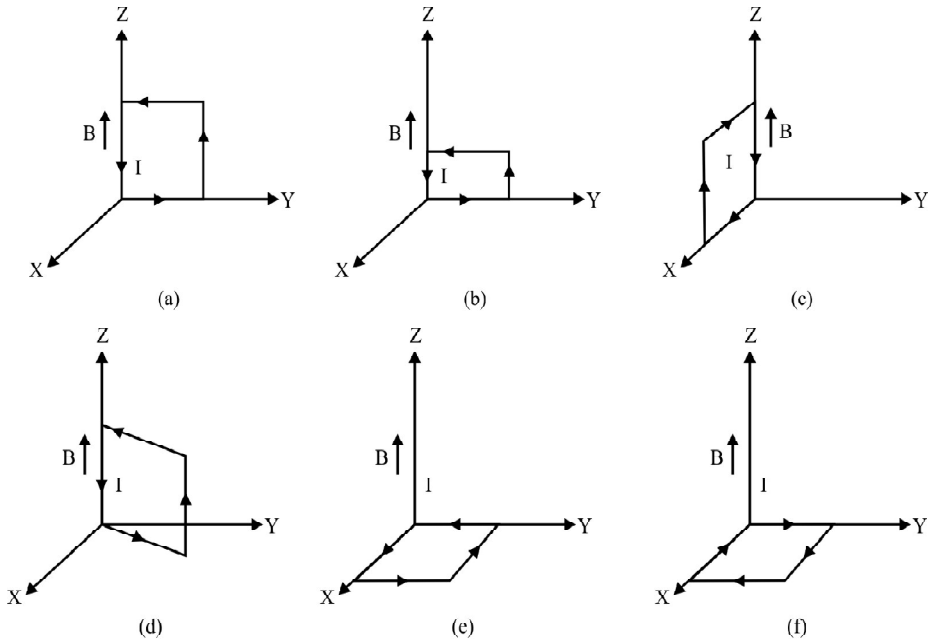
Solution :

The torque, on a current (= i) carrying loop, placed in a magnetic field **B**, is given by

$$\boldsymbol{\tau} = i(\mathbf{A} \times \mathbf{B})$$

where A is the area of the loop.

We use this result for the different cases shown here.



(a) $\tau = i\,AB\sin\theta$

$= i\,AB\sin 90^0 = i(AB)$

$\therefore \quad \tau = 12\times 0.10\times 0.05\times 30\times 10^{-2}\,N-m \qquad \left(\because\ 3000G = 3000\times 10^{-4}\,T\right)$

$= 1.8\times 10^{-2}\,Nm$

The direction of this torque is along the negative z axis.

(b) Same as in (a)

(c) 1.8×10^{-2} N–m along negative *x* direction.

(d) 1.8×10^{-2} (N–m), at an angle of 270^0, with the positive *x* direction.

(e) In this case all the four arms all perpendicular to the field. Hence the torque is zero.

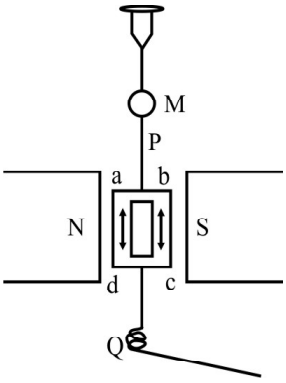
(f) The torque in this case in also zero.

It is clear that only the positions (e) and (f), for which the torque is zero, can be positions of equilibrium.

Moving Coil Galvanometer

A moving coil galvanometer makes use of the torque, on a coil, situated in a uniform magnetic field, to measure the current flowing in the wires of the coil. It consists of a rectangular coil, of N turns of wire, suspended by a thin phospher bronze wire P, between the pole pieces, N and S, of a permanent magnet which provides a uniform field. P carries a small concave mirror, M, to measure, with ease and procision

the small defletion of the coil, using a lamp and scale arrangement. Current enters and leaves the coil via suitable terminals, connected to P and a fine spring, Q.

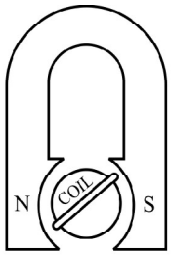


Couple, acting on the coil due to the passage of the current i, is $Ni\,AB\cos\theta$. This couple is balanced by the restoring couple, set up due to the twist in the suspension fibre. This equals $k\alpha$, where k is the torsional couple per unit twist.

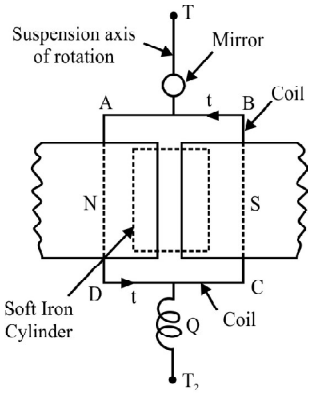
Hence $ni\,AB\cos\theta = k\alpha$

or $$i = \frac{k}{n\,AB\cos\theta}\alpha$$

As such, the instrument will, therefore, have a non linear scale. To achieve a linear scale we use a radial magnetic field. Such a field is obtained by using a permanent magnet with cylindercal concave pole pieces coaxial with the coil, and a soft iron cylinder, kept concentric with the coil. This configuration produces a field whose flux lines are (nearly) along the extension of radii of the soft iron cylinder in the air gap between it and the concave pole pieces.



(a) Top view of the pole pieces and the soft iron cylinder



(b) Front view of the rectangular coil

We then have $\theta = 0^0$ and $\cos\theta = 1$

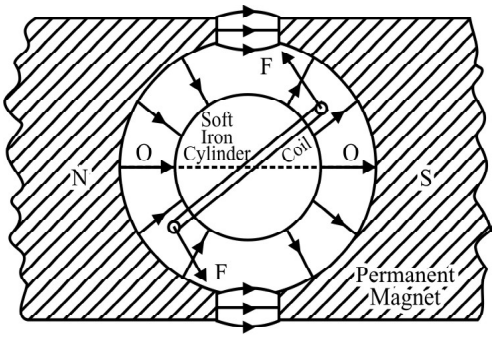
$\therefore \quad Ni\,AB = k\alpha$

or $$\therefore \quad i = \left(\frac{k}{NAB}\right)\alpha = c\alpha$$

The radial field set up thus enables us to get a linear scale. We define

$$\frac{d\alpha}{di} = \frac{I}{c} = \frac{NAB}{k}$$

as 'sensitivity' of the galvanometer.



Air Gap

Example–10 :

A rectangular coil, of area $5\times10^{-4}m^2$ and having 60 turns, is pivoted about one of the vertical sides. The coil is in a radial horizonal field of 90G. Find the torsional constant of the hair springs, connected to the coil, if a current of 0.2 mA, produces are angular deflection of 18^0.

Solution :

For equilibrium we have

$$in\,BA = c\theta$$

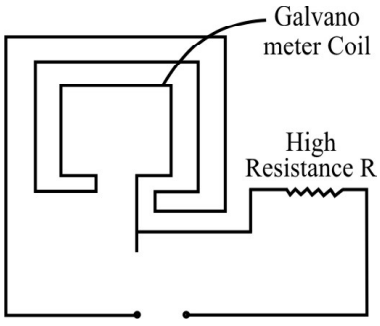
where the symbols have their usual meanings.

Hence $c = \dfrac{ni\,BA}{\theta}$

$$\therefore \quad c = \frac{60\times 0.20\times 10^{-3}\times 90\times 10^{-4}\times 5.0\times^{-4}}{18} \qquad \frac{(N-m)}{degree}$$

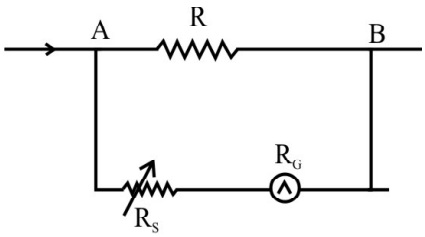
$$= 3\times 10^{-9}\ N\text{–}m \text{ per degree}$$

Voltmeter



A voltmeter is nothing but a high resistance (idealy infinite) galvanometer. It is used to measure the potential difference between two points in an electrical circuit. However to measure this p.d, a small current has to flow-through the voltmeter coil. The movement of the coil, in the magnetic field, can be calihrated to read the potential difference directly. However, the current drawn by the voltmeter coil for its functioning, reduces the current across the two points of the circuit and thus the potential difference (between two points) which was to be measured gets reduced. The voltmeter, therefore does not read the 'true value'. To minimize this error, the voltmeter has to have a high resistance. (so that only a very small fraction, of the current flowing between the two points, goes into it).

We can convert a galvanometer into a voltmeter of a desired range say (0 to V) volts. Let I_g be the maximum permissible current through the galvanometer. It is clear that if the p.d to be measured exceeds $I_g G$, we must connect a suitable resistance R_S, in series with the galvanometer coil. The value of R_S should be such that the current, through the galvanometer is still I_g. Thus R_S is given by



$$\frac{V}{R_S + G} = I_S$$

$$\therefore \quad R_S = \frac{V}{I_g} - G$$

Example–11 :

A moving coil galvanometer has a resistance of 10Ω and gives a full scale deflection for a current of 0.01 A. How can it be converted into a voltmeter that can read up to 150 volts?

Solution :

In order to convert the galvanometer into a voltmeter an external resistance R should be connected in series with G so that when 150 V are connected across the arrangement, the current through the galvanometer is only 0.01 A. Hence

Maximum voltage drop across $G = 0.01 \times 10V = 0.1\ V$

$\therefore$ Voltage drop across $R = (150 - 0.1)\,V = 149.9\,V$

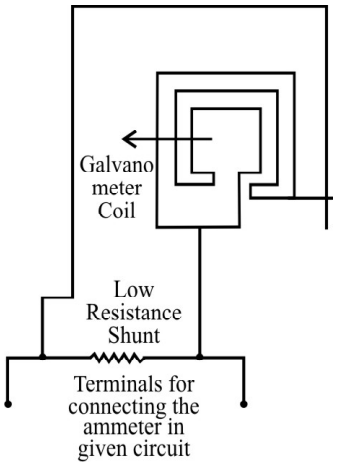
We thus have $0.01 \times R = 149.9$

$$\therefore \quad R = \frac{149.9}{0.01}\ \Omega = 14990\ \Omega$$

We, therefore, need to connect a resistance of $14{,}990\,\Omega$ in series with the galvanometer coil.

Ammeter

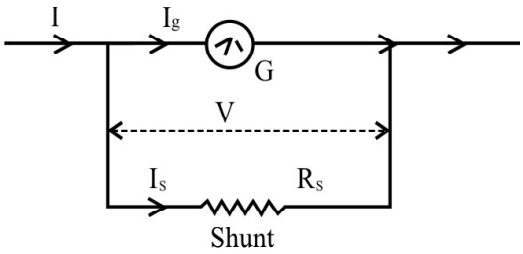
The ammeter is an instrument for meauring a current in an electical circuit. An ammeter is obtained by connecting a low resistance (called shunt) in parallel with a moving coil galvanometer. Hence the resistance of an ammeter is very low. To measure a current in a part of the circuit, the ammeter has to be connected in that part of the circuit and a current flows through the ammeter coil. This current carrying coil, situated in a magnetic field, consequently gets deflected and can be calibrated to read current directly in ampere's. The ammeter is connected in series with the given circuit path, thus increasing the resistance of that path by an amount equal to the resistance of the ammeter itself. This changes the current in the path, to be measured. Obviously this change should be as low as possible, so the resistance of an ammeter has to be very small indeed! Let us see low we can convert a galvanometer of resistance any G into an ammeter of range (0–I) A. To obtain this we have to shunt the galvanometer coil with a resistance R_S. Let I_G be the maximum (permissible) curent through the galvanometer coil.



We then have $V = I_g G = I_S R_S$

But $I_S = (I - I_g)$

$$\therefore \quad R_S = \frac{I_g}{I_S} G = \left(\frac{I_g}{I - I_g} \right) G$$



Example–12 :

A galvanometer, having 30 divisions, has a current senstivity of 20 μA/division. It has a resistance of 25 Ω. How will you convert into an ammeter reading up to 1A?

Solution :

$$\text{Current sensitivity} = 20 \ \frac{\mu A}{\text{div}} = 20 \times 10^{-6} \ \frac{A}{\text{div}}$$

$\therefore$ I_g = Current for full scale deflection = $20 \times 10^{-6} \times 30 A = 600 \ \mu A$

To convert into an ammeter, of range 0 to 1A, the shunt required is

$$S = \frac{I_g}{I - I_g} G = \left[\frac{6 \times 10^{-4} \times 25}{1 - 6 \times 10^{-4}} \right] \Omega$$

$$= 0.105 \Omega$$

We, therefore, need to put a resistance of 0.105Ω in parallel with the galvanometer coil.

Magnetism

Introduction

Historically the subject of magnetism was developed from the point of view of magnetic monopoles in a way completely analogous to the concept of (positive and negative) charges, and Coulomb's law, in electrostatics.

However, it was established later on, that monopoles do not exist. Even if we divide a bar magnet indefinitely, we do not end up with isolated poles; we always obtain a dipole. With developments in our understanding of the structure of matter, we now study magnetic phenomenon as one of the effects of current. However, Bar Magnets still have their own importance. We list some important facts in this context:

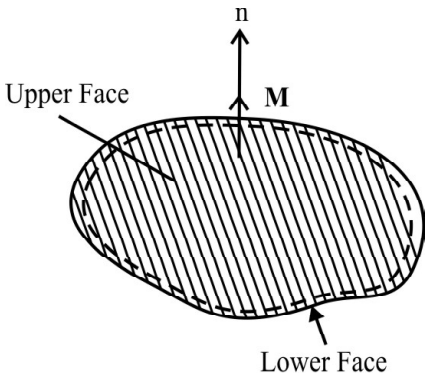
1. The earth apprears to behave like a magnet.
2. A freely suspended (isolated) bar magnet points along the North-South direction.
3. Like poles of two magnets, repel each other while unlike poles attract each other.

4. North and South poles of a bar magnet cannot be isolated.

5. We can make magnets from iron and its arrays.

A Current Loop as a Magnetic Dipole

As shown in figure let us consider a plane loop carrying a current. With reference to its upper face, let the current be anticlockwise, as already explained, this face, then, has a North polarity. This current would appear be clockwise if we look from the lower face of the coil. Hence this face will have a South polarity. The current loop, having a north polarity at one face, and a South polarity at the other face, can, therefore, be regarded as equivalent to a magnetic dipole.



The equivalent magnetic dipole moment, of a current carrying loop, can be shown to have value M, where.

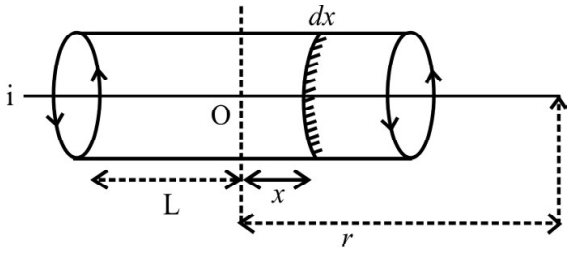
$$\mathbf{M} = iA$$

Here i is the current flowing through the loop and A is the area of the loop.

Similarity of a Current Carrying Solenoid with a Bar Magnet

We have already explained how to calculate the field of a solenoid at a point on its axis. We have seen that the field due to a small element of the solenoid, at a point, on its axis is

$$|\mathbf{dB}| = \frac{\mu_0 i a^2 (n dx)}{\left[2(r-x)^2 + a^2\right]^{3/2}}$$



For r >> a and r >> x, the denommeter becomes of the order of r^3.

$$\therefore \quad \mathbf{dB} \simeq \frac{\mu_0 i a^2 (n dx)}{2r^3}$$

Total field at (a far-off point) say P, would then be

$$B = \frac{\mu_0 n i a^2}{2r^3} \int_{-L}^{L} dx$$

or $$B = \frac{\mu_0 n i a^2}{2r^3}(2L)$$

The equivalent magnetic moment M of the solenoid is

$$M = \left[n(2L) \right] \times i \times \left[\pi a^2 \right]$$

Hence $B = \frac{\mu_0}{4\pi}\left(\frac{2M}{r^3}\right)$

Magnetism, developed from the point of view of the pole concept, showed that the magnetic field, on the axis line of a (short) bar magnet,is also given by the above mentioned result. Also, we have already seen the similarity of the magnetic field lines of a solenoid and a bar magnet. We can, therefore, think of a current carrying solenoid as the equivalent of a bar magnet.

A Magnetic Dipole in a Uniform Magnetic Field

Let us consider a small magnetic needle of magnetic moment **m**, and moment of inertia I, placed in a uniform magnetic field **B**. If the needle is suspended in the field, and allowed to oscillate, then it will experience a restoring couple $\tau = mB\sin\theta$.

Hence, we can write

$$I\frac{d^2\theta}{dt^2} = -mB\sin\theta$$

For small value of θ, we have $\sin\theta \approx \theta$.

$$\therefore \quad I\frac{d^2\theta}{dt^2} \simeq -mB\theta$$

or $$\frac{d^2\theta}{dt^2} = -\left(\frac{mB}{I}\right)\theta = -\omega^2\theta \qquad \left(\text{where } \omega = \frac{mB}{I}\right)$$

Thus the (small amplitude) oscillations (of a magnetic needle) are simple harnonic with a time period given by

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mB}}$$

$$\therefore \quad B = \left(\frac{4\pi^2 I}{mT^2}\right)$$

This expression is often used for an accurate determination of B.

Proceeding, exactly on the same lines, as we studied in electrostatics, we can obtain the magnetic potential energy, U_m, of a magnet, in a magnetic field. We then have

$$U_m = \int \tau(\theta)\,d\theta = \int mB\sin\theta\,d\theta$$

or $$U_m = -mB\cos\theta = -\mathbf{m}.\mathbf{B}$$

Terrrestrial Magnetism

The fact, that a freely suspended magnetic needle, at any point on the surface of earth, always point in the north south direction, shows that a magnetic field exists at all points on the surface of earth. The describe the field at any point we must determine (i) the plane in which earth's field lies and (ii) the intensity of earth's field **B**. This is generally achieved by determining the following three quantities.

(i) The 'declination' or the variation from true north.

(ii) The 'inclination' or DIP to the horizontal, of the total magnetic field of the earth.

(iii) The horizontal component of earth's magnetic field.

These three quantities are called the Magnetic Elements of Earth.

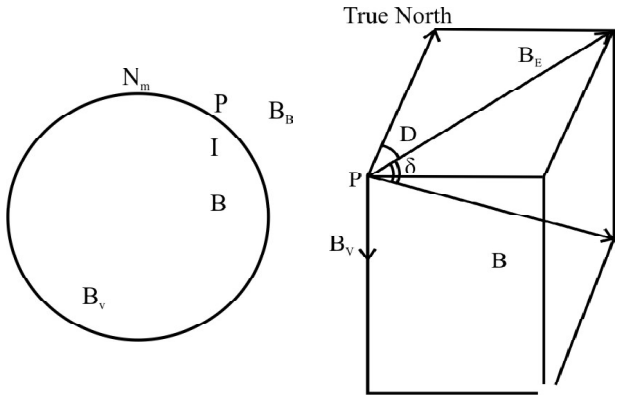
B – the earth's magnetic field

B_E – horizontal component of earth's field

B_v – the vertical component of earth's field

D – the declination

δ – angle of dip



(i) Declination, at a place, is the angle between the geographical and magnetic meridian there.

(ii) The DIP is the angle between the direction of the total magnetic intensity, and the horizontal component of earth's magnetic field.

(iii) The horizontal component of earth's field is the component of the earth's field along the horizontal.

Denoting by δ, the dip at a place and by B_E and B_V, the horizontal and vertical components of earth's field, we can write.

$$B_E = B\cos\delta$$

or $$B_V = B\sin\delta$$

[B = Total (Magnetic) intensity of earth's (magnetic) field.]

It follows that $\tan\delta = \dfrac{B_V}{B_E}$

Example–13 :

A ship is sailing due east, according to the Mariner's compass. If the declination of the place be 10^0 east, what is the true direction of the ship?

Solution :

As the declination is 10^0 east, the north as pointed by Mariner's compass, will be 10^0 east of true north. Hence the east, as shown by the compass, will be actually 10^0 ahead, i.e., 10^0 south of east.

Thus the true direction of the ship is 10^0 south of east.

Magnetisation and Magnetic Intensity

We have shown that the magnetic field due to a long solenoid at a point, on its axis near its centre, is given by

$$B_0 = \mu_0 n i$$

If the interior of the solenoid is filled will a magnetic substance then under the influence of $\mathbf{B}_0$, the equivalent dipoles, of the magnetic material, will set up a secondary field of their own ($\mathbf{B}_s$). The total field can then be written as

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_S$$

The field, $\mathbf{B}_S$, truns out to be proportional to the magnetisation vector **M** of the material and can be written as

$$\mathbf{B}_S = \mu_0 \mathbf{M}$$

The magnetisation vector **M**, of a material, equals the net magnetic dipole moment per unit volume, induced in it by the action of the (external) magnetic field ($\mathbf{B}_0$, in this case).

Detailed analysis of the problem show that it is convenient to introduce a new vector **H** which is defined as

$$\mathbf{H} = \left(\frac{\mathbf{B}}{\mu_0} - \mathbf{M} \right)$$

H can be related to the current flowing in the solenoid.

The vector **M**, is of course, dependent on the nature of magnetic material. We express this fact by writing

$$\mathbf{M} = \chi \mathbf{H}$$

χ is known as susceptibiety of the magnetic material

From above definition of **H**, we get $\mu_0 \mathbf{H} + \mu_0 \mathbf{M} = \mathbf{B}$

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) = \mu_0 (\mathbf{H} + \chi \mathbf{H})$$

$$\mathbf{B} = \mu_0 \mathbf{H} (1 + \chi)$$

Putting $1 + \chi = \mu_r$

We have $\mathbf{B} = \mu_0 \mu_r \mathbf{H}$

$\therefore \quad \mathbf{B} = \mu \mathbf{H} \qquad (\mu = \mu_0 \mu_r)$

[For vaccum $\mu = 1$ and **B** and **H** become one entity as it should be.]

Classification and Properties of Magnetic Materials

Magnetic materials are generally classified into three main categories. They are (i) Diamagnetic (ii) Paramagnetic and (iii) Ferromagnetic. We list below their characterstic properties.

1. Paramagnetic Materials

(a) In a magnetic field, the paramagnetic always move towards the region of increasing magnetic field.

(b) The magnetisation, **M**, varies linearty with **H**

$$\mathbf{M} = \chi \mathbf{H}$$

χ is khown as the susceptibility of the material and is positive for paramagnetics.

(c) χ of paramagnetic varies inversely with temperature.

$$\chi = \frac{c}{T}$$

where c is a constant and T is the absolute temperature. This relation, for paramagnetics, is known as CURIE's Law.

(d) Since $\mathbf{B} = \mu_0 (1 + \chi) \mathbf{H}$ and as $\chi > 0$ it follows that $\mathbf{B} > \mu_0 \mathbf{H}$

This implies that, for paramagnetics the magnetisms field, produces magnetisation in its own direction.

(e) Since $\mu_r = 1 + \chi_m$ it follows $\mu_r > 1$

It is interesting to know that the cause of paramagnetic behaviour can be associated with materials whose atoms / molecules have net (non-zero) permanent magnetic dipole moment associated with them. Paramagnetic materials are therefore, similar to 'Polar dielectrics'.

2. Diamagnetic Materials

(a) The diamagnetic move towards the region of decreasing field strength in a magnetic field.

(b) The relation, $\mathbf{M} = \chi \mathbf{H}$, also holds for diamagnetics. However $\chi < 0$ and is a small quantity.

(c) For diamagnetics χ is (almost) independent of temperature.

(d) As $\mathbf{B} = \mu_0 (1 + \chi)\mathbf{H}$ it follows that for diamagnetics $\mathbf{B} < \mu_0 \mathbf{H}$.

(e) It is easy to see that the permeability $\left(\mu_r = \mu_0 (1 + \chi)\right)$, of diamagnetics, is less than 1.

(f) Diamagnetism is a universal effect. However diamagnetism is a very week effects and it easily get masked when the material is a paramagnetics or a ferromagnetic.

(g) The atoms / molecules of a diamagnetics material do not have a net magnetic dipole moment associated with them. Diamagnetic materials are, therefore, similar to non-polar, dielectric.

3. Ferromagnetic Materials

(a) The magnetising effects, associated with ferromagnetics, are about 10^6 times as large as the corresponding effects for diamagnetics or paramganetics.

(b) Unlike diamagnetics or paramagnetics, **M** of a ferromagnetic is not directly proportional to **H**. **M** is now not a definate function of **H** but depends upon the previous magnetic, history internal and mechanical treatment of the speciman. This charactertics, of ferromagnetics, is known as Hystreresis.

(c) Curie's law for ferromagnetics is

$$\chi = \frac{c}{T - \theta}$$

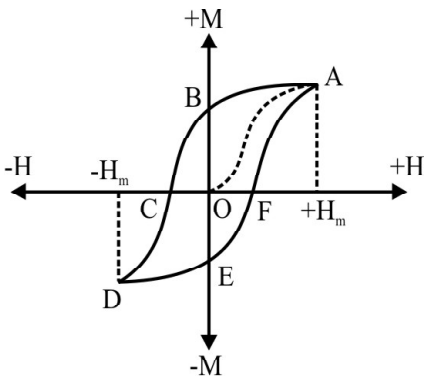
Here, θ is a magnetic transformation temperature, known as the curie temperature. For $T < \theta$, the substances behaves like a ferromagnetic and for $T > \theta$, the substance behaves like a paramagnetic.

Hysteresis Curve

We now discuss the 'Hysteresis behaviour' of ferromagnetic substances.

In the graph shown, point O corresponds to the unmagnetized state of the ferromagnetic material. The magnetisaction, M, at first increases with increasing field. Point A corresponds to the state when magnetisation reaches a satutation value (corresponding to a field H_m). If the applied field is now decreased, in steps, the points, corresponding to the curve AB, are obtained. It is seen that at point B the field has become zero but the (ferromagnetic) material is still (strongly) magnetised. This shows that **M** is lagging behind **H**. On reversing the field, and numerically increasing it in step, it is seen that |**M**| reaches zero, at some point C (corresponding to some reversed value of **H**). On increasing the field, in reverse direction, the portion CD

is obtained. On reversing the field again, we get the portion DE, EF and FA of the complete loop. If the field be now varied cyclically, between the values H_m and $-H_m$, the curve ABCDEFA is obtained. The curve ABCDEFA shows that **M** is always lagging behind **H**. This phenomenon (of **M** lagging behind **H**) is known as hysteresis. The curve (ABCDEFA) known as a hysteresis loop.



The point B, representing the retained or residual magnetisation, is known as the retentinty of the material. The (reverse) magnetic field at the point D, for which the material gets completely demagnetised, is known as the coearcivity.

PROBLEMS FOR PRACTICE

1. Find the magnetic field at the centre of a regnlar hexagon of side h canying a current i.

   Ans.: $\sqrt{3}\left[\frac{\mu_0 i}{\pi h}\right]$

2. An equilaterial triangle is formed from a uniform wire. If the current enters and leaves at the two corners, of a side of the triangle, show that the magnetic field at the centre of the triangle is zero.

3. Calculate the magnetic field, at a point distant 2m, on the axis of a circular coil, of radius I mm, carrying a current of 0.5 A.

   Ans.: 0.393×10^{-15} wb/m^2

4. A circular coil, of radius 2 cm, is concentric and coplaner with a circular coil of radius 7cm. Each coil has 100 turns. With a current of 5A in thelarger coil, find the current needed, in the smaller coil, to give zero magnetic field at the common centre of the two coils.

   Ans.: 143 A in the opposite direction

5. Considering two parallel co-axial circular coils, of equal radius R, and equal number of turns N, carrying equal currents in the same direction, and seperated by a distance R, show that the field on the axis, around the midpoint between the two coils, is (nearly) uniform over a distance that is small compared to R; and its magnitude is given by

   Ans.: $B \approx \left(0.72\ \frac{\mu_0 iN}{R}\right)$

6. A solenoid has 400 turns, of wire wound on a cylinder of radius 5 cm and length 40 cm. Calculate the magnetic field at the (a) midpoint of the solenoid and (b) a point, on the axis, 2cm from one end, of the solenoid, when a current of 1A flows through it.

   Ans.: (a) $1.219\times10^{-3}\ wb/m^2$, (b) $0.3905\times10^{-3}\ wb/m^2$

7. Two short solenoids, each of radius 6cm and length 13 cm, uniformly wound with 30 turns of wire per cm, are placed co-axially with a gap of 6cm between then. If these are connected in series. FInd the magnetic field, at the centre of the gap, due to a current of 1A through the combination.

   Ans.: $1.8\times10^{-2}\ wb/m^2$

8. A long wire, carries a current of 20A along the axis of a long solenoid. The field due to the solenoid is 4mT at a point distance 3cm from its axis. Find the resultant field at this point.

   Ans.: 4.2 mT, 18.4^0 with the axis of the solenoid

9. An electron beam, passes through a transverse magnetic field of 2×10^{-3} T, and an electric field (also transverse) of 3.4×10^{4} Vm^{-1}, both acting simultaneously. If the path of the beam remains undeviated. Calculate the speed of the electrons.

   Ans.: 7×10^{7} m/s

10. After being accelerated through a potential difference of 10,000 votl, an alpha particle moves in a circle of radius 50 cm, in a magnetic field. Calculate the magnitude of the field.

    (Given $M_{\alpha} = 6.68 \times 10^{-27}$ kg)

    Ans.: 4.09×10^{-2} T

11. Deutrons are to be accelerated in a cyclotron. The value of the uniform magnetic field is $4.4\ {wb}/{m^2}$.

    Find the period of revolution of the deutron (given mass of the deutron is 3.3×10^{-27} kg).

    Ans.: 2.946×10^{-8}s

12. If protons are to be accelerated in a cyclotron (i) What should be the frequency of the alternating potential difference applied. (ii) How much energy will a proton acquire after 50 revelutions? The p.d. across the dees is 10^{5} volts and the applied field is $1.5\ {wb}/{m^2}$. Given $m_p = 1.1\times10^{-27}$ Kg.

    Ans.: (i) 2.24×10^{7} Hz, (ii) 1.6×10^{-12} J

13. Two straight wire, each 10 cm long, are parallel to one another and seperated by a distance of 2 cm. They carry currents, of 30 A and 40 A, respectively. calculate the force experienced by either of the wire.

    Ans.: 1.2×10^{-3}N

14. Find the force in kgwt, per decimeter length, between two conductors, spaced 4 cm apart, and carrying currents of 2000 A and 4000 A, respectively. (Take of $g \simeq 10\ ms^{-2}$)

    Ans.: 1.2×10^{-3}N

15. A 100 turn, closely wound circular coil, of mean radius 10cm, carries a current of 3.2 A. The coil is placed in a vertical plane and is free to rotate about a horizontal direction which coinsides with its diameter. A uniform magnetic field, of 2T, in the horizontal direction, exists such that, initially, the axis of the coil is in the direction of the field. The coil rotates through an angle of 90^0 under the influence of the magnetic field. Calculate the torque in the initial and final position of the coil.

    Ans.: zero, 20 N–m

16. A rectangular coil, of sides 8 cm and 6 cm, having 2000 turns and carrying a current of 200 mA, is placed in a uniform magnetic field of 0.2 directed along the positive *x* axis. Calculate the maximum torque the coil can experience.

    Ans.: 0.384 N–m

17. A regtangular coil, of area $5.0\times10^{-4}m^2$, and 90 turns, is free to rotate about one of its vertical sides. The coil is in a radial magnetic field of 60×10^{-4} T. Calculate the torisonal constant of the springs, attached to the coil, if a current of 0.20×10^{-3} A produces on angular displacement of $\pi/10$ radins.

Ans.: 3×10^{-9} (N–m) per Degree

20. A glavanometer is placed in a radial magnetic field of $9.2\,wb/m^2$. The coil of the galvanometer has 200 turns and its area is $1.6\times10^{-4}m^2$. The restoring torsional constant, of the suspension fibre, is 10^{-6} Nm/degree. Calculate the maximumm current that can be measured by this galvanometer if the scale can accomodule up to 45^0 deflection.

Ans.: 7×10^{-4}A

21. A rectangular coil, of area 10^2 m^2, is suspended in a radial magnetic field $5\times10^{-2}\,wb/m^2$. What current should be made to pass through it to keep it in equilibrium at an angle of 30^0 with the field? [Given that the restroing couple per unit twist of the suspension fibre is 10^{-8} (N–m/degree)].

Ans.: 6×10^{-4}A

22. A voltmeter, which deflects full scale for a potential difference of 5.0 V, across it is to be made by connecting a resistance R in series with a galvanometer. The $80\,\Omega$ galvanometer deflects full scale for a potential of 20 mV across it. Calculate R.

Ans.: $19.92\,k\Omega$

23. A galvanometer, with a resistance of $12\,\Omega$, shows full scale deflection for a current of 0.0025 A. How can it be converted into a voltmeter of range 0 to 10V?

Ans.: Connect a $3988\,\Omega$ resistor in series.

24. A 120V, d.c. source, is connected across series combination of a large resistance R and voltmeter, of resistance $10000\,\Omega$. If the voltmeter reads 4V, calculate the value of R.

Ans.: $290\,k\Omega$

25. A galvanometer needs a current of 1 mA for full scale deflection. What is the resistance of the shunt needed to convert into an ammeter reading up to 1A.

Ans.: $5\times10^{-3}\,\Omega$

26. A galvanometer with a resistance of $12\,\Omega$ shows full scale deflection for a current of 2.5 mA. How will you convert it an ammeter of range 0–7.5 A?

Ans.: $0.004\,\Omega$

27. A galvanometer of resistance 20 Ω needs a current of 50 mA for full scale deflection. How it can be converted into an ammeter of range (0–10)A.

Ans.: 0.1 Ω

28. In the magnetic meridian of a place P, the horizontal compoent of the earth's magnetic field is 0.26 gauss and the dip angle is 60^0. What is the total magnetic field of earth at this location?

29. The horizontal component of earth's magnetic field, at a certain place is 3×10^{-5}T and the direction of the field is from geographic south to geographic north. A very long cylinder straight conductor is carrying a stready current of 1A. What is the force per unit length on it when it is placed on a horizontal table and the direction of the current is

    (i) east to west and

    (ii) south to north

    Ans.: (i) 3×10^{-5} Nm^{-1} (ii) zero

30. A current of 2A flows through 10^4 turns of an insulated wire, wound uniformly around an iron anclor ring of radius 0.1m. If permeability of iron is 2000, calculate the magnetic flux through the ring.

    Ans.: 80 SI Units

QUESTION BANK

Key Learning Points

1. Biot and Savart law is mathematically expressed by the equation.

$$|d\mathbf{B}| = k\frac{id\ell \sin\theta}{r^2}$$

The constant of proportionality depends on the system of units. In the S.I system of units $K = \frac{\mu_0}{4\pi}$

where μ_0 is a constant of free space and its value is $4\pi \times 10^{-7}\ \frac{\text{weber}}{\text{ampere} - \text{meter}}$.

2. We can write expression for d**B** in vector rotation as

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\ \frac{id\ell \times \mathbf{r}}{r^3}$$

3. The magnetic field due to a infinite straight wire carrying a current i is given by

$$\mathbf{B} = \frac{\mu_0 i}{2\pi r}\left(\frac{wb}{m^2}\right)$$

4. The magnetic field, due a circular coil of radius a, carrying a current i at its centre is given by

$$\mathbf{B} = \frac{\mu_0 i}{2a}\left(\frac{wb}{m^2}\right)$$

and at a point on the axis of the coil at a distance *x* from its centre is given by

$$\mathbf{B} = \frac{\mu_0 i}{2}\ \frac{a^2}{\left(a^2 + x^2\right)^{3/2}}\left(\frac{wb}{m^2}\right)$$

5. The magnetic field due to a straight solenoid at any point P (but near the end points) given by

$$\mathbf{B} = \frac{\mu_0 Ni}{L}\left(\frac{wb}{m^2}\right)$$

6. At far off points, a circular coil behaves as a dipole of moment **m** wheere m equals the product of the current flowing and the area of the coil. The value of **m** must be regarded as directed normal to the plane of the coil and in the sense of a right handed scrow rotated in the direction of the flow of current.

7. We can generalize the above equality for a plane current loop of any orbitrary shape.

8. According to Ampere's circuital thorem the line intergral of magnetic field around any closed cuve is equal to μ_0 time the total current I enclosed by the closed curve viz.

$$\oint \mathbf{B}.\mathbf{d}\ell = \mu_0 I$$

9. One can use Ampere's circuital a theorem to calculate the magnetic field due to a long straight solenoid and a toreidal solenoid.

10. Ampere's circuital theorem

$$\oint \mathbf{B}.\mathbf{d}\ell = \mu_0 I$$

straight forwardly tells us that mangetic field is a non conservative field.

11. The force experienced by a charged particle when it is moving with a velocity v making angle θ with the direction of **B** is given by

$$\mathbf{F}_B = q(\mathbf{v} \times \mathbf{B})$$

12. The force experienced by a charged particle when it is moving in an electric field is given by **F**=e**E** and is indepedent of the velocity with which the charged particle is moving.

13. The forced acting on a charged particle simultaneously being acted upon by a electric as well as a magnetic field as given by

$$\mathbf{F}_L = q\mathbf{E} + q(\mathbf{v} \times \mathbf{B})$$

and is known as Lorentz force.

14. Mathematically the magnetic field vector **B** is defined through the relation

$$\mathbf{B} = \frac{\mathbf{F}}{qv \sin\theta}$$

The unit of **B** is newton per coulomb meter per second or weber/meter^2 or tesla.

15. A current carrying conductor placed in a magnetic field experiences a force given by

$$\mathbf{F} = i(\mathbf{L} \times \mathbf{B})$$

16. A plane rectangular coil of N turns, carrying a current I, experiences a torque when placed in a uniform magnetic field which is given by

$$\boldsymbol{\tau} = (\mathbf{m} \times \mathbf{B})$$

17. Force between two parallel long current carrying wires is given by

$$\mu_0 \frac{I_R I_S L}{2\pi d}$$

where I_R and I_S are the current flowing respectively in two long wires of length L and seperated by a distance d.

18. A moving coil galvanometer is essentially based on the fact that a rectangular coil placed in a uniform magnetic field experiences a couple. When a current is passed through it this couple is balanced by a restoring couple due to the twist in suspension wire of the galvanometer.

i.e. $Ni\,AB\cos\theta = k\,\theta$

or $$i = \frac{k\theta}{NAB\cos\theta}$$

19. We use a radial magnetic field to provide a linear scale for the measurements by the galvanometer.

20. A voltmeter is a converted galvonometer with a very high resistance (ideally infinite) in series with the galvanometer.

21. An ammeter is a shunted low resistance (ideally tending to zero) galvanometer

22. The earth behaves as a magnet.

23. A freely suspended bar magnet points in the north south pole direction.

24. Like poles repel each other whiile unlike poles attract each other.

25. North and south poles of a bar magnet can not be isolated.

26. A current loop behaves as a magnetic dipole.

27. The magnetic field lines of a bar magnet and a solenoid have a similar pattern.

28. We define the following three quantities to understand the behaviour of earth's magnetism.

(i) The declination

(ii) The dip

and (iii) The horizontal component of earth's magnetic field.

29. Just as in electrostatic we define a magnetisation vector **M** in the study of the behaviour of magnetic materials.

30. We also define a vector **H** by the relation

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - M$$

which further enable us to define a relation.

$$\mathbf{B} = \mu_0 \left(\mathbf{H} + \mathbf{M} \right)$$

31. The magnetic materials can be classified as

(i) Paramagnetics

(ii) Diamagnetics

and (iii) Ferromagnetics

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Biot-Savart Law*

1. A straight wire, carrying a current I, is located along the y-axis. The magnetic field, dB, at a (field) point (a, b, c) due to a general element, $d\ell$, of this straight wire, can be written as

(1) $$d\mathbf{B} = \frac{\mu_0}{4\pi} I \left[\frac{\left(dy\hat{j}\right) \times \left(a\hat{i} + (b-y)\hat{j} + c\hat{k}\right)}{\left(a^2 + (b-y)^2 + c^2\right)^{3/2}} \right]$$ (Correct)

(2) $$d\mathbf{B} = \frac{\mu_0}{4\pi} I \left[\frac{\left(dy\hat{j}\right) \times \left(a\hat{i} + b\hat{j} + c\hat{k}\right)}{\left(a^2 + b^2 + c^2\right)^{3/2}} \right]$$ (Incorrect)

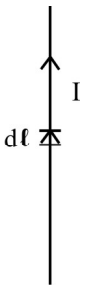
(3) $$d\mathbf{B} = \frac{\mu_0 I}{4\pi} \left[\frac{\left(dy\hat{j}\right) \times \left(a\hat{i} + (b-y)\hat{j} + c\hat{k}\right)}{\left[a^2 + (b-y)^2 + c^2\right]} \right]$$ (Incorrect)

(4) $$d\mathbf{B} = \frac{\mu_0 I}{4\pi} \left[\frac{\left(dy\hat{j}\right) \times \left(a\hat{i} + b\hat{j} + c\hat{k}\right)}{\left(a^2 + b^2 + c^2\right)} \right]$$ (Incorrect)

Correct answer: (1)

Solution:

We need to use Biot-Savart Law.



$$d\mathbf{B} = \frac{\mu_0 I}{4\pi} \frac{d\ell \times \mathbf{r}}{|\mathbf{r}|^3}$$

Here $d\ell$, general current element, can be considered to be located at a general point (0, y, 0), on the y–axis. Also since the wire lies along the y–axis, we can write $d\ell = \left(dy\right)\hat{j}$

The vector **r**, in the Biot-Savart law, is the position vector of the field point (a, b, c), with respect to the point (0, y, 0) where the current element is located. Hence

$$\mathbf{r} = (a-0)\hat{i} + (b-y)\hat{j} + (c-0)\hat{k}$$

and $|\mathbf{r}| = \left[a^2 + (b-y)^2 + c^2\right]^{1/2}$

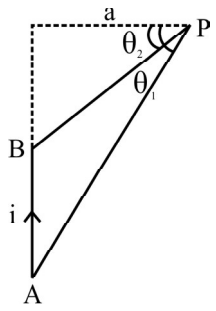
$\therefore \quad d\mathbf{B} = \dfrac{\mu_0}{4\pi} I \dfrac{\left(dy\hat{j}\right) \times \left(a\hat{i} + (b-y)\hat{j} + c\hat{k}\right)}{\left(a^2 + (b-y)^2 + c^2\right)^{3/2}}$

Hence option (1) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Magnetic Field due to Straight Current Carrying Conductor*

2. A straight wire, AB carries a current I; the ends of the wire subtends angles θ_1 and θ_2, at point P, as shown in figure. The magnetic field, at point P, is given by



(1) $\dfrac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$ (Correct)

(2) $\dfrac{\mu_0 I}{4\pi a}(\sin\theta_1 + \sin\theta_2)$ (Incorrect)

(3) $\dfrac{\mu_0 I}{4\pi a}(\cos\theta_1 - \cos\theta_2)$ (Incorrect)

(4) $\dfrac{\mu_0 I}{4\pi a}(\cos\theta_1 + \cos\theta_2)$ (Incorrect)

Correct answer: (1)

Solution:

The magnetic field, at a distance 'a', from a current carrying straight wire, is given by

$$B = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 + \sin\theta_2)$$

Here $\theta_1 = \theta_1$

$$\theta_2 = -\theta_2$$

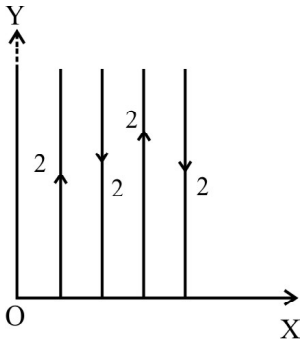
$$\Rightarrow B = \frac{\mu_0 I}{4\pi a}\left(\sin\theta_1 - \sin\theta_2\right)$$

Hence correct option is (1)

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Magnetic Field due to Straight Current Carrying Conductor*

3. Equal currents, of value 1A, are allowed to flow through a sequence of long wires, parallel to y–axis and located at, x = 1m, 2m, 4m and so on. If the currents flow in opposite directions in alternate wires, the value of magnetic field at the origin, O, is



(1) $\frac{\mu_0 i}{3\pi}$ along z–axis (Correct)

(2) $\frac{3\mu_0 i}{4\pi}$ along x–axis (Incorrect)

(3) $\frac{i}{4\pi/\mu_0}$ along z–axis (Incorrect)

(4) $\frac{\mu_0 i}{3\pi}$ along x–axis (Incorrect)

Correct answer: (1)

Solution:

We know that the field at O, due to current i through one wire, is

$$|\mathbf{B}| = \frac{\mu_0 i}{2\pi d}$$

Hence, for the whole sequence of wires, the desired field is

$$\mathbf{B} = \frac{\mu_0 i}{2\pi}\left[\left(1+\frac{1}{4}+\frac{1}{16}+.....\right)\hat{k}-\left(\frac{1}{2}+\frac{1}{8}+.....\right)\hat{k}\right]$$

$$= \frac{\mu_0 i}{2\pi}\left[\frac{1}{(1-4)}\hat{k}-\frac{1}{2}\frac{1}{\left(1-\frac{1}{4}\right)}\hat{k}\right]$$

$$= \frac{\mu_0 i}{3\pi}\,\hat{k}$$

Hence option (1) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Easy *Mangetic Field due to Current Carrying Conductor*

4. A conductor, in the shape of a square of edge length ℓ = 0.4m; carries a current of 10A. Now this conductor is formed into a single circular turn and carries the same current. The difference of magnetic fields, produced in two cases, at the respective centres, will be.

(1) 10.5 μT (Incorrect)

(2) 24.7 μT (Incorrect)

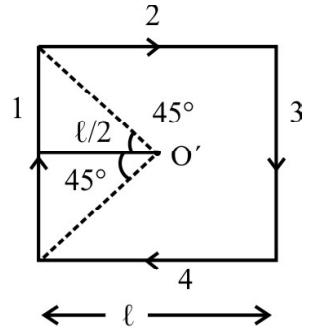
(3) 3.6 μT (Correct)

(4) 20.4 μT (Incorrect)

Correct answer: (3)

Solution:

Magnetic field at the centre 'O' of the current carrying square loop.



$$\mathbf{B} = \mathbf{B_1}+\mathbf{B_2}+\mathbf{B_3}+\mathbf{B_4}$$

$$= 4 \times \mathbf{B_1}$$

$$= 4\times\frac{\mu_0 I}{4\pi\left(\ell/2\right)}\left(\sin 45^0+\sin 45^0\right)$$

$$B = \frac{2\sqrt{2}\mu_0 i}{\pi\ell} = 28.3\ \mu T\,(\text{inward})$$

Now, when wire is bent-into circular loop of single turn.

$$B = \frac{\mu_0 i}{2a}$$

where 'a' is the radius of the circular loop.

$$2\pi a = 4\ell \Rightarrow a = \frac{2\ell}{\pi}$$

$$\therefore \quad B' = \frac{\pi\mu_0 i}{2\ell} = 24.7\ \mu T\,(\text{inward})$$

Hence difference of magnetic fields $\Delta B = (B - B')$

$$= 3.6\ \mu T$$

Hence option (3) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Magnetic Field due to Straight Conductor*

5. Two long parallel transmission lines, 40.0 cm apart carry 25A and 75A current. The ratio of the distances (from the conductor carrying current of 25A), where the net mangetic field is zero, when the two lines carry current in (i) same direction and (ii) in opposite diretions, will be:

(1) 1 : 1 (Incorrect)

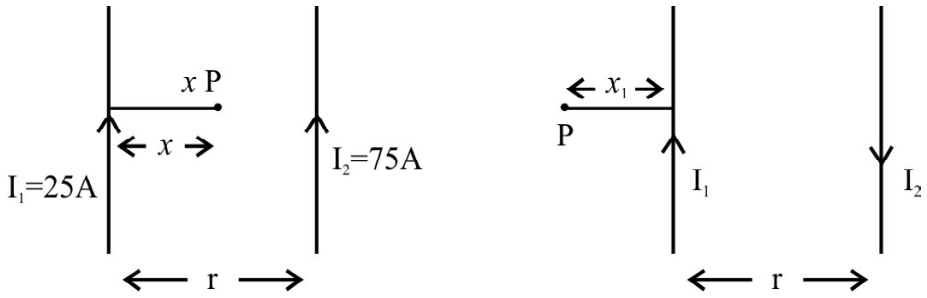
(2) 2 : 1 (Incorrect)

(3) 1 : 2 (Correct)

(4) 3 : 1 (Incorrect)

Correct answer: (3)

Solution:



Magnetic field at point P

$$B = \frac{\mu_0 i}{2\pi x} - \frac{\mu_0 I_2}{2\pi(r-x)} = 0$$

$$\frac{I_1}{x} = \frac{I_2}{(r-x)}$$

$$\frac{25}{x} = \frac{75}{r-x}$$

$$3x = r - x$$

$$4x = r = 40.0$$

$$\Rightarrow x = 10 \text{ cm}$$

$$\therefore \text{ Ratio } \frac{x}{x_1} = \frac{10}{20} = 1:2$$

$$\frac{I_1}{x_1} = \frac{I_2}{(r+x_1)}$$

$$\frac{I_1}{I_2} = \frac{x_1}{(r+x_1)}$$

$$\frac{1}{3} = \frac{x_1}{(40+x_1)}$$

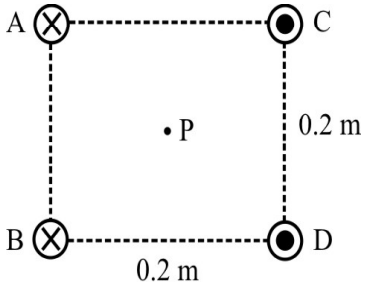
$$x_1 = 20 \text{ cm}$$

Hence option (3) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Magnetic Field due to a Current Carrying Conductor*

6. Four long parallel conductors carry equal currents of 5.0A. The direction of current is into the plane of the page at points A and B, and out of the page at C and D. The magnitude and direction of the magnetic field at point P, located at the centre of the square, will be:



(1) 20 μT downward (Correct)

(2) 10 μT upwards (Incorrect)

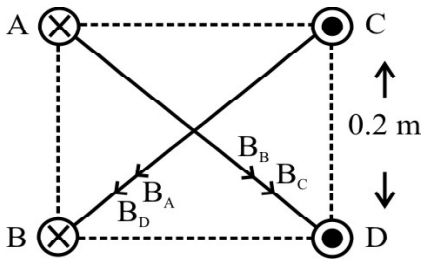
(3) 10 μT downward (Incorrect)

(4) 20 μT upwards (Incorrect)

Correct answer: (1)

Solution:

$$B_A = B_B = B_C = B_D = \frac{\mu_0 2I}{4\pi r} = \frac{10^{-7} \times 2 \times 5\sqrt{2}}{0.2} = 5\sqrt{2} \times 10^{-6} T$$



(Here I = 5A, $r = \frac{a}{\sqrt{2}} = \frac{0.2m}{\sqrt{2}} = 0.14m$)

Net magnetic field at point P.

$$B = \sqrt{(B_A + B_D)^2 + (B_B + B_C)^2} = \sqrt{(2B_A)^2 + (2B_B)^2}$$

$$= 2 \times \sqrt{2}\ B_A$$

$$= 2 \times \sqrt{2} \times 5\sqrt{2} \times 10^{-6}\ T$$

$$= 20\,\mu T \text{ downward}$$

Hence option (1) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Difficult *Magnetic Field due to Straight Current Carrying Conductor*

7. A regular polygon of n-sides is formed by bending a wire of total length $2\pi r$. If this wire now carries a current i, the magnetic field B, at the centre of this polygon, would be

(1) $\frac{\mu_0 i n^2 \sin^2(\pi/n)}{2r^2 \pi \cos(\pi/n)}$ (Incorrect)

(2) $\frac{\mu_0 i n^2 \cos^2(\pi/n)}{2\pi^2 r \sin(\pi/n)}$ (Incorrect)

(3) $\frac{\mu_0 i n^2 \sin^2(\pi/n)}{2\pi^2 r \cos(\pi/n)}$ (Correct)

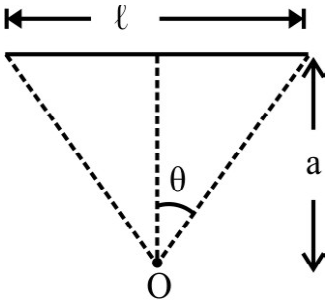
(4) zero (Incorrect)

Correct answer: (3)

Solution:

Length of each side $\ell = \frac{2\pi r}{n}$

and (half) angle $\theta = \frac{\pi}{n}$



Perpendicular distance $(a) = \frac{\ell}{2}\cot\theta = \frac{\pi r}{n}\cot\theta$

Mangetic field $B = n \times \frac{\mu_0 i}{4\pi a} 2\sin\theta$

$$= \frac{\mu_0 i\left(2n^2\right)\sin\theta}{4\pi(\pi r)\cot\theta}$$

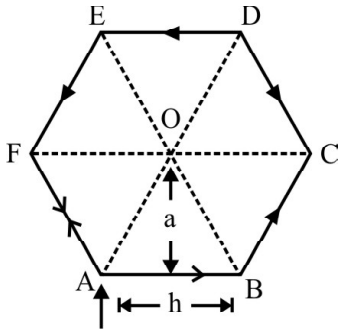
$$= \frac{\mu_0 i\, n^2 \sin^2\left(\pi/n\right)}{2\pi^2 r\cos\left(\pi/n\right)}$$

Hence option (3) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Difficult *Magnetic Field due to a Straight Conductor*

8. A regular hexagon, of side h, is carrying a current i as shown, the magnetic field at the centre of hexagon is



(1) $6\left[\frac{\mu_0 i}{\pi h}\right)$ (Incorrect)

(2) $6\left[\frac{\mu_0}{4\pi}\frac{i^2}{h}\right)$ (Incorrect)

(3) $\frac{\mu_0 2}{2\sqrt{3}\,\pi h}$ (Incorrect)

(4) $\sqrt{3}\left(\frac{\mu_0 2}{\pi h}\right)$ (Correct)

Correct answer: (4)

Solution:

The magnetic field (at the centre) due to all the six wires, forming a hexagon, are in the same direction, perpendicular tothe plan of page and directed outwards.

Due to a finite wire the magnetic field, at a point distant a from it, is

$$B = \frac{\mu_0 2}{4\pi a}\left[\frac{\ell_1}{\ell_1^2 + a^2} + \frac{\ell_2}{\sqrt{\ell_2^2 + a^2}}\right]$$

where ℓ_1 and ℓ_2 are the distances of the two ends of the wire, from the foot of perpendicular, drawn from the field point on the wire.

In the case of the hexagon we have

$$\ell_1 = \ell_2 = \frac{h}{2} \text{ and } a = \left(\frac{h}{2}\right)\tan 60^0 = \frac{\sqrt{3}}{2}h$$

The magnitude of the field, at 0, due to each wire is

$$B' = \frac{\mu_0 2}{4\pi\left(\frac{\sqrt{3}}{2}h\right)}\left[\frac{\frac{h}{2}}{\left(\frac{h}{2}\right)^2 + \left(\frac{\sqrt{3}h}{2}\right)^2} + \frac{\frac{h}{2}}{\left(\frac{h}{2}\right)^2 + \left(\frac{\sqrt{3}h}{2}\right)^2}\right] = \frac{\mu_0 2}{2\sqrt{3}\,\pi h}$$

Hence the total field at centre of hexagon is 6B'

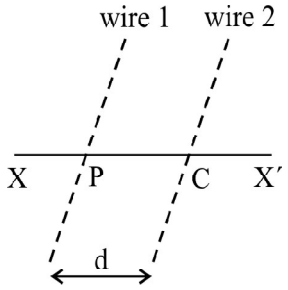
or $$B = 6B' = \sqrt{3}\left[\frac{\mu_0 2}{\pi h}\right]$$

Hence option (4) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Difficult *Magnetic Field due to Long Straight Conductor*

9. Two long parallel wires are at a distance d apart. They carry stready equal currents flowing out of the plane of paper. The graph which would represent the variation of their magnetic field, along the line XX´ ; would be the graph labelled as graph

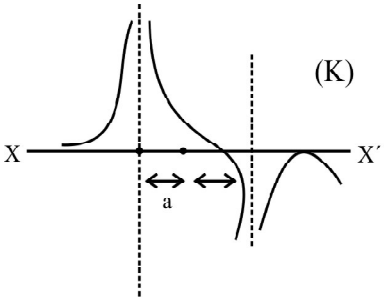


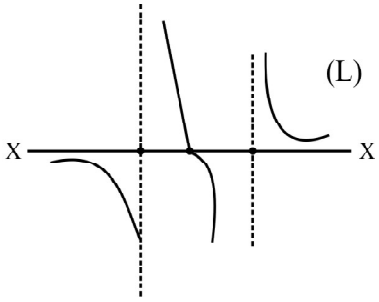
(1) K (Incorrect)

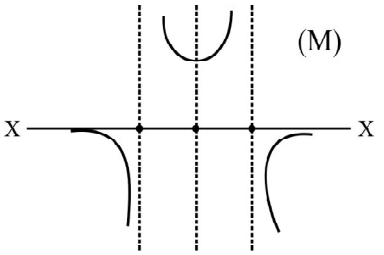
(2) L (Correct)

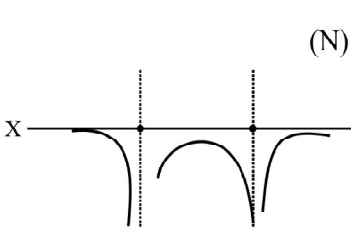
(3) M (Incorrect)

(4) N ncorrect)







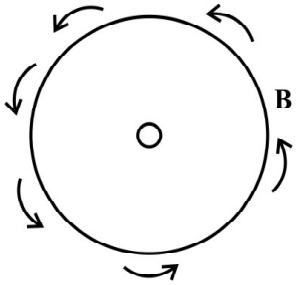


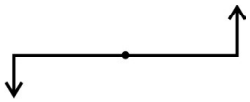
Correct answer: (2)

Solution:

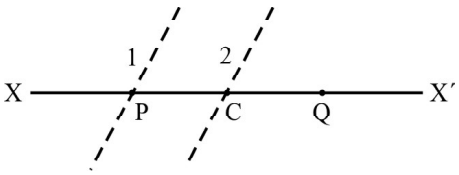
If the current flow out of the paper, the magnetic field at points to the right of the wire, will be upwards and to the left will be downwards (in the plane of the peper).

Net magnetic field at point O:





Magnetic field, in the region QX′ will be upward and positive, because all points, lying in this region, are to the right of both wires.



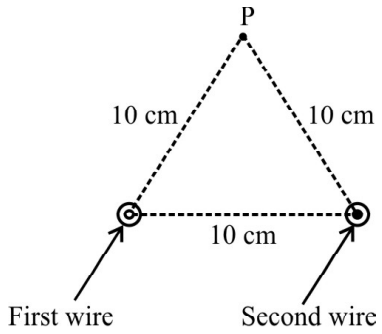
Magnetic field in the region PC will be upwards because points are closer to wire 1 compared to wire 2. Similarly the magnetic field in the region CQ will be downwards. The variation of the magnetic field, due to a long straight wire, is as $\frac{1}{r}$ ($\because$ $B = \frac{\mu_0 I}{2\pi r}$; r = distance of field point.)

All these requirements are met by graph (L). Hence option (2) is the correct option.

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Magnetic Field due to Current Carrying Conductors*

10. Two straight, infinitely long and thin parallel wires are spaced 0.1m apart and carry a current of 10 ampere each. The ratio of the magnetic fields, at a point at a distance of 0.1m from both wires, for the two cases. When the currents are in (1) same direction and (ii) opposite directions; would be



(1) $1:1$ (Incorrect)

(2) $\sqrt{3}:1$ (Correct)

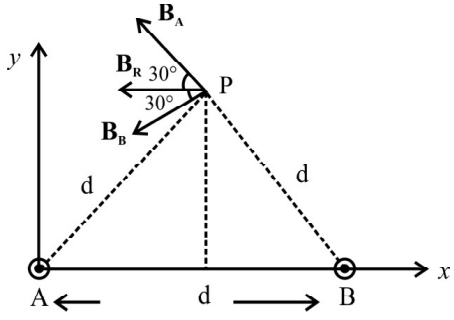
(3) $1:\sqrt{3}$ (Incorrect)

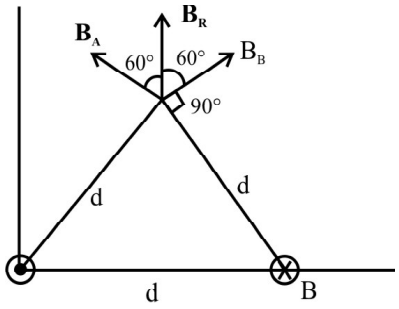
(4) $\sqrt{2}:1$ (Incorrect)

Correct answer: (2)

Solution:

Since point 'P' is situated equidistant from the wires A and B, hence magnitude of magnetic field at point P due to both the wires will be same.





$$B_A = B_B = B = \frac{\mu_0 i}{2\pi d} = 2\times10^{-7}\times\frac{10}{0.1} = 2\times10^{-5}\ T$$

(i) When wires carry currents in same direction (Fig. 1)

$$B_R = 2B\cos\theta\left(-\hat{i}\right) = 2\sqrt{3}\times10^{-5}\,T\ \left(-\hat{i}\right)$$

(ii) When wires carry currents in opposite direction (Fig. 2)

$$B_R = \cos\theta\left(+\hat{j}\right)\ = 2B\cos 60° = 2\times10^{-5}\ T\ \left(\text{along} + \hat{j}\right)$$

Hence ratio of magnetic fields in two cases

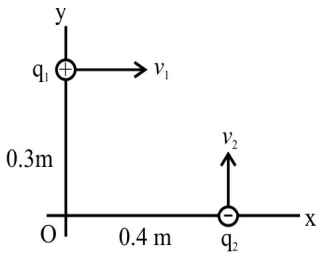
$$= \frac{\sqrt{3}\times2\times10^{-5}\,T}{2\times10^{-5}\,T} = \sqrt{3}:1$$

Hence option (2) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Magnetic Field due to Moving Charged Particle*

11. A pair of point charges $q_1 = +4\mu C$ and $q_2 = -1.5\mu C$ are moving in a frame of reference with the velocities v_1 and v_2 $\left(v_1 = 2\times10^5\,m/s,\ v_2 = 8\times10^5\,m/s\right)$, as shown.The magnitude and direction of the magnetic field produced at the origin will be given by



(1) 3.24×10^{-6} T outwards (Incorrect)

(2) 1.64×10^{-6} T outwards (Correct)

(3) 3.24×10^{-6} T inwards (Incorrect)

(4) 1.64×10^{-6} T inwards (Incorrect)

Correct answer: (2)

Solution:

$$\mathbf{B}_1 = \frac{\mu_0}{4\pi}\frac{q_1\left(\mathbf{v}_1\times\mathbf{r}_1\right)}{r_1^3}$$

$$= \frac{10^{-7} \times 4 \times 10^{-6} \left[2 \times 10^{5} \hat{i} \times \left(+0.3 \hat{j} \right) \right]}{(0.3)^3} \text{T} \quad = + \left(8.89 \times 10^{-7} \hat{k} \right) \text{T}$$

$$\mathbf{B}_2 = \frac{10^{-7} \times \left(-1.5 \times 10^{-6} \right) \left[8 \times 10^{5} \hat{j} \times 0.4 \hat{i} \right]}{(0.4)^3} \text{T} \quad = \left[\left(7.5 \times 10^{-7} \right) \hat{k} \right] \text{T}$$

Total magnetic field $\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2$

$$= \left[\left(8.89 \times 10^{-7} + 7.5 \times 10^{-7} \right) \hat{k} \right] \text{T} \quad = \left[16.4 \times 10^{-7} \right] \hat{k} \text{ T}$$

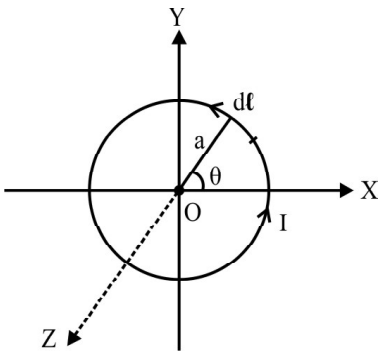
$= 1.64 \times 10^{-6}$ T outwards

Hence option (2) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Biot-Savart Law*

12. A circular coil, of radius a, carrying a current I, is located in the xy plane, with its centre as the origin. The magnetic field B, due to this coil, at a general point (0, 0, c) on its axis, can be expressed as



(1) $\mathbf{B} = \dfrac{\mu_0 I}{4\pi \left(a^2 + c^2 \right)^{3/2}} \left[\int_0^{2\pi} (a d\theta) \left(\hat{j} - \hat{i} \right) \times \left(a \cos\theta \hat{i} + a \sin\theta \hat{j} + c \hat{k} \right) \right]$ (Incorrect)

(2) $\mathbf{B} = \dfrac{\mu_0}{4\pi} I \left[\dfrac{\int_0^{2\pi} \left(-a \sin\theta \hat{i} + a \cos\theta \hat{j} \right) \times \left(a \cos\theta \hat{i} + a \sin\theta \hat{j} + c \hat{k} \right)}{\left(a^2 + c^2 \right)} \right]$ (Incorrect)

(3) $\mathbf{B} = \frac{\mu_0}{4\pi} I \left[\int_0^{2\pi} \frac{(a d\theta)(\hat{i} - \hat{j}) \times (a\cos\theta \hat{i} + a\sin\theta \hat{j} + c\hat{k})}{(a^2 + c^2)} \right]$ (Incorrect)

(4) $\mathbf{B} = \frac{\mu_0 I}{4\pi} \left[\int_0^{2\pi} \frac{(-a\sin\theta \hat{i} + a\cos\theta \hat{j}) d\theta \times (-a\cos\theta \hat{i} - a\sin\theta \hat{j} + c\hat{k})}{(a^2 + c^2)^{3/2}} \right]$ (Correct)

Correct answer: (4)

Solution:

We need to use the Biot-Savart law:

$$\mathbf{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\ell \times \mathbf{r}}{r^3}$$

A current element, $d\ell$, of this circular coil, may be considered to be located around the point (x, y). We have $d\ell = dx\,\hat{i} + dy\,\hat{j}$

Now, as seen from the figure, $x = a\cos\theta$, $y = a\sin\theta$

$\therefore \quad dx = -a\sin\theta\, d\theta; \quad dy = a\cos\theta\, d\theta$

The vector **r**, in the Biot-Savart law, is the position vector of the field point (0, 0, c) with respect to the location, (x, y, 0), of the current element.

Hence $\mathbf{r} = (0 - x)\hat{i} + (0 - y)\hat{j} + (c - 0)\hat{k}$

$= -a\cos\theta \hat{i} - a\sin\theta \hat{j} + c\hat{k}$

$\therefore \quad |r| = (a^2\cos^2\theta + a^2\sin^2\theta + c^2)^{1/2} = (a^2 + c^2)^{1/2}$

The variable θ varies from o to 2π. Hence

$$\mathbf{B} = \frac{\mu_0 I}{4\pi(a^2 + c^2)^{3/2}} \left[\int_0^{2\pi} (-a\sin\theta \hat{i} + a\cos\theta \hat{j}) d\theta \times (-a\cos\theta \hat{i} - a\sin\theta \hat{j} + c\hat{k}) \right]$$

Unit–13 : Magnetic Effects of Current and Magnetism

Difficult *Magnetic Field due to a Circular Coil*

13. We have two parallel co-axial circular coils of radius R, and number of turns N, carrying equal currents in the same direction. The two coils are separated by a distance R. The magnetic field, on their axis, around the midpoint between the coils, is (nearly)

(1) $\frac{\mu_0 i R^2 N}{2}$ (Incorrect)

(2) $0.36\,\frac{\mu_0 N i}{R}$ (Incorrect)

(3) $0.08\,\frac{\mu_0 N i}{R}$ (Incorrect)

(4) $0.72\,\frac{\mu_0 N i}{R}$ (Correct)

Correct answer: (4)

Solution:

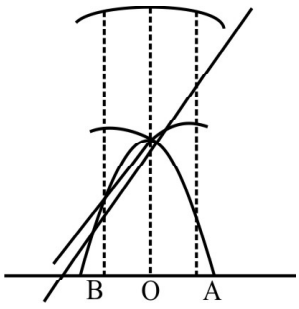
We know that the magnetic field, due to a circular coil, at an axial point distance z from its centre is given by

$$B = \frac{\mu_0 i R^2 N}{2\left(z^2 + R^2\right)^{3/2}}$$

The field, due to first coil, at a point on its axis, at a distance of $\frac{R}{2}$ from the centre, is

$$B_1 = \frac{\mu_0 i N R^2}{2\left(\frac{R^2}{4} + R^2\right)^{3/2}} = \frac{\mu_0 N i}{2\left(\frac{5}{4}\right)^{3/2} R^3}$$

$$\cong \frac{\mu_0 N i}{2R \times 1.4} \equiv \frac{\mu_0 N i}{2.8R} = \frac{0.36\mu_0 N i}{R}$$



Similarly the field, due to the second coil at a point on its axis, at a distance $\left(\frac{R}{2}\right)$ from its centre is, given by $B_2 = \frac{0.36\mu_0 N i}{R}$

Resultant field $\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2$

The two fields, being in the same direction, we have

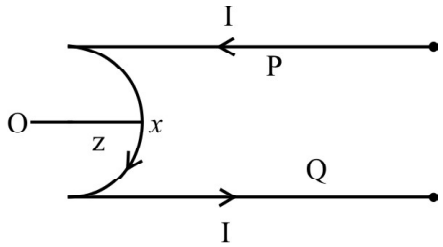
$$\therefore \quad |\mathbf{B}| \cong 0.72\frac{\mu_0 N i}{R}$$

Hence option (4) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Magnetic Field due to Straight and Circular Coils*

14. Figure shows a semi circular coil of radius z along with long straight wires. When a current I flows through this set up, the magnetic field, at O, the centre of semi circular arc, equals



(1) $\frac{\mu_0 i}{4\pi z}\left(\frac{\pi}{2}-2\right)$ outwards (Incorrect)

(2) $\frac{\mu_0 i}{4\pi z}(\pi+2)$ outwards (Incorrect)

(3) $\frac{\mu_0 i}{4\pi z}(\pi-2)$ into the plane of the paper (Correct)

(4) $\frac{\mu_0 i}{4\pi z}\left(\frac{\pi}{2}+2\right)$ into the plane of the paper (Incorrect)

Correct answer: (3)

Solution:

Magnetic field, due to semi circular portion, is given by

$$B_1 = \frac{1}{2}\frac{\mu_0}{4\pi}\frac{(2\pi i)}{z} = \frac{\mu_0 \pi i}{4\pi z} \text{ directed inwards}$$

Now the the point O is at a distance z from the straight conductor P of infinite length.

$\therefore \quad |\mathbf{B}_2| = \dfrac{\mu_0 i}{4\pi z}$ directed outward

Also, the magnetic field at point O due to infinite long straight conductor Q, is given by

$\mathbf{B}_3 = \dfrac{\mu_0 i}{4\pi z}$ directed outward

Hence the resultant field

$$B = B_1 - B_2 - B_3$$

or $\quad B = \dfrac{\mu_0 \pi i}{4\pi z} - \dfrac{\mu_0 i}{4\pi z} - \dfrac{\mu_0 i}{4\pi z} = \dfrac{\mu_0 i}{4\pi z}(\pi - 2)$, into the plane of the paper.

Hence option (3) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Magnetic Field due to a Circular Coil*

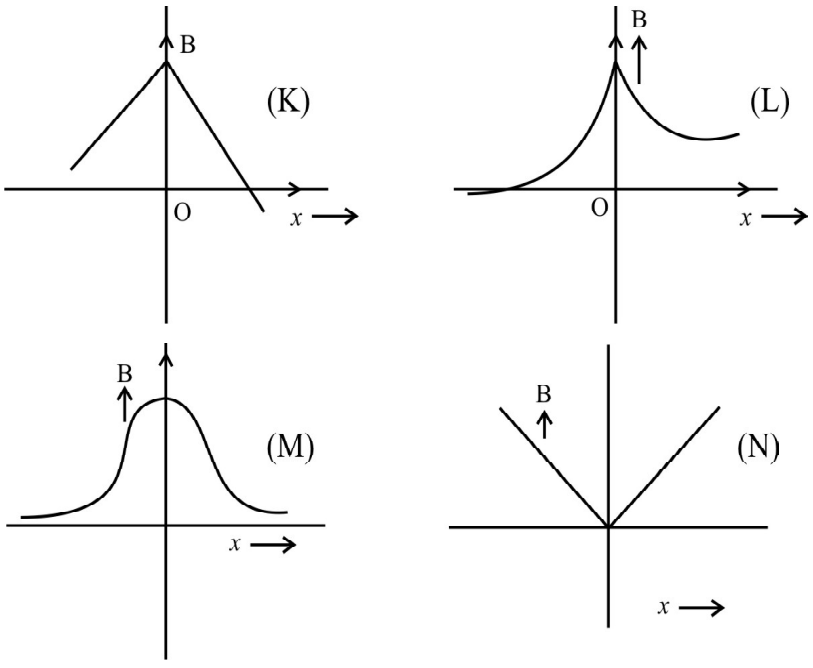
15. The graph which represents the variation of magnetic field, |B|, at axial points for a circular coil, with distance, *x* from the centre of the circular coil, is the graph labelled as graph

(1) K (Incorrect)

(2) L (Incorrect)

(3) M (Correct)

(4) N (Incorrect)



Correct answer: (3)

Solution:

The field due to a circular coil, of radius a, at an axial point distance *x* from centre, is given by

$$|\mathbf{B}| = \frac{\mu_0 n i}{2} \frac{a^2}{\left(a^2 + x^2\right)^{3/2}}$$

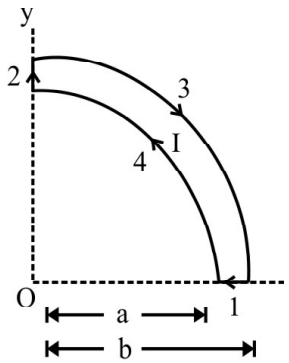
From above relation it is clear that |**B**| will have maximum value at the

centre O ($x = 0$). From this value it decreases symmetrically along both the positive and negative x–axis. Hence option (3) is the correct option.

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Magnetic Field due to Current Loop*

16. The figure shows a wire frame, in xy plane, carrying a current I, the magnetic field at point 'O' is



(1) $\frac{\mu_0 I}{8}\left[\frac{1}{a}-\frac{1}{b}\right]\hat{k}$ (Correct)

(2) $\frac{\mu_0 I}{8}\left[\frac{1}{b}+\frac{1}{a}\right]\hat{k}$ (Incorrect)

(3) $\frac{\mu_0 I}{4}\left[\frac{1}{a}-\frac{1}{b}\right]\hat{k}$ (Incorrect)

(4) $\frac{\mu_0 I}{4}\left[\frac{1}{b}+\frac{1}{a}\right]\hat{k}$ (Incorrect)

Correct answer: (1)

Solution:

Magnetic field at point O

$$\mathbf{B}_0 = \mathbf{B}_1 + \mathbf{B}_2 + \mathbf{B}_3 + \mathbf{B}_4$$

$$\mathbf{B}_1 = 0 = \mathbf{B}_2$$

$$\mathbf{B}_4 = \frac{\mu_0 I}{8a}\hat{k}$$

$$\mathbf{B}_3 = \frac{\mu_0 I}{8b}\left(-\hat{k}\right)$$

$$\Rightarrow B_0 = \frac{\mu_0 I}{8a}\hat{k} - \frac{\mu_0 I}{8b}\hat{k}$$

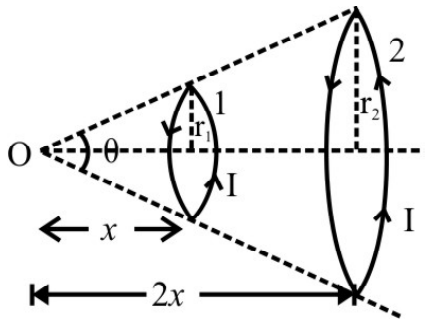
$$\text{Thus } B_0 = \frac{\mu_0 I}{8}\left[\frac{1}{a} - \frac{1}{b}\right]\hat{k}$$

Hence option (1) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Difficult *Magnetic Field on the Axis of a Circular Loop*

17. The figure shows two coaxial circular loops 1 and 2, which form the same angle θ at point 'O'. If B_1 and B_2 are the magnetic fields produced at the point 'O', due to loops 1 and 2, respectively. Then



(1) $\frac{B_1}{B_2} = 1$ (Incorrect)

(2) $\frac{B_1}{B_2} = 2$ (Correct)

(3) $\frac{B_1}{B_2} = 4$ (Incorrect)

(4) $\frac{B_1}{B_2} = 8$ (Incorrect)

Correct answer: (2)

Solution:

$$B_1 = \frac{\mu_0 I A_1}{2\left(x_1^2 + r_1^2\right)^{3/2}} = \frac{\mu_0 2I\pi r_1^2}{4\pi\left(x_1^2 + r_1^2\right)^{3/2}}$$

Similarly $B_2 = \frac{\mu_0 2I\pi r_1^2}{4\pi\left(x_2^2 + r_2^2\right)^{3/2}}$

$$\frac{B_1}{B_2} = \frac{r_1^2\left(x_2^2 + r_2^2\right)^{3/2}}{r_2^2\left(x_1^2 + r_1^2\right)^{3/2}}$$

But $r_1 = x\tan\theta/2$ and $x_1 = x$

$r_2 = 2x\tan\theta/2$ and $x_2 = 2x$

$$\therefore \quad \frac{B_1}{B_2} = \frac{x^2\tan^2\theta/2\left(4x^2 + 4x^2\tan^2\theta/2\right)^{3/2}}{4x^2\tan^2\theta/2\left(x^2 + x^2\tan^2\theta/2\right)^{3/2}}$$

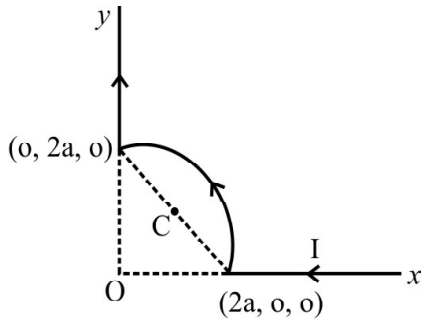
$$= \frac{1}{4} \times \frac{(4)^{3/2}\left(1 + \tan^2\theta/2\right)^{3/2}}{\left(1 + \tan^2\theta/2\right)^{3/2}} = 2$$

Hence option (2) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Difficult *Magnetic Field due to Straight Conductor and Circular Loop*

18. Current I flows through the wire as shown in figure, the semicircular part of the wire is perpendicular to *x*–y plane and out of the plane of paper. The mangetic field, at the centre 'C', of the semi circle, would be



(1) $\frac{-\mu_0 I}{4a}\left[\frac{\hat{i}+\hat{j}}{2}+\frac{\left(2-\sqrt{2}\right)}{\pi}\hat{k}\right]$ (Correct)

(2) $\frac{-\mu_0 I}{4\pi a}\left[\frac{\hat{i}+\hat{j}}{2\pi}+\frac{\left(2-\sqrt{2}\right)}{1}\hat{k}\right]$ (Incorrect)

(3) $\frac{+\mu_0 I}{4a}\left[\frac{\hat{i}+\hat{j}}{2}+\frac{\left(2-\sqrt{2}\right)}{\pi}\hat{k}\right]$ (Incorrect)

(4) $\frac{-\mu_0 I}{4a}\left[\frac{\hat{i}-\hat{j}}{2}+\frac{\left(2-\sqrt{2}\right)}{\pi}\left(-\hat{k}\right)\right]$ (Incorrect)

Correct answer: (1)

Solution:

Magnitude of magnetic field at point 'C' due to straight conductor.

$$|\mathbf{B}_1| = \frac{-\mu_0 I}{4\pi a}\left[\sin 90^0 - \sin 45^0\right] = \frac{\mu_0 I}{4\pi a}\left(1-\frac{1}{\sqrt{2}}\right)$$

$$= \frac{-\mu_0 I}{4\pi a}\left[\frac{\sqrt{2}-1}{\sqrt{2}}\right] \text{ along } (-z) \text{ direction}$$

Magnetic field due to semicircular part

$$B_2 = \frac{\mu_0 I}{4\sqrt{2}a} \text{ along CO}$$

$\left(\text{Radius of semicircle} = \frac{2\sqrt{2}\,a}{2} = \sqrt{2}a\right)$ and Unit vector along $\text{CO} = -\frac{1}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)$

$\therefore$ Resultant magnetic field at C

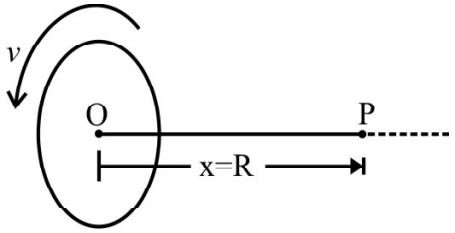
$$\mathbf{B} = 2\frac{\mu_0 I}{4\pi a}\left(\frac{1}{\sqrt{2}}-1\right)\hat{k} - \frac{\mu_0 I}{4a}\frac{\left(\hat{i}+\hat{j}\right)}{2} = \frac{-\mu_0 I}{4a}\left[\frac{\hat{i}+\hat{j}}{2}+\frac{\left(2-\sqrt{2}\right)}{\pi}\hat{k}\right]$$

Hence option (1) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Magnetic Field on the Axis of Current Carrying Loop*

19. A uniformly charged ring, (having a charge q), of radius R, is rotated about its axis with constant linear speed v for each of its particles. The ratio of the electric to magnetic field, at a point P, on the axis of the ring, distance x = R from the centre of the ring, is ('c' is the speed of light)



(1) $\frac{c^2}{v}$ (Correct)

(2) $\frac{v^2}{c}$ (Incorrect)

(3) $\frac{v}{c}$ (Incorrect)

(4) $\frac{c}{v}$ (Incorrect)

Correct answer: (1)

Solution:

At point P

$$E = \frac{1}{4\pi\varepsilon_0}\frac{qx}{\left(R^2+x^2\right)^{3/2}} = \frac{1}{4\pi\varepsilon_0}\frac{qR}{\left(R^2+x^2\right)^{3/2}}$$

$$B = \frac{\mu_0}{4\pi}\frac{2iA}{\left(R^2+x^2\right)^{3/2}}$$

Here $i = \frac{q}{T} = \frac{qv}{2\pi R}$ and Area $A = \pi R^2$

$$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{qvR}{\left(R^2+x^2\right)^{3/2}}$$

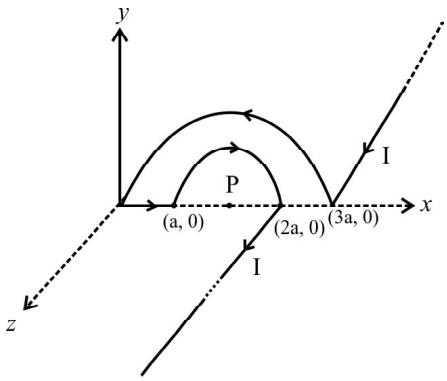
$$\therefore \quad \frac{E}{B}=\frac{1}{\mu_0\varepsilon_0}.\frac{1}{v}\Rightarrow\frac{c^2}{v} \qquad \left(\because \quad C=\frac{1}{\sqrt{\mu_0\varepsilon_0}}\right)$$

Hence option (1) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Magnetic Field due to Current Carrying Circular Loop*

20. In the given figure, the magnetic field at point P is



(1) $\frac{2\mu_0 I}{3\pi a}\sqrt{4-\pi^2}$ (Incorrect)

(2) $\frac{\mu_0 I}{3\pi a}\sqrt{4+\pi^2}$ (Correct)

(3) $\frac{2\mu_0 I}{3\pi a}\left(4+\pi^2\right)$ (Incorrect)

(4) $\frac{2\mu_0 I}{3\pi a}\left(4-\pi^2\right)$ (Incorrect)

Correct answer: (2)

Solution:

Total magnetic field at point P

$$\mathbf{B}_P=\left(\mathbf{B}_1\right)_P+\left(\mathbf{B}_2\right)_P+\left(\mathbf{B}_3\right)_P+\left(\mathbf{B}_4\right)_P+\left(\mathbf{B}_5\right)_P$$

$\left(\mathbf{B}_1\right)_P=\frac{\mu_0 I}{4\pi\left(\frac{3a}{2}\right)}\left(-\hat{j}\right)$ semi infinite wire

$$(\mathbf{B}_2)_P = \frac{\mu_0 I}{4\left(\frac{3a}{2}\right)}\left(+\hat{k}\right)$$

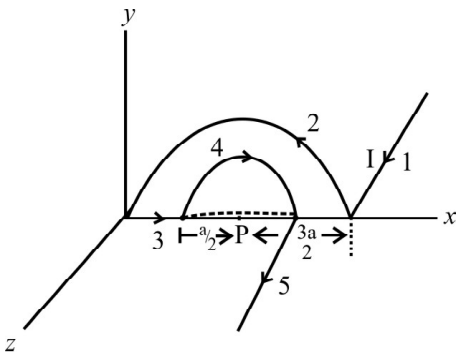
$(\mathbf{B}_3)_P = 0$ (P is along the wire)

$$(\mathbf{B}_4)_P = \frac{\mu_0 I}{4\left(\frac{a}{2}\right)}\left(-\hat{k}\right)$$ semi inifinte wire

$$(\mathbf{B}_5)_P = \frac{\mu_0 I}{4\pi\left(\frac{a}{2}\right)}\left(-\hat{j}\right)$$

$$\mathbf{B}_P = -\frac{2\mu_0 I}{3a}\left[\frac{2}{\pi}\hat{j} + \hat{k}\right] \quad \frac{\mu_0 I}{3\pi a}\sqrt{4+\pi^2}$$

Hence option (2) is correct

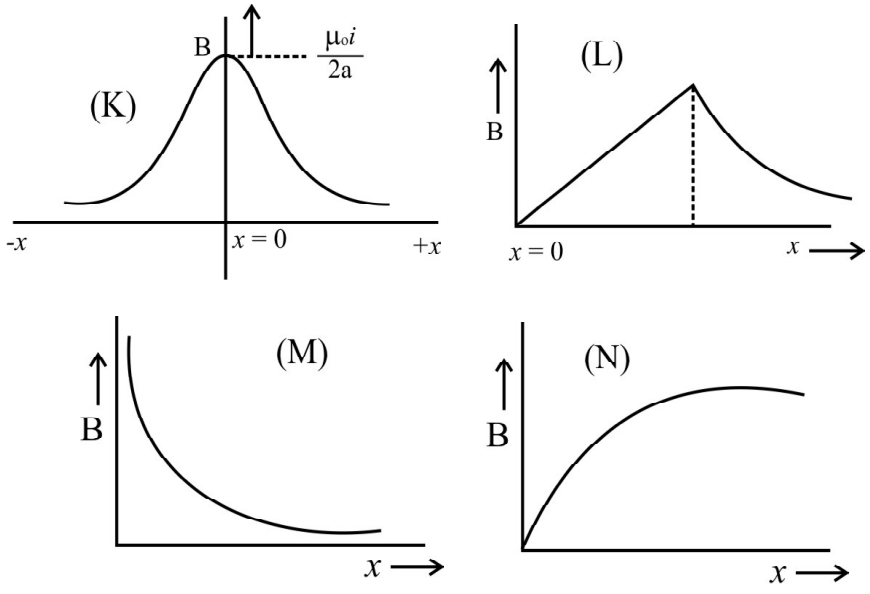


Unit–13 : Magnetic Effects of Current and Magnetism

Average/Easy

Magnetic Field on the Axis of a Current Carrying Circular Loop

21. A current carrying circular loop, of radius a, produces a magnetic field. The variation of the axial magnetic field with distance (on the axis) from centre, will be depicted by the graph, labelled as graph.



(1) K (Correct)

(2) L (Incorrect)

(3) M (Incorrect)

(4) N (Incorrect)

Correct answer: (1)

Solution:

B, on the axis, at distance x, is given by

$$B = \frac{\mu_0 i a^2}{2\left(a^2 + x^2\right)^{3/2}}$$

This shows that B decreases, in a (non-linear) symmetrical way, from its maximum value (at $x = 0$). It is graph (K) that is in accordance with these requirements.

Hence option (1) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Difficult *Magnetic Field on the Axis of Current Carrying Circular Coil*

22. A charge of one coulomb is placed at one end of a non conducting rod of length 0.6 m. The rod is rotated in a vertical plane about a horizontal axis (passing through the other end of the rod) with an angular frequency $\pi \times 10^6$ radian/second.

If half of the charge be removed from one end and placed on the other end, the rod is again rotated in a vertical plane, but now about the horizontal axis passing through the mid point of the rod, (with the same angular frequency).

The ratio of the magnetic field, in the two cases, at a point on the axis [whose distance from the centre, in first case, is 0.8 m and in second case 0.4 m], would be

(1) 1 : 1 (Incorrect)

(2) 2 : 1 (Incorrect)

(3) 1 : 2 (Correct)

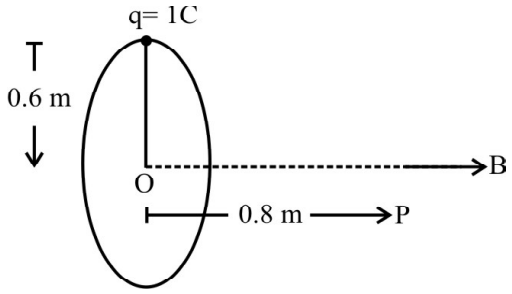
(4) 2 : 3 (Incorrect)

Correct answer: (3)

Solution:

Revolving charge is equivalent to current I = qv, the field, at a distance 'd', from centre, on the axis of a current carrying circular coil, is

$B = \dfrac{\mu_0 i a^2}{2\left(a^2 + d^2\right)^{3/2}}$ where 'a' is the radius of the coil.

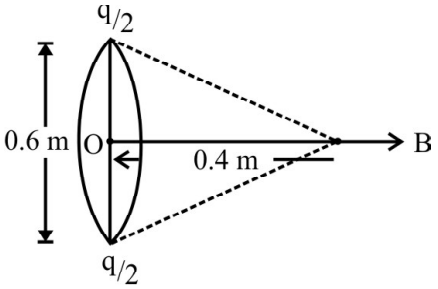


In first case we have $I = \dfrac{q\omega}{2\pi} = 5\times10^{3}\,A$

$$\therefore\ B = \frac{4\pi\times10^{-7}\times5\times10^{3}\times(0.6)^2}{2\left[(0.6)^2+(0.8)^2\right]^{3/2}} = 1.13\times10^{-3}\,T$$

In the second case we have the equivalent current, for each half,

$$I' = \frac{qv}{2} = \frac{5}{2}\times10^{+3}\,A$$



Here a = 0.3m, d = 0.4m

$$B' = 2\times\left[\frac{4\pi\times10^{-7}\times\frac{5}{2}\times10^{3}\times(0.3)^2}{2\left[(0.3)^2+(0.4)^2\right]^{3/2}}\right]$$

$\therefore$ Ratio of magnetic field, produced in first case, to that in the second case, is

$$\frac{B}{B'} = \left(\frac{0.6}{0.3}\right)^2\times\frac{\left[(0.3)^2+(0.4)^2\right]^{3/2}}{\left[(0.6)^2+(0.8)^2\right]^{3/2}} = 4\times\frac{25\times5}{100\times10} = \frac{1}{2}$$

$= 1 : 2$

Hence option (3) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Magnetic Field due to a Current Loop*

23. Consider a co-axial cable which consists of an inner wire of radius 'a', surrounded by an outer shell, of inner and outer radii b and c, respectively. The inner wire carries a current I and the outer shell carries an equal and opposite current. The magnetic field at a distance *x* from the axis, where b < *x* < c, is

(1) $\frac{\mu_0 I\left(c^2-b^2\right)}{2\pi x\left(c^2-a^2\right)}$ (Incorrect)

(2) $\frac{\mu_0 I\left(c^2-x^2\right)}{2\pi x\left(c^2-a^2\right)}$ (Incorrect)

(3) $\frac{\mu_0 I\left(c^2-x^2\right)}{2\pi x\left(c^2-b^2\right)}$ (Correct)

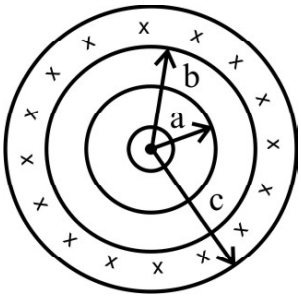
(4) $\frac{\mu_0 I\left(c^2-b^2\right)}{2\pi x\left(c^2-x^2\right)}$ (Incorrect)

Correct answer: (3)

Solution:

Consider an amperian loop of radius 'x' ($b < x < c$)

Threaded current I', by this amperian loop, is



$$I' = I + \left(\frac{x^2-b^2}{c^2-b^2}\right)(-I) = \left(\frac{c^2-x^2}{c^2-b^2}\right)I$$

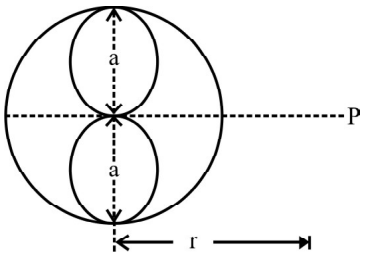
$$\therefore \quad B = \frac{\mu_0 I'}{2\pi x} = \frac{\mu_0 I\left(c^2-x^2\right)}{2\pi x\left(c^2-b^2\right)}$$

Hence option (3) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Difficult *Magnetic Field due to Cylindrical Conductor Carrying Current*

24. A long cylindrical conductor, of radius 'a', has two cylindrical cavities of diameter 'a', through its entire length, as shown in cross section in figure. A current I is directed out of the page and is uniform throughout the cross section of the conductor; the magnitude and direction of magnetic field at point P will be



(1) $\frac{\mu_0 I}{\pi r}\left[\frac{2r^2+a^2}{4r^2+a^2}\right]$ upwards (Correct)

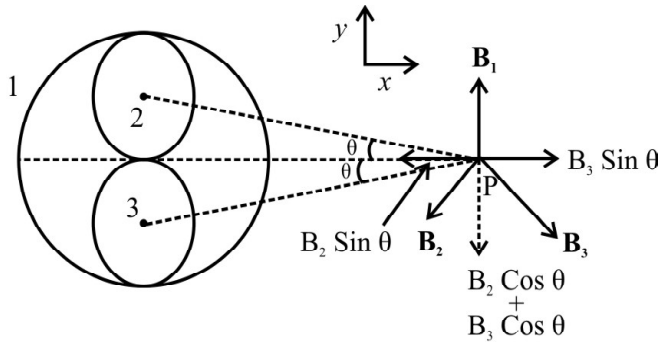
(2) $\frac{\mu_0 I}{2\pi r}\left[\frac{2r^2+a^2}{4r^2+a^2}\right]$ upwards (Incorrect)

(3) $\frac{\mu_0 I}{\pi r}\left[\frac{2r^2+a^2}{4r^2+a^2}\right]$ downwards (Incorrect)

(4) $\frac{\mu_0 I}{2\pi r}\left[\frac{2r^2+a^2}{4r^2+a^2}\right]$ downwards (Incorrect)

Correct answer: (1)

Solution:



Let B_1, B_2 and B_3 be the magnetic fields, due to complete cylinder and due to the upper and lower cavity respectivley.

$$\mathbf{B}_1 = \frac{\mu_0}{4\pi}\frac{2I_1}{r}\hat{j} = \frac{\mu_0 I}{\pi r}\hat{j}$$

$$\mathbf{B}_2 = \frac{\mu_0}{4\pi}\frac{2I_2}{\sqrt{r^2+\frac{a^2}{4}}}\left[-\sin\theta\,\hat{i}-\cos\theta\,\hat{j}\right] = \frac{-\mu_0 I}{2\pi\sqrt{4r^2+a^2}}\left(+\sin\theta\,\hat{i}+\cos\theta\,\hat{j}\right)$$

$$\mathbf{B}_3 = \frac{\mu_0}{4\pi}\frac{2I_3}{\sqrt{r^2+\frac{a^2}{4}}}\left(\sin\theta\,\hat{i}-\cos\theta\,\hat{j}\right) = \frac{\mu_0 I}{2\pi\sqrt{4r^2+a^2}}\left(\sin\theta\,\hat{i}-\cos\theta\,\hat{j}\right)$$

$$\text{Total } \mathbf{B} = \mathbf{B}_1+\mathbf{B}_2+\mathbf{B}_3 = \frac{\mu_0 I}{2\pi}\left[\frac{2}{r}-\frac{2\cos\theta}{\sqrt{4r^2+a^2}}\right]\hat{j}$$

But $\cos\theta = \dfrac{r}{\sqrt{r^2 + \dfrac{a^2}{4}}} = \dfrac{2r}{\sqrt{4r^2 + a^2}}$

$$B = \frac{\mu_0 I}{2\pi}\left[\frac{2}{r} - \frac{4r}{\left(4r^2 + a^2\right)}\right]\hat{j} = \frac{\mu_0 I}{2\pi r}\left[2 - \frac{4r^2}{4r^2 + a^2}\right]\hat{j}$$

$$= \frac{\mu_0 I}{2\pi r}\left[\frac{8r^2 + 2a^2 - 4r^2}{4r^2 + a^2}\right]\hat{j} = \frac{2\times\mu_0 I}{2\pi r}\left[\frac{2r^2 + a^2}{4r^2 + a^2}\right]\hat{j}$$

$$= \frac{\mu_0 I}{\pi r}\left[\frac{2r^2 + a^2}{4r^2 + a^2}\right]\hat{j}$$

Hence option (1) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Ampere's Circuital Law*

25. The current density, in a wire of radius 'a', varies with 'r' according to $j = Kr^2$, where K is a constant and 'r' is the distance from the axis of the wire, the magnitude of magnetic field at a point (inside the wire), at distance 'r' ($0 < r < a$) from the axis, will be

(1) $\dfrac{\mu_0 K a^4}{4r}$ (Incorrect)

(2) $\dfrac{\mu_0 K r^2}{4}$ (Incorrect)

(3) $\dfrac{\mu_0 K r^3}{4}$ (Correct)

(4) zero (Incorrect)

Correct answer: (3)

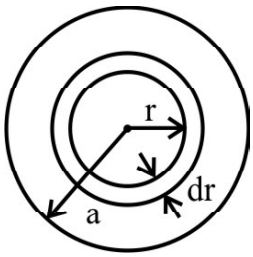
Solution:

Current passing through the area enclosed by (the Amperian) path

$$dI = jdA = Kr^2\left(2\pi r dr\right)$$

$$I = K\int_0^r 2\pi r^3 dr = \frac{\pi K r^4}{2}$$

Using Ampere's circuital law



$$\oint \mathbf{B}.d\ell = \mu_0 i_{enclosed}$$

$$B \times 2\pi r = \frac{\mu_0 K \pi r^4}{2}$$

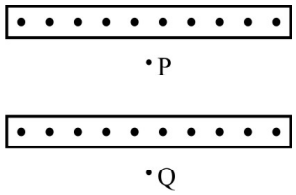
$$B = \frac{\mu_0 K r^3}{4}$$

Hence option (3) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Difficult *Ampere's Circuital Law*

26. Two infinite plates, shown in cross section in figure, carry i amperes of current and out of page per unit width of the plates. The magnetic field, at points P and Q, will be, respectively,



(1) 0 and $\mu_0 i$ (Correct)

(2) 0 and $\frac{1}{2}\mu_0 i$ (Incorrect)

(3) $\frac{1}{2}\mu_0 i$ and $\mu_0 i$ (Incorrect)

(4) $\mu_0 i$ and 0 (Incorrect)

Correct answer: (1)

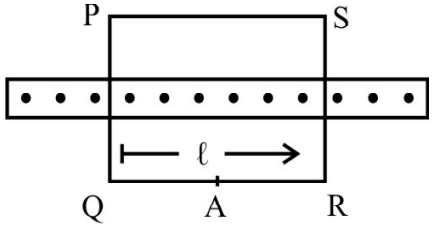
Solution:

To calculate the magnetic field due to current carrying infinite plate at point A.

Choose PQRS as an Amperian loop.

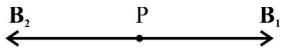
$$\oint_{PQRS} \mathbf{B}.d\ell = \int_{P\to Q} \mathbf{B}.d\ell + \int_{Q\to R} \mathbf{B}.d\ell + \int_{R\to S} \mathbf{B}.d\ell + \int_{S\to P} \mathbf{B}.d\ell$$

$$= 0 + B\ell + 0 + B\ell = 2B\ell$$



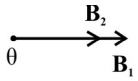
Hence $2B\ell = \mu_0 i\ell$

$$B = \frac{1}{2}\mu_0 i$$



$\therefore$ At point P

Net $\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 = 0$



At point Q

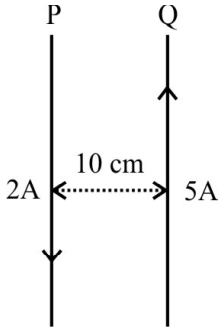
$$\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 = \mu_0 i$$

Hence option (1) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Easy *Force between two long Parallel Current Carrying Conductors*

27. Two long parallel straight wires P and Q, seperated by a distance of 10cm in air, carry current of 2A and 5A, respectively, in opposite directions. Then the force, on 20cm length of wire Q, is



(1) 2×10^{-7} N, perpendicular to the length of Q and acts towards P (Incorrect)

(2) 2×10^{-6} N, perpendicular to the length of Q and acts away from P (Correct)

(3) 2×10^{-6} N, and acts towards P (Incorrect)

(4) 2×10^{-7} N, and acts away from P (Incorrect)

Correct answer: (2)

Solution:

$$|\mathbf{F}| = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r} \times L$$

$$\frac{\mu_0}{4\pi}\frac{2\times 2\times 5\times 0.2}{0.1}\,N = 2\times 10^{-6}\,N$$

The direction of **F** will be perpendicular to the length Q and it acts away from P. (**F** between 'unlike currents', is a force of repulsion)

Hence option (2) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Easy *Motion of Changed Particle in a Magnetic Field*

28. A particle of charge q and mass m, moves in a circular orbit of radius r, with angular speed ω. The ratio of the magnitude of its magnetic moment, to that of its angular momentum, is given by

(1) $\frac{q}{2m}$ (Correct)

(2) $\frac{\omega}{2m}$ (Correct)

(3) $\frac{q\omega}{2m}$ (Incorrect)

(4) $q\omega$ (Incorrect)

Correct answer: (1)

Solution:

The magnetic moment $|\mathbf{M}| = iA = q\left(\frac{\omega}{2\pi}\right)\left(\pi r^2\right) = \frac{q\omega r^2}{2}$

Angular momentum = $I\omega = \left(mr^2\omega\right)$

Hence $\frac{M}{L} = \frac{\frac{q\omega r^2}{2}}{mr^2\omega} = \frac{q}{2m}$

Hence option (1) is correct.

Unit–13 : Magnetic Effects of Current and Magnetism

Easy *Force of Charged Particle in Magnetic Field*

29. An electric current 'i' enters, and leaves, a uniform circular wire of radius 'a' through diametrically opposite points, A charged particle q, moving along the axis of circular wire, passes through its centre at speed 'v'. The magnetic force acting on the particle, when it passes through the centre, has a magnitude

(1) $qv\frac{\mu_0 i}{2a}$ (Incorrect)

(2) $qv\frac{\mu_0 i}{4a}$ (Incorrect)

(3) $qv\frac{\mu_0 i}{2\pi a}$ (Incorrect)

(4) zero (Correct)

Correct answer: (4)

Solution:

Explanation: When current enters into wire, it gets divided into two equal parts; these meet at a point, diametrically opposite, when leaving the circular wire. Hence the two halves of this wire produce (equal magnitude) magnetic fields at the centre, which act in opposite directions. Therefore the net magnetic field at centre is zero. Hence the particle, while passing through centre, experiences no force; [$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$].

Hence option (4) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Motion of Charged Particle in Electric and Magnetic Fields*

30. A uniform magnetic field, 0.75 T, is set up over a small region by using a pair of co-axial circular coils. In the same region a uniform electrostatic field of magnitude 9×10^5 V/m is maintained in a direction normal to the axis of coils. A narrow beam, of charged particles, all accelerated through 15 KV, enters the region in a direction perpendicular to both the axis of coil and the electrostatic field. Under these conditions the beam remains undeflected. Then the particles present in the beam, could be

(1) electrons (Incorrect)

(2) ionized hydrogen atoms (Incorrect)

(3) deutron ions (Correct)

(4) alpha particles (Incorrect)

Correct answer: (3)

Solution:

Under the specified conditions the beam will remain undefined when

$$Bqv = qE \qquad \text{or} \qquad v = \frac{E}{B}$$

When a charged particle, of charge q, is accelerated through a potential V, the gain in the energy of the particle is qV.

Hence $\frac{1}{2}mv^2 = qV$

$$\therefore \quad \frac{q}{m} = \frac{1}{2}\,\frac{v^2}{V}$$

Substituting the given values, the value of $\frac{q}{m}$ obtained is $4.8\times10^7\ \frac{C}{kg}$

Now we have to see which particle has the same value of $\frac{q}{m}$

For deutrion ions, we have

$$\frac{q}{m} = \frac{1.6\times10^{-19}}{\left(2\times1.67\times10^{-27}\right)}\ \frac{C}{kg} = 4.8\times10^7\ \frac{C}{kg}$$

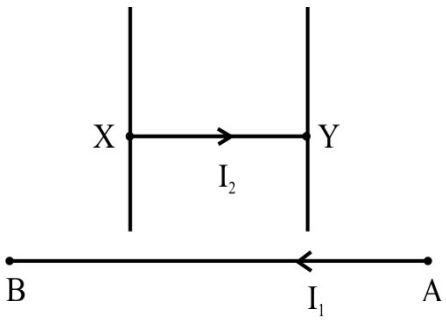
Hence the particle in the beam could be deuterium ions.

Hence option (3) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Force between two long Parallel Current Carrying Conductors*

31. A long wire AB is placed on a table. A wire *x*y (mass = m), of length, z, is kept parallel to, and above AB. xy can slide up and down on two vertical wires. The two wires are carrying currents I_1 and I_2 respetively. The wire *x*y will settle at a distance d above the wire AB, where d equals (nearly).



(1) $\left(\frac{\mu_0 I_1 I_2}{2\pi mg}\right) z$ (Correct)

(2) $\left(\frac{\mu_0 I_1 I_2}{2\pi z}\right)(mg)$ (Incorrect)

(3) $\frac{4\pi z}{\mu_0 I_1 I_2} \times \frac{1}{mg}$ (Incorrect)

(4) $\left(\frac{2\pi z}{mg}\right)(\mu_0 I_1 I_2)$ (Incorrect)

Correct answer: (1)

Solution:

Let the wire *xy* settle at a distance d above the wire AB. The magnetic force, on wire *xy*, of length z, due to current in AB is (nearly).

$$|\mathbf{F}| = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} z$$

Since the currents in wires AB and *xy* are in opposite directions, this force, on *xy*, will be upwards.

The gravitational force on *xy* acting vertically downwards will be mg

Hence $\frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} z = mg$

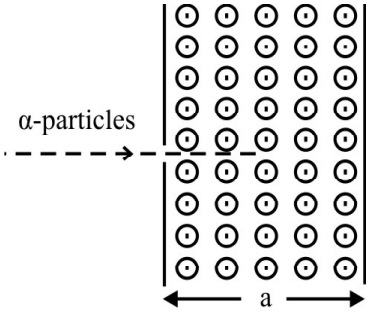
$\therefore \quad d = \left(\frac{\mu_0}{2\pi}\frac{I_1 I_2}{mg}\right) z$

Hence option (1) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Motion of a Charged Particle in a Magnetic Field*

32. A beam of alpha particles, accelerated through a potential V, enters a region, between two long parallel plates, separated by a distance a. A uniform magnetic field, B, directed normally outwards from the plane of the page, exists in the region between these two plates. The minimum value of |B|, that can just prevent the α–particles from colliding with the second plate, equals.



(1) $\frac{1}{a}\left(\frac{Vm_\alpha}{e}\right)^{1/2}$ (Correct)

(2) $\frac{2}{a}\left(\frac{2Vm_\alpha}{e}\right)^{1/2}$ (Incorrect)

(3) $\frac{1}{a}\left(\frac{2Vm_\alpha}{e}\right)^{1/2}$ (Incorrect)

(4) $\frac{2}{a}\left(\frac{Vm_\alpha}{2e}\right)^{1/2}$ (Incorrect)

Correct answer: (1)

Solution:

Let v be the velocity acquired by the alpha particles. We then have

$$(2e)V = \frac{1}{2}m_\alpha v^2$$

$$\therefore \quad v = \sqrt{\frac{4eV}{m_\alpha}} = 2\left(eVm_\alpha^{-1}\right)^{1/2}$$

The (normal) magnetic field would bend the path of these α-particles into a circle of radius R, where

$$\frac{m_\alpha v^2}{R} = (2e)vB$$

$$\therefore \quad R = \frac{m_\alpha v}{2eB} = \frac{(eVm_\alpha)^{1/2}}{eB}$$

$$\therefore \quad B = \left(\frac{Vm_\alpha}{e}\right)^{1/2}.\frac{1}{R}$$

To prevent collision with the second plate, R should not exceed a. Hence the minimum value of B, needed, is

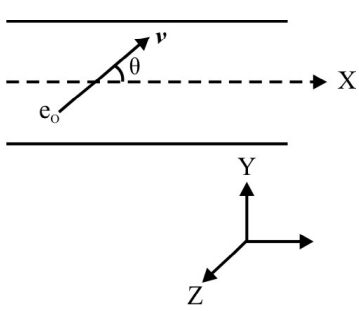
$$B_{min} = \frac{1}{a}\left(\frac{Vm_{\alpha}}{e}\right)^{1/2}$$

Hence option (1) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Difficult *Motion of a Charged Particle in a Magnetic Field*

33. A long thin solenoid, having its axis along the x–axis, has an insulated wire wound on its core. The solenoid carries a current I and has n turns of wire per unit length.



An electron beam (charge = e, mass = m) from a source, moves with a velocity $\mathbf{v} = v\cos\theta\hat{i} + v\sin\theta\hat{j}$, in the (nearly) uniform magnetic field region of this solenoid. The path of these electrons, inside this region, would be a

(1) circular path in the yz plane, of radius $r = \frac{mv\cos\theta}{e\mu_0 nI}$ (Incorrect)

(2) helix (with its axis along the x–axis) of radius $r = \frac{mv\sin\theta}{e\mu_0 nI}$ and pitch $p = \frac{2\pi mv\cos\theta}{e\mu_0 nI}$ (Correct)

(3) helix (with its axis along the y–axis) of radius $r = \frac{mv\cos\theta}{e\mu_0 nI}$ and pitch $p = \frac{2\pi mv\sin\theta}{(e\mu_0 nI)}$ (Incorrect)

(4) circular path, in the z–*x* plane, of radius $r = \frac{mv\sin\theta}{e\mu_0 nI}$ (Incorrect)

Correct answer: (2)

Solution:

The long thin solenoid, produces a (nearly) uniform magnetic field, **B**, around its midpoint, where

$$\mathbf{B} = (\mu_0 ni)(\pm\hat{i})$$

Due to this field, there is no force, on the electron, due to the *x*–component $(= v\cos\theta)$ of its velocity. Hence this component, of the electron's velocity remains unaffected by the field. Due to the y–component $(= v\sin\theta)$ of its velocity, the electrons would describe a circular path, of radius r, where

$$\frac{m(v\sin\theta)^2}{r} = e(v\sin\theta)B$$

Hence $r = \frac{mv\sin\theta}{eB} = \frac{mv\sin\theta}{e\mu_0 nI}$

The time period, for completing this circular path, is

$$T = \frac{2\pi r}{v\sin\theta} = \frac{2\pi m}{e\mu_0 nI}$$

The pitch, p, equals the distance, moved along the x–axis, in a time T.

Hence $p = v\cos\theta . T = \frac{2\pi mv\cos\theta}{e\mu_0 nI}$

Hence option (2) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Difficult *Force on Current Carrying Conductor*

34. A conductor, of length 'ℓ', is placed normally to a horizontal uniform magnetic field B. When a certain amount of charge is suddenly passed through it is found to jump to a height 'h'. The amount of charge that passed through the conductor is

(1) $\frac{m\sqrt{gh}}{B\ell}$ (Incorrect)

(2) $\frac{m\sqrt{gh}}{2B\ell}$ (Incorrect)

(3) $\frac{m\sqrt{2gh}}{B\ell}$ (Correct)

(4) $\frac{m\sqrt{2gh}}{3B\ell}$ (Incorrect)

Correct answer: (3)

Solution:

Force × time = impulse = Change in linear momentum

$\int I\ell B.dt = mv - 0$ (v = velocity acquired by the conductor)

$\ell B\int dq = mv$

$\int dq = q = \frac{mv}{B\ell}$

Now v is related to h by $v = \sqrt{2gh}$

$\therefore \quad q = \frac{m\sqrt{2gh}}{\ell B}$

Hence option (3) is correct.

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Motion of Charged Particle in Magnetic Field*

35. A charged particle is accelerated through a potential difference of 12 KV and acquires a speed of 10^6 m/s. It is then injected perpendicularly into a magnetic field of strengh 0.2 T. The radius of circle would be

(1) 12 cm (Correct)

(2) 8.33 cm (Incorrect)

(3) 0.833 cm (Incorrect)

(4) 1.2 mm (Incorrect)

Correct answer: (1)

Solution:

Given accelerating voltage = 12 KV = 12×10^3 V

$F = qE = \frac{qV}{\ell}$

$\therefore$ acceleration $a = \frac{qV}{m\ell}$

Now $v^2 = u^2 + 2as$

$\left(10^6\right)^2 = 0 + 2\left(\frac{q}{m}\right).\frac{12\times10^3}{\ell}\times\ell$

$\Rightarrow \frac{q}{m} = \frac{10^{12}}{2\times12\times10^3} = 24\times10^{-9}$

$\therefore$ radius $r = \dfrac{mv}{qB} = \dfrac{24\times10^{-9}\times10^{6}}{0.2T} = 12$ cm

Hence option (1) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Force Experienced by a Current Carrying Conductor*

36. A wire is bent in the form of an equilateral triangle of side 10cm and carries a current of 5.0A. It is placed in a magnetic field B of magnitude 2.0T directed perpendicular to the plane of the triangle. The force, experienced by each side of the triangle, and the net force experienced by the wire, will be

(1) 0.87 N and 2.6 N, respectively (Incorrect)

(2) 0.50 N and 1.5 N, respectively (Incorrect)

(3) 1.0 N and zero N, respectively (Correct)

(4) 1.0 N and 3.0 N, respectively (Incorrect)

Correct answer: (3)

Solution:

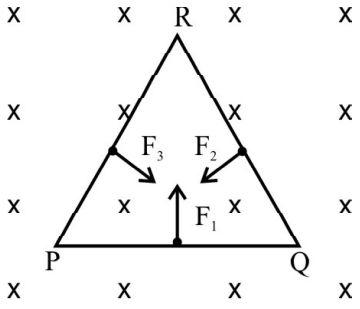
The magnetic field is directed into the plane of paper and wire is in the plane of paper and carries current as shown in the figure.

Directions of forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ will be as shown in Fig.

$|\mathbf{F}_1| = |\mathbf{F}_2| = |\mathbf{F}_3| = I\ell \times \mathbf{B} = I\ell B\sin\theta$

$= I\ell B\sin 90^0$

$= 5.0\times0.10\times2.0$ N $= 1.0$ N



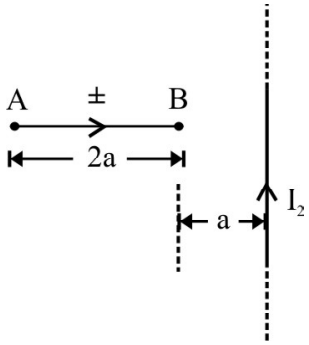
Using vector rule, we get

$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$ N

Unit–13 : Magnetic Effects of Current and Magnetism

Difficult *Force on a Current Carrying Conductor*

37. The segment, AB of a wire, carrying a curent I_1 is placed perpendicular to a long straight wire carrying current I_2 as shown in figure. The magnitude of force, experienced by wire AB, is



(1) $\frac{\mu_0 I_1 I_2}{2\pi} \ell og_e 3$ (Correct)

(2) $\frac{\mu_0 I_1 I_2}{2\pi} \ell og_e 2$ (Incorrect)

(3) $\frac{\mu_0 I_1 I_2}{\pi}$ (Incorrect)

(4) $\frac{\mu_0 I_1 I_2}{2\pi}$ (Incorrect)

Correct answer: (1)

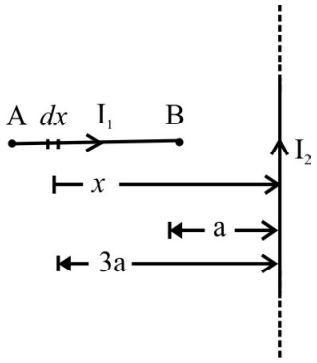
Solution:

Consider a small element of length 'dx' at distance x from end B of the wire.

Force on this portion $dF = I_1 dx B$

$$dF = I_1 dx \times \frac{\mu_0 I_2}{2\pi(x+a)} = \frac{\mu_0}{4\pi}\frac{2I_1 I_2}{(a+x)} dx$$

Total force $F = \int_a^{3a} dF = \frac{\mu_0 2I_1 I_2}{4\pi}\int_a^{3a}\frac{dx}{(a+x)}$



$$= \frac{\mu_0 2I_1 I_2}{4\pi}\left[\ell n(a+x)\right]_a^{3a}$$

$$= \frac{\mu_0 2I_1 I_2}{4\pi}\left[\ell n 3a - \ell n a\right]$$

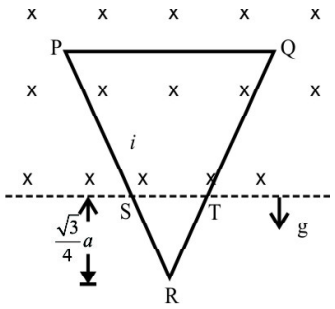
$$= \frac{\mu_0 I_1 I_2}{2\pi}\ell n 3 = \frac{\mu_0 I_1 I_2}{2\pi}\ell og_e 3$$

Hence option (1) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Force due to a Magnetic Field*

38. An equilateral triangle frame, PQR of mass M and side 'a', is kept under the influence of (i) a magnetic force, due to an inward perpendicular magnetic field B, and (ii) The gravitational field, as shown in the figure. The magnitude, and direction of current that should flow in the frame, so that it remains at rest, is



(1) $I = \frac{2mg}{aB}$, anticlockwise (Incorrect)

(2) $I = \frac{2mg}{aB}$, clockwise (Correct)

(3) $I = \frac{mg}{aB}$, anticlockwise (Incorrect)

(4) $I = \frac{mg}{aB}$, clockwise (Incorrect)

Correct answer: (2)

Solution:

Effective length of the wire = ST = $\ell = 2 \times \frac{\sqrt{3}}{4} a \cot 60^0 = \frac{a}{2}$

For equilibrium $I\ell B = Mg$

$I = \frac{2mg}{aB}$ $\left(\text{as } \ell = \frac{a}{2}\right)$ Clockwise

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Motion of Charged Particle in Electric & Magnetic Fields*

39. Uniform electric and magnetic fields, with strength E and B, are directed along y–axis. A particle with specific charge q/m leaves the origin in direction of x–axis with an initial

velocity v_0. The angle α, between the particles velocity vector, and the y–axis at the moment of leaving will be

(1) $\alpha = \tan^{-1}\left(\frac{Bv_0}{2\pi nE}\right)$ (Correct)

(2) $\alpha = \tan^{-1}\left(\frac{Bv_0}{\pi nE}\right)$ (Incorrect)

(3) $\alpha = \tan^{-1}\left(\frac{2Bv_0}{\pi nE}\right)$ (Incorrect)

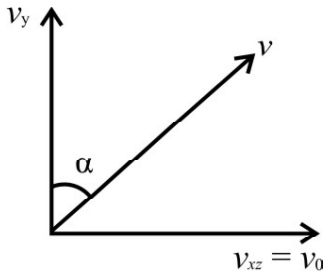
(4) $\alpha = \tan^{-1}\left(\frac{2Bv_0}{3\pi nE}\right)$ (Incorrect)

Correct answer: (1)

Solution:

The path of the particle is helix of increasing pitch, the axis is parallel to y–axis (parallel to **E**) and plane of circle of helix is the XZ plane (perpendicular to **B**) the particle will cross the y–axis after time.

$$t = nT = n\left(\frac{2\pi m}{qB}\right)$$



$$\therefore \quad y = \frac{1}{2}a_y t^2 = \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{2\pi mn}{qB}\right)^2$$

$$= \frac{2n^2 mE\pi^2}{qB^2}$$

Velocity (y–component) at this moment

$$v_y = a_y t = \left(\frac{qE}{m}\right)\left(\frac{2\pi mn}{qB}\right) = 2\pi n\left(\frac{E}{B}\right)$$

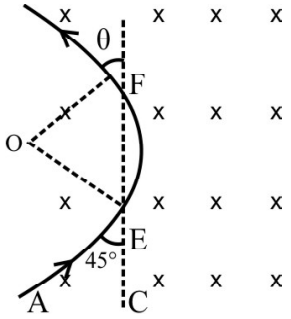
$$\alpha = \tan^{-1}\left(\frac{v_{xz}}{v_y}\right) = \tan^{-1}\left(\frac{v_0 B}{2\pi nE}\right)$$

Hence correct option is (1)

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Motion of Charged Particle in Magnetic Field*

40. A proton enters a region of uniform magnetic field at point E with the speed of 1.0×10^7 m/s, as shown in figure, the strength of magnetic field is 1.0T and is directed into the plane of paper. Proton leaves the region of field at point 'F' the distance, EF and angle θ, will respectively, be equal to (nearly)



(1) 15 cm and 45^0 (Correct)

(2) 7.5 cm and 22.5^0 (Incorrect)

(3) 15 cm and 22.5^0 (Incorrect)

(4) 7.5 cm and 45^0 (Incorrect)

Correct answer: (1)

Solution:

The radius, OE, of the proton's circular path is given by $\frac{mv^2}{2} = qvB$

$\therefore \quad r(=OE) = \frac{mv}{qB}$

We also have;

$\angle AEO = 90^0$ (Radius and tangent)

$\angle AEC = 45^0$ (Given)

We have $\angle OEF = 45^0$

OF and OE are radii of the circular arc, we have $\angle OFE = 45^0$

$\therefore \quad \text{angle}\,\theta = 45^0$

Now EF = 2 OE cos 45^0;

$$\therefore \quad EF = 2\times\left(\frac{mv}{qB}\right)\times\frac{1}{\sqrt{2}}$$

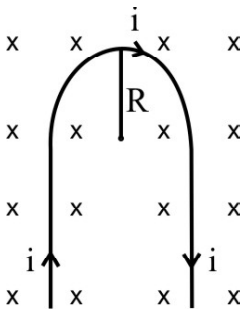
$$=\frac{2\times1.67\times10^{-27}\times1.0\times10^{7}}{1.6\times10^{-19}\times1.0}\times\frac{1}{\sqrt{2}}\,m$$

$$=14.8\,cm\approx15\,cm$$

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Force on Current Carrying Conductor in Magnetic Field*

41. A rigid wire consists of semicircular portion of radius 'R' and two straight sections as shown in figure. It is present in a uniform perpendicular magnetic field 'B'. The magnetic force on the wire, when it carries a current 'i', would be



(1) (2i RB) upward (Correct)

(2) (2iRB) downward (Incorrect)

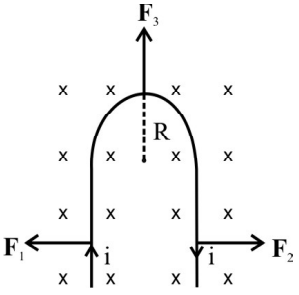
(3) $(i\pi RB)$ upward (Incorrect)

(4) $(i\pi RB)$ downward (Incorrect)

Correct answer: (1)

Solution:

The directions of forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ is marked as per the Fleming's Left hand rule.



Since force $\mathbf{F}_1$ and $\mathbf{F}_2$ are acting in opposite directions and are equal in magnitude, they cancel each other's effect.

Hence the total force on the given loop is due to the semi circular portion,

which is equal to

$$\mathbf{F}_3 = i\ell \times \mathbf{B}$$

$\therefore \quad |\mathbf{F}_3| = i(2R)B\sin 90^0$

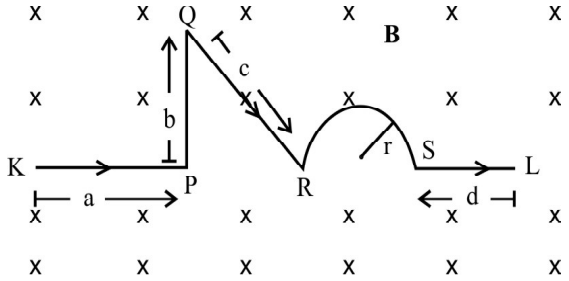
$= 2iRB$ acting upward on the given loop.

Hence option (1) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Force on a Current Carrying Conductor in Magnetic Field*

42. The force on the current carrying conductor KL, in a niform (normal) magnetic field, B, will be



(1) $IB\left(a+\sqrt{c^2-b^2}+2r+d\right)$, downwards in the plane of paper (Incorrect)

(2) $IB(a+b+c+\pi r+d)$, upwards in the plane of paper (Incorrect)

(3) $IB\left(a+\sqrt{c^2-b^2}+\pi r+d\right)$, upwards in the plane of paper (Incorrect)

(4) $IB\left(a+\sqrt{c^2-b^2}+r+d\right)$, upwards in the plane of paper (Correct)

Correct answer: (4)

Solution:

Net force from K to L

We have; $dF = I(d\ell \times \mathbf{B})$

$$\therefore \quad F = \int_K^L dF = \int_K^P I(d\ell_1 \times \mathbf{B}) + \int_P^Q I(d\ell_2 \times \mathbf{B}) + \int_Q^R I(d\ell_3 \times \mathbf{B}) + \int_R^S I(d\ell_4 \times \mathbf{B}) + \int_S^L I(d\ell_5 \times \mathbf{B})$$

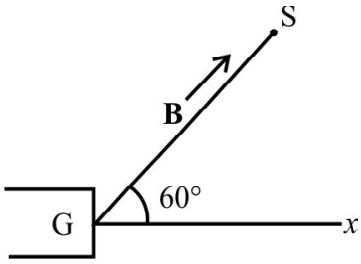
$\mathbf{F}_{net} = IB\left(a + \sqrt{c^2 - b^2} + 2r + d\right)$ upwards in the plane of paper.

Hence option (4) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Difficult/Average *Motion of Charged Particle in Magnetic Field*

43. An electron gun G emits electrons of energy 2 KeV travelling along the positive *x*–direction. The electrons are required to hit the spot of S where GS = 0.1m and the line GS makes an angle of 60^0 with the x–axis as shown in figure. A uniform magnetic field B, parallel to GS, exists in the region outside the electron gun. The minimum value of magnetic field, B, needed to make the electron hit at S, well be



(1) 2.37×10^{-4}T (Incorrect)

(2) 4.73×10^{-4}T (Correct)

(3) 1.58×10^{-4}T (Incorrect)

(4) 9.46×10^{-4}T (Incorrect)

Correct answer: (2)

Solution:

For the electron to hit the target, distance GS should be a multiple of the pitch of its helical path

$GS = np$

where n = integer, p = pitch

For minimum distance n = 1

$$\therefore \quad GS = p = \left(\frac{2\pi m}{qB}\right)(v\cos\theta) = \frac{2\pi}{qB}(mv)\cos\theta$$

$$\therefore \quad p = \frac{2\pi\sqrt{2mk}\cos\theta}{qB} \qquad \left(\because\ mv = \sqrt{2mk}\right)$$

$$\text{Hence, } B = \frac{2\pi\sqrt{2mk}\cos\theta}{qp}$$

$$= \frac{2\times 3.14\sqrt{2\times 9.1\times 10^{-31}\times 2\times 10^{3}\times 1.6\times 10^{-19}}\times\frac{1}{2}}{1.6\times 10^{-19}\times 0.1}$$

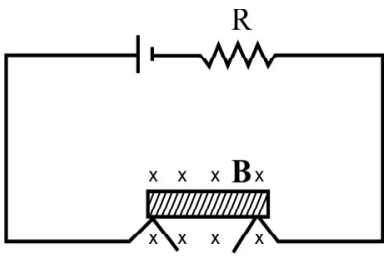
$$= 4.73\times 10^{-4}\,\text{T}$$

Hence option (2) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Force due to Magnetic Field*

44. A thin, 50cm long metal bar, with mass 750 g rests on, but in not attached to, two metallic supports in a 0.450T magnetic field as shown in figure. A battary and resistance $R(=25\Omega)$, in series, are connected to the support. The maximum voltage, the battery can have without breaking the circuit at the supports, will be



(1) 600 V (Incorrect)

(2) 817.5 V (Correct)

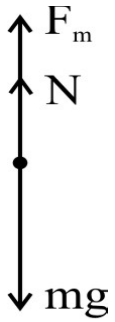
(3) 824.5 V (Incorrect)

(4) 807.25 V (Incorrect)

Correct answer: (2)

Solution:

FBD of metal bar, for metal to be in equilibrium.



$$F_m + N = mg$$

$$F_m = mg - N$$

$$I\ell B = mg - N$$

$$\frac{V}{R}\ell B = mg - N$$

$$V = \frac{R}{\ell B}(mg - N)$$

For maximum voltage, N = 0

$$\therefore\quad V_m = \frac{R}{\ell B}mg$$

$$= \frac{25\times 750\times 10^{-3}\times 9.8}{50\times 10^{-2}\times 0.450}\text{V}$$

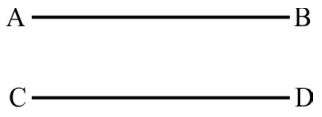
$$= 817.5\text{ V}$$

Hence option (2) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Difficult *Force between two Parallel Current Carrying Conductors*

45. A long horizontal wire AB, which is free to move in a vertical plane, and carries a steady current of 20A, is in equilibrium at a height of 0.01m over another parallel long wire CD. The wire CD is fixed in a horizontal plane and carries a steady current of 30A. As shwon in figure. If wire AB is very slightly depressed and released, the time period of its simple harmonic oscillations, will be



(1) 9.86 second (Incorrect)

(2) 3.14 second (Incorrect)

(3) 0.1 second (Incorrect)

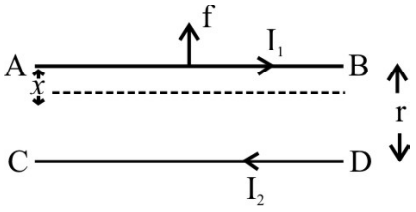
(4) 0.2 second (Correct)

Correct answer: (4)

Solution:

Force per unit length on conductor AB

$$f = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}$$



For equilibrium $f = \frac{m}{\ell}g = \lambda g$

Hence $\lambda g = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}$

Net force on unit length of wire AB

$\lambda a = \lambda g - f'$ where 'a' is the acceleration

or $\lambda a = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r} - \frac{\mu_0}{4\pi}\frac{2I_1I_2}{(r-x)}$

$$\lambda a = -\frac{\mu_0}{4\pi}\frac{2I_1I_2x}{r(r-x)}$$

When $x << r$ $\lambda a = -\frac{\mu_0}{4\pi}\frac{2I_1I_2x}{r^2}$

$$a = -\left(\frac{\mu_0 I_1 I_2}{2\lambda\pi r^2}\right)x$$

$\therefore$ For the motion to be simple harmonic we must have $a = -\omega^2 x$

Hence $\omega^2 = \frac{\mu_0 I_1 I_2}{2\lambda\pi r^2}$

$$\omega = \frac{1}{r}\sqrt{\frac{\mu_0 I_1 I_2}{2\lambda\pi}}$$

Now $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2\lambda\pi r^2}{\mu_0 I_1 I_2}} = 2\pi\sqrt{\frac{2\lambda\pi r^2}{2\pi r\lambda g}} = 2\pi\sqrt{\frac{r}{g}}$

Substituting values,

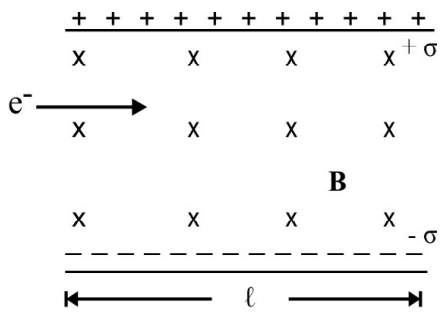
$T = 2\times 3.14\sqrt{\frac{0.01}{9.8}}$ second $= 0.2$ second

Hence option (4) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Force on Charged Particle in Magnetic Field & Electric Field*

46. An electron moves straight inside a charged parallel plate capacitor of uniform surface charge density σ. The space, between the plates, has a constant magnetic field B, as shown in figure. Neglecting gravity, the time taken to cover straight line distance, 'ℓ', by as electron, moving with a constant velocity v, in the capacitor, will be



(1) $\frac{\sigma}{\varepsilon_0 \ell B}$ (Incorrect)

(2) $\frac{\sigma}{\varepsilon_0 B}$ (Incorrect)

(3) $\frac{\varepsilon_0 \ell B}{\sigma}$ (Correct)

(4) $\frac{\varepsilon_0 B}{\sigma}$ (Incorrect)

Correct answer: (3)

Solution:

Net electric field between the plates of capacitor

$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$

$$= \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

$\therefore$ Force on the electron due to electric field

$$\left|F_e\right| = eE$$

Force on the particle due to magnetic field

$$F_m = e\left(\mathbf{v} \times \mathbf{B}\right) = evB$$

For unaccelerated straight line motion $\mathbf{F}_{net} = \mathbf{F}_e + \mathbf{F}_m = 0$

$$\text{Hence } eE = evB \text{ which gives } v = \frac{E}{B} = \frac{\sigma}{\varepsilon_0 B}$$

Hence, time of motion, inside the capacitor, is

$$t = \frac{\ell}{v} = \frac{\varepsilon_0 \ell B}{\sigma}$$

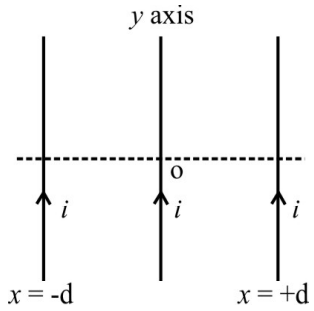
Hence option (3) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Difficult *Force on Current Carrying Conductors*

47. Three infinitely long thin wires, each carrying current i, in the same direction, are in the ***x***–**y plane of gravity free space. The central wire is along the y–axis while other two are along** $x = \pm d$ **. If the central wire is displaced, along the z–direction, by a small amount, and**

released, it starts executing simple harmonic motion. If the linear density of wire is λ. The frequency of oscillations will be



(1) $\frac{i}{2\pi d}\sqrt{\frac{\mu_0}{\pi\lambda}}$ (Correct)

(2) $\frac{i}{d}\sqrt{\frac{\mu_0}{\pi\lambda}}$ (Incorrect)

(3) $\frac{i}{\pi d}\sqrt{\frac{\mu_0}{\pi\lambda}}$ (Incorrect)

(4) $\frac{2i}{\pi d}\sqrt{\frac{\mu_0}{\pi\lambda}}$ (Incorrect)

Correct answer: (1)

Solution:

Let the +y–axis be perpendicular (into) the plane of paper *x* and z axis are in the plane of the paper.

After displacing the central wire, along the z axis a little, through a distance, 'z', say

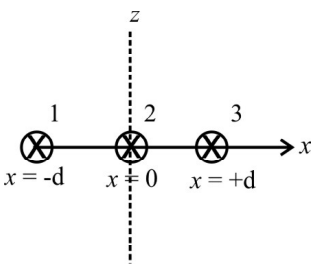
Force/length, acting on the central wire, is $F_{net} = 2F\cos\theta = 2\times\frac{\mu_0 i^2}{2\pi r}\times\frac{z}{r}$

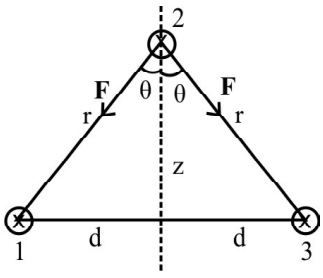
$$F_{net} = \frac{\mu_0 i^2}{\pi r^2}z = \frac{\mu_0 i^2}{\pi\left(d^2+z^2\right)}z$$

If $z << d$, then $F_{net} = -\frac{\mu_0 z^2}{\pi d^2}.\ z$

Let 'a' be the acceleration in the wire in this position then

$$F_{net} = \lambda a = -\frac{\mu_0 i^2}{\pi d^2}z \qquad \therefore\ a = -\frac{\mu_0 i^2}{\pi\lambda d^2}z$$





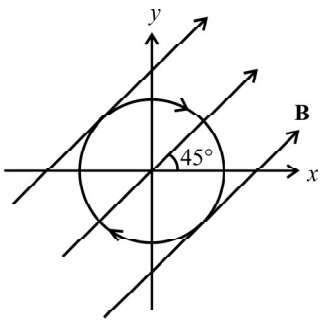
Hence $\omega^2 = \frac{\mu_0 i^2}{\pi\lambda d^2} \Rightarrow \upsilon^2 = \frac{\mu_0 i^2}{4\pi^3\lambda d^2} \Rightarrow \upsilon = \frac{i}{2\pi d}\sqrt{\frac{\mu_0}{\pi\lambda}}$

Hence option (1) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Torque Acting on a Current Loop*

48. A circular loop, of radius 20cm, is placed (in the x–y plane), in a uniform magnetic field of strength 2.0 tesla, that is directed as shown, in the xy plane. The loop carries a current 1.0A in the direction shown in figure. The torque acting on the loop will be.



(1) $0.18\left(\hat{i}-\hat{j}\right)$ (Correct)

(2) $0.18\left(\hat{i}+\hat{j}\right)$ (Incorrect)

(3) $0.18\left(\hat{i}+\hat{k}\right)$ (Incorrect)

(4) $0.18\,\hat{k}$ (Incorrect)

Correct answer: (1)

Solution:

The magnetic field can be expressed as

$$\mathbf{B} = 2\cos 45^0\hat{i} + 2\sin 45\,\hat{j} = \sqrt{2}\left(\hat{i}+\hat{j}\right)\text{T}$$

Magnetic moment is always perpendicular to the plane of loop. In the given figure its direction will be into the plane of paper.

$\therefore$ $\mathbf{M} = \text{Ni}\mathbf{A} = \left[(1)(1.0)(0.2)^2\left(-\hat{k}\right)\right]\text{Am}^2 = 0.04\pi\left(-\hat{k}\right)\text{Am}^2$

$\therefore$ Torque $\vec{\tau} = \mathbf{M}\times\mathbf{B}$

$= 0.04\pi\left(-\hat{k}\right)\times\sqrt{2}\left(\hat{i}+\hat{j}\right) = -0.04\pi\sqrt{2}\left(\hat{j}-\hat{i}\right)$

$= 0.04\times 3.14\times 1.414\left(\hat{j}-\hat{i}\right) \simeq 0.18\left(\hat{i}-\hat{j}\right)\text{N}-\text{m}$

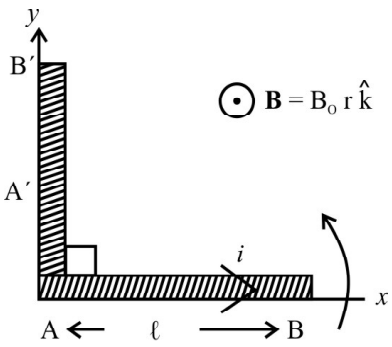
Hence option (1) is correct.

Unit–13 : Magnetic Effects of Current and Magnetism

Average

Force on Conductor

49. A conductor, of length 'ℓ', lies along the x–axis. It carries a current i (along the *x*–axis). The magnetic field, acting in the region is $\mathbf{B} = B_0 r\hat{k}$, where r is the distance of the field point from the origin. The amount of work done in turning it at a constant angular speed, through a right angle, (as shown in figure), will be



(1) $\frac{IB_0\ell^2\pi}{3}$ (Incorrect)

(2) $\frac{IB_0\ell^3\pi}{4}$ (Incorrect)

(3) $\frac{IB_0\ell^3\pi}{6}$ (Correct)

(4) $\frac{IB_0\ell^3\pi}{2}$ (Incorrect)

Correct answer: (3)

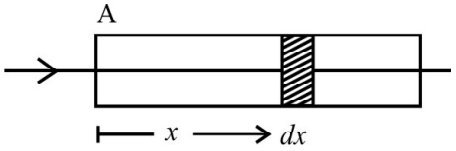
Solution:

Force acting on small element of the conducting rod at distance *x* from end A

$dF = IdxB_0x$

Torque about A due to this force

$d\tau = xdF = IdxB_0x^2$



Total torque $\tau = IB_0\int_0^{\ell} x^2dx = \frac{IB_0\ell^3}{3}$

External torque required to rotate $= = \frac{IB_0\ell^3}{3}$

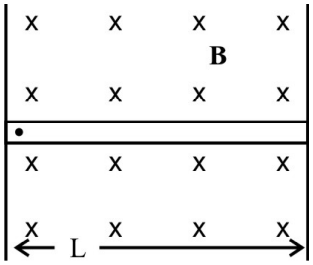
Hence work done $\omega = \tau\theta = \frac{IB_0\ell^3}{3} \times \frac{\pi}{2} = \frac{I\pi B_0\ell^3}{6}$

Hence option (3) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Difficult *Torque / Force on a Conductor*

50. A straight conductor, of mass 'm', and carrying a current i, is hinged at one end and placed in a plane perpendicular to the magnetic field of intensity B as shown in figure. At any moment, if the conductor is let free then the angular acceleration of the conductor (neglecting the effects of gravity) will be



(1) $\frac{2iB}{3m}$ (Incorrect)

(2) $\frac{3iB}{2m}$ (Correct)

(3) $\frac{iB}{2m}$ (Incorrect)

(4) $\frac{3iB}{4m}$ (Incorrect)

Correct answer: (2)

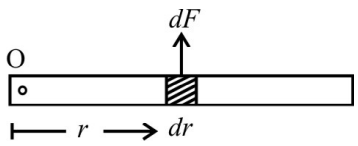
Solution:

Force acting on the elementary portion of the current carrying conductor

$$dF = i(dr)B\sin 90^0 = idrB$$

Torque of this force, on this element, about '0'

$$d\tau = rdF$$



Hence total torque about $0 = \int d\tau = \int r(iBdr)$

$$\tau = IB\int_0^L r\,dr = \frac{iBL^2}{2}$$

Angular acceleration $\alpha = \frac{\tau}{MI} = \frac{\tau}{\text{Moment of inertia}}$

Here Moment of inertia $= \frac{mL^2}{3}$

Hence $\alpha = \frac{3IBL^2}{2.mL^2} = \frac{3iB}{2m}$

Hence option (2) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Easy *Moving Coil Galvanometer*

51. A galvanometer coil is would on the same frame but with a wire of one fourth of the original diameter and 10 times the original number of turns. The ratio of the new deflections to the 'old' deflection, per mA of current the same current as before is supplied to it will be.

(1) $\frac{5}{2}$ (Incorrect)

(2) $\frac{5}{8}$ (Correct)

(3) $\frac{5}{4}$ (Incorrect)

(4) $\frac{5}{16}$ (Incorrect)

Correct answer: (2)

Solution:

Let dashed quantities denote the new values and undashed quantities the 'old' values.

$\therefore \quad d' = \frac{d}{4} \qquad n' = 10n$

The deflection, θ, is related to the current I by the relation $I = \frac{c}{nAB}\theta$

$\therefore \quad I = \frac{4c}{n\lambda d^2 B}\theta$

and $I = \frac{4c\theta'}{10n\pi d^2 B} = \frac{64c\theta'}{10\pi nBd'^2}$

$$\therefore \quad \frac{4c\theta}{n\pi Bd^2} = \frac{64c\theta'}{10\pi nBd^2}$$

or $$\frac{\theta'}{\theta} = \frac{5}{8}$$

Hence option (2) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Easy *Moving Coil Galvonometer*

52. A galvanometer is sensitive if it shows as large a deflection as possible when a very small current is passed through it. To achieve improved sensitivity of the galvanometer, the manifacturers prefer to

(1) increase n as well as k (Incorrect)

(2) decrease k and increase A (Incorrect)

(3) increase n and A (Incorrect)

(4) increase B and decrease k (Correct)

Correct answer: (4)

Solution:

The value of n cannot be increased beyond a certain limit. This can make galvanometer bulky also its increase the resistance of the galvanometer.

Also the area A cannot be increased beyond a limit. It will make galvanometer unmanagable in size and the coil will then not be in auniform field. How ever **B** can be increased using a powerful source of magnetic field and k can be decreased by using materials like quartz or phospher bronze as suspension strips.

Unit–13 : Magnetic Effects of Current and Magnetism

Easy *Moving Coil Galvanometer*

53. In the design of a moving coil galvanometer a soft iron cylinder is kept concentric with cylinderical concave pole pieces and enclosed by them. The role of the soft iron cylinder is to

(1) provide stability to the suspension system of the galvanometer (Incorrect)

(2) provide a uniform field between the pole pieces of the magnet used (Incorrect)

(3) to provide a stronger field (Incorrect)

(4) provide (along with the curved pole pieces) a radial field between the hole pieces of the magnet. (Correct)

Correct answer: (4)

Solution:

Informative

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Moving Ciol Galvanometer*

54. A moving coil galvanomeer is placed in a radial magnetic field of 0.2 wb/m^2 . The coil of the galvanometer has 200 turns and its area is $1.6\times10^{-4}m^2$. The restoring couple constant of the suspension fibre is 10^{-6} (N-m)/degree. The maximum current this galvanometer can measure if its scale can accomodate only up to 45^0 deflection, P would be (nearly)

(1) 7.0 mA (Correct)

(2) 7.0 μA (Incorrect)

(3) 14.0 mA (Incorrect)

(4) 14.0 μA (Incorrect)

Correct answer: (1)

Solution:

The maximum current, that the galvanometer can measure is the one which can produce a deflection of 45^0.

For the galvanometer, we have

$$n\,iBA = c\,\theta$$

$$\therefore \quad i = \left(\frac{c}{nBA}\right)\theta$$

$$\text{Thus } i_{max} = \frac{10^{-6}\times 45}{200\times 0.2\times 1.6\times^{-4}}\,A \simeq 7\times10^{-3}\,A$$

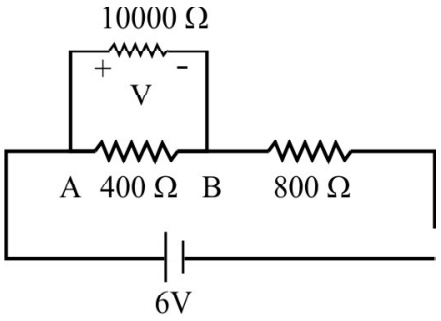
$$\approx 7\,mA$$

Hence option (1) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Voltmeter*

55. Two resistor 400 Ω and 800 Ω are connected in series will a 6V battry. A voltmeter of 10000 resistance is used to measure the potential difference across 400 Ω resistor. The reading of the voltmeter would be



(1) 2.25 V (Incorrect)

(2) 1.95 V (Correct)

(3) 1.75 V (Incorrect)

(4) 1.55 V (Incorrect)

Correct answer: (2)

Solution:

Since 400 Ω and 10000 Ω resistors are in parallel, there equivalent resistance is

$$R_{eq} = \frac{400 \times 10000}{400 + 10000}\Omega = \frac{5000}{13}\Omega$$

Total resistance of the circuit

$$R = \left(\frac{5000}{13} + 800\right)\Omega = \frac{15400}{13}\Omega$$

Current in the circuit $I = \left[\frac{(6)}{\left(\frac{15400}{13}\right)}\right]^{A} = \frac{39}{7700}A$

Potential difference across A and B is given

$$= \frac{39}{7700} \times \frac{5000}{13} = 1.95\,\text{V}$$

Hence option (2) is correct.

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Voltmeter*

56. A voltmeter reads 5 volts at full scale deflection and is graded (according to its resistance per volt at full scale deflection) as 5000 Ω/V . It is desired to convert it into a voltmeter that reads 20V at full scale deflection by connecting a appropriate resistance R in seires with its coil. Then

(1) The value of R should be $10^5\,\Omega$ and its grading will become 2000 Ω/V (Incorrect)

(2) We need to have an additional resistance of $75000\,\Omega$ to be connected in series and the grading will remain the same (Correct)

(3) We need to have an addtional resistance of $75000\,\Omega$ to be connected in series and the grading will become 2000 Ω/V (Incorrect)

(4) We need a resistance of $10^5\,\Omega$ and the grading will be 5000 Ω/V (Incorrect)

Correct answer: (2)

Solution:

The grading of a voltmeter, in terms of resistance/volt, is done by dividing its resistance by the voltage that produces a full scale deflection in it.

Thus the resistance of the given voltmeter is

$$R_v = \left(\frac{5000}{1\,\text{volt}} \times 5\,\text{volt}\right) = 25000\ \Omega$$

$\therefore$ The current that will produce a full scale deflection is given by

$$i_{max} = \frac{5}{25000}\,\text{A} = 2 \times 10^{-4}\,\text{A}$$

When the voltmeter has to have a range, of 20v, its resistance should be

$$R_v = \frac{20}{i_{max}}\Omega = \frac{20}{2 \times 10^{-4}}\Omega = 10^5\ \Omega$$

Hence an additional resistance of $\left(10^5 - 25000\right)\ \Omega$ i.e., $75000\,\Omega$ has to be added in series to effect the conversion of the voltmeter to have a range of 20v.

The grading of the desired converted voltmeter is $\left(\frac{10^5}{20}\right)\Omega/V = 5000\ \Omega/V$

Hence the option (2) is correct.

Unit–13 : Magnetic Effects of Current and Magnetism

Difficult *Ammeter*

57. A galvanometer, having a resistance of $100\,\Omega$ gives a full scale deflection for a current of 1 mA. The resistance, which can convent this galvanometer into an ammeter, giving full scale deflection for a current of 10A, is R. This converted meter, when connected across the terminals of a bettery (emf = E) reads 4A. The current drops to 1. 0A when a resistance of $1.5\,\Omega$ is connected in series with this converted galvanometer. The values of R, E and r (= internal resistance of battery) are given by

(1) $0.01\,\Omega$, 2V and $0.98\,\Omega$ (Incorrect)

(2) $0.1\,\Omega$, 1V and $0.49\,\Omega$ (Incorrect)

(3) $0.01\,\Omega$, 1V and $0.49\,\Omega$ (Correct)

(4) $0.001\,\Omega$, 0.2V and $0.49\,\Omega$ (Incorrect)

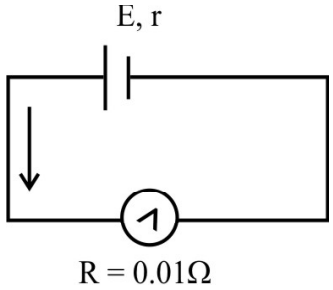
Correct answer: (3)

Solution:

The value of R is obtained from the relation

$$\frac{G}{R} = \frac{9.999}{0.001}$$

$$\therefore\ R = \frac{100 \times 1}{999.9} = 0.01\,\Omega$$



From figure shown, we have

$$4 = \frac{E}{4 + 0.01}$$

and $$1.0 = \frac{E}{0.01 + 1.5 + r}$$

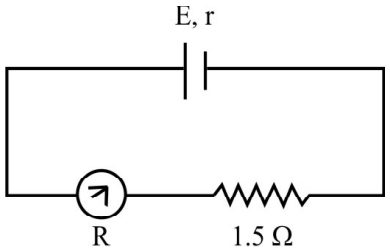
or $1.51 + r = E$

Solving above equations we get

$E = 2$ volt

and $r = 0.49\,\Omega$

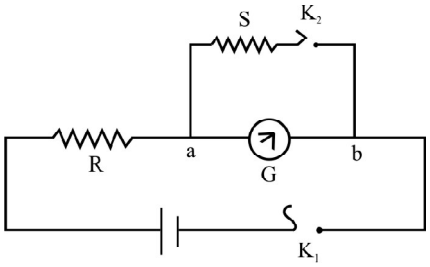
Hence option (3) is correct.



Unit–13 : Magnetic Effects of Current and Magnetism

Easy *Ammeter*

58. An electrical circuit is made as shown. The galvanometer has a resistance G. With the key K_2 open, the current in the circuit is i. The current, in the galvanometer with key K_2 is closed and S = 0, would be



(1) ∞ (Incorrect)

(2) zero (Correct)

(3) $\frac{i}{2}$ (Incorrect)

(4) $\frac{E(S+G)}{R(S+G)+SG}$ (Incorrect)

Correct answer: (2)

Solution:

When the key K_2 open, the resistance of the circuit is (R+G). Hence

$$i = \frac{E}{R+G}$$

When the key K_2 is open and S = 0, all the current reaching point a would go via the zero resistance path aK_2 b. Hence the current through the meter would be zero.

Hence option (2) is correct.

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Ammeter*

59. The resistance of a pivoted type galvanometer is 8Ω and the current for full scale deflection on it is 0.01A. This galvanometer is to be converted into an ammeter of 5A range. The only shunt available is 0.02Ω. The resistance, that needs to be connected in series with the galvanometer coil, is

(1) 02Ω (Incorrect)

(2) 1.98Ω (Correct)

(3) 2.0Ω (Incorrect)

(4) 0.198Ω (Incorrect)

Correct answer: (2)

Solution:

We must have

$$I_g\left(G+R\right)=\left(I-I_g\right)5$$

or $0.01\ (8+R) = (5-0.01)\ (0.02)$

Hence,

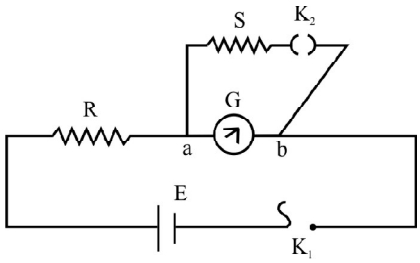
$$R=\left\{\left[\frac{4.99\times0.02}{0.01}\right]-8\right\}\Omega=1.98\Omega$$

Hence option (2) is correct.

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Meters*

60. An electrical circuit is shown in the figure. The meter shows a current i when the key K_2 is open. The resistance of the meter is G. Neglecting the internal resistance of the battery the value of S, to make the current in the meter $\frac{i}{2}$, is equal to



(1) $\left(\frac{RG}{R+G}\right)$ (Correct)

(2) $\frac{\left(\frac{G}{2}\right)}{(R+G)}$ (Incorrect)

(3) $\frac{\left(R+\frac{G}{2}\right)}{(RG)}$ (Incorrect)

(4) G (Incorrect)

Correct answer: (2)

Solution:

Denoting by X, the equivalent resistance of the parallel combination of S and G we have

$$X=\frac{SG}{S+G}$$

As the total resistance of the circuit is now R + X, hence the current i flowing in the circuit now, is given by

$$i'=\frac{E}{R+X}=\frac{E(S+G)}{R(S+G)+SG}$$

This current i', divides inself into two paths (a k_2 b) and aGb (i_2). If the currents in these two paths are i_1 and i_2, we have

$$i' = i_1 + i_2$$

Since the p.d between a and b is one and the same we have

$$i_1 S = i_2 G$$

$$\frac{i_2}{i_1}=\frac{S}{G} \qquad \therefore i_2=\frac{S}{S+G}i'$$

As per question we want $i_2 = i/2$

So using above derived equations we get

$$\left(\frac{S}{S+G}\right)\left[\frac{E(S+G)}{R(S+G)+SG}\right]=\frac{1}{2}\left(\frac{E}{R+G}\right)$$

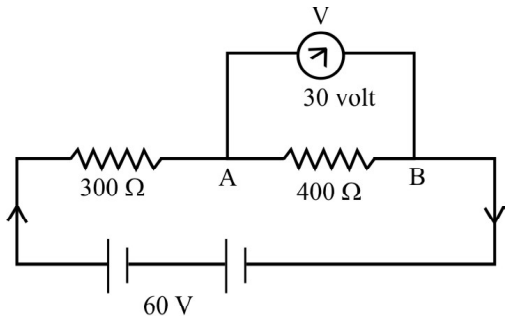
This gives $S = \dfrac{RG}{R+G}$

Hence option (1) is correct.

Unit–13 : Magnetic Effects of Current and Magnetism

Difficult *Voltmeter*

61. A voltmeter is connected across a $400\,\Omega$ resistance and reads 30 volt. If the same voltmeter is connected across the $300\,\Omega$ resistance, then its reading will be



(1) 30 volt (Incorrect)

(2) 22.5 volt (Correct)

(3) 24 volt (Incorrect)

(4) $\left(90/7\right)$ volt (Incorrect)

Correct answer: (2)

Solution:

Let R_v be the resistance of the voltmeter. Then R_v and $400\,\Omega$ resistance, when connected in parallel would have an equivalent resistances given by

$$R_1 = \frac{400 \times R_v}{\left(R_v + 400\right)}$$

and the current in the circuit with correspond to

$$\frac{60}{300 + \left[\dfrac{400\,R_v}{R_v + 400}\right]} = \frac{60\left(R_v + 400\right)}{300\left(R_v + 400\right) + 400R_v}$$

According to question potential drop across AB is 30 volt so

$$30 = \frac{60(R_v + 400)}{300(R_v + 400) + 400R_v} \times R_1 = \frac{60(400R_v)}{300(R_v + 400) + 400R_v}$$

Solving we obtain $R_v = 1200\,\Omega$

Now when the voltmeter is connected across $300\,\Omega$ the equivalent resistance of their parallel combination, is

$$\frac{1200 \times 300}{1200 + 300}\,\Omega = 240\,\Omega$$

Now the total resistance of the circuit would be $(240+400)\,\Omega = 640\,\Omega$ and the corresponding current is $\left(60/640\right)$

Hence in this case reading of the voltmeter would be

$$\frac{60}{640} \times 240 = 22.5\,\text{Volt}$$

Hence option (2) is correct.

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Conversion of Galvanometer into Voltmeter and Ammeter*

62. A given galvanometer can be converted into a (i) a voltmeter of range (0 to V) volt by connecting a resistance, R, in series with its coil, (ii) an ammeter of range (0 to I) ampere by connecting a shunt resistance, S, across its coil.
The resistance (G) of the galvanometer, and the current (i_g), needed to produce a full scale deflection in it, are then given by

(1) $G = \left(\frac{RI - V}{V - SI}\right)S;\ i_g = \left(\frac{V - SI}{R - S}\right)$ (Correct)

(2) $G = \left(\frac{V - SI}{RI - V}\right)S;\ i_g = \left(\frac{V + SI}{R + S}\right)$ (Correct)

(3) $G = \left(\frac{V + SI}{RI + V}\right)S;\ i_g = \left(\frac{V - SI}{R - S}\right)$ (Incorrect)

(4) $G = \left(\frac{RI + V}{V + SI}\right)S;\ i_g = \left(\frac{V + SI}{R + S}\right)$ (Incorrect)

Correct answer: (1)

Solution:

We know that $R = \left(\frac{V}{i_g} - G\right)$ and $S = \left(\frac{i_g}{I - i_g}\right)G$

$\therefore \quad Ri_g = \left(V - i_g.G\right) \quad$ or $\quad \left(R + G\right)i_g = V$

and $SI = \left(G + S\right)i_g \quad$ or $\quad (G + S)i_g = SI$

$$\therefore \quad \frac{R + G}{G + S} = \frac{V}{SI}$$

This gives $G = \left(\frac{RI - V}{V - SI}\right)S$

We then get $i_g = \left(\frac{V - SI}{R - S}\right)$

Hence option (1) is correct.

Unit–13 : Magnetic Effects of Current and Magnetism

Easy *Magnetic Moment of a Current Loop*

63. A loop of irregular shape, of area 10^{-5} m^2, carries a current of 8A. The flow of current appears to be clockwise to an observer. The magnetic moment of current loop is

(1) 0.8×10^{-4} Am^2 and is directed away from the observer (Incorrect)

(2) 0.8×10^{-4} Am^2 and is in the plane of the loop (Incorrect)

(3) 0.8×10^{-4} Am^2 and is directed towards the observer, perpendicular to the palne of the loop (Incorrect)

(4) 0.8×10^{-4} Am^2 and is directed away from the observer perpendicular to the plane of the loop (Correct)

Correct answer: (4)

Solution:

Magnetic moment of a current loop is $\mathbf{M} = \left(iA\right)\hat{n}$

$\therefore \quad \left|\mathbf{M}\right| = 8\times10^{-5}\,\text{Am}^2 = 0.8\times10^{-4}\,\text{Am}^2$

From right hand rule, the direction, of **M**, is away from the observer and perpendicular to the plane of the loop.

Hence option (4) is correct.

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Potential Energy of a Bar Magnet in a Magnetic Field*

64. A short bar magnet, placed with its axis at 30^0 to a uniform external magnetic field of 0.16T, experience a torque of magnitude 0.032 N–m. Its potential energy, in the field, for its two positions of equilibriums, is

(1) 0.064 J for both stable and unstable equilibrium. (Incorrect)

(2) 0.064 J for stable equilibrium and zero for unstable equilibruim (Incorrect)

(3) 0.064 J for stable equilibrium and –0.064 J for unstable equilibrium (Incorrect)

(4) –0.064 for stable equilibrium and +0.064 for unstable equilibrium (Correct)

Correct answer: (4)

Solution:

Potential energy of a bar magnet is given by, $V = -mB\cos\theta$

We are given that

$|\zeta|_{30} = mB\sin 30^0 = 0.032\ N-m$

$\therefore \quad mB = 0.064\ N-m\ = 0.064\ J$

For stable equilibrium $\theta = 0$ and for unstable equilibrium $\theta = \pi$

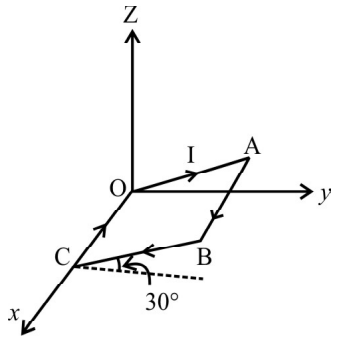
Subsituting the values, the P.E. V – 0.064 J for stable equilibrium and + 0.064 J for unstable equilibrium

Hence option (4) is correct.

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Magnetic Moment*

65. A current carrying loop $OABCO$, carrying current 4.0 A, dimensions OA = 20 cm and AB =10 cm, is placed as shown in figure. The magnetic moment of the loop is



(1) $\left(0.07\hat{j}+0.04\hat{k}\right)Am^2$ (Incorrect)

(2) $\left(0.04\hat{j}+0.07\hat{k}\right)Am^2$ (Incorrect)

(3) $\left(0.07\hat{j}-0.04\hat{k}\right)Am^2$ (Incorrect)

(4) $\left(0.04\hat{j}-0.07\hat{k}\right)Am^2$ (Correct)

Correct answer: (4)

Solution:

Magnetic moment $\mathbf{M} = I.(\mathbf{OA}\times\mathbf{AB})$

where $\mathbf{OA} = OA\cos\theta\hat{j}+OA\sin\theta\hat{k}$

$\mathbf{AB} = AB\hat{i}$

$\therefore \mathbf{M} = I.\left(OA\cos\theta\hat{j}+OA\sin\theta\hat{k}\right)\times AB\hat{i}$

$= I.\left[OA\times AB\cos\theta\left(-\hat{k}\right)+OA\times AB\sin\theta\hat{j}\right]$

$= 4.0\left[200\times10^{-4}\cos30^0\left(-\hat{k}\right)+200\times10^{-4}\sin30°\hat{j}\right]$

$= 4.0\left[-\sqrt{3}\times10^{-2}\hat{k}+10^{-2}\hat{j}\right]$

$= 10^{-2}\left[-6.928\hat{k}+4\hat{j}\right] = \left(-0.07\hat{k}+0.04\hat{j}\right)Am^2$

Hence option (4) is the correct answer.

Unit–13 : Magnetic Effects of Current and Magnetism

Difficult *Magnetic Moment*

66. A charge 'q' is uniformly distributed on a non-conducting disc, of radius R, it is rotated with angular speed 'ω' about an axis passing through the centre of mass of the disc and perpendicular to its plane. The magnetic moment of the disc will be

(1) $\frac{1}{4}\omega qR^2$ (Correct)

(2) $\frac{1}{2}\omega qR^2$ (Incorrect)

(3) ωqR^2 (Incorrect)

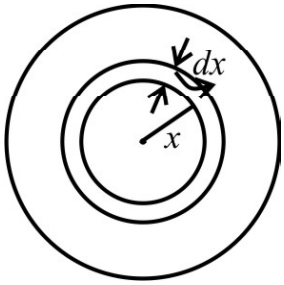
(4) $\frac{1}{8}\omega qR^2$ (Incorrect)

Correct answer: (1)

Solution:

Consider the disc to be made up of large number of elementary concentric rings, consider one such ring of radius '*x*' and thickness '*dx*'.

Charge on this ring $dq = \frac{q}{\pi R^2}.dA = \frac{q}{\pi R^2}(2\pi x dx) = \frac{2qxdx}{R^2}$



Current on this ring $di = \frac{dq}{T} = \frac{\omega dq}{2\pi} = \frac{\omega qxdx}{\pi R^2}$

Magnetic moment of this ring $(dM) = \frac{\omega qxdx}{\pi R^2}\pi x^2 = \frac{\omega q x^3 dx}{R^2}$

Hence magnetic moment of the entire disc $(M) = \int dM = \frac{\omega q}{R^2}\int_0^R x^3 dx$

$= \frac{1}{4}\omega qR^2$

Hence option (1) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Difficult *Magnetic Moment*

67. A solid conducting sphere, of radius R and total charge q, rotates about its diametric axis with a constant angular speed 'ω'. The equivalent magnetic moment of the sphere is

(1) $\frac{1}{3}qR^2\omega$ (Incorrect)

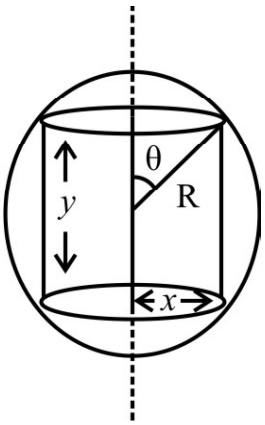
(2) $\frac{2}{3}qR^2\omega$ (Incorrect)

(3) $\frac{1}{5}qR^2\omega$ (Correct)

(4) $\frac{2}{5}qR^2\omega$ (Incorrect)

Correct answer: (3)

Solution:



Consider the sphere to be made up of large number of hollow, coaxial cylinder of different height and radii consider one such cylinder of radius 'x' height 'y' and thickness dx.

$y = 2R\cos\theta, \quad x = R\sin\theta, \quad dx = R\cos\theta\, d\theta$

Charge on this cylinder is $dq = \dfrac{q}{\frac{4}{3}\pi R^3} 2\pi y x dx$

$= 3q\cos^2\theta\sin\theta\, d\theta$

Current, associated with this cylinder, is

$$di = \frac{dq}{T} = \frac{\omega dq}{2\pi} = \frac{3\omega q}{2\pi}\cos^2\theta\sin\theta\, d\theta$$

Hence magnetic moment associated with this cylinder

$$(dM) = di\, A = \left[\frac{3\omega q}{2\pi}\cos^2\theta\sin\theta\, d\theta\right]\pi x^2$$

$$= \frac{3}{2}R^2\omega q A\cos^2\theta\sin^3\theta\, d\theta$$

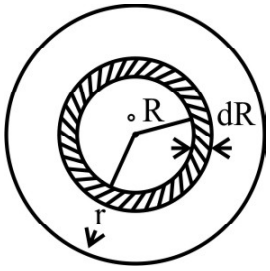
Total magnetic moment (M) $M = \int dM = \dfrac{3}{2}R^2 q\omega\int_{\pi/2}^{\theta}\cos^2\theta\sin^3\theta\, d\theta$

This gives $|M| = \dfrac{1}{5}R^2\omega q$

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Magnetic Moment*

68. A uniformly charged disc, whose total charge has a magnitude 'q', and whose radius is 'r', rotates with a constant angular velocity of magnitude 'ω'. The magnetic dipole moment of the ring is



(1) $\frac{q\omega r^2}{4}$ (Correct)

(2) $\frac{q\omega r^2}{2}$ (Incorrect)

(3) $\frac{q\omega r^2}{8}$ (Incorrect)

(4) $q\omega r^2$ (Incorrect)

Correct answer: (1)

Solution:

Surface charge density $= \frac{q}{\pi r^2}$

$\therefore$ Charge within the ring of radius R and width dR

$$dq = \frac{q}{\pi r^2}(2\pi R dR) = \frac{2q}{r^2} R dR$$

Current carried by this ring

$$di = \frac{dq}{T} = \frac{\omega dq}{2\pi} = \frac{q\omega}{\pi r^2} R dR$$

Hence magnetic moment contributed by this ring (dM) $di \times Area = \frac{q\omega}{\pi r^2} R dR \times \pi R^2$

$$= \frac{q\omega}{r^2} R^3 dR$$

Total magnetic moment of the ring

$$M = dM = \int_0^r \frac{q\omega}{r^2} R^3 dR = \frac{q\omega}{r^2}\left(\frac{R^4}{4}\right)_0^r = \frac{q\omega r^2}{4}$$

Hence option (1) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Cyclotron*

69. The (angular) frequency, ω, of the alternating p.d applied across the dees, and the strength, B, of the (normal) magnetic field, used, in a given cyclotron, have been adjusted to values (say ω_0 and B_0), that would enable this cyclotron to accelerate protons.

If the same cyclotron were to be used for accelerating alpha particles, say, the new value of

(i) ω, when B is left unchanged at its earlier value (B_0) and (ii) B, when ω is left unchanged at its earlier value (ω_0), would equal, respectively

(1) $\frac{\omega_0}{2}$ and $\frac{B_0}{2}$ (Incorrect)

(2) $\frac{\omega_0}{2}$ and $2B_0$ (Correct)

(3) $2B_0$ and $\frac{\omega_0}{2}$ (Incorrect)

(4) $2\omega_0$ and $2B_0$ (Incorrect)

Correct answer: (2)

Solution:

The radius of the circular path, of the charged particle, in a magnetic field, is given by $\frac{mv^2}{r} = qvB$

Hence $r = \frac{mv}{qB}$

The time period, for one cycle of the motion of the charged particle, in this circular path, is given by

$$T = \frac{2\pi r}{v} = \frac{2\pi mv}{v.qB} = \frac{2\pi m}{qB}$$

The corresponding angular frequency, ω, equals $\frac{2\pi}{T} = \frac{qB}{m}$

In the adjusted cyclotron, the (angular) frequency of the applied alternating potential equals this (cyclotron) frequency.

Hence for the protons, we have $\omega_0 = \frac{eB_0}{m}$

For the α–particle, the required (angular) frequency, say, ω_α, is given by

$$\omega_\alpha = \frac{(2e)B_\alpha}{(4m)} = \frac{eB_\alpha}{2m}$$

(i) When there is no change in the value of B, we would have $\omega_\alpha = \frac{eB}{m} \times \frac{1}{2} = \frac{\omega_0}{2}$

(ii) When there is no change in the value of ω_0, we would have $\omega_0 = \frac{eB_\alpha}{2m}$

or $\frac{eB_0}{m} = \frac{eB_\alpha}{2m}$ $\left(\therefore\ B_\alpha = 2B_0\right)$

Hence option (2) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Difficult *Cyclotron*

70. A cyclotron's oscillation frequency is 10 MHz. If the radius of its dee is 60 cm than the kinetic energy of the proton beam, accelerated by the cyclotron is
($e = 1.6\times10^{-19}e$, $m_p = 1.67\times^{-27}$).

(1) 7.5×10^3 MeV (Incorrect)

(2) 7.5 MeV (Correct)

(3) 3.75 MeV (Incorrect)

(4) 7.5 keV (Incorrect)

Correct answer: (2)

Solution:

The charged particles orbit round the circular path, with a frequency, f, given by $f = \frac{Be}{2\pi m}$

Hence $B = \frac{2\pi mf}{e}$

Substituting the given values, we get, B = 0.66 T

The period of revolution of the particles, is $T = \frac{2\pi m}{Be}$

If R is the radius of the dee's of the cyclotron, than the velocity v of the protons, at the point of exit, is

$$v = \frac{2\pi R}{T} = \frac{2\pi R \times Be}{2\pi m} = \frac{BeR}{m}$$

Kinetic energy of the protons produced is $\frac{1}{2}mv^2 = \frac{B^2e^2R^2}{2m}$ =7.5 MeV

Hence option (2) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Easy *Tessestral Magnetism*

71. Angle of dip at a place is 30°. If the verticle component of earth's magnetic field at that place is $\left(0.16\sqrt{3}\right)\times10^{-4}\,\text{T}$, the value of the horizontal component would be

(1) $48\times10^{-4}\,\text{T}$ (Incorrect)

(2) $0.048\times10^{-4}\,\text{T}$ (Incorrect)

(3) $144\times10^{-4}\,\text{T}$ (Incorrect)

(4) $0.48\times10^{-4}\,\text{T}$ (Correct)

Correct answer: (4)

Solution:

We know than $\delta = \dfrac{B_V}{B_H}$

Hence $B_H = \left(\dfrac{B_V}{\tan\delta}\right)$

$$\therefore\quad B_H = \frac{\left(0.16\times\sqrt{3}\right)\times10^{-4}}{\tan 30^0} = \frac{\left(0.16\times\sqrt{3}\right)\times10^{-4}}{\left(\dfrac{1}{\sqrt{3}}\right)}\,\text{T} = 0.48\times10^{-4}\,\text{T}$$

Hence option (4) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Easy *Terrestrial Magnetism*

72. At the magnetic poles, earth's total magnetic field is

(1) Perpendicular to surface of the earth (Correct)

(2) Parallel to the surface of the earth (Incorrect)

(3) zero (Incorrect)

(4) will be at a dip of 60^0 (Incorrect)

Correct answer: (1)

Solution:

Denoting by B, B_H and B_V, the total feild of earth, horizontal component of earth's field and vertical component of earth's field, we have

$$B_V = B\sin\delta$$

where δ is the angle of dip.

At magnetic poles $\delta = 90^0$ and hence $B_H = 0$

At the poles, the earth's totoal field is, therefore, the same as its vertical component. It is, therefore, perpendicular to the surface of earth at the poles.

Hence option (1) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Easy *Terrestrial Magnetism*

73. If we represent earth's magnetic field at the equator, by B, the dipole moment of earth (radius = R)'s equivalent magnet is given by

(1) $\frac{4\pi R^3 B}{\mu_0}$ (Correct)

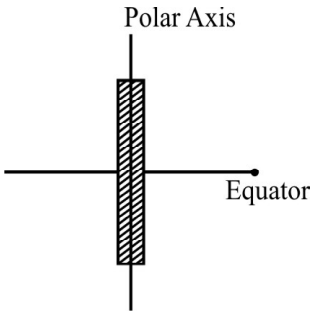
(2) $\frac{4\pi R^2 B}{\mu_0}$ (Incorrect)

(3) $\frac{2\pi R B^2}{\mu_0}$ (Incorrect)

(4) $\frac{2\pi R B}{\mu_0}$ (Incorrect)

Correct answer: (1)

Solution:



We can assume earths magnetic field as due to a (short) bar magnet, centered at the centre of earth and held along its polar axis. We can then write:

$$B_{equator} = \frac{\mu_0 M}{4\pi R^3}$$

where M is the dipole moment of earth's equivalent magnet.

Hence $M = \frac{4\pi R^3 B}{\mu_0}$

Hence option (1) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Terrestrial Magnetism*

74. For a partical plane, other than the magnetic meridian , i.e., a vertical plane at some angle θ, with the magnetic meridian,

(1) angle of dip decreases (Incorrect)

(2) angle of dip increases (Correct)

(3) angle of dip remain uneffected (Incorrect)

(4) the apparent value of the angle of dip becomes zero (Incorrect)

Correct answer: (2)

Solution:

For a vertical plane at an angle θ with the magnetic meridian.

$$B_H' = B_H \cos\theta \quad \text{and} \quad B_V' = B_V$$

Therefore, observed angle of dip δ, in a vertical plane, making an angle θ, with the magnetic meridian, would be

$$\tan\delta' = \frac{B_V'}{B_H'} = \frac{B_V}{B_H \cos\theta} = \frac{\tan\delta}{\cos\theta} \qquad \left(\because \quad \frac{B_V}{B_H} = \tan\delta\right)$$

$$\therefore \quad \tan\delta = \frac{\tan\delta}{\cos\theta}$$

Here δ represents the true value of dip and δ' is the apparent value of dip.

For a vertical plane other than magnetic meridian,

$$\theta > 0^0 \quad \text{and} \quad \cos\theta < 1$$

$$\therefore \quad \delta' > \delta$$

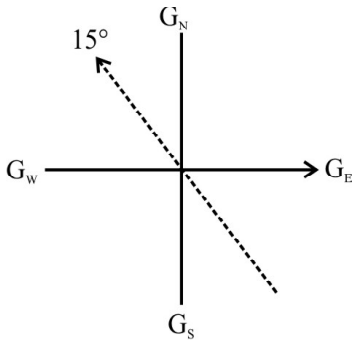
Hence the angle of dip : increases

Hence option (2) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Terrestrial Magnetism*

75. At a certain location a compass points 15^0 west of geographic north. The north tip of the magnetic needle, of a dip circle placed in the plane of the magnetic meridian, points 60^0 above the horizontal. If the horizontal component of earth's magnetic field is measured to be 0.2G, the magnitude and direction of the earth's field at that location, are



(1) 0.4G, directed upward of the horizontal line, in the vertical plane, 15^0 west of geographic north plane (Correct)

(2) 0.4G, directed downward of the horizontal line, in the vertical plane, 15^0 west of geographic north plane (Incorrect)

(3) 0.4G, directed upward of the horizontal line, in the vertical plane, 15^0 east of geographic north plane (Incorrect)

(4) 0.4G, directed downward of the horizontal line, in the vertical plane, 15^0 east of geographic north plane (Incorrect)

Correct answer: (1)

Solution:

We have $B_H = B\cos\delta$

$$\therefore \quad B = \frac{B_H}{\cos\delta} = \frac{0.2}{\cos 60}T = \frac{0.2}{0.5}T = 0.4\,T$$

As shown by the compass needle, B points upward of the horizontal line, 15^0 west of geographic north plane.

Hence option (1) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Average *Terrestrial Magnetism*

76. A ship is to reach a place 7° south of west. If the declination of the plane at the place is 15° of north west the ship should be steered along

(1) east of magnetic north at an angle of 82^0 (Incorrect)

(2) west of magnetic south at an angle of 82^0 (Incorrect)

(3) west of magnetic north at an angle of 82^0 (Correct)

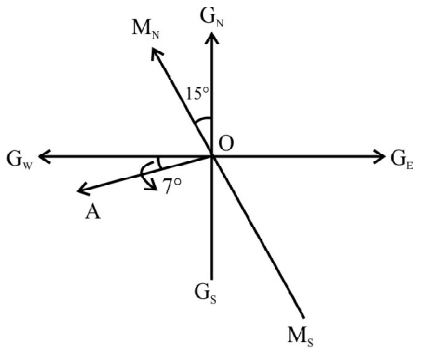
(4) west of magnetic north at an angle of 115^0 (Incorrect)

Correct answer: (3)

Solution:

As the ship is to read 7^0 at a place south of west along 0A), it should be steered west of magnetic north at an angle of $\left(\frac{\pi}{2}-15^0+7^0\right)=82^0$

Hence option (3) is correct



Unit–13 : Magnetic Effects of Current and Magnetism

Average *Elements of Earth's Magnetic Field*

77. The angle of dip, and the horizontal component of earth's magnetic field, at a given place, equal α and B_H, respectiely. If a charged particle, of charge q and mass m, were to move with a speed v, horizontally northwards at that place, the magnitude of the acceleration, caused on this charged particle, would equal

(1) $\frac{qvB_H\cot\alpha}{m}$ (Incorrect)

(2) $\frac{qvB_H\cos\alpha}{m}$ (Incorrect)

(3) $\frac{qvB_H\tan\alpha}{m}$ (Correct)

(4) $\frac{qvB_H\sin\alpha}{m}$ (Incorrect)

Correct answer: (3)

Solution:

Since the charged particle is moving horizontally north wards, it would experience a force, only due to the vertical component (say B_V) of the earth's (total) magnetic field (= B say). The angle of dip being α, we can write

$$B_H = B\cos\alpha \quad \text{and} \quad B_v = B\sin\alpha$$

$$\therefore \quad \frac{B_V}{B_H} = \tan\alpha \quad \text{or} \quad B_V = B_H \tan\alpha$$

The force, due to the earth's magnetic field, on the charged particle, has a magnitude F, where

$$F = \left|q\left(\mathbf{v}\times\mathbf{B}\right)\right| = qvB_V$$

$\therefore$ The acceleration caused, has a magnitude

$$a = \frac{F}{m} = \frac{qvB_V}{m} = \frac{qvB_H\tan\alpha}{m}$$

Hence option (4) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Difficult *Classification of Magnetic Materials*

78. An iron rod (of density 8×10^3 kg/m^3 and specific heat capacity 0.462×10^{-3} J/kg^{0}C) is subjected to cycles of magnetissation at the rate of 50 cycles/s. If the area enclosed by B.H curve corresponds to an energy of 10^{-2} joule, the rise of temperature per minute (assuming there are no radiation losses) would be

(1) 81^0C (Incorrect)

(2) 8.1^0C (Correct)

(3) 0.81^0C (Incorrect)

(4) 16.2^0C (Incorrect)

Correct answer: (2)

Solution:

The area enclosed by the B–H curve, equals the energy loss per unit volume per cycle.

$\therefore$ The energy loss per unit volume in one second = $10^{-2}\times50$ J

Hence Q, the heat produced per unit volume in one minute, equals

$$Q = \left(10^{-2}\times50\times60\right)\text{J} = 30\,\text{J}$$

Let θ be the rise in temperature per unit volume. The mass per unit volume equals ρ, the density. Hence

$$Q = \rho s\theta$$

Hence $8\times10^{3}\times0.462\times10^{-3}\times\theta = 30$

or $\theta = \dfrac{30}{0.462\times8}\,^{0}C = 8.1^{0}C$

Hence option (2) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Difficult *Classification of Magnetic Materials*

79. An iron rod 0.2m long, 10 mm in diameter and of permeability 10^3, is placed inside a long solenoid wound with 300 turns/meter. If a current of 0.5A is passed through the rod, total magnetic moment of the rod would be (nearly)

(1) 2335 SI Unit (Incorrect)

(2) 233.5 SI Unit (Incorrect)

(3) 23.35 SI Unit (Incorrect)

(4) 2.335 SI Unit (Correct)

Correct answer: (4)

Solution:

M, the magnetisation vector (magnetic moment per unit volume is given by)

$$\mathbf{M} = \left(\frac{\mathbf{B}}{\mu_0}\right) - \mathbf{H} = \frac{\mu\mathbf{H}}{\mu_0} - \mathbf{H} = (\mu_r - 1)\mathbf{H}$$

$\therefore\ \mathbf{M} = (\mu_r - 1)\mathbf{H}$

For a long soleniod $|\mathbf{H}|$ is given by

$$|\mathbf{H}| = ni = 300\times0.5 \text{ SI Units} = 150 \text{ SI Units}$$

$\therefore\ |\mathbf{M}| = (1000-1)\times150 \simeq 0.15\times10^{6}$ SI Unit

Total magnetic moment of rod $= |\mathbf{M}| \times V = M\times\pi r^2 L$

$= 0.15\times10^{6}\times3.142\times(5\times10^{-3})^{2}\times0.2$

≈ 2.335 SI units

Hence option (4) is correct

Unit–13 : Magnetic Effects of Current and Magnetism

Difficult *Terrestrial Magnetism*

80. Let δ_1 and δ_2 be the angle fo dip in any two vertical planes (other than the magnetic meridian) that are at right angles to each other. If, δ is the true value of the angle of dip, we would have

(1) $\tan^2\delta_1 + \tan^2\delta_2 = \tan^2\delta$ (Incorrect)

(2) $\dfrac{\tan^2\delta_1}{\tan^2\delta_2} = \cot^2\delta$ (Incorrect)

(3) $\cot^2\delta_1 + \cot^2\delta_2 = 1$ (Incorrect)

(4) $\cot^2\delta = \cot^2\delta_1 + \cot^2\delta_2$ (Correct)

Correct answer: (4)

Solution:

Let the vertical plane, in which angle of dip is δ_1 subtends an angle θ with the magnetic meridian. Their other plane then subteneds an angle $\left(\dfrac{\pi}{2}-\theta\right)$ with the magnetic meridian.

We have, $\tan\delta = \dfrac{B_{V_1}}{B_{H_1}} = \dfrac{B_V}{B_H\cos\theta}$

$\therefore \quad \cot\delta_1 = \dfrac{B_H\cos\theta}{B_V}$

Similarly $\cot\delta_2 = B_H\dfrac{\cos\left(\dfrac{\pi}{2}-\theta\right)}{B_V} = \left(\dfrac{B_H\sin\theta}{B_V}\right)$

Hence $\cot^2\delta_1 + \cot^2\delta_2 = \left(\dfrac{B_H}{B_V}\right)^2\left(\cos^2\theta + \sin^2\theta\right)$

$= \left(\dfrac{B_H}{B_V}\right)^2 \times 1 = \cot^2\delta \qquad \left(\because \quad \dfrac{B_V}{B_H} = \tan\delta\right)$

Hence, $\cos^2\delta = \cot^2\delta_1 + \cot^2\delta_2$

Hence option (4) is correct

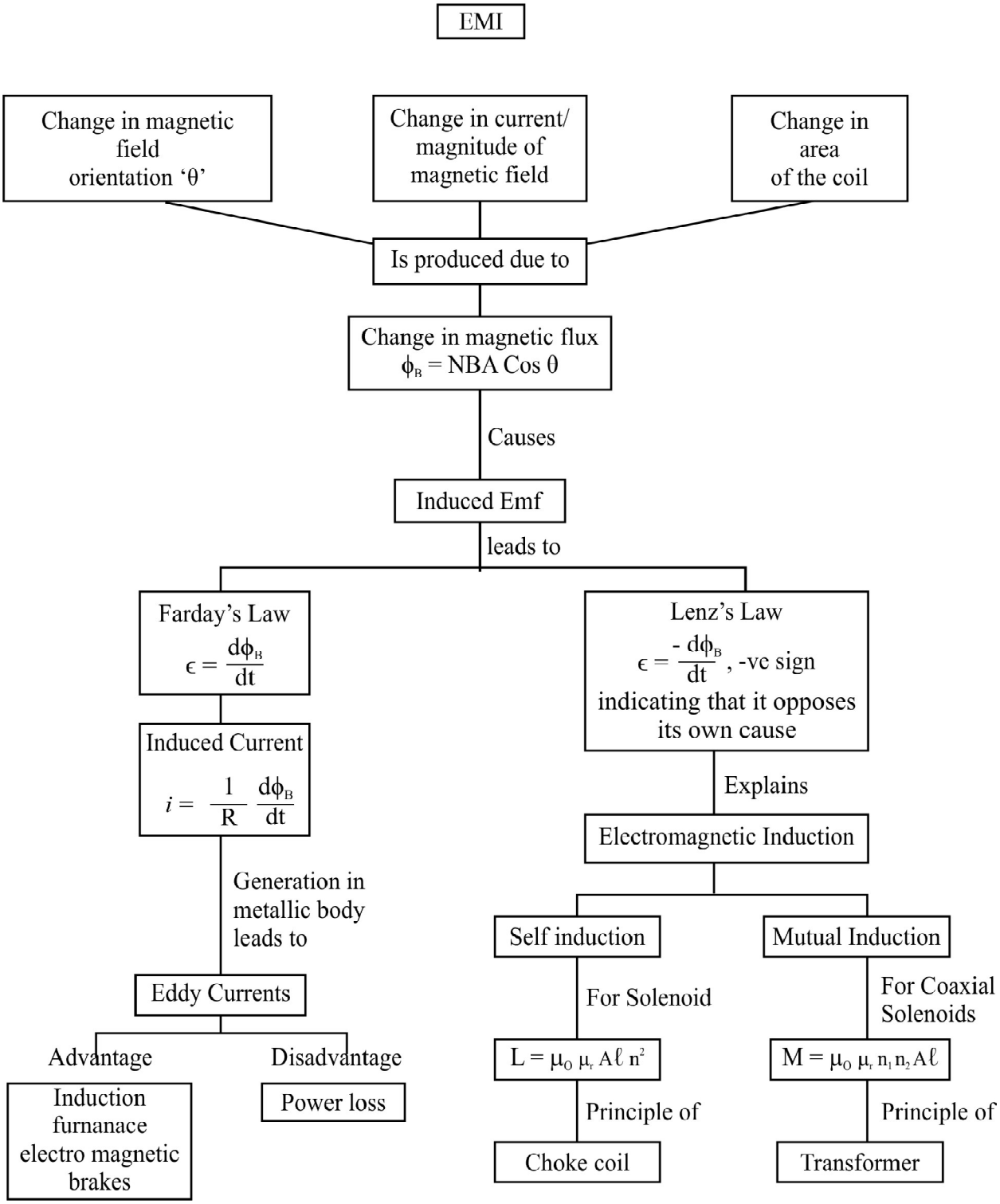
UNIT–14 : ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENTS

Learning Objectives

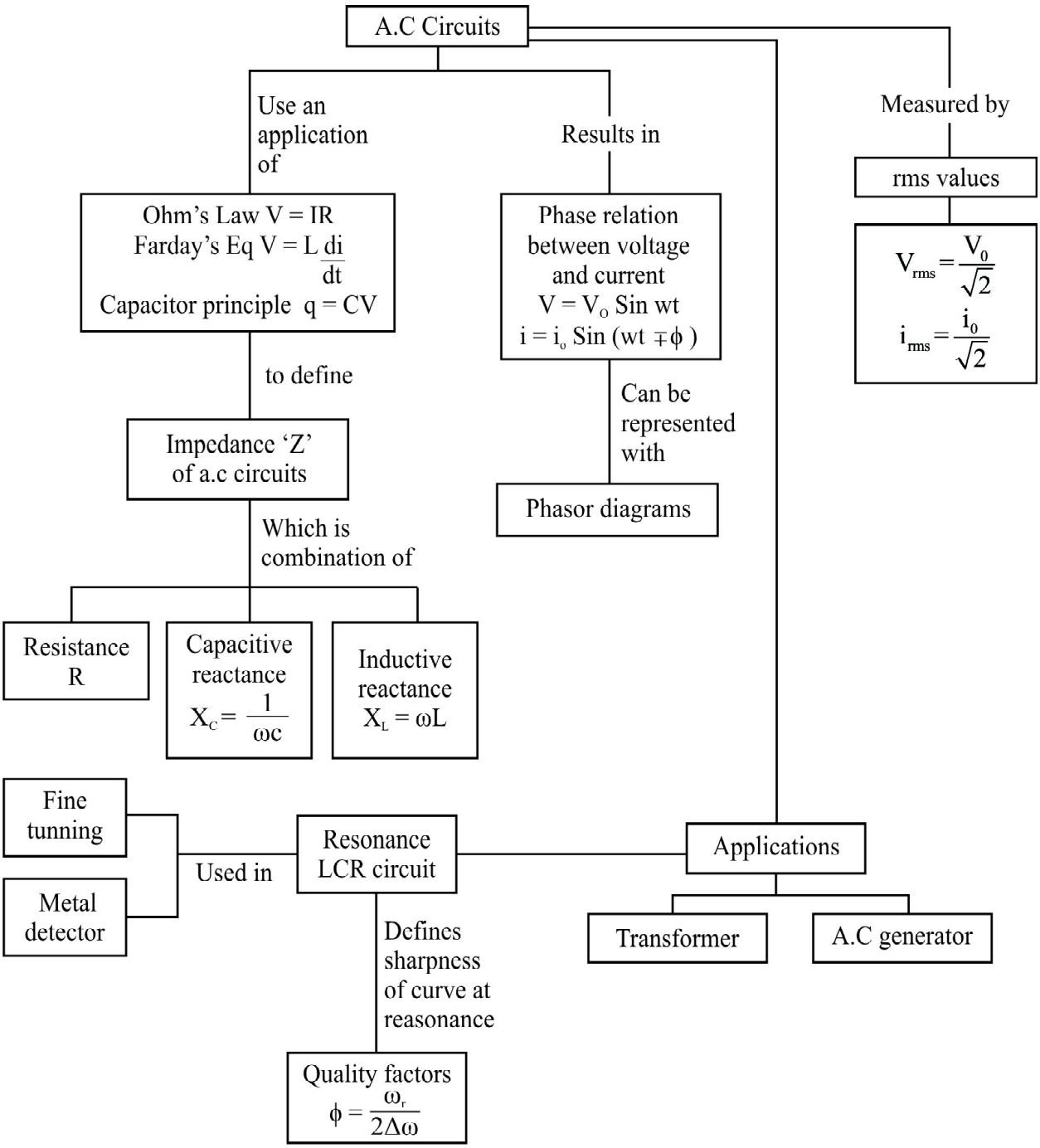
After going through this, unit you will be able to understand, appreciate and apply the following concept:

- Understand the methods, or ways, to induce emf in a coil.
- Understand the Faraday's law of induction and Lenz's Law.
- Calculate emf, current and magnetic flux using Faradays' Law.
- Explain the physical meaning of Lenz's Law.
- Derive the formula for motional emf and power spent by external agency in moving a coil with constant velocity.
- Explain the magnitude and direction of an induced eddy current, and the effect this will have on the object it is induced in.
- Describe the several applications of eddy currents.
- Explain the self induction and back emf.
- Calculate the inductance, energy stored and emf generated in a coil / inductor.
- Explain the mutual induction between the neighbouring coils.
- Calculate the mutual inductance of coaxial solenoids.
- Sketch (draw phasor diagrams) voltage and current versus time in simple inductive, capacitive and resistive circuits.
- Calculate the reactance, impedance, current and voltage in simple series a.c. circuits having L, C and R.
- Derive the phase angle, resonant frequency, power, power factors in series LCR circuit.
- Explain the significance of series resonant circuit, resonant freqeuncy and quality factor.

Concept Map of EMI



Concept Map of Alternating Current



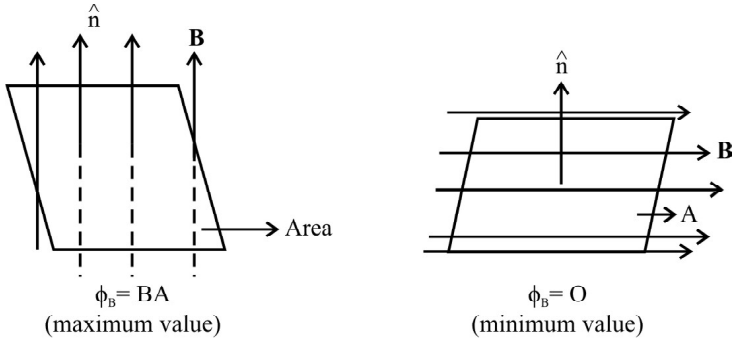
Electromagnetic Induction

Magnetic Flux

The number of magnetic field lines, or magnetic flux 'ϕ_B', which pass through a given cross-sectional area 'A', placed in a uniform magnetic field 'B', can be written as

$$\phi_B = \mathbf{B}.\mathbf{A} = BA\cos\theta$$

where θ is the agnle between the direction the magnetic field and the (outward) unit vector normal aera $\left(A = A\hat{n}\right)$. The SI unit of magnetic flux is weber (Wb). It also equals tesla metere squared (Tm^2). It is a scalar quantity.



The figures, given above, show the positions (of the area A, with respect to the direction of a uniform magnetic field) for which the magnetic flux, linked with the area, has its

(i) maximum value (= BA)

(ii) minimum value (= 0)

Electromagnetic Induction

We knwo that moving electric charges, or currents, can produce magnetic fields. Is the conserve effect possible? Can moving magnets produce electric currents? In the year 1830, Faraday conducted many experiments to demonstrade that electric currents can be induced in closed coils when they are present in a changing magnetic field or when changing the magnetic flux, linked with them, is made to change.

From the experiemental observations of Faraday, it was concluded that whenever the number of magnetic field lines or magnetic flux, passing through a coil changes, an emf is induced in that coil. If the circuit is closed, a current flows through it. The emf, and the current so produced, are called. 'induced emf' and 'induced current' respectivley. They last only as long as the (linked) magnetic flux is changing. This phenomenon is now known as 'electromagnetic induction'.

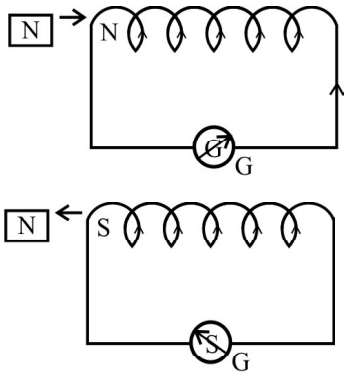
Ways of Changing the (Linked) Magnetic Flux

The magnetic flux, through a circuit, may be changed in a number of ways. This can be done (i) by moving a magnet relative to the circuit (ii) by changing the current in a neighbouring circuit (iii) by changing the current in the same circuit and (iv) by rotating a coil in a uniform magnetic field.

Let us no consider two experiments demonstrating the phenomenon of electromagnetic.

Experiment–1 :

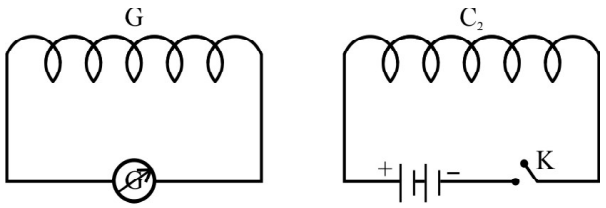
When a bar magnet is pushed towards or pulled away, from the coil, the pointer in the galvanometer deflects. This indicates the presence of electric current in the coil.



The deflection, in the two cases are, however, in opposite senses.

- It is also observed that faster the motion of the magnet, larger the deflection.
- When the bar magnet is held fixed and the coil is moved towards or away from the magnet, similar effects are observed.
- This shows that it is the existence of a relative motion, between the magnet and the coil, that is responsible for inducing electric current in the coil.

Experiment–2 :



- It is observed that the galvanometer shows a deflection (momentary) when the current in C_2 is started by closing key 'K'. The pointer in the galvnometer in (C_1) returns to zero immediately.
- When the key 'K'is released, there is again a momentary curent in C_1 but in opposite direction.
- Similar effects are observed by increasing or decreasing the current in the coil C_2.
- It is also observed that the deflection increases, quite significantly, when an iron rod is inserted into the coils along their axis.
- Hence a changing current in one coil, is a source of induced current in another coil (neighbouring).

Faraday's Law of Induction

Faraday stated these experimental observations in the form of a Law, called 'Faraday's Law of Electricmagnetic Induction'. The law is stated as follows:

"The magnitude of the induced emf, in a circuit, is equal to the time rate of change of the magnetic flx linked with the circuit." Mathematically, the magnitude of induced emf is given by

$$|\varepsilon| = \frac{d\phi_B}{dt}$$

For a closely wound coil of N turns, the total induced emf is given by

$$|\varepsilon| = N\frac{d\Phi_B}{dt}$$

where Φ_B = (Average) magnetic flux lined with 'one turn'.

The product $\left(N\phi_B\right)$ is called the 'number of magnetic flux linages' in the coil. Its unit is 'weber-turns'.

Lenz's Law

The polarity, of the induced emf, is given by a rule, known as lenz's law. The statement of the law is as follows:

"The polarity of induced emf is always such that it tends to produce a curernt, which opposes the change in the magnetic flux that induced it.

We can, therefore, combine Faraday's Law and Lenz's Law to write

$$\varepsilon = -\frac{d\phi_B}{dt}$$

The negative sign indicates the direction of the induced emf 'ε'.

It the circuit is closed, a current $i\left(=\frac{\varepsilon}{R}\right)$ is set up in it. Here R is the resistance of the circuit.

$$\therefore \quad i = -\frac{1}{R}\ \frac{d\phi_B}{dt}$$

Lenz's law is in accordance with the principle of conservation of energy. It enables us to realize that it is the mechanical work, needed to be done to 'move the magnet (towards the coil)', that gets converted into the induced) eletrical energy.

Induced Current and Induced Charge

If, in a coil of N turns, the rate of change of magnetic flux (linked with each turn) be $\frac{d\phi_B}{dt}$, then

$$i = -\frac{1}{R}\left(\frac{N d\phi_B}{dt}\right)$$

Here R is the total resistance of the circuit. The charge, flowing through the circuit, in a time-interval, will be given by $\Delta q = i\,dt$

$$= \frac{1}{R}\left(\frac{N d\phi_B}{dt}\right) \times dt$$

$$\Rightarrow \Delta q = \frac{N}{R}\left(d\phi_B\right)$$

The above equation shows that the total induced charge (that flows) does not depend upon the time interval 'dt'. Whether the change in magnetic flux, is rapid or slow, the total charge that flows in the circuit, remains the same.

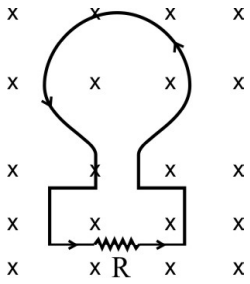
Example–1 :

The magnetic flux, through a coil, present in a magnetic field, directed perpendicular to its plane, varies with time (in second) according to the relation.

$\phi_B = 6t^2 + 7t + 1$ (ϕ_B in miliweber)

Find the magnitude of the induced emf in the coil at t = 2s. Also find the current, and its direction, in the resistance R, if $R = 10\,\Omega$

Solution :



We have $|\varepsilon| = \frac{d\phi_B}{dt} = \frac{d}{dt}\left(6t^2 + 7t - 1\right)$

$= (12t + 7)\ mV$

$\therefore$ At t = 2s, we have

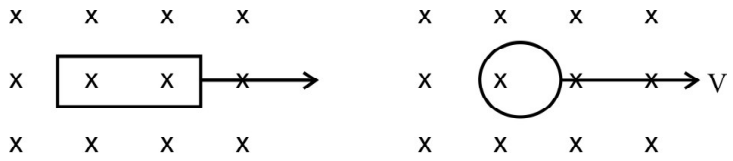
$|\varepsilon| = (12 \times 2 + 7)\,mV = 31\,mV$

Also $i = \frac{|\varepsilon|}{R} = \frac{31}{10}\,mA = 3.1\,mA$

The direction of current, in the resistance R, will be as shown, according to lenz's rule, i.e., from left to right. (As the magnetic flux is increasing with time, induced current has to oppose its cause. For this, it has to set up its own magnetic field, in a direction opposite to the applied field. This would require the induced current to flow in an anticlockwise sense in the coil).

Example–2 :

A rectangular loop, and a circular loop, are moving out of a uniform magnetic field region, to a field free region, with a constant velocity 'V'. In which loop would the induced emf be constant during its pass age out of field region. (The magnetic field is directed normal to the plane of both the loops).

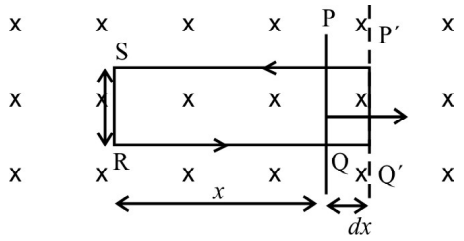


Solution :

In case of circular loop, the rate of change of area of the loop, during its passage out of field region, is not constant. Hence induced emf will varry accordingly. In the case of the rectangular loop ($\frac{dA}{dt}$ and, therefore) $\frac{d\phi_B}{dt}$ is constant. Hence induced emf can be expected to be constant for the rectangular loop.

Motion of a Conductor in a Uniform Magnetic Field

Let us consider a straight conductor, PQ, present in a, uniform and time independent magnetic field. As shown in the figure conductor 'PQ' is free to move on a rectangular conductor frame work, PQRS. Let the rod, PQ, moved towards right with a constant velocity V. As PQRS forms a closed circuit, there in a change in area as the rod PQ moves.



If the length RQ = x and RS = ℓ, then the instantaneous magnetic flux, linked with PQRS, is

$$\phi_B = BA = B\ell x$$

Since x is changing with time, the change of flux, will induce an emf, ε, given by

$$\varepsilon = -\frac{d\phi_B}{dt} = -B\ell\frac{dx}{dt}$$

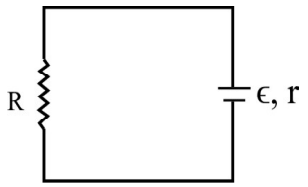
But $\left(\frac{dx}{dt}\right) = v$ the velocity of the conductor PQ, towards right.

Hence $\varepsilon = B\ell v$

The induced emf, $BV\ell$, is called motional emf. Thus we can also produce an induced emf by moving a conductor (instead of varying the magnetic field) in a uniform magnetic field.

We can consider the above rod 'PQ' as a battery of emf, ε, having some internal resistance 'r'. 'R' be the total resistance of the (other) three arms (RS, RQ, PS).

Then, the induced current in the circuit will then be



$$i = \frac{\varepsilon}{R+r} = \frac{BV\ell}{R+r}$$

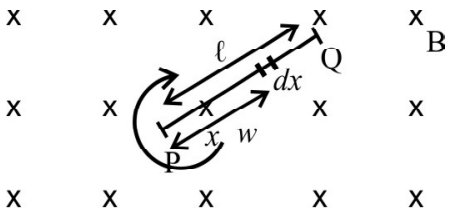
From the above, we conclude that when a straight conductor, of length 'ℓ', moves with a velocity 'V', perpendicular to a uniform magnetic field B, (this 'cutting' its magnetic flux lines) an induced emf, of magnitude $BV\ell$, is induced between the ends of that conductor.

Special Case :

If the direction of the velocity **v**, of the conductor, makes an angle θ with the direction of the magnetic field **B**, the component of **V**, perpendicular to **B**, will be $V\sin\theta$. Therefore in this case the emf induced between the ends of the conductor, will be $BV\ell\sin\theta$. It follows that if a conductor moves parallel to the field, ($\theta = 0$) no emf will be induced between its ends. It can be shown that: in general, motional (induced) emf, can be written as $\varepsilon = (\mathbf{V}\times\mathbf{B}).\ell$.

Situation–1 :

Consider a conducting rod PQ, of length ℓ, being rotated about its one end (in the plane of the page) in a uniform magnetic field **B** (directed normal to the plane of the page), with an angular speed 'ω' as shown.



Consider a small element, of length dx, at a distance x from the confined end, i.e. the end P (about which the rod is rotating).

The induced emf '$d\varepsilon$', across this elemental length, will be = B (dx)V

where $V = \omega x$

Thus, $\varepsilon = \int_{x=0}^{x=\ell} d\varepsilon = B\omega \int_{x=0}^{x=\ell} x\,dx$

$$= \frac{1}{2}B\ell^2\omega$$

If the rod moves about its centre, we would have

$$\varepsilon = \int_{-\ell/2}^{+\ell/2} d\varepsilon = \frac{1}{2} B\omega \left| x^2 \right|_{-\ell/2}^{+\ell/2}$$

$$= 0$$

Energy Considerations : Let 'r' be the resistance of the moveable arm PQ, (the moving rectangular condutor) in the above figure. We assume that the remaining arms, QR, RS and SP, have negligible resistance. In that case

$$i = \frac{\varepsilon}{r} = \frac{BV\ell}{r}$$

On account of the presence of the magnetic field, there will be a lorentz force, on the arm 'PQ', given by $F = Bi\ell$. This force would be directed inwards in a direction opposite to the velocity of the rod.

The magnitude of this force is

$$F = B\left(\frac{BV\ell}{r}\right)\ell$$

$$= \frac{B^2\ell^2 V}{r}$$

This force acts like a retarding force. Hence power is spend by an external agency to move the rod with a constant velocity 'V'.

The power required to do this is given by

$$P = \mathbf{F}.\mathbf{V}$$

$$= \frac{B^2\ell^2 V^2}{r} \qquad \text{(This is in the form of mechanical work)}$$

This power, spent by an external agent, is dissipated as Joule heat. The Joule heat produced is given by

$$P = i^2 r = \left(\frac{BV\ell}{r}\right)^2 r = \frac{B^2\ell^2 V^2}{r}$$

Example–3 :

A 10 meter long wire is kept in east west direction. It is falling down with a speed of 5 m/s, perpendicular to the horizontal component of earth's magnetic field of 0.30×10^{-4} Wb/m^2. (i) what is the momentary p.d induced between the ends of the wire? (ii) which end of the wire will be at a higher potential?

Solution :

(i) If a wire, of length ℓ, move perpendicular to a magnetic field B, with a velocity v, the emf induced will be given by

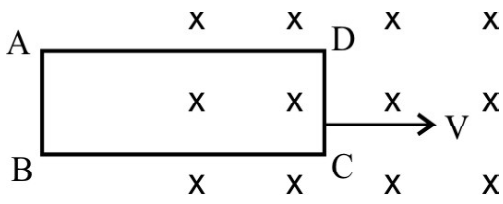
$$\varepsilon = BV\ell$$

$$\therefore \quad \varepsilon = 0.30\times10^{-4}\times5\times10\,V = 1.5\,mV$$

(ii) According to fleming left hand rule, the magnetic force on the electrons, in the wire, in the magnetic field will act towards west. Hence electrons will more to the western end of the wire, therefore the eastern end of the wire will be a higher (positive) potential.

Example–4 :

A horizontal metal frame, ABCD, moves with a uniform (horizontal) velocity of 0.2 m/s, into a uniform field of 10^{-2}T, acting vertically downwards. AB = 0.1m and AD = 0.2m and the resistance R of the frame is 5Ω. As shown, the sides AB and CD enter the field in a direction normal to the field boundary. Find the magnitude of current induced in the metal frame when (i) CD just enters the field (ii) the whole frame is moving through the field (iii) CD just moves out of the field on the other side? Draw a sketch graph showing the variation of the induced current, the force required to move the frame with a constant velocity, and the power spend by the external agency.



Solution :

(i) The sides AD and BC are parallel to **V**. Hence no emf in induced in them. For the side CD, moving in the field,

$$|\varepsilon| = BV\ell$$

$$= 10^{-2}\times0.1\times0.2\ V$$

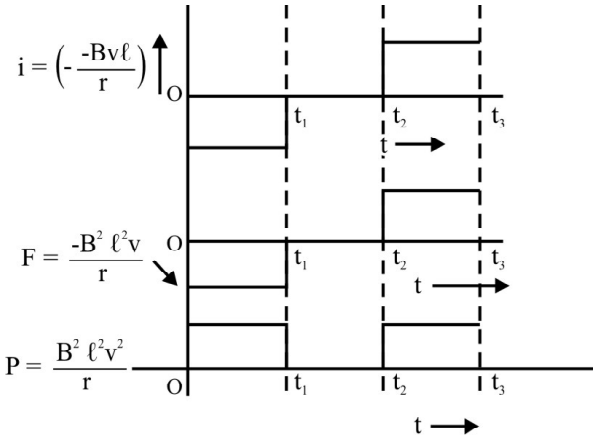
$$2\times10^{-4}\ V$$

$$\therefore \quad i = \frac{|\varepsilon|}{R} = \frac{2\times10^{-4}}{5}\,A = 4\times10^{-5}\,A$$

(ii) As the whole frame in moving through the field from time t_1 to t_2 say the flux through the frame is constant. Hence there is no induced current during this interval. [Alternatively the induced emf in the arms AB and CD are in opposite directions, the resultant emf in the frame is zero.]

(iii) When CD just moves out of the field region, on the other side, the magnitude of the induced emf, in AB, moving through the field, is again equal to 2×10^{-4} V. Hence magnitude of induced current, in the

frame, is same, but its direction is now opposite to that in the first case (The flux is now decreasing with time). The required sketches, therefore, have the forms shown.



Example–5 :

An air plane, with 20 m total wing span is flying with a speed of 250 m/s, straight south, parallel to the earth's surface. The earth's magnetic field has a horizontal component of 2×10^{-5} Wb m^{-2} and the dip angle is 60^0. Calculate the emf induced between the plane tips.

Solution :

Here $\varepsilon = B_V \ell V$ where B_V is vertical component of earth's magnetic field.

We have $\frac{B_V}{B_H} = \tan\delta$

$\therefore \quad B_V = B_H \tan\delta$

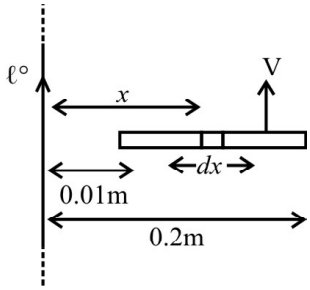
$\therefore \quad \varepsilon = \left(B_H \tan\delta\right)\ell V$

$= 2\times10^{-5}\times (\tan 60^0)\times 20\times 250$ V

$= 10^{-1}\times\sqrt{3}$ V $= 0.173$ V

Example–6 :

A copper rod, of length 0.19m, kept parallel to a long straight wire carrying a current of 5A is with a uniform velocity of 10 m/s. The rod is perpendicular to the wire. The two ends of rod are at distance of 0.01m and 0.2m, from the wire, (as shown in the figure). Calculate the emf induced in the rod.



Solution :

The magnetic field due to the long straight wire, at a distance 'x', from it is

$$B_x = \frac{\mu_0 i}{2\pi x}$$

This field is directed into the page (downwards). If '$d\varepsilon$' is the emf induced, over the element 'dx' of the rod, we have

$$d\varepsilon = BV(dx)$$

$$= \frac{\mu_0 i}{2\pi x} V dx$$

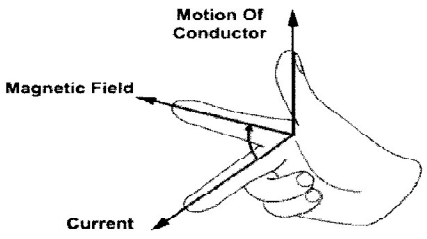
$$\therefore \text{ Total emf, } \varepsilon = \int d\varepsilon = \frac{\mu_0 iV}{2\pi} \left| \log_e x \right|_{0.01m}^{0.02m}$$

$$= \frac{4\pi \times 10^{-7} \times 5 \times 10}{2\pi} \times (2.303 \log_{10} 20)$$

$$= 3 \times 10^{-5} \text{ V}$$

Flemings's Right Hand Rule

The direction of the induced current, when a conductor moves in a uniform magnetic field, is given by Fleming's right hand rule. This rule states that if we stretch the right hand thumb and the first two fingers, perpendicular to one another, and if the forefinger points in the direction of magnetic field, and the thumb in the direction of motion of conductor, the middle finger will point in the direction of the induced current.



Eddy Currents

Facoult, in 1895, discoverd that when a solid mass of metal is placed in a changing magnetic field, (or moved in a magnetic field), causing a change in the magnetic flux linked with it, induced currents are set up throughout the volume of the metal. These currents are known as 'eddy currents' or 'facault currents'. The direction of circulation of these currents is such as to oppose the motion of the metal or change in magnetic flux (in accordance with Lenz's law).

The resistance of thick metal being very low, the eddy currents are generally quite large in magnitude and produce considerable heating effect in accordance with 'Joule's heating' law. This heating effect of eddy

currents in undesirable in the interior of iron cores of rotating armatures of motors, dynamos and transformers etc. To minimize these currents, the cores are not taken as a single piece of soft iron but are made of many thin (insultated from each other) laminas of soft iron. This type of core is called a 'laminated core'.

Eddy currents are, however, also used to advantage in certain applications. These include:

(i) **Electromagnetic Damping :** Certain galvanometers have a fixed core made of a non magnetic metallic material. When the coil oscillates, the eddy currents, generated in the core opposes the motion and bring the coil to rest quickly.

(ii) **Induction Furance :** The metal to be heated, or melted, is placed in a high frequency changing magnetic field. Strong eddy currents, induced in it, cause heating.

(iii) **Magnetic Brakes in Trains :** Strong electromagnets are kept above the rails in some electrically powered trains. When the electromagnets are activated the eddy currents, induced in the rails, opposes the motion of the train.

Self Induction

We know that, when a current flows through a coil, it produces a magnetic field and hence a magnetic flux, which can be linked with the coil itself. It is possible that emf is induced in a single isolated coil due to change of flux through the coil itself by means of varying the current through the same coil. This phenomenon is called self-induction. The induced emf is called 'back emf".

Thus, when the current in a coil is switched on, the (induced) back emf opposes the growth of current. Similarly, when the current is switched off, the back emf opposes the decay of the current.

Let us consider a coil of N turns carrying a current i. Let ϕ_B the magnetic flux linked with each turn of the coil. The total flux, linked with the coil, is proportional to the current i through the coil, and can be expressed as

$$N\phi_B \propto i$$

$$N\phi_B = Li$$

Here L is a constant called the coefficient of self induction, or 'self inductance', of the coil. Physically, the self inductance plays the role of 'inertia' in electrical circuits. It is defined as the magnetic flux, linked with the coil, when a unit current is flowing through the coil itself.

When the current in the coil is varied, the flux linked with the coil also changes and an emf is induced in the coil. This is given by

$$\varepsilon = -\frac{d}{dt}(N\phi_B)$$

$$= -L\frac{di}{dt}$$

The negative sign indicates that the self induced emf (or back emf) always opposes any change (increase or decrease) of current in the coil. From the above formula, we have

$$L = \frac{\varepsilon}{\left(\frac{di}{dt}\right)}$$

Hence the coefficient of self induction or self inductance of a coil is numerically also equal to the emf induced in coil when the rate of change of current, in the coil in unity.

The S.I unit of 'L' is 1 Henry = 1 H

We have $H = \frac{\text{Volt}}{(\text{ampere / second})} = \text{ohm - second}$

Also H = Wb/ampere (from $\phi = Li$)

Calculation of Self Inductance

It is possible to calculate the self in ductance for circuits with simple geometries.

Let us consider a long straight solenoid of cross section area A and length ℓ, having 'n' turns per unit length. When the solenoid carries a current I, the total flux linking with the solenoid

$$\phi_B = N\phi_B = NBA$$

$$= (n\ell)(\mu_0 ni)A$$

By definition, self inductance $L = \frac{(N\phi_B)}{i}$

Hence $L = \mu_0 n^2 A\ell$

If we fill the inside of the solenoid with a material of relative permability μ_r, then, $L = \mu_0\mu_r n^2 A\ell$

This, the self inductance of the coil depends on its geometry and on the permability of the medium.

The corresponding expression for a (small) plane circular coil of radius 'r' will be

$$L = \frac{\mu_0 n^2 \ell^2 \pi r}{2}$$

Energy Associated with Self Inductance

Let $d\omega$ be the work done in establishing a current 'i' in the solenoid, in a time dt. Work has to be done, against the back emf, in establishing this current. This work is stored as magnetic potential energy.

$$d\omega = Vdq = (-\varepsilon)(idt)$$

$$= \left(-L\frac{di}{dt}\right)dt\, i$$

$$d\omega = -Li\, di$$

$\therefore$ Total amount of work done in establishing the current I is, then

$$\omega = \frac{1}{2}LI^2$$

This work is stored in the form of magnetic potential energy $\left(U_m = \frac{1}{2}LI^2\right)$ in the magnetic field of the solenoid. This expression can be considered as equalivalent to $\frac{1}{2}mV^2$ for mechanical kinetic energy of a particle of mass M. It shows that L is analogus to m (L is the 'electrical inertia' which opposes the growth and decay of current in the circuit).

The above expression for the magnetic energy, stored in the magnetic field of solenoid, can be rewritten as

$$U_m = \frac{1}{2}Li^2$$

$$= \frac{1}{2}L\left(\frac{B}{\mu_0 n}\right)^2 \qquad (\because (B = \mu_0 n\, i) \text{ for a solenoid})$$

$$= \frac{1}{2}\left(\mu_0 n^2 A\ell\right)\left(\frac{B}{\mu_0 n}\right)^2 \qquad \left(\because\ L = \mu_0 n^2 A\ell\right)$$

$$= \frac{1}{2}\frac{B^2}{\mu_0}(A\ell)$$

$\therefore$ The magnetic energy, per unit volume, is

$$U_B = \frac{U_m}{V} = \frac{1}{2}\frac{B^2}{\mu_0}$$

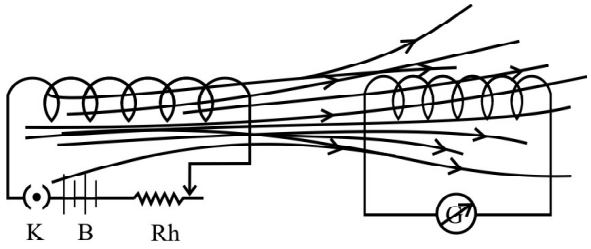
This expression is similar to electrostatic potential energy stored per unit volume in the electric field. In both cases energy is propotional to the square of the (corresponding) field strength.

An ideal solenoid, or coil, whose inductance (L) is much greater than its winding resistance (R) [L >> R) is considered to be an inductor. Hence inductor is a circuit element (like the capacitor) which stores magnetic energy in the magnetic field of a current carrying coil.

We have derived the formula for energy density, for the special case of the uniform magnetic field of an ideal solenoid (inductor), the result is a geenral one and is valid for any region of space in which a magnetic field exists.

Mutual Induction

Let there be two coils in close proximity of each other. When the current in one of these coils changes, the magnetic field due to it over the other coil changes. Hence the magnetic flux linked with the other coil also changes and an emf is induced in this neighbouring coil. Such a phenomenon is known as mutual induction. The (first) coil, in which the current changes (leading to change in magnetic flux linked with the second coil), is called the primary coil. The second coil, in which induction takes place, is called the secondary coil.



Primary Coil Secondary Coil

Let ϕ_B be the magnetic flux linked with the secondary coil, when a current i_P flows through the primary coil. Then

$$\phi_B = (N_S \phi_S) \propto i_P$$

or $$\phi_B = (N_S \phi_S) = M i_P$$

where M is a constant of proportionlity.

'M' is called the 'Mutual inductance' or 'Coefficient of mutual inductance', for the given pair of coils. The emf induced in the secondary coil is given by

$$\varepsilon_S = -\frac{d\phi_B}{dt} = -\frac{d}{dt}(Mi)$$

$$\therefore \quad \varepsilon_S = -M\frac{di}{dt}$$

$$\therefore \quad M = -\frac{\varepsilon_S}{\left(\frac{di}{dt}\right)}$$

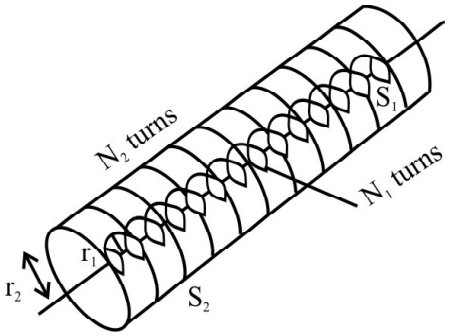
Hence mutual inductance, for a pair of coil, is equal to the induced emf, set up in one coil due to unit rate of change of current in the other coil.

The SI Unit of M = 1H = 1 Henry

= 1 (ohm-second) = 1 (Wb/ampere)

The value of mutual inductance of a given pair of coils, depends upon the number of turns of the coils their geometrical shape and size, their relative separation as well as their relative orientation. It also depends upon the permiability of the medium in the space between the two coils.

Consider two long co-axial solenoids each of length 'ℓ'.



When a current i_2 is set up in the outer coil S_2, the corresponding flux linkage with $S_1 = N_1\phi_1 = M_{12}i_2$

Also $N_1\phi_1 = N_1B_2A_1$

$$= (n_1\ell)(\mu_0 n_2 i_2)(\pi r_1^2)$$

This implies that

$$M_{12} = \mu_0 n_1 n_2 i_2 \pi r_1^2 \ell$$

If we consider the reverse case, the flux linkage, due to current i_1 in S_1, can be assumed to be confined solely inside S_1. Since solenoids are (usually) very large. The corresponding flux linkage with S_2,

$N_2\phi_2 = M_{21}i_1$

$N_2\phi_2 = N_2B_1A_1$

This implies that $= (n_2\ell)(\mu_0 n_1 i_1)\pi r_1^2$

We thus see that $M_{21} = \mu_0 n_1 n_2 \pi r_1^2 \ell$

$$M_{12} = M_{21} = M$$

Hence, in general one can write that, for a pair of solenoidal coils

$$M = \frac{\mu_0 \mu_r N_S N_P A}{\ell}$$

Here N_S and N_P are number of turns in the primary and secondary coils, having a cross sectional area A, and a length ℓ.

Special Case–I :

Consider the general case of currents flowing simultaneously in two nearby coils. The flux linked with one coil will be the sum of two fluxes which exist independently. We can write

$$N_1\phi_1 = M_{11}I_1 + M_{12}I_2$$

Here M_{11} represent the inductance effects, due to the same coil. It is therefore, its self inductance ; 'L_1'

$$\therefore \quad N_1\phi_1 = L_1I_1 + M_{12}I_2$$

Hence the total induced emf, in coil 'I', will be its self induced emf (back emf) plus the 'mutual induced emf', due to change in current in coil '2'. This can be expressed as $\varepsilon_1 = \left(-L_1 \frac{dI_1}{dt}\right) + \left(-M_{12} \frac{dI_2}{dt}\right)$

Special Case–II :

(a) When two ideal solenoids (inductors), of self inductances L_1 and L_2 are connected in seires, then their equivalent self inductance will be

$$L_{eq} = L_1 + L_2 \pm 2M \qquad \text{(If the coils are coaxial or very close)}$$

If flux, of one coil, favours the other, then $L_{eq} = L_1 + L_2 + 2M$

and if flux, of one coil, opposes the other, then $L_{eq} = L_1 + L_2 - 2M$

(b) If the coils are seperated by a large distances, M becomes negligible. Hence, in such a case, for the two coils.

$$L_{eq} = L_1 + L_2$$

(c) If two inductor are connected in parallel, turns out that

$$L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 \pm 2M} \qquad \text{(For coaxial coils or nearby coils)}$$

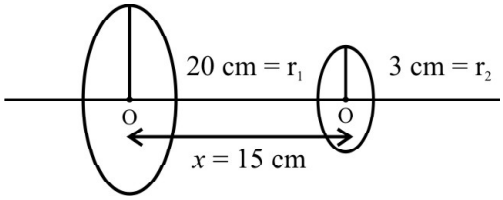
For isolated coils (or for large separation of two coils)

$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

(d) The coefficient of coupling, K, of two coils is a measure of the magnetic coupling of the coils. It is given by $K = \sqrt{\frac{M}{L_1 L_2}}$. Here L_1 and L_2 are the self inductances of the two coils and M is their mutual inductance. K is always less than one.

Example–7 :

A small circular loop, of radius 3cm, and a bigger loop, of radius 20cm, are placed coaxially, at a distance of 15cm as shown in the figure. A current of 2A flows through the smaller loop. Find the flux linked with the bigger loop and the mutual inductance of the two loops.



Solution :

By symmetry, the flux linked with the bigger loop, due to current in the smaller loop, will be same as the flux linked with the smaller loop, due to the same current in the bigger loop ($\therefore$ $M_{12} = M_{21} = M$).

Now the magnetic field, at the centre of smaller loop, due the to a current, i, in the bigger loop, is

$$B_1 = \frac{\mu_0 i r_1^2}{2\left(r_1^2 + x^2\right)^{3/2}}$$

$$= \frac{4\pi \times 10^{-7} \times 2 \times (0.20)^2}{2\left((0.20)^2 + (0.15)^2\right)^{3/2}}$$

$$= 3.2 \times 10^{-6} \text{ Wbm}^{-2}$$

The field B, is along the axis. The flux linkage, with the smaller loop due to B_1 this field will be (nearly)

$$\phi_2 = B_1 A_2 = B_1 \pi r_2^2$$

$$= 3.2 \times 10^{-6} \times 3.14 (0.030)^2 \text{ Wb}$$

$$= 9 \times 10^{-9} \text{ Wb}$$

This is the same as the flux ϕ_1, linked with the bigger coil, due to same current in the smaller loop. We, therefore, have

$$M = \frac{\phi_2}{i_1} = \frac{9\times10^{-9}\,\text{Wb}}{2\text{A}} = 4.5\times10^{-9}\,\text{H}$$

$$= 4.5\ \text{nH}$$

Example–8 :

A very small circular loop of area $5\times10^{-4}m^2$, resistance 2 ohm and negligible inductance, is initially coplanar and cocentric, with a much larger fixed circular loop of radius 0.1m. A constant current of 1 ampere is passed through the bigger loop and the smaller loop is rotated with an angular velocity 'ω' rad/s, about a diameter. Calculate the induced emf and induced current, as a function of time, in the smaller loop.

Solution :

The magnetic field, at the centre of larger loop, is

$$B_1 = \frac{\mu_0 i}{2r} = \frac{4\pi\times10^{-7}\times1}{2\times0.1}\,\text{T} = 2\pi\times10^{-6}\,\text{T}$$

Magnetic flux, linked with the smaller loop (of area A_2), placed at centre will be (nearly)

$$\phi_2 = B_1 A_2\cos\theta$$

$$= B_1 A_2\cos\omega t$$

$$= \left(2\pi\times10^{-6}\times5\times10^{-4}\right)\cos\omega t$$

$$= \left(\pi\times10^{-9}\right)\cos\omega t$$

The induced emf, in the smaller

$$\varepsilon_2 = \frac{-d\phi_2}{dt} = -\pi\times10^{-9}\left(\frac{d}{dt}\cos\omega t\right)$$

$$= \left[\left(\pi\times10^{-9}\omega\right)\sin\omega t\right]\text{Volt}$$

$\therefore$ Induced current

$$i_2 = \frac{\varepsilon_2}{R}$$

$$\therefore \quad i_2 = \left[\left(\frac{\pi}{2}\times 10^{-9}\omega\right)\sin\omega t\right] \text{ ampere}$$

Induced emf in a planer coil,due to a change in its orientation in a uniform magnetic field

Let a planer coil, having N turns, of area A each, rotate in a uniform magnetic field, B, with a constant angular velocity 'ω', about an axis coinciding with the plane of the coil and perpendicular to the uniform field. As a result, the effective area of the coil, exposed to the magnetic field lines, changes with time. The flux, at any time t, is

$$\phi_B = N(BA\cos\omega t) \quad (\because \text{ effective area} = A\cos\theta = A\cos\omega t)$$

From Faraday's law, induced emf in the rotating coil, is

$$\varepsilon = -\frac{d\phi_B}{dt} = -NBA\frac{d}{dt}(\cos\omega t)$$

$$= NBA\omega\sin\omega t$$

This is the instantaneous value of induced emf. Clearly $\varepsilon_0\,(= NBA\omega)$ in the maximum value of this induced emf; this occurs when $\sin\omega t = \pm 1$. We can write the above equation as

$$\varepsilon = \varepsilon_0 \sin\omega t \text{ where } \varepsilon_0 = NBA\omega.$$

Hence induced emf, in the coil, varies with time in a sinusoidal way. When the plane of the coil is parallel to **B** ($\sin\omega t = 1$), the induced emf is maximum. When this plane is perpendicular to B ($\sin\omega t = 0$), the induced emf is zero.

This method, of producing a flux change, due to a change in the loop's orientation, (or change in its effective area ($A\cos\theta$)) is the principle of operation of a simple a.c. generator.

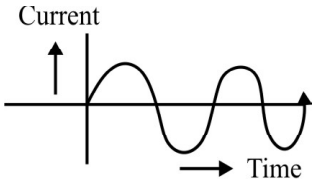
An a.c. generator converts mechanical energy into electrical energy. In commercial generators, the mechanical energy, required for rotation of the coil, is provided either by water falling from heights in dams (hydro-electric generators) or by steam at high pressure, produced in thermal generators. Nuclear energy is now also being increasingly used, for this purpose, in the so called nuclear energy powered electric generators.

Alternating Current

Alternating Current

Alternating current is that current which continuously varies in magnitude and periodically reverse its direction. The same is true for alternating emf's or voltages.

The simples type of alternating current is one which varies with time is a simple harmonic (sinusoidal) way. It is represented by $i = i_0 \sin \omega t$



where i is the instantaneous value of curernt, at time t, i_0 is the maximum (or peak) value of the curernt. i_0' may be called as the 'current amplitude'. 'ω' is called the angular frequency, of the alternating current (a.c.), and in given by

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Here T is the time period and f is the (linear) frequency of the alternating current. In the same manner, the instantaneous value of a sinusoidal alternating emf (or voltage) is represent by

$$v = v_0 \sin \omega t$$

Mean (or Average) Value of A.C.

An alternating current (or voltage) flows, during one half cycle, in one direction, and during the other half cycle in opposite direction. Hence for one complete cycle the mean value of the a.c. would be zero. This is why there is no (net) deflection in a moving coil galvanometer when an alternating current passes through it. Average value over full cycle

$$i_{av} = \frac{\int_0^T i_0 \sin \omega t \, dt}{\int_0^T dt} = 0$$

However, the mean value, of a sinusoidal a.c, over a half cycle, has a finite value.

Mean, or average value over half a cycle, is given by

$$i_{av} = \frac{\int_0^{T/2} i_0 \sin \omega t \, dt}{\int_0^{T/2} dt} = \frac{2i_0}{\pi}$$

$$= 0.637 \, i_0$$

$$\left(\text{Similarly } v_{av} = \frac{2v_0}{\pi} = 0.637 \, v_0 \right)$$

We thus notice that the mean value, of a simusoidal a.c., over a half cycle, is 63.7% of its peak value.

Root mean Square Value of a.c

The root mean square (r.m.s) value of a.c is defined as the square root of the average value of i^2, over a complete cycle.

Now, average value of i^2 , over a full cycle is given by,

$$\overline{i^2} = \frac{1}{T} \int_0^T i^2 dt$$

$$= \frac{1}{T} \int_0^T i_0^2 \sin^2 \omega t \, dt$$

This comes out to be $\overline{i^2} = \frac{i_0^2}{2}$

Hence, the root mean square value of a sinosoidal a.c, is given by

$$i_{r.m.s} = \sqrt{\overline{i^2}} = \frac{i_0}{\sqrt{2}}$$

$$= 0.707 \, i_0$$

Thus, the root mean square value of an a.c, is 70.7% of its peak value.

In a similar way,

$$v_{r.m.s} = \frac{v_0}{\sqrt{2}}$$

Physical Significance of r.m.s Value

If an a.c, $i = i_0 \sin \omega t$, passes through a resistance R, the instantaneous rate of heat production is

$$P = i^2 R$$

The 'average' rate of production of heat, over one complete cycle of current, is then given by

$$P_{av} = \overline{i^2} R = i_{rms}^2 R$$

Now if one passes a direct current, of strength i_{rms}, in a resistance R, the rate of heat production will again be $i_{rms}^{2'} R$. Thus the root mean square value, of an a.c., is equal to that direct current, which produces the same (total) heat, in a given resistance (in a time equal to the time period of the a.c.) as the given alternating current does over its one complete cycle.

Hence, the root mean square value, of an a.c., is also called as the 'effective value' or the 'virtual value' of the given a.c. The measurable value of an a.c. is thus its r.m.s value, or 'effective (or virtual) value'. The ammeter or voltmeter designed to measure a.c., would directly give the relevant r.m.s value. In our houses, alternating current is supplied at 220 V. It means that

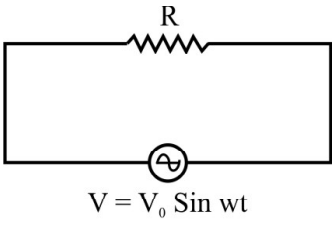
$V_{rms} = 220$ V. The peak value $\left(v_0\right) = \sqrt{2}\, v_{rms} = 311$ V

'Phasors' or Phasor Diagrams

In an a.c. circuit, the frequency of the alternating current and alternating emf is same. However, it is not necessary that they both be in the same phase. Usually, when the current in the circuit in maximum, the emf is not maximum and vice-versa. The phase difference between the two, depends upon the type of circuit. Phase of a.c may be defined as the time period that has elapsed. Since the current (or emf) last passed its zero value in the positive direction.

The study of a.c circuits is much simplified if we treat alternative current or alternating emf as (kind of) vectors. The angle between the 'vectors' representing current and voltage is taken equal to the phase difference between them. The current and emf 'vectors' are better called 'phasors'. A diagram, representing alternating current and emf as 'phasors', showing the phase angle between them, is called a 'Phasor Diagram'.

AC Voltage applied to a Resistor



Let an alternating voltage $v = v_0 \sin \omega t$ be applied across a pure resistance 'R'. The applied voltage will be equal to the p.d across R at any instant of time t. Thus

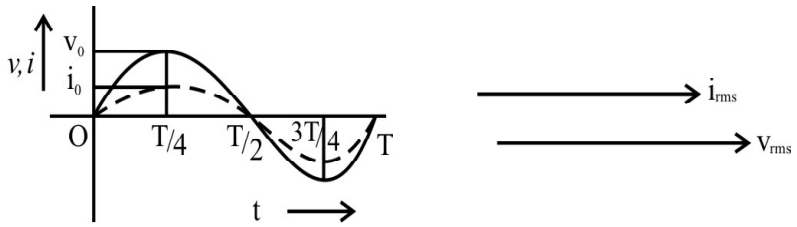
$$v\left(t\right) = i\left(t\right) R$$

$\therefore \quad v_0 \sin \omega t = iR$

Hence $i = \frac{v_0}{R} \sin \omega t = i_0 \sin \omega t$

where $i_0 \left(= \frac{v_0}{R} \right)$ is the peak value of current in the circuit.

The above expression shows that in a pure resistance, the current is always in phase with the applied voltage.



The above graphical representation shows this. It shows that both v and i reach their zero, minimum and maximum values at the same time; there is thus no phase difference between v and i. The instantaneous power disipated in resistor is

$$P = i^2 R = \left(i_0^2 \sin^2 \omega t \right) R$$

The average value of P, over a complete cycle, is

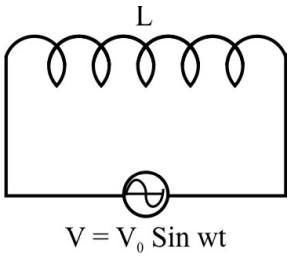
$$P_{av} = \frac{\left(\int_0^T i_0^2 \sin^2 \omega t \, dt \right)}{T} \quad R = \frac{i_0^2 R}{T} \int_0^T \sin^2 \omega t \, dt$$

$$= \frac{1}{2} i_0^2 R = \left(\frac{i_0}{\sqrt{2}} \right)^2 R$$

$\therefore \quad P_{av} = i_{rms}^2 R = \frac{v_{rms}^2}{R}$

$\left(\because \quad v_{rms} = i_{rms} R \right)$

AC Voltage applied to a Pure Inductor



Let an alternating voltage, $v = v_0 \sin \omega t$, be applied across a pure inductor of inductance L (it is supposed to have zero resistance). As the current i, in the inductor varies with time, an opposing back emf $\left(= L\frac{di}{dt} \right)$ (self induced), gets induced in the inductor. We, therefore, have, as per Kirchoff's law.

$$v - L\frac{di}{dt} = 0$$

$$\therefore \quad v_0 \sin \omega t = L\frac{di}{dt}$$

$$\therefore \quad di = \left(\frac{v_0}{L} \sin \omega t \right) dt$$

On integrating both sides, we get an expression for the instantaneous current i in the circuit. We have

$$i = \left(-\frac{v_0}{\omega L} \right) \cos \omega t$$

or $$i = \left(\frac{v_0}{\omega L} \right) \sin \left(\omega t - \frac{\pi}{2} \right)$$

or $$i = i_0 \sin \left(\omega t - \frac{\pi}{2} \right)$$

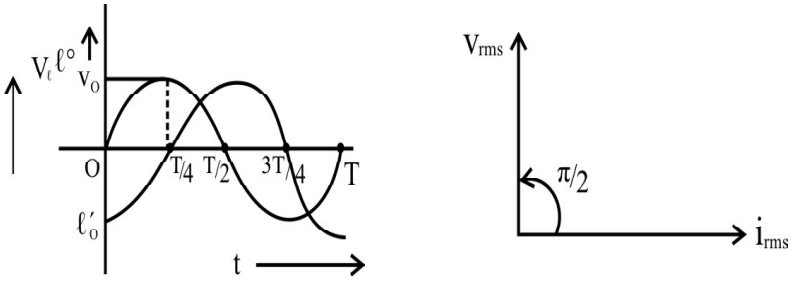
Here $i_0 \left(= \frac{v_0}{\omega L} \right)$ is the maximum, or peak, value of curernt in the circuit.

A comparison of this equation with the voltage equation, shows that, in a pure inductor, the current lags behind the applied voltage by a phase angle of $\frac{\pi}{2}$. Voltage thus reaches its maximum value $\left(\frac{T}{4} \right)$ seconds earlier than the current in the circuit.

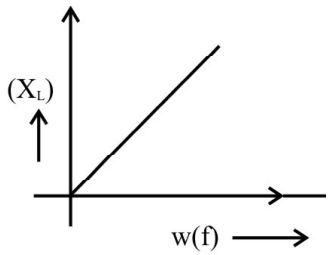
As $i_0 = \frac{v_0}{\omega L}$, we find that the product ωL must have the same dimension as those of resistance. This term, therefore, represents the effective opposition of the inductor to the flow of alternating current. It is known

as the 'reactance' of the inductance, or inductive reactance. It is denoted by the symbol X_L. Thus

$$X_L = \omega L = 2\pi fL$$



The inductive reactance increases with frequency, in a linear, or proportinoal way.



We can also write, $v_{rms} = i_{rms} X_L$.

The instantaneous power supplied to the inductor is

$$P = vi = (v_0 \sin \omega t)\left(i_0 \sin\left(\omega t - \frac{\pi}{2}\right)\right)$$

$$= -v_0 i_0 \sin \omega t \cos \omega t$$

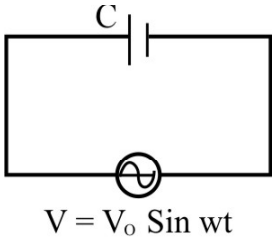
The average power, over a complete cycle, is given by

$$P_{av} = \frac{1}{T}\int_0^T vi\, dt$$

This comes out to be zero i.e. $P_{av} = 0$. This means that in alternate quater cycles, energy is absorbed from the source by the inductor and then returned back to the source. This is because the 'back emf' opposes both the growth and decay of the current in the circuit. Over a complete cycle, the inductor thus acts as a 'wattless' circuit element.

AC Voltage applied to a Pure Capacitor

Let alternating voltage, $v = v_0 \sin \omega t$, be applied to a pure capacitor of capacitance 'C'. The capacitor gets alternatively charged, and discharged, as the current reverses each half cycle. Let q be the charge on the capacitor plates, at any instant of time 't'. We have



$$q(t) = cv(t)$$

$$\therefore \quad v(t) = \frac{q}{c}(t)$$

$$\therefore \quad \frac{dv}{dt} = \frac{1}{c}\frac{dq}{dt}$$

The instaneous current in the circuits is, therefore, given by

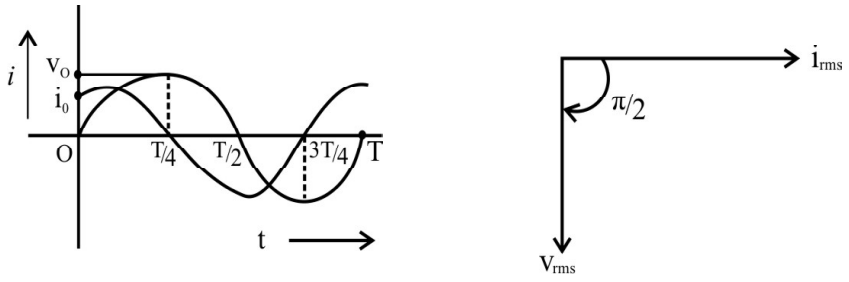
$$i = \frac{dq}{dt} = c\frac{d}{dt}\left(v_0 \sin \omega t\right)$$

$$\therefore \quad i = cv_0\omega \cos\ \omega t$$

or
$$i = \frac{v_0}{\left(\frac{1}{\omega c}\right)}\cos \omega t = \frac{v_0}{\left(\frac{1}{\omega c}\right)}\sin\left(\omega t + \frac{\pi}{2}\right)$$

where $\left(= \frac{v_0}{\left(\frac{1}{\omega c}\right)}\right)$ is the maximum, or peak value, of current in the circuit.

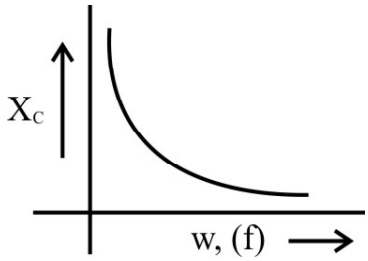
A comparison of this equation with the voltage equation shows that, in a pure capacitor, current is ahead of voltage by a phase angle of $\frac{\pi}{2}$. The current, therefore, reaches its maximum value $\frac{T}{4}$ seconds earlier than the voltage applied across the capacitor.



As $i_0 = \frac{v_0}{\left(\frac{1}{\omega c}\right)}$, we find that the product $\frac{1}{\omega c}$ must have the same dimension as those of resistance. This term thus represents the 'effective opposition' of the capacitor to the flow of alternating current. It is known as 'reactance of the circuit' or 'capacitive reactance'. It is denoted by the symbol X_C.

$$X_C = \frac{1}{\omega c} = \frac{1}{2\pi fc}$$

Thus capacitive reactance decreases with increase in freqeuncy. As $X_C \propto \frac{1}{\omega}$, the graph of X_C, against ω (or f), has the form shown here.



We can also write, $v_{rms} = i_{rms} X_C$.

The instantaneous power supplied to the capacitor is

$$P = vi = (v_0 \sin \omega t)\left(i_0 \sin\left(\omega t + \frac{\pi}{2}\right)\right)$$

$$= v_0 i_0 \sin \omega t \cos \omega t$$

The average power, over a complete cycle,

$$P_{av} = \frac{1}{T}\int_0^T vi\, dt$$

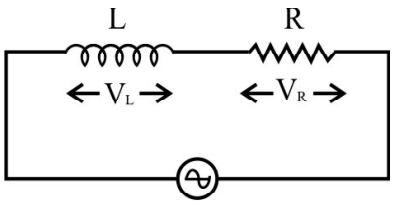
This comes out to be zero.

This means that energy is absorbed from the source during one quater cycle (when capacitor is getting charged). In the next quater cycle, energy gets returned to the source, (as capacitor gets discharged over a complete cycle), the capacitor thus acts like a 'wattless' element.

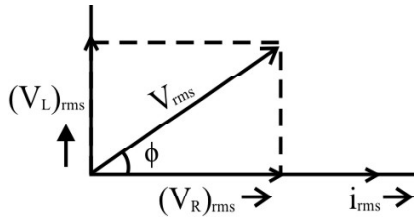
To summarise:

Pure R	**Pure L**	**Pure C**
$v = v_0 \sin \omega t$	$v = v_0 \sin \omega t$	$v = v_0 \sin \omega t$
$i = i_0 \sin \omega t$	$i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right)$	$i = i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$
$v_{rms} = i_{rms} R$	$v_{rms} = i_{rms} X_L$	$v_{rms} = i_{rms} X_C$
$P_{av} = i_{rms}^2 R$	$P_{av} = 0$	$P_{av} = 0$
v, i are in same phase	v is ahead of i (in phase) by $\frac{\pi}{2}$	v lags behind i (in phase) by $\frac{\pi}{2}$

AC Voltage applied to Inductor and Resistor in Series (LR Series Circuit)



Let an alternating voltage, $v = v_0 \sin \omega t$ be applied to a circuit containing a pure L and a pure R, joined in series. The same current '*i*' will flow both in L and R. However, in R, it is in phase with the applied a.c voltage but in L, it lags behind the applied a.c voltage by a phase angle of $\frac{\pi}{2}$. Thus rms potential drop across R, $(V_R)_{rms}$, and the rms potential drop across L, (V_L) rms, must be shown at right angles to each other in a phasor diagram. The phasor diagram is, therefore, drawn as shown.



$$\therefore \quad v_{rms} = \sqrt{(V_R)^2_{rms} + (V_L)^2_{rms}}$$

or $$v_{rms} = i_{rms}\sqrt{R^2 + X_L^2}$$

The term $\sqrt{R^2 + X_L^2}$ is therefore, the 'effective resistance' (series combination of the resistance of R and the reactance of L) of the circuit. It has been given the name 'impedance of the circuit'. We use the symbol z for impedance.

Thus, here $z = \sqrt{R^2 + X_L^2}$

$$= \sqrt{R^2 + (\omega L)^2}$$

Also, $v_{rms} = i_{rms}\, z$

The phasor diagram shows that, in a series LR circuit, the voltage (still) leads the current, now by a phase angle $\phi\left(0 < \phi < \frac{\pi}{2}\right)$. This phase angle is given by

$$\tan\phi = \frac{(V_L)_{rms}}{(V_R)_{rms}} = \frac{i_{rms}(X_L)}{i_{rms}(R)} = \frac{\omega L}{R}$$

The inductive effect dominates in a series LR circuit. This means that the voltage reaches its maximum value $\left(\frac{T}{2\pi}\phi\right)$, or $\left(\frac{\phi}{\omega}\right)$, seconds earlier than the current in the circuit. (There is a time – lag of $\Delta t = \frac{T\phi}{2\pi} = \frac{\phi}{\omega}$ seconds, between the current and the voltage, vis-a-vis reaching their peak values).

Example–9 :

A students connects a long air cored coil, of manganin wire, to a 100 V d.c source and records a current of 1.5 A. When the same coil is connected across. (100 V; 50 Hz) a.c source, the current reduces to 1A. Why? Also calculate the reactance of the coil.

Solution :

In d.c circuit the impedance of the coil, is only due to its resistance 'R'. Hence

$$i_{d.c} = \frac{V}{R}$$

$$\therefore \quad R = \frac{100}{1.5}\Omega = 66.67\,\Omega$$

When the same coil is connected to an a.c source, the impedance of the coil is due to both its resistance R and its inductance L. The coil now acts like a LR series circuit, having a impedance

$$z = \sqrt{R^2 + (\omega L)^2}$$

The value of current, in the a.c circuit decreases, because of this increase in impedance of the coil.

$$\text{Now } i_{rms} = \frac{v_{rms}}{\sqrt{R^2 + (\omega L)^2}}$$

$$\therefore \quad 1 = \frac{(100)^2}{\sqrt{(66.67)^2 + (2\pi fL)^2}}$$

$$\therefore \quad (2\pi fL)^2 = (5555)$$

$$\Rightarrow X_L (= 2\pi fL) = \sqrt{5555}\,\Omega$$

$$= 74.53\,\Omega$$

Example–10 :

An inductor 'L', and a resistor 'R', are connected in series to an a.c source. The potential drop across inductor is 160 watts and that across the resistance 'R' is 120 volt. Find the virtual value of the applied voltage. Is algebric sum of the potential drops across L and R more than this value? If yes, resolve the paradox.

Solution :

We have $v_{rms} = \sqrt{(V_R)_{rms}^2 + (V_L)_{rm}^2}$

$= \sqrt{(160)^2 + (120)^2}$

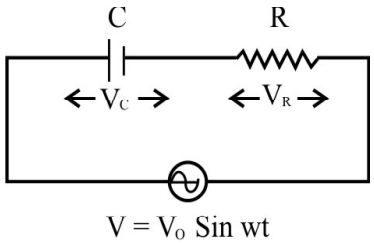
$= 200$ V

Hence algebric sum of p.d. across L and R ((160 + 120) V = 280 V) is more than the virtual or rms value of applied voltage. This is due to the phase relation between $(V_L)_{rms}$ and $(V_R)_{rms}$. The potential drop, across the inductor, is ahead of potential drop across resistor by a phase angle of $\frac{\pi}{2}$. Therefore, they cannot be added like pure numbers. We need to add them by using the rule for 'vector'. This type of 'addition' gives

$v = \left[\sqrt{(120)^2 + (160)^2}\right] V$

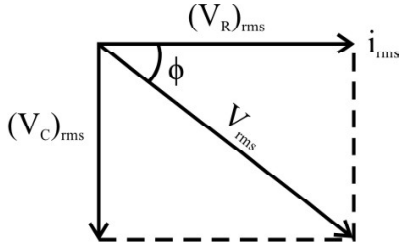
$= 200$ V

AC Voltage applied to a Capacitor and a Resistor in Series (RC Series Circuit)



Let an alternating voltage, $v = v_0 \sin \omega t$, be applied to a circuit containing a pure C an da Pure R, joined in series. The same current i will flow in both C and R. However in R it is in phase with the applied a.c voltage, whereas in C, it leads the a.c.voltage in phase by $\frac{\pi}{2}$. Thus rms potential drop, across the resistance, is ahead ot the rms potential drop across the capacitor by a phase angle of $\frac{\pi}{2}$. The phasor diagram is, therefore drawn as shown.

Thus, $v_{rms} = \sqrt{(V_R)_{rms}^2 + (V_C)_{rms}^2}$



or $v_{rms} = i_{rms}\sqrt{R^2 + X_C^2}$

Here $\sqrt{R^2 + X_C^2}$ is the 'effective resistance' of the (series CR) circuit.
It is called the impedance of this circuit and is denoted by z.

$\therefore \quad z = \sqrt{R^2 + X_C^2}$

$$= \sqrt{R^2 + \left(\frac{1}{\omega c}\right)^2}$$

and $v_{rms} = i_{rms} z$

The phasor diagram shows that in the RC circuit, the voltage lags behind the current by a phase angle $\phi\left(0 < \phi < \frac{\pi}{2}\right)$. As seen from the phasor diagram, the phase angle, ϕ, is given by

$$\tan\phi = \frac{(V_C)_{rms}}{(V_R)_{rms}} = \frac{X_C}{R} = \frac{1}{\omega CR}$$

The capacitive effect dominates in this circuit. This means that the current reaches its maximum value $\left(\frac{T}{2\pi}\phi\right)$, or $\left(\frac{\phi}{\omega}\right)$, seconds earlier value the voltage in the circuit. There is a time lag of $\Delta t = \frac{T}{2\pi} = \frac{\phi}{\omega}$ seconds between the voltage and the current, vis-a-vis reaching their peak values.

Example–11 :

A $100\ \mu F$ capacitor is connected in seires with a $40\,\Omega$ resistor to a (110 V, 60 Hz) supply. Find the time lag between the instants at which the current and voltage reach their, respective, maximum values.

Solution :

Time lag $\Delta t = \frac{T}{2\pi}\phi$

Here $\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{1}{\omega CR}\right)$

$$= \left[\tan^{-1} \left(\frac{1}{2 \times 3.14 \times 60 \times 40} \right) \right]$$

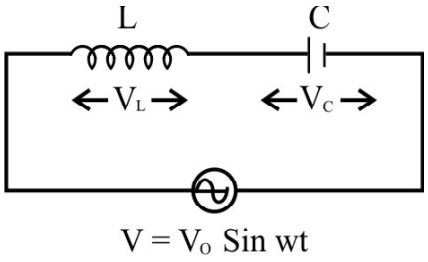
$$= 33.56^0$$

Now $\Delta t = \frac{T}{360^0} \times \left(33.56^0\right) \left(T = \frac{1s}{60} \right)$

$$\therefore \quad \Delta t = \left[\frac{33.56°}{360°} \times \frac{1}{60} \right] s = 1.55 \times 10^{-3} s$$

$$= 1.55 \text{ ms}$$

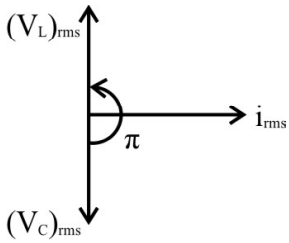
AC Voltage applied to a Pure Inductor and a Pure Capacitor in Series (LC Series Circuit)



In this case, voltage across L will lead the current in the circuit by $\frac{\pi}{2}$; while the voltage across Cwill lag behind the current by $\frac{\pi}{2}$. Thus phase difference between the two voltages is π.

Hence $v_{rms} = \left| \left(V_L\right)_{rms} - \left(V_C\right)_{rms} \right|$

$$= i_{rms} \left| X_L - X_C \right|$$



Here $\left| X_L - X_C \right|$ is the effective reactance of the circuit.

Special Case:

(i) If, in this circuit, $X_L = X_C$, the amplitude of current in the circuit, would tend to be infinite. It is therefore, the condition of 'electrical resonance in this circuit.

When $X_L = X_C$, we have

$$\omega L = \frac{1}{\omega C}$$

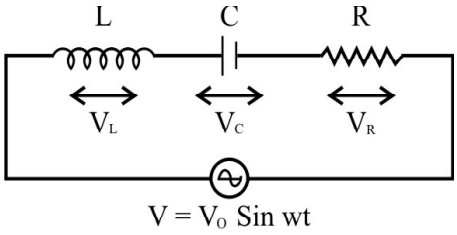
or $\omega = \frac{1}{\sqrt{LC}}$, or $f = \frac{1}{2\pi\sqrt{LC}}$

This frequency value (= f), is called the 'resonant frequency' of the given (series) LC circuit.

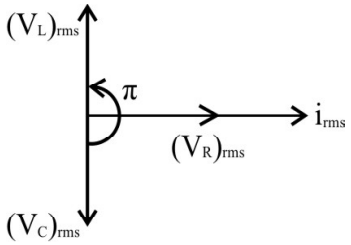
(ii) If $X_L > X_C$, the inductive effect dominates. The current lags behind the voltage by a phase angle of $\frac{\pi}{2}$ in this a.c circuit. However the peak value, or amplitude of current, increases as effective value of reactance is lower than that of either L or C.

(iii) If $X_C > X_L$, the capacitive effect dominates.The current leads the voltage by a phase angle of $\frac{\pi}{2}$ in the a.c circuit. However the peak value of amplitude, of current increases as the effective value of reactance is lower than that of either L or C.

AC Voltage applied to Series LCR Circuit



Let an alternating voltage, $v = v_0 \sin \omega t$, be applied to an a.c circuit, having L, C and R, joined in series. The same current i, will flow in all the three elements. As discussed earlier, the phasor diagram for this circuit will be drawn as shown here.



Case (i) :

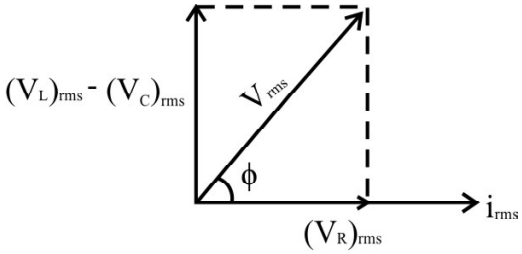
If $(V_L)_{rms} > (V_C)_{rms}$, the resultant, of these two, will be $[(V_L)_{rms} - (V_C)_{rms}]$.

We, therefore, have

$$v_{rms} = \sqrt{(V_R)_{rms}^2 + \left((V_L)_{rms} - (V_C)_{rms}\right)2}$$

$$= i_{rms}\sqrt{R^2 + (X_L - X_C)^2}$$

$$= i_{rms} z$$



Here, the impedance of the circuit, 'z' is given by

$$z = \sqrt{R^2 + (X_L - X_C)^2}$$

Here $(V_L)_{rms} > (V_C)_{rms}$

$$\Rightarrow i_{rms}X_L > i_{rm}X_C$$

$\therefore \quad X_L > X_C$

Hence the inductive effect dominates in such a circuit. The applied voltage, V, leads the current, *i*, by a phase angle ϕ where

$$\tan\phi = \frac{\left[(V_L)_{rms} - (V_C)_{rms}\right]}{(V_R)_{rms}}$$

$$= \left(\frac{X_L - X_C}{R}\right)$$

This means that the voltage reaches its maximum value $\left(\frac{T}{2\pi}\phi\right)$ or $\left(\frac{\phi}{\omega}\right)$ seconds eariler than the current reaching its maximum value, in the circuit. The value of instantaneous current in the circuit is, $i = i_0 \sin(\omega t - \phi)$

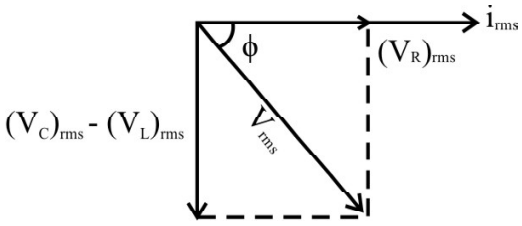
Case (ii) :

If $(V_C)_{rms} > (V_L)_{rms}$, the resultant of the two will be $[(V_C)_{rms} - (V_L)_{rms}]$

We, therefore, have

$$V_{rms} = \sqrt{(V_R)_{rms}^2 + \left((V_C)_{rms} - (V_L)_{rms}\right)^2}$$

$$= i_{rms}\sqrt{R^2 + (X_C - X_L)^2}$$



$= i_{rms}\, Z$

Here the impedance of the circuit, 'Z', is given by

$$Z = \sqrt{R^2 + \left(X_C - X_L\right)^2}$$

Here $(V_C)_{rms} > (V_L)_{rms}$

$\Rightarrow i_{rms}\, X_C > i_{rms}\, X_L$

or $X_C > X_L$ Hence capacitive effect dominates in such a circuit. The applied voltage, V, lags behind the current i, by a phase angle ϕ; where

$$\tan\phi = \frac{\left[\left(V_C\right)_{rms} - \left(V_L\right)_{rms}\right]}{\left(V_R\right)_{rms}}$$

$$= \frac{\left[X_C - X_L\right]}{R}$$

This means that voltage reaches its maximum value $\left(\frac{T}{2\pi}\phi\right)$ or $\left(\frac{\phi}{\omega}\right)$ seconds later than the current reaching its maximum value in the circuit.

The instantaneous current i in such circuit is, $i = i_0 \,\mathrm{Sin}\left(\omega t + \phi\right)$

Case (iii) :

An interesting characteristics of the series LCR circuit is the phenomenon of resonance.

For an LCR circuit, driven with applied voltage of amplitude V_0 and (angular) frequency ω, we know that the current amplitude equals

$$i_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + \left(X_L - X_C\right)^2}}$$

Here $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$. If ω is varied, at a particular frequency ω_0, we can have $X_C = X_L$. [or $(V_C)_{rms} = (V_L)_{rms}$]

The impedance of the circuit is then at its minimum value $\left(z_{min} = \sqrt{R^2 + 0} = R\right)$. This (angular) frequency

ω_0, is called the resonant (angular) frequency. It is given by as $X_C = X_L$, $\frac{1}{\omega_0 C} = \omega_0 L$

$$\therefore \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

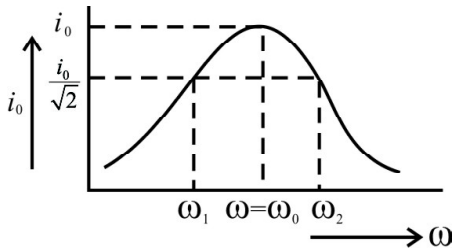
At resonant frequency the current amplitude is maximum $(i_0) = \frac{V_0}{Z_{min}} = \frac{V_0}{R}$

For values of ω, other than ω_0, the amplitude of current in less than its maximum value. The maximum value of current, however, depends on the resistance R in the circuit.

Resonant circuits have a variety of applications. These include, among others, the tunning circuits of a radio or T.V. set, and the working of metal detectors.

Sharpness of Resonance

Suppose we choose a value of ω for which the current amplitude is $\frac{1}{\sqrt{2}}$ times its maximum value. At this frequency, the power, dissipated by the circuit, becomes half of its maximum value.



From the graph between i and ω, we see that there are two frequncies, ω_1 and ω_2, one greaer and the other smaller than ω_0. These two frequencies are located symeterically about ω_0; for the angular frequency, the current amplitude is $\frac{i_0}{\sqrt{2}}$. These two frequencies are referred to as the half power frequencies.

We can write $\omega_1 = \omega_0 + \Delta\omega$

$\omega_2 = \omega_0 - \Delta\omega$

The difference, $(\omega_1 - \omega_2)(= 2\Delta\omega)$, is often called the 'bandwidth' of the circuit. The quantity $\left(\frac{\omega_0}{2\Delta\omega}\right)$ is regarded as a measure of the sharpness of resonance. The smaller $\Delta\omega$ is the sharper, or narrower, is the

resonance. The value of $\Delta\omega$ comes out to be equal to $\frac{R}{2L}$. (by solving the expression for impedances for $\omega=\omega_0$ and $\omega=\omega_2$ or ω_1)

The sharpness of resonance is, therefore, given by

$$\frac{\omega_0}{2\Delta\omega}=\frac{\omega_0 L}{R}$$

The ratio $\left(\frac{\omega_0 L}{R}\right)$ is called the 'quality factor', Q of the circuit. Thus

$$Q=\frac{\omega_0}{(\omega_2-\omega_1)}=\frac{\omega_0}{\Delta\omega}=\frac{\omega_0 L}{R}=\frac{1}{\omega_0 CR}=\frac{1}{R}\sqrt{\frac{L}{C}} \qquad \left(\because\ \omega_0=\frac{1}{\sqrt{LC}}\right)$$

Example–12 :

An a.c. source of frequency 50 Hz is applied to a series circuit of resistance 20Ω, an inductance of 100 mH and a capacitance of $30\,\mu F$. Does the current lag or lead the applied voltage? If so, by what (phase) angle?

Solution :

Here inductive reactance $XL=\omega L$

$$=2\pi fL$$

$$=2\times3.14\times50\times0.1\Omega=31.4\Omega$$

Capacitive reactance $X_C=\frac{1}{\omega C}$

$$=\frac{1}{2\pi fC}=\frac{1}{2\times3.14\times60\times30\times10^{-6}}\Omega$$

$$=106.2\Omega$$

Thus $X_C > X_L$. Hence the current leads the applied voltage by an angle ϕ where

$$\tan\phi=\frac{(X_C-X_L)}{R}=\frac{106.2-13.4}{20}$$

$$=3.74$$

$$\therefore\ \phi=\tan^{-1}(3.74)\simeq750$$

Example–13 :

An LCR series circuit, with resistance $100\,\Omega$ is connected to an a.c. source of 200 V and angular frequency 300 rad/s. When the capacitance is removed from the circuit, current lags behind the voltage by 60^0. When the inductance is removed from the circuit, the current leads the voltage by 60^0. Calculate the current in the circuit.

Solution :

We have $\tan 60^0 = \dfrac{\omega L}{R}$ (when 'C' is removed) and

$\tan 60^0 = \dfrac{1}{\omega CR}$ (when 'L' is removed)

$\therefore\ \omega L = \dfrac{1}{\omega C}$

or $\qquad \omega = \dfrac{1}{\sqrt{LC}}$

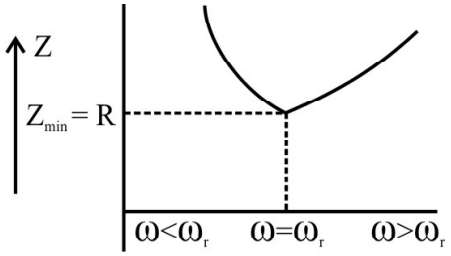
This implies that circuit is at resonance.

$\therefore\ i_{rms} = \dfrac{V_{rms}}{Z_{min}} = \dfrac{V_{rms}}{R}$

$= \dfrac{200}{100} A = 2A$

Example–14 :

Sketch the variation of impedance (z) with angular frequency (ω), for a series LCR (a.c.) circuit.



Solution :

We have $Z^2 = R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2$

At $\omega = \omega_r$, $Z = R$

Z is more than R both for $\omega < \omega_R$ and for $\omega > \omega_R$. The curve is symmetrical.

Further,

For $\omega < \omega_r$, capacitive effect dominates in the circuit.

$\omega > \omega_r$, inductive effect dominates in the circuit.

Power in a Series LCR AC Circuit

When a voltage, $v = v_0 \sin \omega t$, is applied to a series LCR circuit, the current in the circuit is given by

$i = i_0 \sin(\omega t \mp \phi)$. Where $i_0 = \frac{v_0}{z}$ and $\phi = \tan^{-1}\left(\frac{|X_L - X_C|}{R}\right)$

[negative sign for inductive effect and positive sign for capacitive effect]

Therefore, the instantneous power 'P' supplied, by the source, is

$$P = vi$$

$$= (v_0 \sin \omega t)(i_0 \sin(\omega t \mp \phi))$$

The average power over a full cycle is given by

$$P_{av} = \frac{1}{T}\int_0^T vi\, dt$$

$$= \frac{v_0 i_0}{T}\int_0^T (\sin \omega t)(i_0 \sin(\omega t \mp \phi))\, dt$$

$$= \frac{1}{2} v_0 i_0 \cos\phi$$

$$= v_{rms} i_{rms} \cos\phi$$

This can be written as

$$P_{av} \equiv i_{rms}^2 z \cos\phi$$

So, the average power dissipated in the circuit depends not only on the rms values of voltage and current

but also on the phase angle, ϕ, between the two.

The quantity 'cos ϕ' is called the power factor (for the series a.c.) circuit.

Case (i)

Resistive Circuit : If the circuit contains only a pure resistor, there voltage and current are in same phase i.e. $\phi = 0$. The power factor 'cos ϕ', is therefore unity. There is maximum power dissipation in such a pure resistive circuit.

Case (ii)

Purely inductive or capacitive or a series LC circuit: If the circuit contains only a pure reactance 'X', the phase diffrence, between voltage and current, is $\left(\pm\frac{\pi}{2}\right)$. Therefore, power factor 'cos ϕ', is zero. Hence no net power is dissipated, over a complete cycle, though current is flowing through the circuit all the time. This current is therefore also referred to as a 'watless current'.

Case (iii)

LCR Circuit : In a series LR, RC or a series LCR circuit, ϕ is non zero $\left(0 < \phi < \frac{\pi}{2}\right)$. Even in such cases, power is dissipated only in the resistor. In ideal case $P_{av} = v_{rms} i_{rms} \cos\phi = i_{rms}^2 R \cos\phi$

Case (iv)

LCR circuit at resonance : At resonance $X_L = X_C$ and $\phi = 0$. Therefore 'cos ϕ' is unity. As $Z_{min} = R$, $P_{av} = i_{rms}^2 R$. Thus maximum power is dissipated in such a circuit (through R) at resonance.

Example–15 :

A series LR circuit draws a power of 550W from a 220V– 50Hz source. The power factor of the circuit in 0.8. The current in the circuit lags behind the voltage. Show that a capacitor, of value about $\left(\frac{1}{42\pi}\times 10^{-2}\right)$F, will have to be connected in the circuit to bring its power factor to unity.

Solution :

We are given that $\cos\phi = 0.8$

$$\therefore \tan\phi = \left(\frac{\omega L}{R}\right) = \frac{\sqrt{1-\cos^2\phi}}{\cos\phi} = \frac{0.6}{0.8} = \frac{3}{4}$$

$\therefore\ \omega L = \frac{3}{4}R$

$\therefore$ Inpedance of the LR circuit $= z = \sqrt{R^2 + (\omega L)^2} = \frac{5}{4}R$

Power dissipated $= \frac{V_{rms}^2}{z}.\cos\phi$

$\therefore\quad 550 = \frac{(220)^2}{(5/4\,R)}.(0.8)$

This gives $R = 56.32\ \Omega$

and $\omega L = \frac{3}{4}\times 56.3\Omega$

The power factor would become unity if a capacitor, C, is connected in the circuit, for which

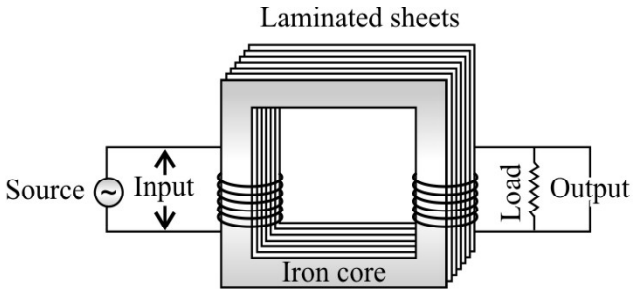
$$\omega L = \frac{1}{\omega C}$$

This given $C = \frac{1}{\omega^2 L} = \frac{1}{\omega.\omega L}$

$$= \frac{1}{(2\pi\times 50)\times 3\times 56.32}\ F \simeq \frac{1}{42\pi}\times 10^{-2}\ F$$

Transformer

It consists of two coils wound on the same core. The alternating current passing through the primary creates a continuously changing flux through the core. This changing flux induces an alternating emf in the secondary.



(1) Transofrmer works on ac only and never on d.c.

(2) It can increase or decrease either voltage or current but not both simultaneously.

(3) Transformer does not change the frequency of input a.c.

(4) There is no electrical connection between the winding but they are linked magnetically.

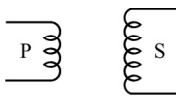
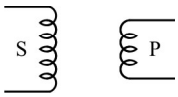
(5) Effective resistance between primary and secondary winking is infinite.

(6) The flux per turn of each coil must be same i.e. $\phi_s = \phi_p ; -\frac{d\phi_s}{dt} = -\frac{d\phi_p}{dt}$

(7) If N_P = number of turns in primary, N_S = number of turns in secondary V_P = applied (input) voltage to primary, V_S = Voltage across secondary (load voltage or output), e_P = induced emf in primary; e_S = induced emf in secondary, ϕ = flux linked with primary as well as secondary, i_P = current in primary, i_S = current in secondary (or load current).

As in an ideal transformer there is no loss of power i.e. $P_{out} = P_{in}$ so $V_S i_S = V_P i_P$ and $V_P \simeq e_P$, $V_S \approx e_S$. Hence $\frac{e_S}{e_P} = \frac{N_S}{N_P} = \frac{V_S}{V_P} = \frac{i_P}{i_s} = k$; k = transformation ratio (or turn ratio)

Types of Transformer

Step up transformer	Step down transformer
It increases voltage and decreases current	It decreases voltage and increases current
	
$V_S > V_P$	$V_S < V_P$
$N_S > N_P$	$N_S < N_P$
$E_S > E_P$	$E_S < E_P$
$i_S < i_P$	$i_S < i_P$
$R_S > R_P$	$R_S < R_P$
$t_S > t_P$	$t_S > t_P$
$k_S > 1$	$k_S < 1$

(8) **Efficiency of transformer (h) :** Efficiency is defined as the ratio of output power to input power i.e.

$$\eta\% = \frac{P_{out}}{P_{in}} \times 100 = \frac{V_S i_S}{V_P i_P} \times 100$$

For an ideal transformer $P_{out} = P_{in}$ so $\eta = 100\%$ (But efficiency of practical transformer _____ 70% – 90)

For practical transformer $P_{in} = P_{out} + P_{losses}$

so $\eta\% = \frac{P_{out}}{P_{in}} \times 100 = \frac{P_{out}}{(P_{out} + P_L)} \times 100 = \frac{P_{in} - P_L}{P_{in}} \times 100$

(9) **Losses in transformer :** In transformers some power is always lost due to, heating effect, magnetic flux leakage, eddy currents, hystersis and humming.

(i) **Cu loos (i^2 R) :** When current flows through the transformer windings some power is lost in the form of heat ($H = I^2 Rt$). To minizie this loss windings are made of thick wires (to reduce resistance)

(ii) **Eddy Current loss :** Some electrical power is wasted in the form of heat due to eddy currents induced in core, to minimize this loss transformers core are laminated and silicon is used as the core material as it increases the resistivity. The material of the core is then called alloy of iron (steel).

(iii) **Hystersis loss :** The alternating current flowing through the coils magnetises and demagneties the iron core again and again. Therefore, during each cycle of magnetisation, some energy is lost due to hysteresis. However, the loss of energy can be minimised by selecting the suitable of core, which has a narrow hysterisis loop. Therefore core of transformer is of soft iron. Now a days it is made of "Permalloy" (Fe-22%, Ni-78%).

(iv) **Magnetic flux Leakage :** Magnetic flux product in the primary winding is not completely linked with secondary becuase few magnetic field lines complete their path in air. To minimize this loss secondary winding is kept inside the primary winding.

(v) **Humming losses :** Due to the passage of alternating current, the core of the transformer starts vibrating and produces humming sound. Thus, some part (may be very small fraction) electircal energy is wasted in the form of humming sound produced by the vibrating core of the transformer.

(10) **Uses of transformer :** A transformer is used in almost all a.c. operation e.g.

(i) In voltage regulators for TV, refregerator, computer, air conditioner etc.

(ii) In the induction furnances.

(iii) Step down transformer is used for welding purposes.

(iv) In the transmission of a.c. over long distance.

PROBLEMS FOR PRACTICE

1. A long solenoid of radius 0.01 m has 600 turns per meter. A smaller coil of 100 turns is wrapped closely around the centre of the bigger solenoid. If the current in the solenoid rises from zero to 2A in 1ms. Find the emf induced in smaller coil.

   Ans.: 47.3 mV

2. A coil of area 500 cm^2 and 2000 turns is kept normal to a magnetic field of 4×10^{-5} Wb/m^2. It is rotated through 180^0 in 0.1 second. (i) Calculate the average induced emf (ii) if the total resistance of circuit be 20 ohm, calculate the induced charge.

   Ans.: (i) 0.08 V, (ii) 4×10^{-4}C

3. In an aeroplane, the distnace between the edges of its wings being 30m. During landing down its velocity is 300 Km/hr and wings of the aeroplane be east-west. Find out the potential difference between the edges of the wings. What happens if wings be north-south then? [Hint : (In first case wings will cut B_H and in second case neither B_H or BV) (Give $B_H = 0.4$ gauss and angle of dip at the place 60^0)]

   Ans.: (i) 01.V, (ii) zero

4. A conducting loop of area 5.0 cm^2 is placed in a magnetic field which varies sinosoudally with time as $B = B_0 \sin\omega t$ where $B_0 = 0.20$ T and ω 300/s, such that the normal to the coil makes an angle of 60^0 with the field.

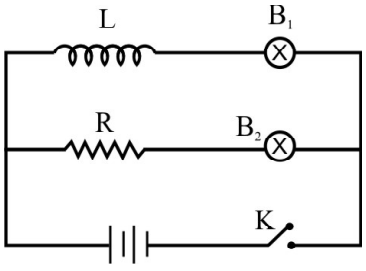
   Find (a) the maximum emf induced in the coil (b) the emf induced at $t = \left(\frac{\pi}{600}\right)s$.

   Ans.: (a) 15 mV, (b) zero

5. An air cored coil has a self inductance of 0.1 H. A soft iron core of relative permability 1000 is introduced and the number of turns is reduced to $\frac{1}{10}$th of the previous value. Find the new value of self inductance.

   Ans.: 1H

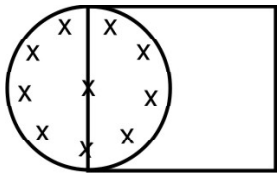
6. In the given circuit, the resistance 'R' is equal to the resistance of the coil that makes the inductance L, and the bulbs B_1 and B_2 are identical. (i) which of the bulbs lights up earlier when k is closed? (ii) Will the bulbs be equally bright after same time?



7. A conducting circular loop of area 1 mm^2 is placed coplanarly with a long, straight wire at a distance of 20 cm from it. The straight wire carries an electric current which changes from 10A to zero in 0.1s. Find the average emf induced in the loop in 0.1 s.

   Ans.: 10^{-10}V

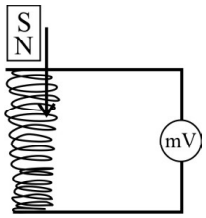
8. A uniform magnetic field B exists in a cylindrical region of radius 0.1 m as shown in the figure. A uniform wire of length 0.80 m and resistance 4.0 Ω is bent into a square frame and is placed with one side along a diameter of the cylindrical region. If the magnetic field increases at a constant rate of 0.010 TS^{-1}. Calculate the current induced in the frame.



   [**Hint:** $|\varepsilon| = \frac{d\phi_B}{dt} = \frac{1}{2}\pi^2 \frac{dB}{dt}$]

   Ans.: 3.925×10^{-5}A

9. Figure shows a bar magnet M falling under gravity through an aircored coil C plot a graph showing the variation of induced emf 'ε' with time 't'. What does the area enclosed by the $\varepsilon - t$ curve depict.



10. Two coils m and n having 10 turns and 15 turns respectively are placed close to each other. When 2A current is passing through coil m, then flux linked with coil n is 1.8×10^{-4} wb per turn. If 3A current passed through coil n, then calculate the flux linked per turn of coil m.

    Ans.: 4.05×10^{-4} wb

11. An alternating voltage v = 200 sin 300t is applied across a series combination of R = 10 Ω and L = 800 mH. Calculate (i) impedance of the circuit (ii) peak current in the circuit and (iii) power factor of the circuit.

    Ans.: (i) 240.2 Ω (ii) 0.83 A (iii) 0.041

12. When and inductor L and a resistor R in series are connected across a 12V–50 Hz supply, a current of 0.5A flows through the circuit. The current differs in phase from the applied voltage by $\frac{\pi}{3}$ radian. Calculate the value of L and R.

    Ans.: 0.066 H (ii) 23 Ω

13 An alternating emf 110V is applied to a circuit containing a resistance of 40 Ω and an inductance L

in series. The current lags behind the emf by an angle $\tan^{-1}\left(\frac{3}{4}\right)$. Find the inductive reatance, impedance of the circuit and current in the circuit. If L = 0.1 H. Find the frequency of applied emf.

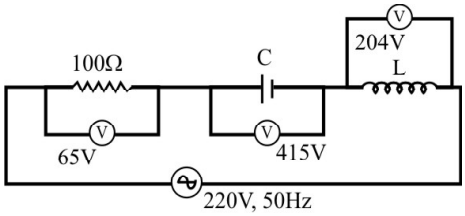
14. The voltage ε and current i of an alternating source are represented by

$$\varepsilon = 100\sin\left(\omega t + \frac{\pi}{6}\right) \text{ and } i = 5\sin\left(\omega t + \frac{\pi}{6}\right)$$

Dtermine the phase difference between ε and i and impedance of the circuit.

Ans.: $\Delta\phi = \frac{\pi}{3},\ z = 22\,\Omega$

15. In the given LCR series circuit fed by 200V–50Hz a.c. mains. Find (i) current in the circuit (ii) value of inductor L (iii) value of capacitor C and (iv) value of C (for same L) to have power factor of the circuit unity.



Ans.: (i) 0.65A, (ii) 1H, (iii) 5 μF (iv) 10 μF

16. A 80 V, 600 W heater is to be operated on a 100V–50Hz supply. Calculate the inductance of the choke required.

Ans.: 0.02H

17. A current of 4A flows in a coil when connected to 12V dV source. If the same coil is connected to a 12V, 50 rad/s a.c. source, a current of 2.4 A flows in the circuit. Determine the inductance of the coil. Also find the power developed in the circuit if a 2500 μF capacitor is connected in series with the coil.

Ans.: 0.08H, 17.28W

18. An LCR circuit has L = 10mH, R = 3 Ω and C = 1 μF connected in series to a source 15 cos ωt volt. Calculate the current amplitude and average power dissipated per cycle at a frequency 10% lower than the resonant frequency.

Ans.: 0.744A, 0.744W

19. A step down transformer having a power output of 10 kW and efficiency of 90% downs the voltage from 11 kV to 220 V. Find the number of turns in the primary if the secondary has 100 turns and the current in the primary.

    Ans.: 5000, 1.01 A

20. An a.c. source of internal resistance 9000 Ω is is to supply current to a load resistor of 10 Ω. Find the ratio of the currents passing through the load and the source?

    [Hint : $V_P i_P = V_S i_S$. Also if transformer is ideal then power dilivered is constant i.e. $\frac{V_S^2}{R_S} = \frac{V_P^2}{R_P}$]

    Ans.: 30

QUESTION BANK

Key Learning Points

1. The magnetic flux through a surface of area A, placed in a uniform magnetic field B is defined as

$$\phi_B = \overline{B}.\overline{A} == BA\cos\theta$$

where θ is the angle between **B** and the normal to the plane of the area.

2. The emf, induced in a coil of N turns, is directly proportional to the rate of change of magnetic flux through it

$$\varepsilon = -N\frac{d\phi_B}{dt}$$

where ϕ_B is the flux linked with one turn. The negative sign indicates that polarity of induced emf is such that it opposes its own cause.

3. When a metal rod of length ℓ is placed normal, to a uniform magnetic field **B**, and moved with velocity **V**, perpendicular to the field, the emf induced (Motional emf) across its ends is $\varepsilon = BV\ell$.

4. For a rod rotating about its one end, in a uniform magnetic filed, about an axis perpendicular to the rod, with the magnetic field parallel to the axis, the emf induced between its two ends, $\varepsilon = \frac{1}{2}B\ell^2\omega$.

5. Changing magnetic fields can set up current loops in nearby metal bodies. They dissipate electrical energy as heat. Such currens are called eddy currents.

6. When a current in a coil changes, it induces back emf in the same coil. The self induced emf is given by

$$\varepsilon = -L\frac{di}{dt}$$

where L is the self inductance of the coil.

7. The self inductance (or inductancee) of a long (ideal) solenoid (the core of which has a permability) is given by $L = \mu_0\mu_r n^2 A\ell$. Here A is the cross-section area of the solenoid ℓ its length and 'n' the number of turns per unit length.

8. A changing current, in one coil, can induce an emf in another (nearly) coil. The emf induced, in the second coil, in given by $\varepsilon_2 = M_{12}\frac{di_1}{dt}$. Here $M_{12} = M_{21} = M$ is called the mutual inductance between the two coils.

   For two coaxial (long) solenoids placed such that one is completely placed well inside the other, the

mutual inductance is given by

$$M = \mu_0 \mu_r n_1 n_2 \pi r^2 \ell$$

where n_1 and n_2 are the number of turns, per unit length, in the two coils. The coils have a radius r and a length 'ℓ'.

9. When a coil, of N turns, and area A, is rotated, at υ revolutions per second, in a uniform magnetic field **B**, then the emf induced is given by

$$\varepsilon = NBA2\pi\upsilon \sin(2\pi\upsilon t)$$

Here, at t = 0, the coil is considered tobe perpendicular to the field. This is the working principle of an a.c generator.

10. The mean, or average, value ofan alternating voltage (or current) over a complete cycle is zero. Over a half cycle, the mean value is 63.7% of its peak value.

$$(i_{AV}) \text{ Half cycle} = \frac{2i_0}{\pi} = 0.637\, i_0$$

11. The r.m.s value of current is given by

$$i_{rms} = \frac{i_0}{\sqrt{2}} = 0.707\, i_0$$

Similarly $v_{rms} = \frac{v_0}{\sqrt{2}} = 0.707\, v_0$

12. An alternating voltage $(v = v_0 \sin \omega t)$, applied to a pure resistor R, drives a current, $i = i_0 \sin \omega t$, where $i_0 = \frac{v_0}{R}$. The current is in phase with the applied voltage.

13. An a.c voltage, $v = v_0 \sin \omega t$, applied to a pure inductor, drives a current $i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right)$ where $i_0 = \frac{v_0}{x_L}$. Here $x_L (= \omega L)$, is called inductive reactance. The current in the inductor lags behind the voltage by $\frac{\pi}{2}$. The average power supplied to a pure inductor over a complete cycle is zero.

14. An a.c voltage, $v = v_0 \sin \omega t$, applied to pure capacitor, drives a current in the capacitor, $i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right)$, where $i_0 = \frac{v_0}{x_C}$. Here $x_C \left(= \frac{1}{\omega C}\right)$, is called the capacitive reactance. The current through the capacitor is $\frac{\pi}{2}$ ahead of the applied voltage. Average power, supplied to a pure capacitor, over a complete cycle, is zero.

15. In a series LCR circuit, if a voltage $\left(v = v_0 \sin \omega t\right)$, is applied, then current in the circuit is given by $i = i_0^{'} \sin\left(\omega t \mp \phi\right)$ where $i_0 = \frac{v_0}{z}$. Here $\left(= \sqrt{R^2 + \left(x_L - x_C\right)^2}\right)$ is called the impedance of the circuit. The phase angle, between the voltage and current, is ϕ. Here $\phi = \tan^{-1} \frac{\left(x_L - x_C\right)}{R}$.

Case (i): $x_L < x_C$

or $\omega > \frac{1}{\sqrt{LC}}$, $\omega > \omega r$ $\left(\omega r = \frac{1}{\sqrt{LC}}\right)$

Inductive effect dominates in the circuit. Voltage leads the current by phase angle ϕ.

Case (ii): $x_C > x_L$

or $\omega > \frac{1}{\sqrt{LC}}$, $\left(\omega < \omega r\right)$

Capacitive effect dominates in the circuit. Current leads the voltage by phase angle ϕ.

16. An interesting characterstics of a series LCR circuit is the phenomenon of resonance. $(x_L = x_C)$. The circuit exhibits resonance at the (angular) frequency $\omega_0 > \frac{1}{\sqrt{LC}}$. The impedance of the circuit is minimum. The amplitude of the current is maximum at resonance. At resonance, maximum power is supplied to the series LCR circuit.

17. The quality factor, Q is defined as

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

'Q' is an indicator of the sharpness of the (i) vs $\left(\omega\right)$ curve around its resonance value.

18. The average power dissipation, or loss of power over a complete cycle, is given by

$$P_{av} = v_{rms} \; i_{rms} \cos\phi$$

The term, cos ϕ is called the power factor. For pure resistive circuit, $\cos\phi$ is unity. For pure reactance (Pure L or pure L), the $\cos\phi$ term is zero.

19. In pure inductive, or capacitive or a series (LC) circuit, the power factor, $\cos\phi$, is zero. Hence no power is dissipated even though current is flowing in the circuit. In such cases current is referred as a 'wattlless current'.

20. A transformer consists of an iron core on which are bound a primary coil of N_p turns and a secondary coil of N_S tunrs. If the primary coil is connected to an a.c cource the primary and secondary voltages are related by

$$V_S = \left(\frac{N_S}{N_P}\right) V_P$$

The currents are related by

$$I_S = \left(\frac{N_P}{N_S}\right) I_P$$

In step up transformer $N_S > N_P$

and in step down transformer $N_P < N_S$

Unit–14 : Electromagnetic Induction and Alternating Currents

Easy *Self Induction*

1. When the current in a coil changes from +2A to –2A in 0.05 second, an emf of 8V is induced in a coil. The coefficient of self induction of the coil is:

(1) 0.1 H (Correct)

(2) 0.2 H (Incorrect)

(3) 0.4 H (Incorrect)

(4) 0.8 H (Incorrect)

Correct answer: (1)

Solution:

$$\varepsilon = -L\frac{di}{dt}$$

$$\delta = -L\frac{(-2-2)}{0.5}$$

$$\Rightarrow L = 0.1\,H$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Induced emf*

2. A conducting circular loop is placed in a uniform magnetic field B = 0.25 T, with its plane perpendicular to the field. The radius of the loop is made to shrink at a constant rate of 1 mm/s. The induced emf, when the radius is 2 cm, would be

(1) $\left(\frac{\pi}{2}\right)\mu V$ (Incorrect)

(2) $(\pi)\mu V$ (Correct)

(3) $(2\pi)\mu V$ (Incorrect)

(4) $(2.5)\pi\,\mu V$ (Incorrect)

Correct answer: (2)

Solution:

$$|\varepsilon| = \frac{d\phi_B}{dt}$$

$$= \frac{d}{dt}\left(B.\pi r^2\right)$$

$$= B\pi(2r)\frac{dr}{dt}$$

$$= 0.025 \times \pi \times 2 \times 2 \times 10^{-2} \times \left(1 \times 10^{-3}\right)$$

$$= \pi \times 10^{-6} \ \mu V$$

$$= (\pi)\mu V$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Difficult *Change in Magnetic Flux*

3. A magnetic field, directed along z axis, varies as $B = B_0 \frac{x}{a}$. where a is a positive constant. A square loop (of side ℓ), made of copper, is placed with its edges parallel to the *x* and y axes. If the loop is made to move with a constant velocity V_0, directed along *x* axis; the emf induced in it, would be

(1) $B_0 V_0 \frac{\ell^2}{a}$ (Correct)

(2) $B_0 V_0 \ell$ (Incorrect)

(3) $B_0 V_0 \frac{\ell^2}{2a}$ (Incorrect)

(4) $B_0 V_0 \frac{\ell^3}{a^2}$ (Incorrect)

Correct answer: (2)

Solution:

We have,

$$d\phi = \mathbf{B}.d\mathbf{A}$$

$$= BdA \qquad (\text{where } dA = \ell dx)$$

$$\text{Initial total flux } \phi = \int_0^{\ell} d\phi$$

$$= \frac{B_0}{a}\ell\int_0^{\ell} dx = \frac{B_0\ell}{a}\left|\frac{x^2}{2}\right|_0^{\ell}$$

$$\phi_{initial} = \frac{B_0\ell^3}{2a}$$

In one second the loop covers a distance of V_0, so that the final flux can be found by integrating $d\phi$ between V_0 and $(V_0 + \ell)$.

This flux comes out to be $\left[\frac{B_0V_0\ell^2}{a} + \frac{B_0\ell^3}{2a}\right]$

$$\phi_{initial} = \frac{B_0V_0\ell^2}{a} + \frac{B_0\ell^3}{2a}$$

$$\Rightarrow \text{Change in flux } \phi = \frac{B_0V_0\ell^2}{a}$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Difficult *Induced emf*

4. A conducting ring, of radius r, is placed in a varying magnetic field perpendicular to the plane of the ring. If the rate at which the magnetic field, varies, is *x*, the (average) electric field intensity, at any point of the ring, would be

(1) $r\,x$ (Incorrect)

(2) $\frac{r\,x}{2}$ (Correct)

(3) $2rx$ (Incorrect)

(4) $\frac{4r}{x}$ (Incorrect)

Correct answer: (2)

Solution:

Let E be the field strength at any point on the circumference of the ring. Then, the emf induced

$$\varepsilon = \oint \mathbf{E}.d\ell = E2\pi r \quad .....(1)$$

Also induced emf $|\varepsilon| = \frac{d\phi_B}{dt} = \pi r^2 \frac{dB}{dt}$

$$= \pi r^2 x \quad .....(2)$$

From (1) and (2)

$$E = \frac{rx}{2}$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Difficult *Power Loss*

5. Magnetic flux, through a stationary loop, with a resistance R, varies during the time internal 'τ', as $\phi = at(\tau - t)$ where a is a constant. The amount of of heat generated in the loop, during time internal, τ, would be

(1) $\frac{a^2\tau^3}{6R}$ (Incorrect)

(2) $\frac{a^2\tau^3}{4R}$ (Incorrect)

(3) $\frac{a^2\tau^3}{3R}$ (Correct)

(4) $\frac{a^2\tau^3}{2R}$ (Incorrect)

Correct answer: (3)

Solution:

We have, $\phi_B = at\tau + at^2$ (given)

$\therefore$ Instantaneous $|\varepsilon| = \dfrac{d\phi_B}{dt} = a\tau - 2at$

Instantaneous power loss / dissipation $= \dfrac{|\varepsilon|^2}{R}$

$$= \frac{(a\tau - 2at)^2}{R}$$

$$\Rightarrow H = \int_0^\tau P dt$$

$$= \frac{1}{R}\left|\frac{(a\tau - 2at)^3}{(3-2a)}\right|_0^\tau$$

$$= \left|\frac{(a\tau - 2at)^3}{-6aR}\right|_0^\tau$$

$$= \frac{a^2\tau^3}{3R}$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Self Induction*

6. The current, through a coil of self inductance. L = 2mH, is given by $I = t^2e^{-t}$ at time t. The time taken, to make the induced emf zero, will be

(1) 1 S (Incorrect)

(2) 2 S (Correct)

(3) 3 S (Incorrect)

(4) 4 S (Incorrect)

Correct answer: (2)

Solution:

We want to have $|\varepsilon| = L\frac{di}{dt} = 0$

$\Rightarrow L\frac{d}{dt}\left(t^2e^{-t}\right) = 0$

$\Rightarrow -t^2e^{-t} + 2te^{-t} = 0$

$\Rightarrow te^{-t}\left(-t+2\right) = 0$

$\Rightarrow t = 2s$

Unit–14 : Electromagnetic Induction and Alternating Currents

Difficult *Induced emf*

7. The magnetic flux ϕ, through a stationary loop of wire, having a resistance R, varies with time as $\phi = (at^2 + bt)$ where a and b are positive constants. The average emf, and the total charge flowing in the loop in the time interval t = 0 to t = τ, would be equal, respectively, to

(1) $(a\tau + b), \left(\frac{a\tau^2 + b\tau}{R}\right)$ (Correct)

(2) $(a\tau + b), \left(\frac{a\tau^2 + b\tau}{2R}\right)$ (Incorrect)

(3) $\left(\frac{a\tau + b}{2}\right), \left(\frac{a\tau^2 + b\tau}{R}\right)$ (Incorrect)

(4) $2(a\tau + b), \left(\frac{a\tau^2 + b\tau}{2R}\right)$ (Incorrect)

Correct answer: (1)

Solution:

We have $|\varepsilon| = \frac{d\phi}{dt} = 2at + b$

$$\therefore \quad \varepsilon_{average} = \frac{\int_0^{\tau} |\varepsilon| \, dt}{\tau \int_0^{} dt} = \left| \frac{at^2 + bt}{t} \right|_0^{\tau}$$

$$= \frac{a\tau^2 + b\tau}{\tau} = a\tau + b$$

$$q = \int_0^{\tau} i dt = \frac{\Delta\phi}{R} = \frac{a\tau^2 + b\tau}{R}$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *LC Oscillations*

8. An LC circuit contains a 20 mH inductor, and a 50 μF capacitor, of the circuit is negligible. Let the instant, the circuit is closed, be t = 0. The (first instant of) time, at which energy stored would be completely magnetic, is

(1) $t = 0$ (Incorrect)

(2) $t = 1.57\,ms$ (Correct)

(3) $t = 3.14\,ms$ (Incorrect)

(4) $t = 6.28\,ms$ (Incorrect)

Correct answer: (2)

Solution:

Total energy, in a LC circuit, is completely magnetic (at first) at $t = \frac{T}{4}$

Here $T = 2\pi\sqrt{LC}$ $\left(\omega = \frac{1}{\sqrt{LC}} \right)$

$$= 2 \times 3.14 \sqrt{50 \times 10^{-6} \times 20 \times 10^{-3}} \text{ s}$$

$$= 2 \times 3.14 \sqrt{10^{-6}} \text{ s}$$

$= 6.28$ ms

$\therefore \quad \frac{T}{4} = 1.57$ ms

Unit–14 : Electromagnetic Induction and Alternating Currents

Easy *Induced emf due to Change in Orientation*

9. A rectangular coil, of 300 turns, has an average area of 25 cm×10 cm. The coil rotates, with a speed of 50 cycles per second, in a uniform magnetic field of 4×10^{-2}T, about an axis perpendicular to the field. The peak value of induced emf (in volt), is

(1) 3π (Incorrect)

(2) 30π (Correct)

(3) 300π (Incorrect)

(4) 3000π (Incorrect)

Correct answer: (2)

Solution:

$\varepsilon_0 = NBA\omega$

$= \left(300\times4\times10^{-2}\times250\times10^{-4}\times2\pi\times50\right)V$

$= \left(30\pi\right)$Volt

Unit–14 : Electromagnetic Induction and Alternating Currents

Easy *Change in Magnetic Flux / Lenz's Law*

10. Two identical coaxial coils, P and Q, carrying equal amount of current, in the same direction, are brought nearer. The current in

(1) P increases while in Q decreases (Incorrect)

(2) Q increases while in P decreases (Incorrect)

(3) both P and Q increases (Incorrect)

(4) both P and Q decreases (Correct)

Correct answer: (4)

Solution:

When both the coils are brought closer. The linked magnetic flux, increases for both of then. Hence induced current (set up in then) will try to decreases the flux (as per Lenz's law). Hence current in both the coils will decrease.

Unit–14 : Electromagnetic Induction and Alternating Currents

Easy *Induced emf*

11. A 50 turns circular coil has as average a radius of 3 cm. It is kept in a magnetic field acting normal to the area of the coil. The magnetic field B is increased from (0.10 T) to (0.35 T) in 2 ms. The average induced emf in the coil, would be

(1) 0.177 V (Incorrect)

(2) 1.77 V (Incorrect)

(3) 17.7 V (Correct)

(4) 177 V (Incorrect)

Correct answer: (3)

Solution:

Change in magnetic flux, linking the coil, is

$$d\phi_B = \left(NA\cos 0^0\right)\left(dB\right)$$

$$= 50\times 3.14\times\left(3\times10^{-2}\right)^2\left(0.35-0.10\right) \text{ Weber}$$

$$\therefore \text{ Induced emf } |\varepsilon| = \frac{d\phi_B}{dt}$$

$$= \frac{50\times 3.14\times\left(3\times10^{-2}\right)^2\left(0.35-0.10\right)}{2\times10^{-3}} \text{ V}$$

$$= \frac{50\times 3.14\times 9\times 0.25\times10^{-4}\times10^{3}}{2} \text{ V}$$

$$= 17.7 \text{ V}$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Induced emf*

12. A conducting circular loop is placed in a uniform magnetic filed (of induction 'B' tesla), with its plane normal to the field. If the radius of the loop were to start shrinking at a constant rate $\frac{dr}{dt}$. The induced emf, at the instant when its radius is 'r', would be

(1) $-\pi rB\left(\frac{dr}{dt}\right)$ (Incorrect)

(2) $-2\pi rB\left(\frac{dr}{dt}\right)$ (Correct)

(3) $-\pi r^2 B\left(\frac{dr}{dt}\right)B$ (Incorrect)

(4) $-\frac{1}{2}\pi r^2 B\left(\frac{dr}{dt}\right)B$ (Incorrect)

Correct answer: (2)

Solution:

The magnetic flux, linking the loop, (when its radius is r) is

$\phi - \mathbf{B}.\mathbf{A} = BA\cos 0^0$

$= B\left(\pi r^2\right)$

$\therefore$ The emf, induced at this instant is

$\therefore \quad \varepsilon = -\frac{d\phi}{dt}$

$= -\frac{d}{dt}\left(B\pi r^2\right) \equiv B\pi\frac{d}{dt}\left(r^2\right)$

$= -2\pi rB\frac{dr}{dt}$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Motion of a Conductor in Magnetic Field*

13. A conducting rod of length L lies along the y–axis with one end at origin and the other end at (0, L, 0). There is a magnetic field $\mathbf{B} = B_0 y\hat{k}$ in that region. If the rod moves with a velocity $V = V_0\hat{i}$, the e.m.f induced between the ends of the rod will be

(1) $\frac{B_0 V_0 L}{2}$ (Incorrect)

(2) $\frac{B_0 V_0 L^2}{2}$ (Correct)

(3) $B_0 V_0 Ld$ (Incorrect)

(4) $\frac{B_0 V_0 L^2}{4}$ (Incorrect)

Correct answer: (2)

Solution:

The induced emf, across a small element dy of the rod is

$$d\varepsilon = (B_0 y)(dy)V_0 \qquad [\because \quad \varepsilon = BV\ell]$$

$\therefore$ The induced emf across the rod of length L will be

$$\varepsilon = \int_0^L (B_0 y)(dy)V_0$$

$$= B_0 V_0 \int_0^L ydy$$

$$= B_0 V_0 \left|\frac{y^2}{2}\right|_0^L = \frac{1}{2} B_0 V_0 L^2$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Induced emf and Current*

14. The magnetic flux, through a circular coil of resistance R, changes by an amount $\Delta\phi$ in a time Δt. Then the total quantity of electric charge 'Q', that passes any point in the coil, during the time Δt, is represented by

(1) $Q = \frac{1}{R}\frac{\Delta\phi}{dt}$ (Incorrect)

(2) $Q = \frac{\Delta\phi}{R}$ (Incorrect)

(3) $Q = \frac{R\Delta\phi}{dt}$ (Incorrect)

(4) $Q = -\frac{\Delta\phi}{R}$ (Correct)

Correct answer: (4)

Solution:

The induced emf in the coil will be given by

$$\varepsilon = -\frac{d\phi}{dt}$$

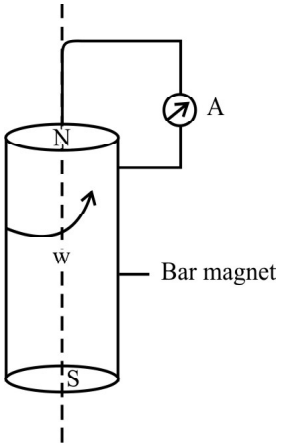
$$\Rightarrow i = -\frac{\varepsilon}{R} = -\frac{1}{R}\frac{\Delta\phi}{\Delta t}$$

$$\therefore \quad Q = i\Delta t = -\frac{\Delta\phi}{R}$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Induced Current*

15. A cylindrical bar magnet is rotated about its axis as shown in the diagram. One end of a wire is connected to apoint on the axis and its other end is made to touch the cylindrical surface through an ammeter. We would then observe



(1) That a direct current flowing in the ammeter A (Incorrect)

(2) That no current flows through the ammeter A (Correct)

(3) That an alternating simusoidal current flows through the ammeter A (Incorrect)

(4) A time varying, non-simusoidal, current flows through the ammeter A (Incorrect)

Correct answer: (2)

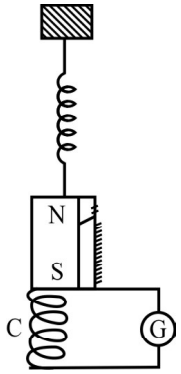
Solution:

As there is no change in the magnetic flux linked with the circuit, no emf gets induced in the circuit. The ammeter 'A' would, therefore, not show any deflection.

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Induced emf and Current*

16. A magnet NS is suspended from a spring. While it oscillates simple harmonically, it moves in and out a coil 'C'. The coil is connected to a galvanometer G.



As the magnet oscillates (Simple harmonically)

(1) G shows no deflection at all (Incorrect)

(2) G shows deflection, alternately, to the left and right, but its amplitude keeps on changing (Correct)

(3) G shows deflection, alternately, to the left and right, with a constant amplitude (Incorrect)

(4) G shows a deflection on one side only (Incorrect)

Correct answer: (2)

Solution:

As the magnet moves in and out of the coil, there is a continuous change in the magnetic flux linked with the coil. The flux first increases (as magnet moves in) and then decreases (as magnet moves out). Hence the induced emf keeps on changing.

$$\varepsilon = -\frac{d\phi_B}{dt}$$

The motion of the magnet is S.H.M. (an accelerated motion). Hence $\frac{d\phi_B}{dt}$ would not be a constant, hence $\frac{d\phi_B}{dt}$ varies.

Unit–14 : Electromagnetic Induction and Alternating Currents

Difficult *Magnetic Flux*

17. A loop, made of straight edges has six corners: at A (0, 0, 0), B (L, 0, 0), C (L, L, 0), D (0, L, 0) E (0, L, L) and F (0, 0, L). A uniform magnetic field, $\mathbf{B} = B_0\left(\hat{i} + \hat{k}\right)$ is present in the region. The magnetic flux, linking the whole loop ('ABCDEFA') is

(1) B_0L^2wb (Incorrect)

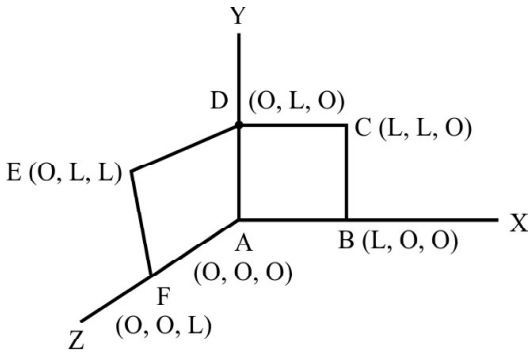
(2) $2B_0L^2$wb (Correct)

(3) $\sqrt{2}B_0L^2$wb (Incorrect)

(4) $4B_0L^2$wb (Incorrect)

Correct answer: (2)

Solution:



Given $\mathbf{B} = B_0\left(\hat{i} + \hat{k}\right)$

The area vector of part ABCD is

$$\mathbf{A}_1 = L^2\hat{k}$$

and that of part DEFA is

$$\mathbf{A}_2 = L^2\hat{k}$$

Total flux linking to the loop will be

$$\phi = \mathbf{B}\left(\mathbf{A}_1 + \mathbf{A}_2\right)$$

$$= \left[B_0\left(\hat{i} + \hat{k}\right)\right].\left[L^2\left(\hat{i} + \hat{k}\right)\right]$$

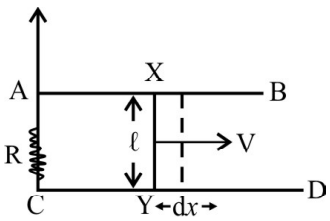
$$= B_0L^2\,(1 + 0 + 0 + 1)$$

$$= 2B_0L^2$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Induced emf*

18. A conducting wire xy, of mass m, (and negligible resistance) slides smoothly on two parallel conducting wires asshown in the diagram. The closed circuit has resistance 'R' due to side AC. Sides AB and CD are perfect conductors. If a time varying magnetic field $B = B(t)\hat{k}$, is present over a large region, the acceleration of the wire xy will be



(1) $-\dfrac{\ell^2B^2}{mR}\left(\dfrac{dx}{dt}\right)$ (Incorrect)

(2) $-\dfrac{\ell^2B^2}{mR}\dfrac{dx}{dt} + \dfrac{\ell^2B}{mR}\dfrac{dB}{dt}\left(x(t)\right)$ (Incorrect)

(3) $-\dfrac{\ell^2B^2}{mR}\dfrac{dx}{dt} - \dfrac{\ell^2B}{mR}\dfrac{dB}{dt}x(t)$ (Correct)

(4) $-\frac{\ell^2 B^2}{mR}\frac{dx}{dt}+\frac{\ell^2 B}{mR}\frac{dB}{dt}x(t)$ (Incorrect)

Correct answer: (3)

Solution:

Let, at time t, the wire xy wire be at $x = x(t)$.

The magnetic flux associated with the area ACYXA, (at time t) will be

$$\phi = \overline{B}.\overline{A}$$

$$= B(t)\left[\ell x(t)\right]$$

This magnetic flux changes with time due to two reasons. There are:

(i) The change of the magnetic field itself with time.

(ii) The motion of the wire on the rods.

We can, therefore, write

$$\varepsilon = -\frac{d\phi}{dt}$$

$$= -\left\{\left[\frac{dB}{dt}.\ell x(t)\right]+\left[B(t).\ell.\frac{dd}{dt}.x(t)\right]\right\} \quad .....(1)$$

The force, on wire *x*y, at this instant, will be

$$F = iB\ell$$

$$= \frac{\varepsilon}{R}B\ell \quad .....(2)$$

From (1) & (2), we get

$$F = ma = \frac{B\ell}{R}\left[-\frac{dB}{dt}.\ell x(t)\right]-\left[B(t).\ell.\frac{dx}{dt}\right]$$

$$\Rightarrow a = \frac{\ell^2 B}{mR}\frac{dB}{dt}(t)-\frac{\ell^2 B^2}{mR}\frac{dx}{dt}$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Motion of a Conductor in Uniform Magnetic Field*

19. In a uniform magnetic field of induction B, a wire, in the form of semicircular of radius 'R', rotates about the diameter of the circle with an angular speed 'ω'. The axis of rotation is perpendicular to the field. If the total resistance of the circuit is R', the near power generated per period of rotation is

(1) $\frac{B\pi R^2\omega}{2R'}$ (Incorrect)

(2) $\frac{(B\pi R^2\omega)^2}{8R'}$ (Correct)

(3) $\frac{(B\pi R\omega)^2}{8R'}$ (Incorrect)

(4) $\frac{B\pi R^2\omega^2}{8R'}$ (Incorrect)

Correct answer: (2)

Solution:

We have $\phi = BA\cos\theta = \left(\frac{B\pi R^2}{2}\right)\cos\omega t$

$\therefore$ Induced emf, $\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}\left(\frac{B\pi R^2\cos\omega t}{2}\right)$

$= -\frac{B\pi R^2\omega\sin\omega t}{2}$

$\therefore$ Instantaneous power generated $= \frac{\varepsilon^2}{R'} = \frac{(B\pi R^2\omega)^2}{4R'}\sin^2\omega t$

The average value of $\sin^2\omega t$, over one complete cycle, is $\frac{1}{2}$.

$\therefore$ Average power over one cycle $= \frac{(B\pi R^2\omega)^2}{8R'}$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Motion of a Conductor in Uniform Magnetic Field*

20. A straight rod PQ, of length L, is rotating about an axis passing through 'O' and perpendicular to its plane. The rod rotates in a uniform (and normal) magnetic field B, with an angular speed ω (The piont 'O' is at a perpendicular distance $\frac{L}{3}$ from P). The potential difference between P and Q, would be the equal to

(1) $\frac{B\omega L^2}{6}$ (Correct)

(2) $\frac{B\omega L^2}{4}$ (Incorrect)

(3) $\frac{B\omega L^2}{3}$ (Incorrect)

(4) $\frac{1}{2}B\omega L^2$ (Incorrect)

Correct answer: (1)

Solution:

The potential difference,between two ends of a rod, confined at one end, is given by $\frac{1}{2}B\omega L^2$

(L = distance of the other end from the confined end)

$$\therefore \quad V_Q - V_O = \frac{1}{2}B\omega\left(\frac{2L}{3}\right)^2$$

and $$V_P - V_O = \frac{1}{2}B\omega\left(\frac{L}{3}\right)^2$$

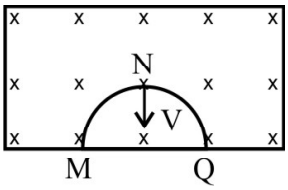
$$\therefore \quad \left(V_Q - V_P\right) = \frac{1}{2}B\omega L^2\left(\frac{4}{9} - \frac{1}{9}\right)$$

$$= \frac{B\omega L^2}{6}$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Difficult *Induced emf*

21. A thin semicircular conducting ring, of radius R, in falling with its plane vertical in a uniform horizontal magnetic field of strength B. At the position MNQ the speed of the ring is V. The potential difference, developed across the ring, would than be



(1) $\frac{1}{2}BV\pi R^2$ with M at highter potential (Incorrect)

(2) πRBV , with Q at higher potential (Incorrect)

(3) 2RBV, with Q at higher potential (Correct)

(4) $2R^2BV$, with M at higher potential (Incorrect)

Correct answer: (3)

Solution:

Rate of decrease of area, of the semicircular ring, in the magnetic field, for the position MNQ, will be

$$\frac{dA}{dt} = (2R)V$$

Now $|\varepsilon| = \frac{d\phi}{dt} = \frac{BdA}{dt} = B(2RV)$

As per Lenz's law, the induced current in the ring must produce magnetic field directed 'outwards'. Thus Q must be at a higher potential (so as to send the required anticlockwise (induced) current in the ring).

Unit–14 : Electromagnetic Induction and Alternating Currents

Easy *Self Induction*

22. A conducting coil of 600 turns has a self inductance of 108 mH. The self inductance of a second identical coil, having of 500 turns, will be

(1) 75 mH (Correct)

(2) 90 mH (Incorrect)

(3) 130 mH (Incorrect)

(4) 155 mH (Incorrect)

Correct answer: (1)

Solution:

We know that

$$L = \frac{\mu_0 N^2 A}{\ell}$$

$$\Rightarrow L \propto N^2 \qquad \text{(for identical coils)}$$

$$\Rightarrow \frac{L_2}{L_1} = \left(\frac{n_2}{n_1}\right)^2$$

$$L_2 = \left(\frac{500}{600}\right)^2 \times 108\,\text{mH}$$

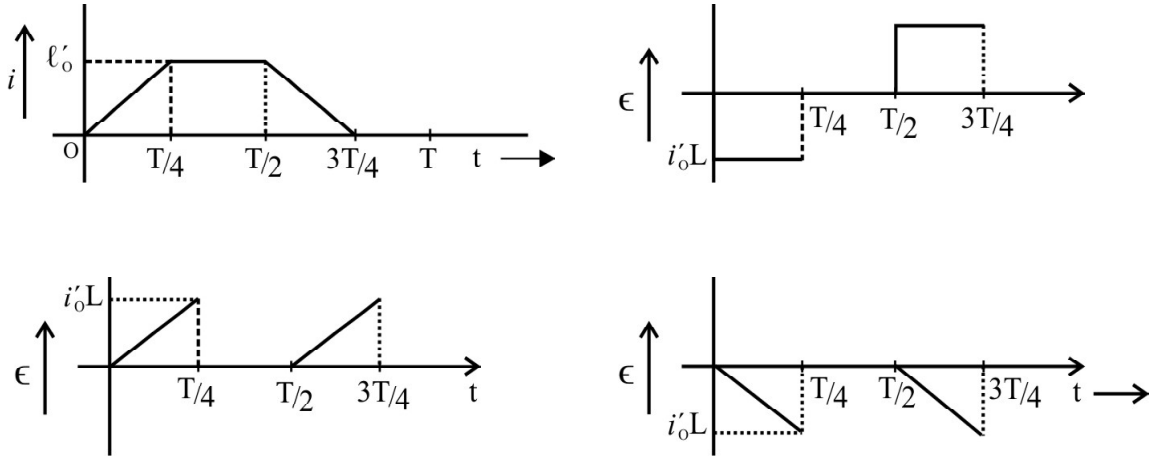
$$= \frac{25}{36} \times 108\,\text{mH}$$

$$= 75\ \text{mH}$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Self Induction*

23. The current 'i', in a coil of self inductance 'ℓ' varies with time as shown in figure (1). The variation of the induced emf 'ε' with time would than be represented by the graph labelled as graph



(1) A (Correct)

(2) B (Incorrect)

(3) C (Incorrect)

(4) D (Incorrect)

Correct answer: (1)

Solution:

We have $\varepsilon_{induced} = -L\frac{di}{dt}$

During 0 to $\frac{T}{4}, \frac{di}{dt}$ =a constant, as $(i-t)$ graph is a straight line with positive slope $(= i_0)$. Hence $\varepsilon_{induced}$ is negative and constant $(=(-i_0L))$.

For time $\frac{T}{4} \le t \le \frac{T}{2} \frac{di}{dt} = 0$

$\Rightarrow |\varepsilon_{induced}| = 0$

For time $\frac{T}{2} \le t \le \frac{3T}{4} \frac{di}{dt}$ is again a constant $(= -i_0)$.

Hence $\varepsilon_{induced}$ is positive and constant $(= i_0L)$.

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Self Induction*

24. A (solenoid) coil is wound on a core of square cross-section. If all the linear dimensions of core are increased 2 times, but the number of turns per unit length of the coil, remains same, the self inductance would increases by a factor of

(1) 2 (Incorrect)

(2) 4 (Incorrect)

(3) 8 (Correct)

(4) 16 (Incorrect)

Correct answer: (3)

Solution:

The self inductance of the coil is given by

$L = \mu_0 \mu_r n^2 A \ell$

For a square core $A = \ell^2$

$\Rightarrow L \propto \ell^3$

$\therefore \quad \frac{L_2}{L_1} = (2)^3 = 8$

The self inductance, therefore, increases by a factor of 8.

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Mutual Induction*

25. A pair of (insulated) coils having of n_1 and n_2 tunrs, are kept close together. When the current, passing through the first, is reduced at a rate r, an emf of 3mV is developed across the other coil. If the second coil were to carry a current which gets reduced at a rate 2r, then induced emf developed across the first coil will be

(1) 1.5 mV (Incorrect)

(2) 2.0 mV (Incorrect)

(3) 3 mV (Incorrect)

(4) 6 mV (Correct)

Correct answer: (4)

Solution:

Induced emf, due to mutual induction between in two coils, is given by

$|\varepsilon| = +M \frac{di}{dt}$

We are given that $\varepsilon_2 = M_{21} \frac{di_1}{dt} = 3\,mV$

$\therefore \quad M_{21} = \frac{3mV}{(r)}$

We know that $M_{12} = M_{21}$

$\therefore \quad \varepsilon_1 = M_{12} \dfrac{di_2}{dt}$

$= \dfrac{3mV}{4} \times 24 = 6mV$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Mutual Induction*

26. The coefficient of mutual inductance of two circuits A and B is 3 mH and their respective resistances are 100 hm and 4 ohm. How much should the currents change, in 0.02 second, in circuit A, so that an induced current 0.006 ampere flows through the circuit B?

(1) 0.16 A (Correct)

(2) 0.18 A (Incorrect)

(3) 0.24 A (Incorrect)

(4) 1.6 A (Incorrect)

Correct answer: (1)

Solution:

We have

$|\varepsilon_B| = M \dfrac{di_A}{dt}$

$\therefore \quad i_B R_B = M \dfrac{di_A}{dt}$

$\Rightarrow 0.006 \times 4 = \dfrac{\left(3 \times 10^{-3}\right) \times di_A}{0.02}$

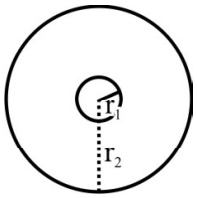
$\Rightarrow di_A = \dfrac{48 \times 10^{-5}}{3 \times 10^{-3}} = 10 \times 10^{-2}$

$= 0.16 \text{ A}$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Mutual Induction*

27. Consider two coplanar, co-centric circular coils having radii r_1 and r_2 ($r_2 > r_1$) as shown in the figure. The mutual inductance between these two coils will be



(1) $\frac{\mu_0 \pi r_1^2}{2r_2}$ (Correct)

(2) $\frac{\mu_0 \pi r_1}{2r_2}$ (Incorrect)

(3) $\frac{\mu_0 \pi r_2^2}{2r_2}$ (Incorrect)

(4) $\frac{\mu_0 \pi r_1^2}{2r_1}$ (Incorrect)

Correct answer: (1)

Solution:

We will consider the bigger coil as the primary coil. In the expression

$$\phi_S = \mathbf{B}_P . \mathbf{A}_S$$

we put the value of B at the centre of the bigger coil.

$$\Rightarrow \phi = \left(\frac{\mu_0 i_1}{2r_2} \right) \pi r_1^2$$

$$\Rightarrow \phi = \left(\frac{\mu_0 \pi r_1^2}{2r_2} \right) i_1$$

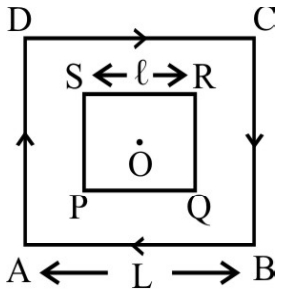
But $\phi = MI$

$$\therefore \quad M = \frac{\mu_0 \pi r_1^2}{2r_2}$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Difficult *Mutual Induction*

28. A small square loop of wire, of side ℓ, is placed inside a large square loop of wire of side L ($L >> \ell$). The loops are coplanar and their centres coincide. The mutual inductance of the system is proportional to



(1) $\frac{\ell}{L}$ (Incorrect)

(2) $\frac{\ell^2}{L}$ (Correct)

(3) $\frac{\ell}{L}$ (Incorrect)

(4) $\frac{L^2}{\ell}$ (Incorrect)

Correct answer: (2)

Solution:

The bigger loop may be taken as primary coil and the smaller one as the secondary coil. (We need to use the expressions for the magnetic field due to a finite wire). The bigger coil can be considered as equivalent to four current carrying long wires. All these wires, it can be seen produce magnetic fields in the same sense. Hence net magnetic field at the centre, will be

$$B_{net} = 4\frac{\mu_0 i}{4\pi R}\left(\sin 45^0 + \sin 45^0\right)$$

$$= \frac{2\sqrt{}\mu_0 i}{\pi L} \qquad \left(\therefore \text{ Here } R = \frac{L}{2}\right)$$

The flux, linked with the smaller loop, will be

$\phi_S = Mi_S$ .....(1)

Also $\phi_S = \sum\left(B_P\left(\Delta A_S\right)\right)$

We can assume the magnetic field to b constant over the whole area of the smaller loop.

or $\therefore \quad \phi_S = \dfrac{2\sqrt{2}\,\mu_0 i \times \ell^2}{\pi L}$ .....(2)

From (1) & (2), we get

$$M = \frac{2\sqrt{2}\,\mu_0}{\pi}\frac{\ell^2}{L}$$

$$M \propto \frac{\ell^2}{L}$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Easy *Self + Mutual Induction*

29. Two (insulated) coils are wound, (one over the other), on a solenoid core. If the number of turns, per unit length, of the first ciol and of the second coil, were to be increased by a factors of 3 and 4, respectively, the self inductance of the two coils (L_1 and L_2) and their mutual inductance (M), would increase (respectively) by factors of

(1) 3, 4 and 12 (Incorrect)

(2) 3, 4 and $\frac{4}{3}$ (Incorrect)

(3) 9, 16 and $\frac{16}{9}$ (Incorrect)

(4) 9, 16 and 12 (Correct)

Correct answer: (4)

Solution:

We know that the self inductance, of a solenoidal coil, is proportional to n^2 (n = number of turns per unit length).

Hence the self inductances, of the two coils will increase by factors of 3^2 and 4^2, i.e. 9 and 16.

The mutual inductance of the two solenoid coils (one over the other) is proportional to the product $n_1 n_2$ (n_1, n_2 are the number of turns per unit length in the two coils).

Hence M, in the given case, would increase by a factor of 3×4, i.e. (12).

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Energy Associated with an Inductor*

30. A spring, of spring constant $100\,\frac{N}{M}$, is compressed through a distance of 4cm. The potential energy, stored in this compressed spring, equals the work done, by the 'source', in establishing a current of 2A in a given coil.

The self inductance of the coil equals

(1) 20 mH (Incorrect)

(2) 40 mH (Correct)

(3) 60 mH (Incorrect)

(4) 80 mH (Incorrect)

Correct answer: (2)

Solution:

The P.E., of a compressed spring $= \frac{1}{2}kx^2$

(k = spring constant; x = distance through which the spring has been compressed).

The work, needed to be done, to establish a current I, in a coil of self inductance L, equals

$$w = \frac{1}{2}LI^2$$

We, therefore, have

$$\frac{1}{2}LI^2 = \frac{1}{2}kx^2$$

$$\therefore \quad L = \frac{kx^2}{I^2} = \frac{100\times\left(4\times10^{-2}\right)^2}{2^2}\,H$$

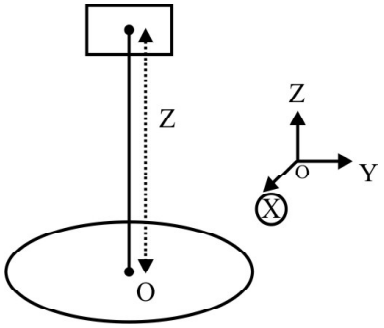
$$= \frac{16\times10^{-2}}{4}\,H = 40\times10^{-3}\,H = 40\,mH$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Induced emf*

31. A circular ciol, of average radius R, having N turns and carrying a current I, is set-up, horizontally with its plane in the *x*–y plane and its axis along the z–axis, so shown.

A small rectangular lamina of sides Δa and Δb, is set up with its centre on the z–axis and its plane parallel to the xy plane. If this lamina is flipped over through an angle π, in time Δt, the magnitude of the emf, momentairly induced, in it, would equal (nearly)



(1) zero (Incorrect)

(2) $\frac{(\Delta a)(\Delta b)}{\Delta t} N \frac{\mu_0 I}{(R)}$ (Incorrect)

(3) $\frac{(\Delta a)(\Delta b)}{\Delta t} N \frac{\mu_0 R^2 I}{\left(R^2+z^2\right)^{3/2}}$ (Correct)

(4) $\frac{(I)(\Delta a)(\Delta b)}{2\Delta t} N \frac{\mu_0 R^2}{\left(R^2+z^2\right)^{3/2}}$ (Incorrect)

Correct answer: (3)

Solution:

The magnetic field, due to the current carrying circular coil, at the centre of the rectangular lamina

$$B=\frac{N\mu o}{2}\frac{R^2 I}{\left(R^2+Z^2\right)^{3/2}}$$

The field can be assumed to have this (nearly) constant magnitude all over the plane of the small lamina. Hence the change in flux through it, when it is 'flipped over' is

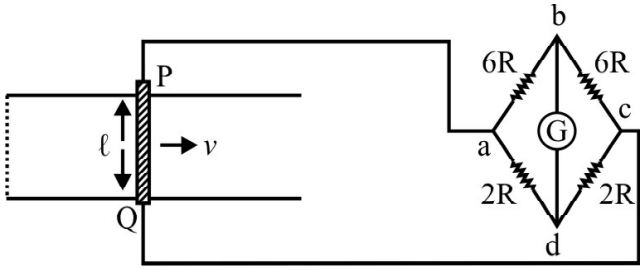
$$|\Delta\phi| = 2.(\Delta a \Delta b).N\frac{\mu_0}{2}\frac{R^2 I}{\left(R^2+z^2\right)^{3/2}}$$

$\therefore$ Magnitude of emf induced $= \dfrac{(\Delta a)(\Delta b)}{\Delta t} = \dfrac{N\mu_0 R^2 I}{\left(R^2+z^2\right)^{3/2}}$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Induced Current*

32. A rod, PQ, of length ℓ, slides with a constant velocity on two conducting rails. The system of the rod and conducting rails, is present in a region where a uniform (normal) magnetic field, of strength B, is present. The two ends of the rod PQ, (rod's resistance = R) are joined to the wheatstone bridge, abcd, shown here. If the current in the arms and and dc, of the wheatstone bridge, equals i_0, the velocity, v, of the sliding rod, equals



(1) $\dfrac{16Ri_0}{3B\ell}$ (Correct)

(2) $\dfrac{16Ri_0}{B\ell}$ (Incorrect)

(3) $\dfrac{B\ell}{(16Ri_0)}$ (Incorrect)

(4) $\dfrac{3B\ell}{(16Ri_0)}$ (Incorrect)

Correct answer: (1)

Solution:

The emf, induced in the rod, equal $B\ell v$ where, v, is the (constant) velocity of the sliding rod.

The wheatstone bridge, abcd, as shown is a balanced bridge. Its equivalent resistance, r, is given by

$$\frac{1}{r} = \frac{1}{(6R+6R)} + \frac{1}{(2R+2R)}$$

This gives r = 3R

The total resistance of the circuit is, therefore (R+3R) = 4R

$\therefore$ The induced current, in rod PQ, equals $\frac{B\ell v}{4R}$

In the wheatstone bridge, the part, i_0, going into the arms (ad and (dc), is $\left(\frac{1}{(4R+12R)}\times 12R\right)$, (i.e.) $\frac{3}{4}$ of this current.

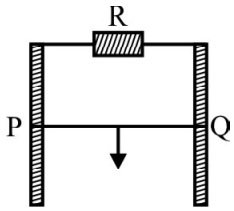
Hence $i_0 = \frac{3}{4}\times\frac{B\ell v}{4R}$

This given $v = \frac{16Ri_0}{3B\ell}$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Force due to Induced Current*

33. The set-up, shown in the figure, is present in a uniform magnetic field, B, directed perpendicular to the plane of this 'set-up'. The rod, PQ, of length L, is allowed to slide down vertically, under its own weight, on the (shaded) conducting rods. The terminal velocity, acquired by the sliding rod (of mass m), would equal



(1) $\left(\frac{m^2g^2R}{B\ell}\right)$ (Incorrect)

(2) $\left(\frac{B\ell}{m^2g^2R}\right)$ (Incorrect)

(3) $\left(\frac{B^2\ell^2}{mgR}\right)$ (Incorrect)

(4) $\left(\frac{mgR}{B^2\ell^2}\right)$ (Correct)

Correct answer: (4)

Solution:

As the rod PQ slides down, the magnitude of the induced emf, in it, at an instant when its velocity is v, equals $B\ell v$. Due to this an induced current $i\left(=\frac{B\ell v}{R}\right)$ flows in it. Because of this induced current, the sliding rod experiences an (mg). This (upward) force equals $Bi\ell = \frac{B^2\ell^2 v}{R}$. When this upward force becomes equal to the weight of the sliding rod, the net force on the rod becomes zero and the rod, subsequently, slides down with a constant velocity, v_T, its terminal velocity. We thus have

$$\frac{B^2\ell^2}{R} V_T = mg$$

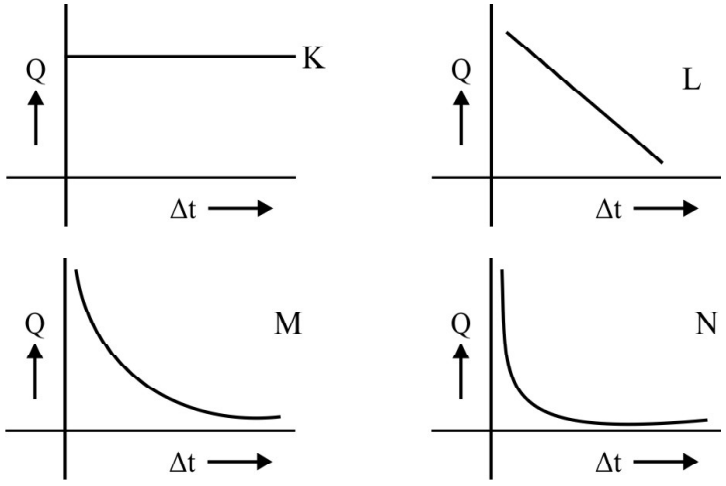
or $$v_T = \left(\frac{mgR}{B^2\ell^2}\right)$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Induced Charge Flow*

34. A small coil, having n turns each of (everage) radius r, is held normal to the magnetic field liens in the region between the (flat) pole pieces of a horse shoe magnet. The terminals ofthe coil are connected to a special galvanometer that can measure the total charge fleming in the circuit. The coil is taken out of the field region in a (small) time Δt and it is observed that a charge Q flows through the coil-galvanometer circuit.

The graph that best shows the dependence of the charge flowing (Q) on the time of withdrawl (Δt) of the coil (from the region of the magnetic field), is the graph labelled as graph.



(1) K (Correct)

(2) L (Incorrect)

(3) M (Incorrect)

(4) N (Incorrect)

Correct answer: (1)

Solution:

Let B be the strength of the magnetic field between the poles of the horse shoes magnet. The magnetic flux, linked with the small coil, when it is present in the field region, is

$$\phi = \left(B.\pi r^2\right)n$$

The linked magnetic flux becomes zero when it is withdrawn from the field. The instantaneous induced emf, would be

$$\varepsilon = -\frac{d\phi}{dt}$$

The instantaneous induced current (k) is $\frac{\varepsilon}{R}$ where R is the resistance of the coil-galvanometer circuit. The total charge flowing is given by

$$Q = \int_0^{\Delta t} idt = \frac{-\pi r^2 n}{R} \int_0^{\Delta t} \frac{dB}{dt} dt$$

$$= \frac{-\pi r^2 n}{R}\left[(B)_{\Delta t} - B(0)\right] = \frac{-\pi r^2 n}{R} B$$

Q, therefore, does not depend on Δt. This is shown in graph K.

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Self Inductance of a Solenoid*

35. A solenoid, having a total of N turns, of insulated wire, would uniformly on an iron core has a (long) length a and a radius r (r << L). The total flux, linked with its cross section, when a current I flows through it, equals ϕ. The relative permeability (μ_r), of iron and the self inductance, L, of the solenoid, expressed in terms, of its dimensions and ϕ, I and μ_0, would then equal (nearly) respectively

(1) $\left(\mu_r = \frac{\mu_0 N^2 . \pi r^2 I}{\phi a^2}\right)$ and $\left(L = \frac{\phi}{I}\right)$ (Incorrect)

(2) $\left(\mu_r = \frac{\phi a^2}{\mu_0 N^2 . \pi r^2 I}\right)$ and $\left(L = \frac{\phi}{I}\right)$ (Correct)

(3) $\mu_r = \left(\frac{\mu_0 . \phi a^2}{\pi r^2 N^2 . I}\right)$ and $\left(L = \frac{\phi}{NI}\right)$ (Incorrect)

(4) $\mu_r = \left(\frac{\mu_0 . \phi a^2 I N^2}{\pi r^2}\right)$ and $L = \left(\frac{N.\phi}{I}\right)$ (Incorrect)

Correct answer: (2)

Solution:

The magnetic field, B, along the axis of this (long) solenoid, equals

$$B = \mu_0 \mu_r \frac{N.I}{a}$$

The flux, ϕ, linked with its cross section, is given by

$$\phi \simeq B.\pi.r^2.\frac{N}{a}$$

We also have

$$\phi = LI$$

We thus get

$$\mu_r = \frac{\phi a^2}{\left(\mu_0 . N^2 . \pi r^2 I\right)}$$

and $L = \left[\mu_0 . \mu_r \frac{\pi r^2}{a^2} a N^2\right] = \left(\frac{\phi}{I}\right)$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Slef + Mutual Inductance*

36. A (long thin) solenoid core has an insulated primary coil, (of n_1 turns per unit length) and an insulated secondary coil, (of n_2 turns per unit length) wound on top of the primary coil. The cross sectional area of the solenoid, equals A.

If L_1 and L_2 are the self inductances of the two coils and M is their coefficient of mutual inductance, we would have (in the ideal case).

(1) $M = \dfrac{\left(L_1^3\right)}{\left(L_2^2\right)}$ (Incorrect)

(2) $M = \sqrt{L_1 L_2}$ (Correct)

(3) $M = \left(\dfrac{L_1^2}{L_2}\right)$ (Incorrect)

(4) $M = \left(\dfrac{L_2^2}{L_1}\right)$ (Incorrect)

Correct answer: (2)

Solution:

The self inductance, L_1 and L_2, of the two solenoid coils, are given by

$$L_1 = \mu n_1^2 A\ell$$

and $L_2 = \mu n_2^2 A\ell$

when μ is the magnetic permeability of the core material and ℓ is the length of the solenoid.

Also M, the coefficient of mutual industance, is going by

$$M = \mu n_1 n_2 A\ell$$

We, therefore, have $L_1 L_2 = M^2$ or $M = \sqrt{L_1 L_2}$

[Alternatively: We know that, in the dieal case, the coefficient of coupling between the two coils, equals unity. Now k is defined as $K = \dfrac{M}{\sqrt{L_1 L_2}}$ $\therefore$ when $k = 1$, we have $M = \sqrt{L_1 L_2}$]

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Mutual Inductance of a Pair of Coils*

37. Two circular loops, of radii R and r (R >> r), respectively, are positioned, parallel to each other, in the yz plane. The centres and axis, of both the loops, lie on the *x*–axis, a distance *x* apart. The larger loop has N turns, while the smaller one has n turns, all of the same average radius, in each case. The mutual inductance M, of this pair of loops, for *x* >> R, equals (nearby)

(1) $\frac{\mu_0\pi}{2x^3}\left(\frac{r^2R^2}{nN}\right)$ (Incorrect)

(2) $\frac{\mu_0\pi}{2x^2}\left(nNrR^2\right)$ (Incorrect)

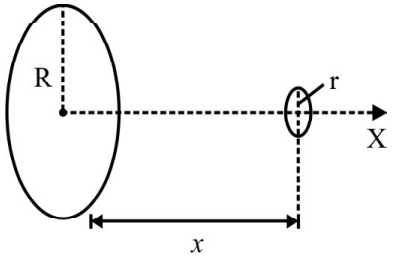
(3) $\frac{\mu_0\pi}{2x^2}\left(nN\right)\left(Rr^2\right)$ (Incorrect)

(4) $\frac{\mu_0\pi}{2x^3}\left(nr^2\right)\left(NR^2\right)$ (Correct)

Correct answer: (4)

Solution:

We imagine a current I to flow through the larger loop. The magnetic field, due to it, at the centre of the small loop, would then equal



$$B = N.\frac{\mu_0}{2}\frac{R^2}{\left(R^2+x^2\right)^{3/2}}I$$

The small coil has a very small radius. We may, therefore, assume that the magnetic flux, linked with it, is (nearly)

$$\phi \simeq B.\left(\pi r^2\right).n = \frac{N\mu_0}{2}\frac{R^2\left(\pi r^2\right).n}{\left(R^2+x^2\right)^{3/2}}I$$

But $\phi = M\,I$

Hence $M \simeq \frac{\mu_0\left(nN\right)}{2}\frac{\pi r^2R^2}{\left(R^2+x^2\right)^{3/2}}$

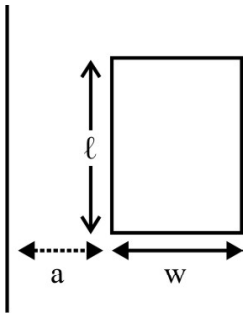
For $x >> R$, we can write

$$M' \simeq \mu_0 \left[\frac{nN.\pi r^2 R^2}{2x^3} \right]$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Difficult *Mutual Inductance*

38. A long straight wire and a planer rectangular loop are present in the vicinity of each other in the *xy* plane. The coefficient of mutual inductance, for this pair, would equal



(1) $\dfrac{\dfrac{\mu_0 \ell}{2\pi}}{\left[\ell n\left(1+\dfrac{\omega}{a}\right)\right]}$ (Incorrect)

(2) $\dfrac{\dfrac{\mu_0 \omega}{2\pi}}{\left[\ell n\left(1+\dfrac{\ell}{a}\right)\right]}$ (Incorrect)

(3) $\dfrac{\mu_0 \ell}{2\pi} \ell n\left(1+\dfrac{\omega}{a}\right)$ (Correct)

(4) $\dfrac{\mu_0 \omega}{2\pi} \ell n\left(1+\dfrac{\ell}{a}\right)$ (Incorrect)

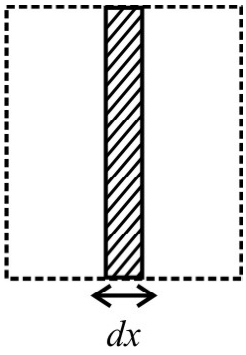
Correct answer: (3)

Solution:

We imagine a current I to flow through the long straight wire. The magnetic field, due to it at any point (in the xy plane), distant *x* from it, has a magnitude $\dfrac{\mu_0 I}{2\pi x}$ and it is directed perpendicular to the plane. Hence

the magnetic flux, linked with a small segmental strip, (of with dx), of the rectangular loop, would be

$$d\phi = (\ell dx).\frac{\mu_0 I}{2\pi x}$$



The total magnetic flux, linked with the rectangular loop, is

$$\phi = \frac{\mu_0 I\ell}{2\pi}\int_{x=a}^{x=a+\omega}\frac{dx}{x}$$

$$= \frac{\mu_0 I\ell}{2\pi}\ell n\left(1+\frac{\omega}{a}\right)$$

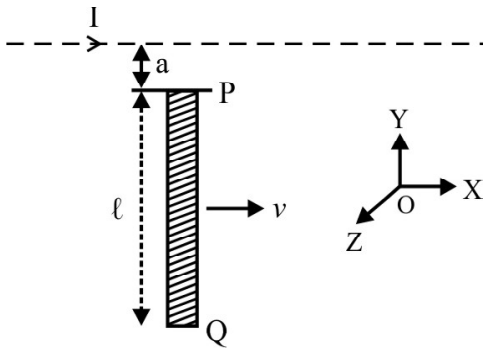
But $\phi = MI$

Hence $M = \frac{\mu_0\ell}{2\pi}\ell n\left(1+\frac{\omega}{a}\right)$

Unit–14 : Electromagnetic Induction and Alternating Currents

Difficult *Motional emf*

39. A (infinite) long straight wire, carrying a current I, is placed parallel to the x–axis. If a straight conductor, of length ℓ, positioned along the y–axis, as shown, starts moving with a uniform velocity $\mathbf{v} = v\hat{i}$, the magnitude of the emf, induced between its two ends, P and Q, would be



(1) zero (Incorrect)

(2) $\frac{2\mu_0 Iv\ell}{(2a+\ell)}$ (Incorrect)

(3) $\frac{\mu_0 Iv}{2\pi}\ell n\left(1+\frac{\ell}{a}\right)$ (Correct)

(4) $\dfrac{\mu_0 Iv}{2\pi}\dfrac{\ell}{\left(a+\dfrac{\ell}{2}\right)}$ (Incorrect)

Correct answer: (3)

Solution:

The magnetic field, due to the long straight current carrying wire, would have different magnitudes on the different elements of the straight conductor PQ. However its direction, at all these elements, is directed along $\left(-\hat{k}\right)$, as seen from the Biot-Savart rule. Hence the magnitude of the (motional) emf, induced in an element of length dy, situated at a distance below the straight wire, is

$$d\varepsilon = |By|(dy)v = \frac{\mu_0 I}{2\pi y}(dy)v$$

y varies from y = a to y = (a + ℓ). Hence the emf induced, between the ends P and Q, is

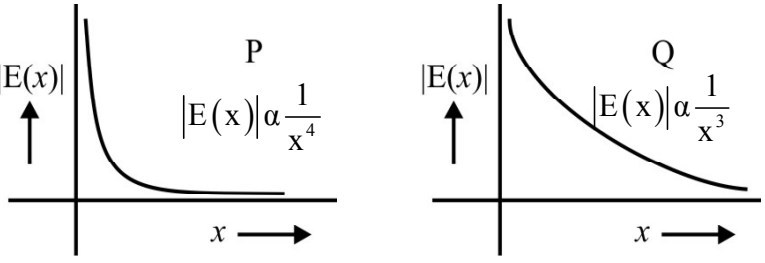
$$\varepsilon = \int_a^{a+\ell} \frac{\mu_0 Iv}{2\pi y}\,dy = \frac{\mu_0 Iv}{2\pi}\ell n\left(1+\frac{\ell}{a}\right)$$

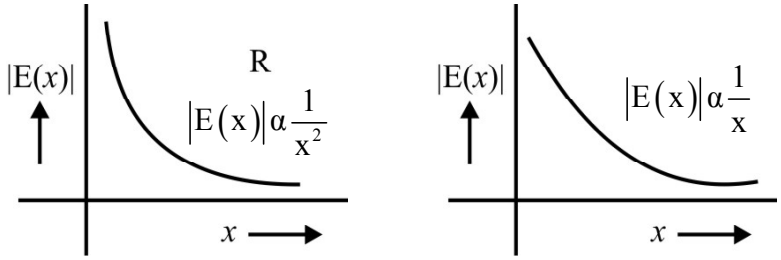
Unit–14 : Electromagnetic Induction and Alternating Currents

Difficult *Induced emf*

40. A circular coil, of radius R, is set up with its plane in the yz plane; its centre at the origin and its axis along the x–axis. It carries a constant current I.

A small rectangular lamina, of area ΔA, is set-up with its plane parallel to the yz plane and its 'centre' on the x–axis. It is made to move, with a uniform velocity $v\left(=v\hat{i}\right)$. The dependence of the instantaneous induced emf $(\varepsilon(x))$, in it (when its centre is to a distance *x* from the origin (*x* >>R), on *x*, is best represented by the graph, labelled as graph





(1) P (Correct)

(2) Q (Incorrect)

(3) R (Incorrect)

(4) S (Incorrect)

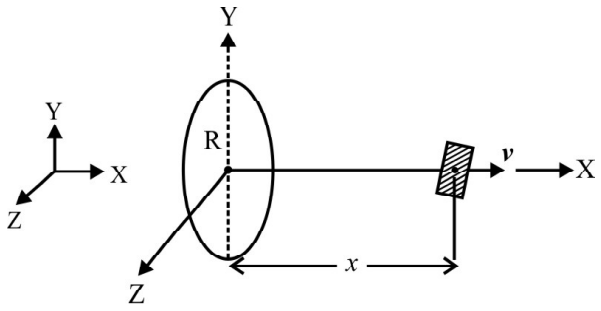
Correct answer: (1)

Solution:

The set-up, of the 'system', is as shown here.

The magnetic field, due to the current carrying circular coil, at the centre of the rectangular lamina (when this centre is at a distance x from the origin), is given by

$$B(x) = \frac{\mu_0}{2} \frac{R^2}{\left(R^2 + x^2\right)^{3/2}} I$$



The rectangular lamina, being a small sized one, the magnetic flux, linked with it, can be written as

$$\phi(x) = \Delta A B(x)$$

$$= \frac{\mu_0}{2} \frac{R^2 (\Delta A) I}{\left(R^2 + x^2\right)^{3/2}}$$

We are given that $x >> R$. Hence

$$\phi(x) \simeq \frac{\mu_0}{2} \frac{R^2 (\Delta A) I}{x^3}$$

The instantaneous induced emf, $\varepsilon(x)$, would equal

$$\varepsilon(x) = -\frac{d\phi(x)}{dt}$$

$$= -\frac{d\phi(x)}{dx}.\frac{dx}{dt}$$

$$= -v\frac{\mu_0}{2}\frac{R^2(\Delta A)I}{x^4}(-3)$$

$$= +\frac{3}{2}\frac{\mu_0 R^2(\Delta A)Iv}{x^4}$$

$$= (\text{const})\frac{1}{x^4}$$

$\therefore\ \varepsilon(x)$ varies as $\left(\frac{1}{x^4}\right)$. This corresponds to graph (P).

Unit–14 : Electromagnetic Induction and Alternating Currents

Difficult *Power Factor and Quality Factor in a Series LCR Circuit*

41. A voltage source, $v = v_0 \sin\omega t$, is connected across a series LCR circuit. The value of L and C, in the circuit, are equal to $\frac{nR}{\sqrt{3}\omega}$ and $\frac{\sqrt{3}}{(n-1)R\omega}$, respectively. The power factor and the quality factor, of the circuit, are equal, respectively, to

(1) $\frac{(n-1)}{(2n^2-2n+1)}$ and $\sqrt{\frac{3}{n(n-1)}}$ (Incorrect)

(2) $\left(\frac{\pi}{6}\right)$ and $\sqrt{\frac{n(n-1)}{3}}$ (Correct)

(3) $\left(\frac{\pi}{6}\right)$ and $\sqrt{\frac{3}{n(n-1)}}$ (Incorrect)

(4) $\frac{(n-1)}{(2n^2-2n+1)^{1/2}}$ and $\sqrt{\frac{n(n-1)}{3}}$ (Incorrect)

Correct answer: (2)

Solution:

The phase angle, Q, (current lagging), between the current flowing and the applied voltage, in a series LCR circuit, is given by

$$\tan\theta = \left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

In the given case, $\omega L = \frac{nR}{\sqrt{3}}$ and $\omega C = \frac{\sqrt{3}}{(n-1)R}$

$$\therefore \tan\theta = \frac{R}{\sqrt{3}}.\frac{1}{R} = \frac{1}{\sqrt{3}}$$

$$\therefore \theta = \left(\frac{\pi}{6}\right)$$

The quality factor, Q, of a series LCR circuit, equals $\frac{\omega_0 L}{R} = \frac{L}{R}\frac{1}{\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}$. In the given case

$$\frac{L}{C} = \frac{nR}{\sqrt{3}\omega}.\frac{(n-1)R\omega}{\sqrt{3}} = \frac{n(n-1)}{3}R^2$$

$$\therefore \quad Q = \frac{\sqrt{n(n-1)}}{3}$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Voltage Magnification in a Series LCR Circuit*

42. A series LCR circuit can bring about a 'voltage magnification', of the voltage of the a.c. source connected across it, provided the frequency, of the a.c. voltage source, equals

(1) $\frac{1}{\sqrt{LC}}$; the voltage magnification factor, then equals $\frac{1}{R}\sqrt{\frac{L}{C}}$ (Incorrect)

(2) $\frac{1}{\sqrt{LC}}$; the voltage magnification factor, then equals $\left(\frac{L}{C}\right)\frac{1}{R^2}$ (Incorrect)

(3) $\frac{1}{2\pi\sqrt{LC}}$; the voltage magnification factor, then equals $\frac{1}{R}\sqrt{\frac{L}{C}}$ (Correct)

(4) $\frac{1}{2\pi\sqrt{LC}}$; the voltage magnification factor, then equals $\left(\frac{L}{C}\right)\frac{1}{R^2}$ (Incorrect)

Correct answer: (3)

Solution:

The resonant frequency, for the series LCR circuit, equals $\frac{1}{2\pi\sqrt{LC}}$.

For a voltage source, having this frequency, the net impedance of the circuit equals R. Hence the source voltage equals the voltage drop V_R (= iR), across the resistor.

The total voltage drop, across the combination of L and C, is zero but the individual voltage drops, across L and C, are NOT zero.

At the resonant frequency, voltage drop across L (equal and opposite to the voltage drop across C) = $i\omega_0 L$

$$\therefore \quad \frac{V_L}{V_R} = \left(\frac{i\omega_0 L}{iR} = \frac{\omega_0 L}{R}\right) = \left(\frac{V_L}{\text{Voltage of source}}\right) = \text{Voltage magnification}$$

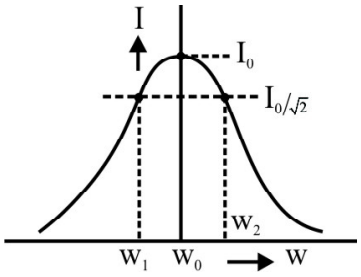
$$\therefore \quad \text{Voltage magnification factor} = \frac{\omega_0 L}{R}$$

$$= \frac{L}{R}.\frac{1}{\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Easy *Sharpness of Resonance*

43. In A.C.voltage source, $v = v_0 \sin \omega t$; having a fixed value for v_0, but of variable frequency, is connected across a given series LCR circuit. The variation of the current I, flowing in the circuit, with ω, (the angular frequency ofthe source) is as shown. The power consumption, in the circuit, at (angular) frequencies ω_1 and ω_2, is



(1) 50% of that at (angular) frequnecy ω_0 and the difference, $(\omega_2 - \omega_1)$, increases with an increase in the Q-factor of the circuit (Incorrect)

(2) 50% of that at (angular) frequnecy ω_0 and the difference, $(\omega_2 - \omega_1)$, decreases with an increase in the Q-factor of the circuit (Correct)

(3) (nearly) 71% of that at (angular) frequency ω_0 and the difference, $(\omega_2 - \omega_1)$, decreases with an increase in the 'Q-factor' of the circuit (Incorrect)

(4) (nearly) 71% of that at (angular) frequency ω_0 and the difference, $(\omega_2 - \omega_1)$, increases with an increase in the 'Q-factor' of the circuit (Incorrect)

Correct answer: (2)

Solution:

The current value, for ω_1 and ω_2, is $\frac{1}{\sqrt{2}}$ times the current value at ω_0. Hence the power consumption (αI^2) would become $\left(\frac{1}{2}\right)$ or 50% of that at ω_0.

Further, the more the value of Q, the sharper is the resonance of the circuit. This implies that there is a decrease, in the difference, $(\omega_2 - \omega_1)$ when the Q-further increases.

Unit–14 : Electromagnetic Induction and Alternating Currents

Difficult *Power Factor and Quality Factor of a Series LCR Circuit*

44. A voltage source, $v = v_0 \sin \omega t$; is connected across a series LCR circuit. The value of L and C, in the circuit, equal $\frac{5R}{\sqrt{3}\omega}$ and $\frac{\sqrt{3}}{4R\omega}$, respectively. The power factor and the quality factor, of the circuit, are equal, respectively, to

(1) $\frac{\sqrt{3}}{2}$ and $\sqrt{\frac{20}{2}}$ (Ccorrect)

(2) $\frac{1}{\sqrt{3}}$ and $\sqrt{\frac{14}{3}}$ (Incorrect)

(3) $\frac{1}{\sqrt{3}}$ and $\sqrt{\frac{14}{3}}$ (Incorrect)

(4) $\frac{\sqrt{3}}{2}$ and $\sqrt{\frac{20}{3}}$ (Incorrect)

Correct answer: (1)

Solution:

The power factor, of a series LCR circuit, equals $\cos\theta$ where

$$\tan\theta = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}$$

For the given circuit : $\omega L = \frac{5}{\sqrt{3}}R$ and $\omega C = \frac{3}{4R}$

$$\therefore \quad \tan\theta = \frac{\frac{1}{\sqrt{3}}R}{R} = \frac{1}{\sqrt{3}}$$

$$\therefore \quad \theta = \frac{\pi}{6}$$

Hence $\cos\theta = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

Also $Q = \frac{\omega_0 L}{R} = \frac{L}{R}.\frac{1}{\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}$

Here $\frac{L}{C} = \frac{5R}{\sqrt{3}\omega} \cdot \frac{4R\omega}{\sqrt{3}} = \left(\frac{20}{3}R^2\right)$

$\therefore \quad Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{R}.R.\sqrt{\frac{20}{3}} = \sqrt{\frac{20}{3}}$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Q-Factor*

45. For a given series LCR circuit, the term

$$\frac{1}{R}\sqrt{\frac{L}{C}}$$

(1) is a dimensionless term that provides a measure of (i) the quality factor (Q) and (ii) the power factor of the circuit at resonance (Incorrect)

(2) is a dimensionless term that provides a measure of (i) the sharpness of resonance (S) and (ii) the voltage magnification of the circuit at resonance (Incorrect)

(3) is a dimensional term that provides a measure of the (i) voltage magnification of the circuit at resonance and (ii) the sharpness of resonance (S) of the ciruit (Incorrect)

(4) is a dimensionless term that provides a measure of (i) the sharpness (S) of resonance the circuit and (ii) the quality factor (Q) of the circuit (Correct)

Correct answer: (4)

Solution:

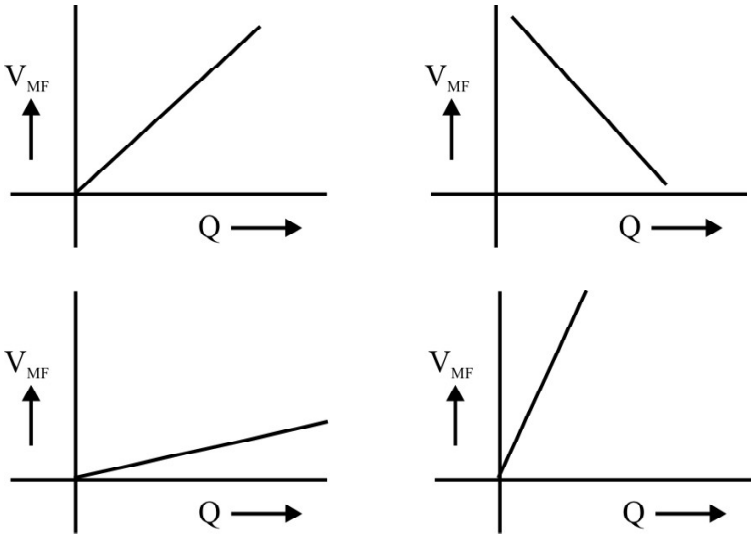
For a series LCR circuit, the sharpness of resonance (S), the quality factor (Q) and the voltage magnification, at resonance, are all measureing through the term $\frac{\omega_0 L}{R}$. The resonant frequency ω_0, being equal to $\frac{1}{LC}$, this term is equivalent to the term $\frac{1}{R}\sqrt{\frac{L}{C}}$.

This term is a dimensionless term (because $\omega_0 L$ as well as R both have the dimensions of impedance). (The power factor, of the circuit, at resonance, equals 1; it is not equal to this term).

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Q-Factor and Voltage Magnification Factor*

46. A series LCR circuit is connected across a given a.c. source, $v = v_0 \sin \omega t$, (where $\omega_0 = \frac{1}{\sqrt{LC}}$). Keeping L and C fixed, the value of R is varied. The graph, showing the relation between the Q-factor of the circuit, and its voltage magnification (V_{MF}), is the graph labelled as graph



(1) A (Correct)

(2) B (Incorrect)

(3) C (Incorrect)

(4) D (Incorrect)

Correct answer: (1)

Solution:

The Q-factor, for a series LCR circuit, equals $\frac{\omega_0 L}{R}\left(=\frac{1}{\omega_0 CR}\right)$

The voltage magnification factor, is,

$$V_{MF} = \frac{V_L\,(=V_C)}{V_R} = \frac{V_L\,(=V_C)}{V_{source}} = \frac{\omega_0 L}{R}\left(=\frac{1}{\omega_0 CR}\right)$$

Hence $V_{MF} = Q$

The graph, between the two, is, therefore, a straight line of slope = 1. The straight line graph should, therefore, be inclined equally (at 45^0) to either of the two axes. Of the four straight line graphs shown here, it is only graph A that has the straight line graph of slope equal to 1.

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Q-factor and Resonant Frequency*

47. A sinusoidal A.C. voltage source, of adjustable frequency, is connected across a series 'LCR' combination. The inductance (L) and the capacitance (C), present in the circuit, can be expresssed in terms of the quality factor (Q) the resonant frequency (ω_0) (of the circuit), and the resistance R, through the relations.

(1) $L = \left(\frac{1}{\omega_0 QR}\right)$ and $C = \left(\frac{QR}{\omega_0}\right)$ (Incorrect)

(2) $L = \frac{QR}{\omega_0}$ and $C = \frac{1}{\omega_0 QR}$ (Correct)

(3) $L = \frac{Q^2R^2}{\omega_0}$ and $C = \frac{1}{\omega_0 Q^2R^2}$ (Incorrect)

(4) $L = \frac{1}{\omega_0 Q^2R^2}$ and $C = \left(\frac{Q^2R^2}{\omega_0}\right)$ (Incorrect)

Correct answer: (2)

Solution:

The resonant frequency, ω_0, and the quality factor, Q, are related, to L and C, as

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \text{ and } \omega_0 = \frac{1}{\sqrt{LC}}$$

These give $L = Q^2R^2C$ and $L = \frac{1}{\omega_0^2 C}$

$$\therefore \quad C = \left(\frac{1}{\omega_0^2 QR}\right)$$

$$\therefore \quad L = \frac{1}{\omega_0^2 C} = \frac{1}{\omega_0^2} \cdot \frac{\omega_0 QR}{1} = \left(\frac{QR}{\omega_0}\right)$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Phase Angle and Quality Factor in a Series LCR Circuit*

48. A voltage source, $v = v_0 \sin \omega t$, is connected across a series LCR circuit. The values of L and C, in the circuit, equal $\frac{9R}{\omega}$ and $\frac{1}{10R\omega}$, respectively. The phase angle (between the current and voltage in the circuit) and the quality factor, Q, of the circuit, are then equal, respectively, to

(1) $\frac{\pi}{4}$ (Current lagging) and $2\sqrt{10}$ (Incorrect)

(2) $\frac{\pi}{4}$ (Current lagging) and $3\sqrt{10}$ (Incorrect)

(3) $\frac{\pi}{4}$ (Current lagging) and $3\sqrt{10}$ (Correct)

(4) $\frac{\pi}{4}$ (Current lagging) and $2\sqrt{10}$ (Incorrect)

Correct answer: (3)

Solution:

The phase angle, θ, (current lagging), between the current flowing and the voltage applied, in a series LCR circuit (connected to a A.C. voltage source) is given by

$$\tan\theta = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R} = \frac{9R - 10R}{R} = (-1)$$

$$\therefore \quad \theta = \left(-\frac{\pi}{4}\right) \text{ or } \left(\frac{3\pi}{4}\right)$$

The quality factor, Q, for a series LCR circuit, equals

$$\frac{\omega_0 L}{R} = \frac{L}{R} \cdot \frac{1}{\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Hence, for the given case,

$$Q = \frac{1}{R}\sqrt{90R^2} = 3\sqrt{10}$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Quality Factor Series LCR Circuits*

49. The following table gives the value of L, C and R, used in three different series LCR circuits.

Circuits No.		Value of	
	L	**C**	**R**
1.	***x* henry**	$\left(\frac{1}{x^3}\right)$ **farad**	$\left(x^3\right)$ **ohm**
2.	**3*x* henry**	$\left(\frac{27}{x^3}\right)$ **farad**	$\left(\frac{1}{3}x^3\right)$ **ohm**
3.	**5*x* henry**	$\left(\frac{125}{x^5}\right)$ **farad**	$\left(\frac{x^4}{5}\right)$ **ohm**

The quality factors, Q_1, Q_2, Q_3 of these circuits would be in the ratio

(1) $1 : x : x^2$ (Incorrect)

(2) $x : 1 : 1$ (Incorrect)

(3) $1 : x : x$ (Correct)

(4) $x^2 : x : 1$ (Incorrect)

Correct answer: (3)

Solution:

We know that $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

$$\therefore \quad Q_1 = \frac{1}{x^3}\sqrt{\frac{x}{\left(\frac{1}{x^3}\right)}} = \frac{1}{x^2}$$

$$Q_2 = \frac{3}{x^3}\sqrt{\frac{3x}{\left(\frac{27}{x^3}\right)}} = \frac{3}{x^2}\times\frac{1}{3}x^2 = \frac{1}{x}$$

$$Q_3 = \frac{5}{x^4}\sqrt{\frac{5x}{\left(\frac{125}{x^5}\right)}} = \frac{5}{x^4}\times\frac{1}{5}x^3 = \frac{1}{x}$$

$$\therefore \quad Q_1 : Q_2 : Q_3 = \frac{1}{x^2} : \frac{1}{x} : \frac{1}{x}$$

$$= 1 : x : x$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Easy *Quality Factor and Resonant Frequency*

50. The inductance, capacitance and resistance in a given series LCR circuit, have values of (n) henry, $\left(\frac{1}{n^3}\right)$ farad and (n^2) ohm. The quality factor, Q, and the resonant frequency (ν_0), of this circuit, equal, respectively,

(1) $Q = \frac{1}{2};\ \nu_0 = \frac{n}{4\pi}$ (Incorrect)

(2) $Q = 1;\ \nu_0 = \left(\frac{n}{2\pi}\right)$ (Correct)

(3) $Q = \frac{4\pi^2}{n^2};\ \nu_0 = \left(\frac{2\pi}{n}\right)$ (Incorrect)

(4) $Q = \frac{8\pi^2}{n^2};\ \nu_0 = \left(\frac{4\pi}{n}\right)$ (Incorrect)

Correct answer: (2)

Solution:

The quality factor, Q, of a series LCR circuit, equals

$$\frac{1}{R}\sqrt{\frac{L}{C}}$$

$\therefore$ In the given case,

$$Q = \frac{1}{n^2}\sqrt{\frac{n}{\left(\frac{1}{n^3}\right)}} = 1$$

Also the resonant frequency, ν_0, is given by

$$\nu_0 = \frac{1}{2\pi}\cdot\frac{1}{\sqrt{LC}}$$

$$\therefore\ \nu_0 = \frac{1}{2\pi}\cdot\frac{1}{\left(n\frac{1}{n^3}\right)^{1/2}} = \left(\frac{n}{2\pi}\right)$$

[**Note:** We can check the consistency of these results by noting that $Q = \frac{\omega_0 L}{R}$

$\therefore$ In the case, $Q = \frac{\left(2\pi\frac{n}{2\pi}n\right)}{n^2} = 1,$ as obtained above].

Unit–14 : Electromagnetic Induction and Alternating Currents

Easy *Sharpness of Resonance of a Series LCR Circuit*

51. It is required to improve the sharpness of resonance of a series LCR circuit by a factor of 4. This requirement can be met by

(1) increasing the resistance to 4 times its present value; leaving L and C values unchanged (Incorrect)

(2) increasing the inductance to eight times to present value; decreasing the capacitance by a factor of 2; and leaving R unchanged (Correct)

(3) increasing the inductance by a factor of 4, leaving C and R unchanged (Incorrect)

(4) decreasing the capacitance by a factor or 4; leaving L and R unchanged (Incorrect)

Correct answer: (2)

Solution:

The sharpness of resonance, of a series LCR circuit, improves by a factor of 4 when its quality factor goes up by a factor of 4. Now

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

It is easy to see that it is only the changes, suggested in option (2), that increase the value of the Q-factor by a factor of 4.

Hence we can improve the sharpness of resonance by a factor of (4) by following the changes suggested in option (2).

[**Note:** The sharpness of resonance improves by a factor of 4 when for the same value of the resonant frequency, the band width, $(\omega_2 - \omega_1)$, goes down by a factor of 4. The ratio $\left(\frac{\omega_2 - \omega_1}{\omega_0}\right)$ equals $\frac{1}{Q}$].

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *LCR Circuit*

52. A series LC circuit is connected across an a.c. voltage source : $(v = v_0 \sin \omega t)$, **for which** $\omega \neq \frac{1}{\sqrt{LC}}$ **and** $\omega L > \frac{1}{\omega C}$. **The instantneous current and the instantaneous power dissipated, in the circuit, are then equal respectively, to**

(1) ∞ and 0 (Incorrect)

(2) $\left[\frac{v_0}{\left(\omega L - \frac{1}{\omega C}\right)}\right] \sin\left(\omega t - \frac{\pi}{2}\right)$ and $\frac{-v_0^2}{\left(\omega L - \frac{1}{\omega C}\right)}(\sin \omega t \cos \omega t)$ (Correct)

(3) $\left[\frac{v_0}{\left(\omega L-\frac{1}{\omega C}\right)}\right]\sin\left(\omega t+\frac{\pi}{2}\right)$ and $\frac{+v_0^2}{\left(\omega L-\frac{1}{\omega C}\right)}(\sin\omega t\ \cos\omega t)$ (Incorrect)

(4) $\left[\frac{v_0}{\left(\omega L-\frac{1}{\omega C}\right)}\right]\sin\left(\omega t+\frac{\pi}{2}\right)$ and zero (Incorrect)

Correct answer: (2)

Solution:

Since $\omega \neq \frac{1}{\sqrt{LC}}$, the net impedance of the circuit is not zero. Also since $\omega L \neq \frac{1}{\omega C}$, the net impedance of the circuit, is $\left(\omega L-\frac{1}{\omega C}\right)$. The inductive impedance being greater the current, in the circuit LAGS in phase with respect to the voltage. Hence the instantaneous current, is the circuit, is given by

$$I=\frac{v_0}{\left(\omega L-\frac{1}{\omega C}\right)}\sin\left(\omega t-\frac{\pi}{2}\right)=\frac{-v_0\cos\omega t}{\left(\omega L-\frac{1}{\omega C}\right)}$$

$\therefore$ The instantaneous power, dissipated in the circuit, is

$$P_{inst}=VI=\frac{-v_0^2}{\left(\omega L-\frac{1}{\omega C}\right)}\sin\omega t\cos\omega t$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Easy *Behaviour of L and C in A.C. Circuits*

53. A student writes the following statements about the 'inductors' and the 'capacitors'.

(a) An inductor behaves almost like an 'open' for very low frequency a.c. sources as well as d.c. sources.

(b) A capacitor behaves like an 'open' for very low frequency a.c. sources as well as d.c. sources.

(c) For very high frequency a.c. sources, a capacitor behaves as an 'open' while an inductor behaves like a 'short'.

(d) For very law frequency a.c. sources (as well as d.c. sources), an inductor behaves like an 'open' while a capacitor behaves like a short.

(1) a (Incorrect)

(2) b (Correct)

(3) c (Incorrect)

(4) d (Incorrect)

Correct answer: (2)

Solution:

An 'open' implies an (almost) infinite impedance while a 'short' implies an impedance tending to zero. Now

(i) Impedance of inductor = ωL

$\therefore$ The inductor is (almost) a 'short' for very low frequency a.c. sources as well as d.c. sources. It behaves like an 'open' for very high frequency a.c. sources.

(ii) Impedance of capacitor = $\frac{1}{\omega C}$

Hence a capacitor behaves in a way opposite to the behaviour of an inductor.

Hence statement (B) is the only 'logically valid' statement, out of the four given statement.

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *LC Circuits*

54. A circuit is made up of an (almost) pure inductor and an (almost) pure capacitor, connected together. The circuit (i) has a resonance frequency ν_0 when it is put in an evacuated container and (ii) has a resonance frequency ν_0' when the container contains a non-reacting gas. The dielectric constant of the gas equals.

(1) $\frac{\left(\nu_0'\right)^2}{\left(\nu_0^2\right)}$ (Incorrect)

(2) $\frac{(\nu_0)^2}{(\nu_0')^2}$ (Correct)

(3) $\left[\frac{(\nu_0')^2}{(\nu_0^2)}-1\right]$ (Incorrect)

(4) $\left(\frac{\nu_0^2}{\nu_0'^2}-1\right)$ (Incorrect)

Correct answer: (2)

Solution:

Let L be the value of the inductance and C that of the capacitor. We then have

$$\nu_0 = \frac{1}{2\pi\sqrt{LC}} \text{ or } LC = \frac{1}{4\pi^2\nu_0^2}$$

In the presence of the gas, of dielectric constant, say, K, the capacitance changes to KC. Hence

$$\nu_0' = \frac{1}{2\pi\sqrt{L.KC}} \text{ or } KLC = \frac{1}{4\pi^2\nu_0^2}$$

$$\therefore \quad K = \frac{\nu_0^2}{(\nu_0')^2}$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Series LCR Circuit*

55. When a given (fixed frequency, fixed voltage) a.c. source is connected across a series LCR circuit, the current in the circuit has its maximum value when

(1) $C = \frac{1}{\omega^2 L}$; it decreases in a (nearly) symmetrical way when $C > \frac{1}{\omega^2 L}$ or $C < \frac{1}{\omega^2 L}$ (Correct)

(2) $C = \frac{1}{\omega^2 L}$; it decreases in a non-symmetrical way when $C > \frac{1}{\omega^2 L}$ and when $C < \frac{1}{\omega^2 L}$ (Incorrect)

(3) $C = \frac{\omega^2}{L}$; it decreases in a (nearly) symmetrical way when $C > \frac{\omega^2}{L}$ or $C < \frac{\omega^2}{L}$ (Incorrect)

(4) $C = \frac{\omega^2}{L}$; it decreases in a non-symmetrical way when $C > \frac{\omega^2}{L}$ and when $C < \frac{\omega^2}{L}$ (Incorrect)

Correct answer: (1)

Solution:

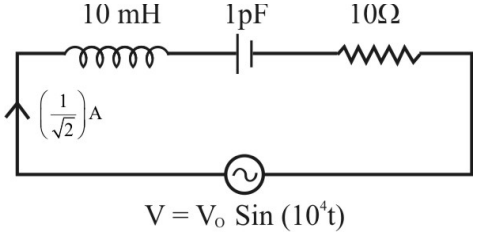
The impedance of a series LCR circuit $\left(z = \left[R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2 \right]^{1/2} \right)$ has its minimum value (= R) when $\omega L = \frac{1}{\omega C}$ or $C = \frac{1}{\omega^2 L}$. The current in the circuit is, therefore, maximum for this value of C.

Because of the presence of the squared term $\left[\left(\omega L - \frac{1}{\omega C} \right)^2 \right]$ in Z, we can expect a (nearly) identical increase in Z when C increases / decreases, around the value $\left(\frac{\omega^2}{L} \right)$, by equal amounts. Hence the decrease in current, in either case, is likely to take palce in a (nearly) symmetrical way.

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *LCR Circuit*

56. For the LCR series circuit shown here, the peak value of the voltage of the source, the phase difference between the applied voltage and the current flowing, and the power factor of the circuit, equal, respectively.



(1) $\sqrt{175}$ volt; $+\frac{\pi}{3}; \frac{1}{2}$ (Incorrect)

(2) $\sqrt{175}$ volt; $-\frac{\pi}{3}$; $\frac{1}{2}$ (Incorrect)

(3) 10 volt; 0^0; 1 (Correct)

(4) $\sqrt{200}$ volt; $\frac{\pi}{4}$; $\frac{1}{\sqrt{2}}$ (Incorrect)

Correct answer: (3)

Solution:

The impedance, z, of the circuit is given by

$$z = \left[R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2 \right]^{1/2}$$

Here $R = 10\,\Omega$, $\omega = \left(10^4\right)^{-1}$ s, $L = 10$ mH, $C = 1$ pF

$\therefore \quad \omega L = 10^4 \times 10 \times 10^{-3}\,\Omega = 10^2\,\Omega$

and $\frac{1}{\omega L} = \frac{1}{10^4 \times 10^{-6}}\,\Omega = 10^2\,\Omega$

$\therefore \quad \omega L - \frac{1}{\omega C} = 0$

$\therefore \quad z = R = 10\,\Omega$

$\therefore$ Peak value of voltage $= \left(i_{rms} \times \sqrt{2}\right) \times (z)$ volt

$$= \left(\frac{1}{\sqrt{2}} \times \sqrt{2} \times 10 \right) \text{ volt} = 10 \text{ volt}$$

Phase difference between current and voltage

$$\theta = \tan^{-1} \left(\frac{\omega L - \frac{1}{\omega C}}{R} \right) = \tan^{-1} 0 = 0^0$$

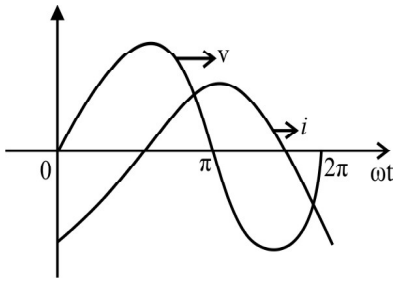
$\therefore \quad$ Power factor $= \cos\theta = \cos 0^0 = 1$

Unit–14 : Electromagnetic Induction and Alternating Currents

Difficult *Adjusting the Value of C in a given LCR Circuit*

57. The graph drawn here, shows, simultaneously the variation of the (alternating) voltage (applied) and the current (flowing) for a given series LCR circuit.

Without making any other change, the power factor of this circuit, can be made unity if another capacitor, C", of value



(1) $\frac{C}{\left(\omega^2 LC-1\right)}$ is put in parallel with a given C (Incorrect)

(2) $\frac{C}{\left(\omega^2 LC-1\right)}$ is put in series with a given C (Correct)

(3) $\frac{\left(\omega^2 LC-1\right)}{C}$ is put in series with a given C (Incorrect)

(4) $\frac{\left(\omega^2 LC-1\right)}{C}$ is put in parallel with a given C (Incorrect)

Correct answer: (2)

Solution:

The given graph shows the current lagging in phase with respect to the (applied) voltage. The power factor of the circuit becomes unity if the circuit is in its resonance state. This requires the capacitor to have a value, C', such that

$$\frac{1}{\omega C'} = \omega L, \text{ i.e. } C' = \frac{1}{\omega^2 L}$$

We need to increase the capacitative impedance to make it equal to ωL. This means that the capacitance needs to be decreases. We, therefore, need to put another capacitor, C", in SERIES with the given capacitor (C) such that

$$\frac{1}{C}+\frac{1}{C''}=\frac{1}{C'}=\omega^2 L$$

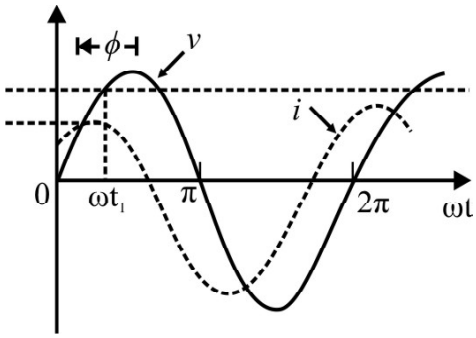
This gives $\frac{1}{C''}=\frac{\left(\omega^2 LC-1\right)}{C}$

$\therefore \quad C''=\frac{C}{\left(\omega^2 LC-1\right)}$

Unit–14 : Electromagnetic Induction and Alternating Currents

Difficult *Adjusting the Value of L in a given LCR Circuit*

58. A given AC source, $v = v_0 \sin \omega t$, is connected across a given series LCR circuit. The graph, showing simultaneously, the variation of the voltage and current in the circuit, with time, has the form shown. The current, flowing in this circuit, has to be made to have its maximum value $\left(=\frac{V}{R}\right)$. This aim can be achieved if, without making any other change in the circuit, we join another inductor (L''), of value



(1) $\left(\frac{1-\omega^2 LC}{\omega^2 C}\right)$ in series with the given inductor (L) (Correct)

(2) $\left(\frac{1-\omega^2 LC}{\omega^2 C}\right)$ in parallel with the given inductor (L) (Incorrect)

(3) $\left(\frac{\omega^2 C}{1-\omega^2 LC}\right)$ in series with the given inductor (L) (Incorrect)

(4) $\left(\frac{\omega^2 C}{1-\omega^2 LC}\right)$ in parallel with the given inductor (L) (Incorrect)

Correct answer: (1)

Solution:

The given graph shows the current leading in phase with respect to the applied voltage. The given condition (current has its maximum value $\left(=\frac{V}{K}\right)$) is met when the circuit is in its resonance state. For this, the net inductance, has to have a value, L', such that $\omega L' = \frac{1}{\omega C}$. This requires $L' = \frac{1}{\omega^2 C}$.

Since the current is leading in phase, we need to increase, the value of the inductance. We, therefore, need to add another inductor (of value L") in series with the given inductor, such that

$$L + L'' = L' = \frac{1}{\omega^2 C}$$

$$\therefore\ L'' = \left[\frac{1}{\omega^2 C} - L\right] = \left(\frac{1-\omega^2 LC}{\omega^2 C}\right)$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Changes in LCR Circuit*

59. A series LCR circuit is connected to an a.c. voltage source $(v = v_0 \sin \omega t)$ and the value of C is such that $\frac{1}{\omega C} \simeq 0$. The phase angle (θ), between the current flowing and the voltage applied, then equals.

(1) $\tan^{-1}\left(\frac{\omega L}{R}\right)$; current leading; θ can be made zero by taking a value of C equal to $\left(\frac{1}{\omega^2 L}\right)$

(Incorrect)

(2) $\tan^{-1}\left(\frac{1}{\omega LR}\right)$; current lagging; θ can be made zero by taking a value of C equal to $\left(\frac{1}{\omega^2 L}\right)$

(Incorrect)

(3) $\tan^{-1}\left(\frac{1}{\omega LR}\right)$; current leading; θ can be made zero by taking a value of C equal to $\left(\frac{1}{\omega^2 L}\right)$

(Incorrect)

(4) $\tan^{-1}\left(\frac{\omega L}{R}\right)$; current lagging; θ can be made zero by taking a value of C equal to $\left(\frac{1}{\omega^2 L}\right)$ (Correct)

Correct answer: (4)

Solution:

The phase angle (correct lagging) between the current flowing, and the voltage applied, in a series LCR circuit is given by

$$\theta = \tan^{-1}\frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}$$

Here $\frac{1}{\omega C} = 0.$

Hence $\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$

We can make $\theta = 0$ by taking $\frac{1}{\omega C} = \omega L$, i.e., $C = \frac{1}{\omega^2 L}$.

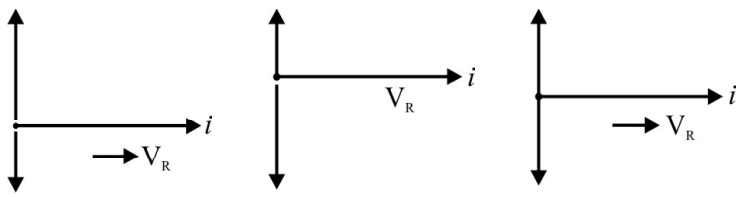
The required value of C is, therefore,

$$C = \left(\frac{1}{\omega^2 L}\right)$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Sereis LCR Circuits*

60. A series LCR circuit is connected across a given a.c. voltage source for which $v = v_0 \sin \omega t$. The threee phasor diagrams, for this circuit (having a total impedance z), correspond, respectively, to the cases



(1) $\left(L > \frac{1}{\omega^2 C}\right); \left(L = \frac{1}{\omega^2 C}\right)$ and $\left(L < \frac{1}{\omega^2 C}\right)$ (Correct)

(2) $\left(L < \frac{1}{\omega^2 C}\right); \left(L = \frac{1}{\omega^2 C}\right)$ and $\left(L > \frac{1}{\omega^2 C}\right)$ (Incorrect)

(3) $\left(L < \frac{1}{\omega^2 C}\right); (z = R)$ and $\left(L > \frac{1}{\omega^2 C}\right)$ (Incorrect)

(4) $\left(L < \frac{1}{\omega^2 C}\right); \left(L = \frac{1}{\omega^2 C}\right)$ and $(z = R)$ (Incorrect)

Correct answer: (1)

Solution:

For the series LCR circuit, (since i has the same value for all three elements), implies that $V_L > V_C$ is equivalent to saying $X_L > X_C$. In the given diagram, the upward phasor corresponds to V_L while the downward phasor corresponds to V_C. Thus diagram (i), (ii), (iii), correspond to $(X_L > X_C)$; $(X_L = X_C)$ and $(X_C < X_L)$, respectively. Since $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$, we have $X_L > X_C$ when $L > \frac{1}{\omega^2 C}$ and vice-crsa. Further z = R is true only when $X_L = X_C$. It is only in option (1), that the given information is in agreement with these requirements (for all the three bits of given information).

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Series LCR Circuit*

61. The graphs, (A) and (B), drawn here, show, respectively, the phase relationship between the voltage and current, in a series LCR circuit (connectd to an a.c. source) when

(1) (A) R and L are both shorted and (B) R and C are both shorted (Incorrect)

(2) (A) R and C are both shorted and (B) R and L are both shorted (Incorrect)

(3) (A) C alone is shorted (B) L alone is shorted (Incorrect)

(4) (A) alone is shorted (B) C alone is shorted (Correct)

Correct answer: (4)

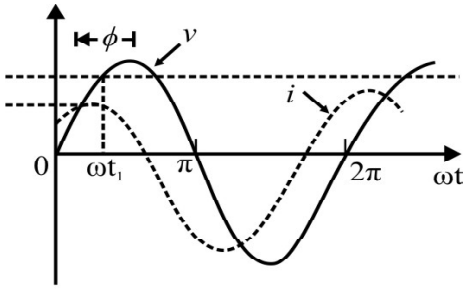
Solution:

For a series LCR circuit, the (phase) angle θ, by which the voltage (V) leads the current (I), is given by

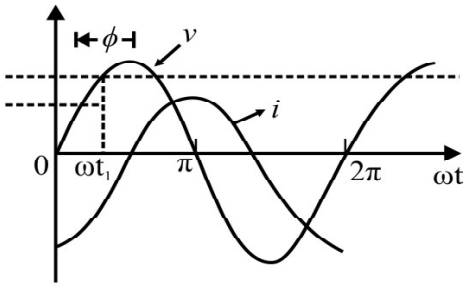
$$\tan\theta = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}$$

(i) When L alone is shorted, θ becomes a negative angle, i.e. the voltage lags behind the voltage in phase. This corresponds to graph A.

(ii) When C alone is shorted, θ becomes a positive angle; i.e. the voltage leads the current in phase. This corresponds to graph (B).



(A)



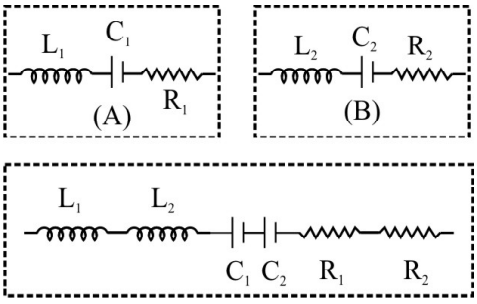
(B)

[Option (1) and (2) are incorrect becuase when R and $\frac{L}{C}$ are both shorted, the circuit becomes a pure $\frac{C}{L}$ circuit. For both these cases, θ would be $\mp\frac{\pi}{2}$; this is not so in the given graph].

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Resonant Frequency of a Series Combination of Two LCR Circuits*

62. The resonance frequency for each of the two individual (LCR) series circuits, [(A) and (B)], shown here, has the same value, say ω_0. For the series LCR circuit, obtained by having the 'series combination' shown, of the circuits (A) and (B), the resonant frequency will be



(1) ω_0 (Correct)

(2) ω_0^2 (Incorrect)

(3) $2\omega_0$ (Incorrect)

(4) $\frac{\omega_0}{2}$ (Incorrect)

Correct answer: (1)

Solution:

We are given that

$$\frac{1}{\sqrt{L_1C_1}} = \omega_0 \text{ and } \frac{1}{\sqrt{L_2C_2}} = \omega_0$$

For the 'combined circuit', the equivalent inductance (L), and the equivalent capacitance (C), have the values given by

$$L = L_1 + L_2 \text{ and } \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \qquad \left(\therefore \quad C = \frac{C_1C_2}{C_1 + C_2}\right)$$

The resonant frequency, say ω', of the 'combined circiuit' is given by

$$\omega' = \frac{1}{\sqrt{LC}}$$

Now $LC = \dfrac{(L_1 + L_2)(C_1C_2)}{C_1 + C_2} = \dfrac{(L_1C_1)C_2 + (L_2C_2)C_1}{(C_1 + C_2)}$

$$= \frac{\frac{1}{\omega_0^2}[C_2 + C_1]}{C_1 + C_2} = \frac{1}{\omega_0^2}$$

$\therefore \quad \omega' = \sqrt{\omega_0^2} = \omega_0$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Phase Angle and Power Factor in a Series LCR Circuit*

63. An A.C. voltage source $(v = v_0 \sin \omega t)$, is connected across a series LCR Circuit. If the values of L and C, in the circuit, equal $\left(\dfrac{2\sqrt{3}\,R}{\omega}\right)$ and $\left(\dfrac{1}{\sqrt{3}(R\omega)}\right)$, respectively; the phase angle (between the current and voltage) and the power factor of the circuit, are equal, respectively, to

(1) $\left(\frac{\pi}{6}\right)$ and $\frac{\sqrt{3}}{2}$ (Incorrect)

(2) $\left(\frac{\pi}{3}\right)$ and $\left(\frac{1}{2}\right)$ (Correct)

(3) $\left(\frac{\pi}{4}\right)$ and $\left(\frac{1}{\sqrt{2}}\right)$ (Incorrect)

(4) $\left(\frac{\pi}{3}\right)$ and $\left(\frac{\sqrt{3}}{2}\right)$ (Incorrect)

Correct answer: (2)

Solution:

The phase angle, θ, between the current and voltage, in a series LCR circuit, is given by

$$\tan\theta = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$\therefore$ In the given case:

$$\tan\theta = \frac{1}{2}\left[\frac{\omega.2\sqrt{3}R}{\omega} - \frac{1}{\omega.\left(\frac{1}{\sqrt{3}R\omega}\right)}\right]$$

$$= \frac{1}{2}\left[2\sqrt{3}R - \sqrt{3}R\right] = \sqrt{3}$$

$\therefore \quad \theta = \frac{\pi}{3}$

$\therefore$ Power factor $= \cos\theta = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Induced emf*

64. A series LCR, connected to an a.c.voltage source $(v = v_0 \sin \omega t)$, has L = 0. The phase angle, θ, between the current flowing, and the voltage applied, then equals

(1) $\tan^{-1}\left(\frac{1}{\omega CR}\right)$; current leading; θ can be made zero by taking a value of L equal to $\left(\frac{1}{\omega^2 C}\right)$

(Correct)

(2) $\tan^{-1}\left(\frac{1}{\omega CR}\right)$; current lagging; θ can be made zero by taking a value of L equal to $\left(\omega^2 C\right)$

(Incorrect)

(3) $\tan^{-1}\left(\frac{\omega C}{R}\right)$; current leading; θ can be made zero by taking a value of L equal to $\left(\omega^2 C\right)$

(Incorrect)

(4) $\tan^{-1}\left(\frac{\omega C}{R}\right)$; current lagging; θ can be made zero by taking a value of L equal to $\left(\frac{1}{\omega^2 C}\right)$

(Incorrect)

Correct answer: (1)

Solution:

The phase angle (current lagging), θ, between the current flowing andthe voltage applied, in a series LCR circuit (connected to an a.c. voltage source) is given by

$$\theta = \tan^{-1}\left[\frac{\left(\omega L = \frac{1}{\omega C}\right)}{R}\right]$$

Here $L = 0$. Hence $\theta = \tan^{-1}\left(\frac{1}{\omega CR}\right)$; The (–ve) sign implies that the current now leads the voltage.

We can make $\theta = 0$ if we select a value of L for which $\omega L = \frac{1}{\omega C}$. The required value of L is, therefore, $\left(L = \frac{1}{\omega^2 C}\right)$.

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *LCR Circuit*

65. A given series LCR circuits is connected across a simusoidal AC voltage source.

$$v = v_0 \sin(\omega t)$$

The inductance, L, in the circuit, has a value equal to $\frac{1}{\omega^2 C}$. The rms voltage drop, across L, C, and R, of this circuit, are equal, respectively, to

(1) $\frac{v_0}{\sqrt{2}R}\cdot\frac{1}{\omega C}; \frac{v_0}{R}\cdot\frac{1}{\omega C}$ and v_0 (Incorrect)

(2) $\frac{v_0}{R}\cdot\frac{1}{\omega C}; \frac{v_0}{R}.\omega L$ and v_0 (Incorrect)

(3) 0, 0 and $\frac{v_0}{\sqrt{2}}$ (Incorrect)

(4) $\frac{v_0}{(\sqrt{2})R}\cdot\frac{1}{\omega C}; \frac{v_0}{(\sqrt{2})R}\cdot\frac{1}{\omega C}; \frac{v_0}{\sqrt{2}}$ (Correct)

Correct answer: (4)

Solution:

We are given that $L = \frac{1}{\omega^2 C}$. This implies that $\omega = \frac{1}{\sqrt{LC}}$, i.e. the given circuit is in its resonance state.

At resonance, net impedance of the circuit equals R. Hence, the rms value of the current in the circuit, is $\frac{v_0}{\sqrt{2}.R}$. The impedance of the inductance and capacitance are both equal to $\frac{1}{\omega C}(\text{or } \omega L)$. Hence the rms voltage drop, across the inductor and the capacitor, are each equal to $\frac{v_0}{\sqrt{2}R}\cdot\frac{1}{\omega C}$. The total of these two voltage drop, at resonance, is zero. The rms voltage drop, across the resistor, equals the rms value of the supply voltage, i.e. $\frac{v_0}{\sqrt{2}}$.

Unit–14 : Electromagnetic Induction and Alternating Currents

Difficult *Adjusting the Value of L in a given LCR Circuit*

66. A given series LCR circuit has an alternating voltage $\left(v = v_0 \sin \omega t\right)$, applied across it. The current, flowing in the circuit, is observed to lag in phase with respect to the applied voltage.

Without making any other change, the value of the (net) inductance, present in the circuit, is to be adjusted such that the voltage of the source, and the voltage drop across the resistor in the circuit, have identical values. For this, we need to put another inductor, L''. of value

(1) $\dfrac{\left(\omega^2 LC - 1\right)}{\left(L\right)}$ in parallel with the given inductor (L), already present in the circuit (Incorrect)

(2) $\dfrac{\left(\omega^2 LC - 1\right)}{\left(L\right)}$ in series with the given inductor (L), already present in the circuit (Incorrect)

(3) $\dfrac{\left(L\right)}{\left(\omega^2 LC - 1\right)}$ in parallel with the given inductor (L), already present in the circuit (Correct)

(4) $\dfrac{\left(L\right)}{\left(\omega^2 LC - 1\right)}$ in series with the given inductor (L), already present in the circuit (Incorrect)

Correct answer: (3)

Solution:

The voltage of the source, and the voltage drop across the resistor, have identical values only when the circuit is in its resonance state. Hence the net inductance (L'), in the circuit must be such that

$$\omega L' = \frac{1}{\omega C} \quad \text{or} \quad L' = \frac{1}{\omega^2 C}$$

Since the current is lagging in phase, the inductance value needs to be decreased. For this, we need to put an inductance L'' in Parallel with the given inductance L, such that

$$\frac{1}{L} + \frac{1}{L'} = \frac{1}{L''} = \omega^2 C$$

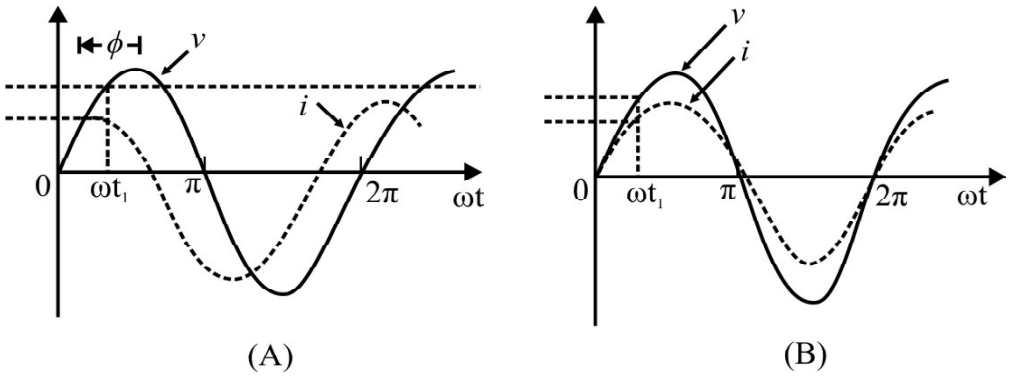
This gives $L'' = \dfrac{L}{\left(\omega^2 LC - 1\right)}$

Unit–14 : Electromagnetic Induction and Alternating Currents

Difficult *Adjusting the Value of C in a LCR Circuit*

67. Graph A, drawn here, shows simulteneously the variation of the (alternating) voltage (applied) and the current (flowing), with time, for a given senses LCR circuit.

Without making any other change in the circuit, the (voltage/current) vs (time) graph, for the circuit, would take the form shown, in graph B, if another capacitor C", of value



(A) (B)

(1) $\left(\frac{1-\omega^2LC}{\omega^2L}\right)$ is put in parallel with the given C (Correct)

(2) $\left(\frac{1-\omega^2LC}{\omega^2L}\right)$ is put in series with the given C (Incorrect)

(3) $\left(\frac{\omega^2L}{1-\omega^2LC}\right)$ is put in parallel with the given C (Incorrect)

(4) $\left(\frac{\omega^2L}{1-\omega^2LC}\right)$ is put in series with the given C (Incorrect)

Correct answer: (1)

Solution:

Graph A shows the current leading in phase w.r.t the voltage. In graph B, the voltage and current are in phase. This means that we need a net value, C', of capacitance, for which

$$\frac{1}{\omega C'} = \omega L \quad \text{or} \quad C' = \frac{1}{\omega^2 L}$$

Since the current is leading in phase, it can be brought 'in phase', by increasing the given value of C. For this, we put a capacitor, C", in parallel with the given capacitance (= C) such that

$$C + C'' = C'$$

$$\therefore \quad C'' = C' - C = \frac{1}{\left(\omega^2 L\right)} - C = \left(\frac{1 - \omega^2 LC}{\omega^2 L}\right)$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *LCR Circuits*

68. The impedance of a given series LCR circuit, at its resonance (angular) frequency $(= \omega_0)$ is X ohms. its impedance becomes Y ohm when the source voltage has a (angular) frequency ω. The values of L, C and R, in the given circuit, are equal, respectively, to

(1) $L = \frac{\omega^2}{\omega_0}\frac{\sqrt{Y^2 - X^2}}{\left(\omega^2 - \omega_0^2\right)};\ C = \frac{\left(\omega^2 - \omega_0^2\right)}{\omega\omega_0^2\sqrt{Y^2 - X^2}};\ (R = X)$ (Correct)

(2) $L = \frac{\omega_0^2}{\omega_0}\frac{\sqrt{Y^2 - X^2}}{\left(\omega^2 - \omega_0^2\right)};\ C = \frac{\omega^2 - \omega_0^2}{\omega\omega_0^2\sqrt{Y^2 - X^2}};\ (R = X)$ (Incorrect)

(3) $L = (\omega\omega_0)\frac{\sqrt{Y^2 - X^2}}{\left(\omega^2 - \omega_0^2\right)};\ C = \frac{\omega^2 - \omega_0^2}{\omega\omega_0^2\sqrt{Y^2 - X^2}};\ (R = X)$ (Incorrect)

(4) $L = \frac{1}{(\omega\omega_0)}\frac{\sqrt{Y^2 - X^2}}{\left(\omega^2 - \omega_0^2\right)};\ C = \frac{\left(\omega^2 - \omega_0^2\right)\omega^2\omega_0}{\sqrt{Y^2 - X^2}};\ (R = X)$ (Incorrect)

Correct answer: (1)

Solution:

The impedance, of a series LCR circuit, at its resonance (angular) frequency (ω_0), equals R. Hence

$$R = X$$

We also have $\omega_0 L - \frac{1}{\omega_0 C} = 0$

and $R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = Y^2$

$\therefore \quad \omega L - \frac{1}{\omega C} = \sqrt{Y^2 - R^2} = \sqrt{Y^2 - X^2}$

Solving the two equations, we get

$$L = \left[\frac{\omega^2}{\omega_0} \cdot \frac{\sqrt{Y^2 - X^2}}{\left(\omega^2 - \omega_0^2\right)}\right]$$

and $C = \frac{\left(\omega^2 - \omega_0^2\right)}{\left[\omega\omega_0^2\sqrt{Y^2 - X^2}\right]}$

Unit–14 : Electromagnetic Induction and Alternating Currents

Difficult *Analysing LCR Circuit*

69. An electrical circuit is made up of an inductance (L), a capacitor (C) and a resistor (R), connected in series. When it is connected across the mains (Voltage = 111.8 V $\simeq \sqrt{12500}$ V), it draws 125W power from these mains. It is also observed that the current lags in phase, with respect to the applied voltage by $\frac{\pi}{4}$. If the inductive impedance is twice the capacitance impedance, the three impedances in the circuit: (i) its resistive impedance; (ii) its inductive impedance and (iii) its capacitative impedance, are (nearly) equal, respectively, to

(1) 142 Ω, 284 Ω, 142 Ω (Incorrect)

(2) 106.5 Ω, 213 Ω, 106.5 Ω (Incorrect)

(3) 89 Ω, 178 Ω, 89 Ω (Incorrect)

(4) 71 Ω, 142 Ω, 71 Ω (Correct)

Correct answer: (4)

Solution:

Let z be the total impedance of the circuit. We then have

$$\text{Power} = P = \frac{V^2}{z}$$

$$\therefore \quad z = \frac{V^2}{P} = \frac{12500}{125} = 100\,\Omega$$

Now phase angle, θ, is given by

$$\tan\theta = \frac{X_L - X_C}{R}$$

Here $\theta = \frac{\pi}{4}$. This means that $(X_L - X_C) = R$

Also $z^2 = R^2 + (X_L - X_C)^2$. Hence $100^2 = R^2 + R^2$

$$\therefore \quad R^2 = 5000\,(\Omega)^2$$

$$\therefore \quad R = \sqrt{5000}\,\Omega \simeq 71\,\Omega$$

Also $X_L - X_C = R = 70\,\Omega$

$$\therefore \quad 2X_C - X_C = R.$$

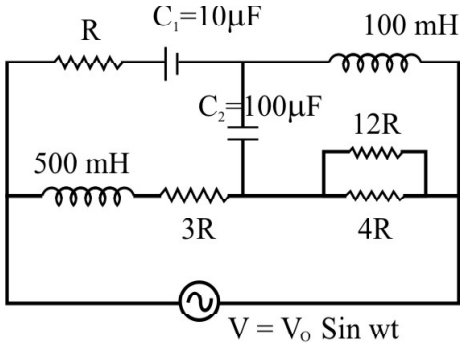
Hence $X_C = 71\,\Omega$

$$\therefore \quad X_L = 142\,\Omega$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Impedance Calculation*

70. The circuit, shown in the figure here, has been connected to an a.c. voltage source $(v = v_0 \sin\omega t)$ of a very high frequency. The effective impedance of this circuit, and the current flowing in it, would then be given by



(1) $4R,\ I = \frac{v_0}{4R}\sin\omega t$ (Correct)

(2) $7R,\ I = \frac{v_0}{7R}\sin\omega t$ (Incorrect)

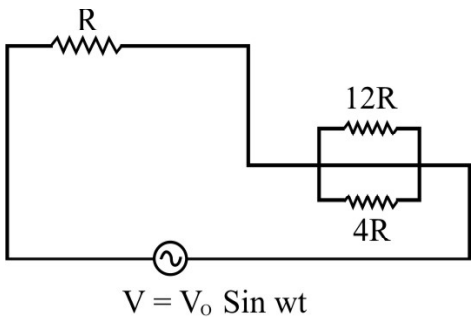
(3) $7R,\ I = \frac{v_0}{7R}\cos\omega t$ (Incorrect)

(4) $4R,\ I = \frac{v_0}{4R}\cos\omega t$ (Incorrect)

Correct answer: (1)

Solution:

At very high frequencies, the effective impedance of the capacitor tends to zero while that of the inductor tends to infinity. The branches, containing inductors, are, therefore, (effectively) not parts of the circuit while those conatining the capacitor, are (effectively) like a simple connecting wire. The circuit, therefore, looks effectively like the one shown here. The effective impedance is, therefore, $\left[R + \frac{4\times12}{16}R\right]$ i.e. 4R. The current flowing, in the circuit, is therefore, in phase with the applied voltage and is given by



$$I = \left(\frac{v_0}{4R}\right)\sin\omega t$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Induced emf in a Rotating Coil*

71. A planar circular coil, of radius a, has N turns and it can be made to rotate about its diameter as axis. The coil has a resistance R and is present in a region where a uniform magnetic field, B, directed perpendicular to the axis of rotation of the coil, exists. If the coil has to be kept rotating with a constant frequency υ, (assuming friction and other dissipative effects to be negligible)

(1) an external torque, of magnitude $2\pi\upsilon NB\sin^2(2\pi\upsilon t)$ must be made to act on it (Incorrect)

(2) an external torque, of magnitude $2\pi\upsilon N\left[\pi a^2 B\sin\left(2\pi\upsilon t\right)\right]$ must be made to act on it (Correct)

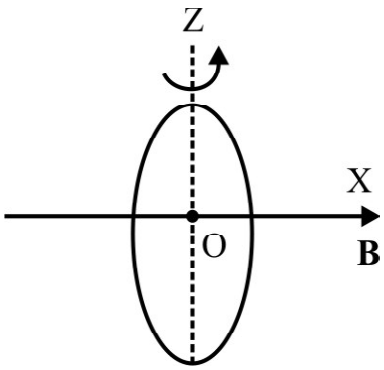
(3) an external torque, of magnitude $2\pi\upsilon N\left[\pi a^2 B\right]^2\sin\left(2\pi\upsilon t\right)$ must be made to act on it (Incorrect)

(4) it just needs to be given the necessary 'spin' at start, it would then keep on rotating at its initial rate, due to its 'rotational inertia'. (Incorrect)

Correct answer: (2)

Solution:

Let the coil rotate about a diameter, taken as the z–axis. The magnetic field isdirected along the *x*–axis. The flux, linked with the coil, at any instant, is



$$\phi = N\left(\pi a^2\right)B\cos\theta$$

$$= N\pi a^2 B\cos\left(2\pi\upsilon t\right)$$

$$\therefore \text{ Induced emf} = \varepsilon - \frac{d\phi}{dt} = 2\pi\upsilon N.\pi a^2 B\sin\left(2\pi\upsilon t\right)$$

$$\text{Instantaneous induced current} = i = \frac{\varepsilon}{R}$$

The magnetic moment, associated with this induced current,

$$M = i.\left(\pi a^2\right)$$

The magnetic field exerts a torque, which has an instantneous value, $\tau = MB\sin\theta$

The external torque is needed to oppose this torque on the coil due to the induced current. Hence external torque = $\tau = MB\sin\theta = 2\pi\upsilon N\pi a^2 B\sin\left(2\pi\upsilon t\right)$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *AC Generator*

72. A circular coil, having N turns of wire, each of mean diameter D, is set up so that it can rotate about a vertical axis, at the rate of p rotations per second. The horizotal component, of the earth's magnetic field, at that place, equals B_H.

The magnitudes of the instantaneous emf, induced in the coil, when its plane

(i) is at right angles to the magnetic meridian.

(ii) makes an angle of 30^0 with the magnetic meridian.

(iii) coincides with the magnetic merician, would equal, respectively.

(1) $(0),\left(\frac{\sqrt{3}}{4}\pi^2D^2NB_Hp\right);\left(\frac{1}{2}\pi^2D^2NB_Hp\right)$ (Correct)

(2) $\left(\frac{1}{2}\pi^2D^2NB_Hp\right);\left(\frac{\sqrt{3}}{4}\pi^2D^2NB_Hp\right),(0)$ (Incorrect)

(3) $(0),\left(\frac{1}{4}\pi^2D^2NB_Hp\right);\left(\frac{1}{2}\pi^2D^2NB_Hp\right)$ (Incorrect)

(4) $(0),\left(\frac{1}{4}\pi^2D^2NB_Hp\right);\left(\frac{1}{2}\pi^2D^2NB_Hp\right)$ (Incorrect)

Correct answer: (1)

Solution:

When the normal, to the plane of the coil, makes an angle θ with the direction of the magnetic field, the instantaneous flux linked with it is

$$\phi = N\left(\frac{\pi D^2}{4}\right)B_H\cos(2\pi pt) \qquad (\theta = \omega t = 2\pi pt)$$

$\therefore$ Instantaneous induced emf is

$$\varepsilon = -\frac{d\phi}{dt} = \frac{N\pi D^2}{4}.B_H(2\pi p)\sin(2\pi pt)$$

$$= \frac{\pi^2D^2NB_Hp}{2}\sin\theta$$

In the cases (i), (ii), (iii), θ (angle betwwen the normal to the plane of the coil and the magnetic field) equals 0^0, 60^0 and 90^0, respectively.

Hence the three values of ε are

$$0,\ \frac{\sqrt{3}}{4}\pi DNB_Hp \text{ and } \frac{\pi^2d^2NB_Hp}{2}, \text{ respectively}$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *AC Generator*

73. A mini generator has a coil of 1000 turns, each of area $10^{-2}m^2$. The coil is placed with its plane perpendicular to a uniform magnetic field of intensity 25 mT and is rotated at a uniform rate of 100 rotations per second. The magnitudes of the average of the emf generated, during the four quarters of its rotation, equal, respectively

(1) 0 volt, 0 volt, 0 volt, 0 volt (Incorrect)

(2) 0 volt, 100 volt, 0 volt, 100 volt (Incorrect)

(3) 100 volt, 0 volt, 100 volt, 0 volt (Incorrect)

(4) 100 volt, 100 volt, 100 volt, 100 volt (Correct)

Correct answer: (4)

Solution:

The flux, linked with the coil, at any instant is

$$\phi = NAB\cos\omega t$$

$\therefore$ Instantaneous induced emf, ε, is given by

$$\varepsilon = \frac{d\phi}{dt} = NAB\omega\cos\omega t$$

Here N = 100, A = $10^{-2}m^2$, B = 25×10^{-3} T; $\omega = (100\times2\pi)s^{-1}$

$\therefore$ $NAB\omega = 10^3\times10^{-2}\times25\times10^{-3}\times2\pi\times10^2$ volt

$= 50\pi$ volt

$\therefore$ $\varepsilon = (50\pi)\sin\omega t$

The average, of this, over success four quarters of the cycle, are given by $\frac{4}{T}$ times the integral of the above, from (i) (0) to $\left(\frac{T}{4}\right)$ (ii) $\left(\frac{T}{4}\right)$ to $\left(\frac{T}{2}\right)$ (iii) $\left(\frac{T}{2}\right)$ to $\left(\frac{3T}{4}\right)$ and (iv) $\left(\frac{3T}{4}\right)$ to (T). Each of there is seen to have a magnitude.

$$\left(\frac{4}{\left(\frac{1}{100}\right)}\times50\pi\times\frac{\cos0^0}{200\pi}\right)\text{volt} = 100\,\text{V}$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Easy *Transformer*

74. A step down transformer (having windings of negligible resistance) is used to step down the mains voltage of 220V, to22V to operate a device of impedance 220V. Assuming an efficienty of 80%, the current flowing in the primary coil of two transformer is

(1) 8 nA (Incorrect)

(2) 10.25 mA (Incorrect)

(3) 12.5 mA (Correct)

(4) 14.75 mA (Incorrect)

Correct answer: (3)

Solution:

The secondary, supplying a voltage of 22V, operates a device of impedance 200 Ohms.

Hence current in the secondary $= \frac{22}{220}\ A = 0.1A$

$\therefore$ Power consumed in the secondary circuit $= 22 \times 0.1,\ W = 2.2\ W$

Efficiency = 80%

Power available in the primary circuit $= 2.2 \times \frac{100}{80}\ W = 2.75\ W$

$\therefore$ If I is the current in the primary coil, we have

$220 \times I = 2.75$

$\therefore\ I = \frac{2.75}{220} A = \frac{2750}{220}\ mA$

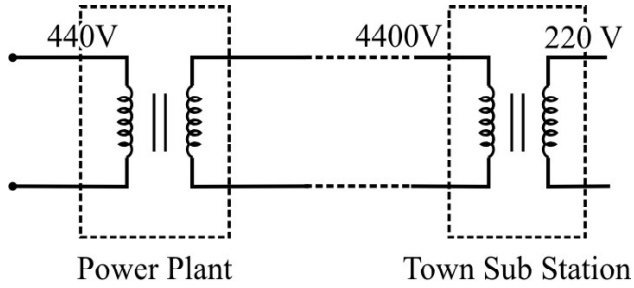
$= 12.5\ mA$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Transformer*

75. The diagram, given here, shows a 'step-up' that may be used to supply power from a 'power plant' to a 'town' via transmission lines of a total (to and fro) length of 50 kM. The power requirement, of the town is 2.2 MW. Assuming that the two transformers, used at the

'power plant' and the 'town substation' are ideal ones and the resistance of the transmission line is 1 ohm/km, the stepped up voltage, available at the secondary coil of the transformer, used at the power plant, should be



(1) 4400 V (Incorrect)

(2) 16900 V (Incorrect)

(3) 20600 V (Incorrect)

(4) 29400 V (Correct)

Correct answer: (4)

Solution:

The power available at the primary coil of the town substation transformer = power requirement of town

$= 2.2\times10^{6}\,W$

Voltage available at its primary = 4400 V

$\therefore$ rms current in the line $= \dfrac{2.2\times10^{6}\,W}{4.4\times10^{3}\,V} = 500\,A$

Total resistance of the line = $50\,\Omega$

Voltage drop across the line = 500 × 50 V = 25000 V

$\therefore$ Voltage that needs to be available at the secondary coil of the transformer of the power plant

= 29400 V

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Transformer*

76. A power station, located at a distance of (3π) km from a town, has to deliver 8.8 of MW of power to the town. The transmission wires are made from a metal of resistivity 2×10^{-8} SI units and have a diameter of 1 cm.

The power station uses a step-up transformer to boost the voltage to 22kV; transmitts power at this voltage and uses a step down transformer, near the town, to bring down voltage to 220V. The percentage of ohmic losses, during transmission, in terms of the power transmitted, equal (nearly)

(1) 13 % (Incorrect)

(2) 8.7 % (Correct)

(3) 6.5 % (Incorrect)

(4) 4.4 % (Incorrect)

Correct answer: (2)

Solution:

The total resistance, of the transmission lines, is

$$R = \rho \frac{\ell}{A} = \frac{2\times10^{-8}\times2\times3\pi\times10^{3}}{\pi\left(0.5\times10^{-2}\right)^{2}}$$

$$= 48\times10^{-1}\Omega = 4.8\Omega$$

The current in the transmission wires

$$I = \frac{\text{Power}}{\text{Voltage}} = \frac{8.8\times10^{6}}{22000}\ \text{A}$$

$$= 400\ \text{A}$$

$\therefore$ Ohmic losses $= I^2R = (4\times10^2)^2\times4.8$ W

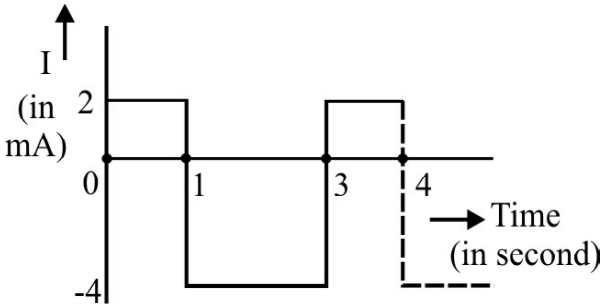
$= 76.8\times10^4$W $= 0.768\times10^6$ W

$\therefore$ Percentage of ohmic losses, in terms of power transmitted $= \frac{0.768\times10^6}{8.8\times10^6}\times100 \simeq 8.7\%$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *RMS Value*

77. The alternating current, in a given set up, varies with time in the manner shown. The rms value of this current, over one cycle, is



(1) 3 A (Incorrect)

(2) 3.17 A (Correct)

(3) 3.28 A (Incorrect)

(4) 3.62 A (Incorrect)

Correct answer: (2)

Solution:

The rms value, by definition, is obtained by taking the average of the squares of all values (over one complete cycle) and then taking the square root of this average.

For the given case, we have

For (t = 0) to (t = 1); sum of the squares of all values = $(2)^3 = 4$

For (t = 1) to (t = 3); sum of the squares of all values = $(-4)^2 \times 2 = 32$

For (t = 3) to (t = 4); sum of the squares of all values = $(2)^2 \times 1 = 4$

$\therefore$ Sum of the squares of all values over one complete cycle (= 4s) = $(4 + 32 + 4)A^2 = 40\,A^2$

$\therefore$ Average of the squared values = $\left(\frac{40}{4}\right)A^2 = 10A^2$

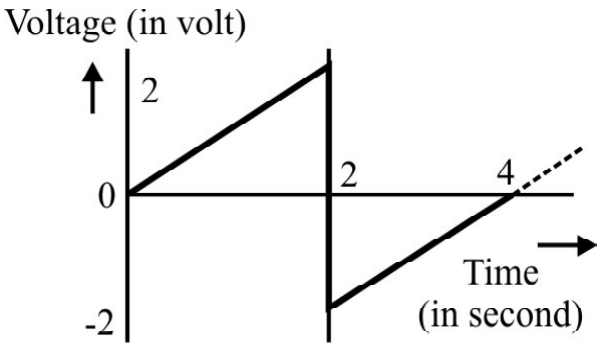
$\therefore$ rms value = $\sqrt{10}$ A

$\simeq 3.17$ A

Unit–14 : Electromagnetic Induction and Alternating Currents

Difficult *Voltage (in Volt)*

78. The rms value, of the saw tooth alternating voltage, shown here, equals (nearly)



(1) 1.732 Volt (Incorrect)

(2) 1.00 Volt (Incorrect)

(3) 1.15 Volt (Correct)

(4) 1.42 Volt (Incorrect)

Correct answer: (3)

Solution:

The rms value, by definition, is obtained by taking the average of the squares of all values (over one complete cycle) and then taking the square root of this average.

We see that for the time period of the alternating voltage is 4V.

For (t = 0) to (t = 2), we have v (t) = 1×t (slope = 1)

$$\therefore \text{ Sum of the squares of all values} = \int_0^2 \left[v(t)\right]^2 dt$$

$$= \int_0^2 t^2 dt = \left(\tfrac{8}{3}\right) \text{volt}^2$$

For (t = 2) to t = 4, we have

v (t) = 1×t – 4

$$\therefore \text{ Sum of the squares of all values} = \int_2^4 (t-4)^2 dt$$

$$= \left|\frac{(t-4)^3}{3}\right|_2^4 = \left[0 - \left(\frac{-8}{3}\right)\right] = \frac{8}{3} v^2$$

$\therefore$ Sum of the squares of all values over one complete cycle

$$=\left(\frac{8}{3}+\frac{8}{3}\right)\text{volt}^2=\frac{16}{3}\text{volt}^2$$

$\therefore$ Average of the squared values $=\frac{16}{3}v^2\times\frac{1}{4}$

$$=\frac{4}{3}v^2$$

$\therefore$ rms value $-\frac{\sqrt{4}}{2}v=\frac{2}{\sqrt{3}}v\simeq 1.15\,v$

Unit–14 : Electromagnetic Induction and Alternating Currents

Average *Induced Current and Resulting Power Loss*

79. A circular coil, of radius a, (having N turns) is made to rotate about its vertical diameter with an angular speed ω. The coil is present in a region where a uniform horizontal magnetic field B is present. If the coil has a resistance R, the rms value of the induced current and the resulting power loss, in it, are given, respectively, by

(1) $\frac{N\pi a^2\omega B}{R}$ and $\frac{\left(N\pi a^2\right)^2\omega B}{R}$ (Incorrect)

(2) $\frac{N\pi a^2\omega B}{\sqrt{2}R}$ and $\frac{2\left(N^2\pi a^2\omega B\right)}{R}$ (Incorrect)

(3) $\frac{N\pi a^2\omega B}{\sqrt{2}R}$ and $\frac{\left(N\pi a^2\omega B\right)^2}{2R}$ (Correct)

(4) $\frac{N\pi a^2\omega B}{R}$ and $\frac{\left(N\pi a^2\omega B\right)}{R}$ (Incorrect)

Correct answer: (2)

Solution:

The instantaneous flux, linked with the coil; is

$$\phi = N\pi a^2 B\cos(\omega t)$$

$\therefore$ Instantaneous induced emf is

$$\varepsilon = -\frac{d\phi}{dt} = N\pi a^2 \omega B \sin \omega t$$

$$= \varepsilon_0 \sin \omega t$$

where $\varepsilon_0 = N\pi a^2 \omega B$

$\therefore$ The rms of the induced current $= \frac{\varepsilon_{rms}}{R} = \frac{\varepsilon_0}{\sqrt{2}\,R}$

$$= \frac{N\pi a^2 \omega B}{\sqrt{2}\,R}$$

$\therefore$ Instantaneous power loss $= \varepsilon_{rms} I_{rms}$

$$= \frac{N\pi a^2 \omega B}{\sqrt{2}} \cdot \frac{N\pi a^2 \omega B}{\sqrt{2}R}$$

$$= \frac{1}{2R}\left[N.\pi a^2 \omega B\right]^2$$

Unit–14 : Electromagnetic Induction and Alternating Currents

Easy *Wattless Curents*

80. For a pure inductor, or a pure capacitor, connected to an a.c. voltage source, one can say that the instantnaeous power.

(1) as well as the average, over a cycle, are both zero (Incorrect)

(2) may not be zero but the average power, over a cycle, is zero (Correct)

(3) can zero but the average power, over a cycle, is hot zero (Incorrect)

(4) as well as the average power, over a cycle, are both not zero (Incorrect)

Correct answer: (2)

Solution:

The current, in a pure inductor, or a pure capacitor, has a phase lag of $\pm\frac{\pi}{2}$ with respect to the applied voltage. Hence if the applied voltage is $(v_0 \sin \omega t)$, the current is $I_0 \sin\left(\omega t \pm \frac{\pi}{2}\right)$, is $\pm I_0 \cos \omega t$. The instantaneous power is, therefore,

$$P_{inst} = \pm v_0 I_0 \sin \omega t \cos \omega t$$

$$= \pm \frac{v_0 I_0}{2} \sin 2\omega t$$

This is not zero at all instant. However, its average, over a complete cycle, is zero.

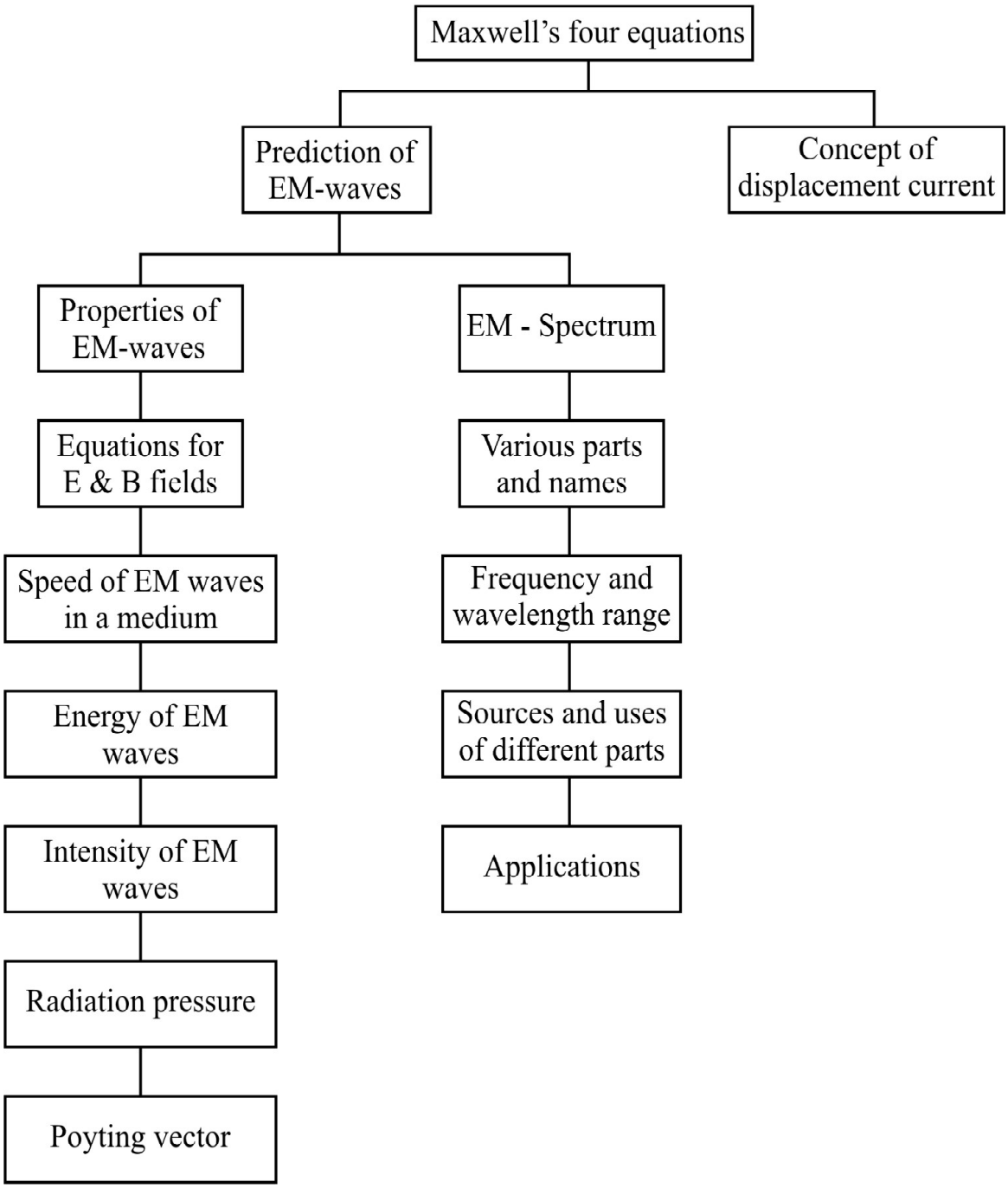
UNIT–15 : ELECTROMAGNETIC WAVES

Learning Objectives

To enable the learners to understand the following concepts / topics, analyse and appreciate them incorporating the previous knowledge, enhance their application skills in various concepts, develop values and life skills required in furthering their overall personality.

- Maxwell's equations and production of electromagnetic waves.
- Concept of displacement current.
- Source / origin of electromagnetic waves.
- Nature of EM waves and their expressions.
- Properties of EM waves.
- Intensity, radiation pressure and momentum of EM-waves.
- Poynting vector (**S**)
- Various types of waves is EM- spectrum, their names, frequencies and uses.
- Applications of EM waves.

Concept Map


Maxwell's four equations
Prediction of EM-waves
Concept of displacement current
Properties of EM-waves
EM - Spectrum
Equations for E & B fields
Various parts and names
Speed of EM waves in a medium
Frequency and wavelength range
Energy of EM waves
Sources and uses of different parts
Intensity of EM waves
Applications
Radiation pressure
Poyting vector

Introduction

James Clark Maxwell, in taking stock of our knowledge of electricity and magnetism, summarised basic results in terms of four equations known as Maxwells equations. He also showed that one of these equations known as Ampere's circuital law needed some modification. This was achieved by introducing the concept of displacement current. Maxwell also shared that the electric and magnetic fields satisfy equation of wave motion and co-exist with one another. He postulated the existence of what is now known as electromagnetic waves. In this unit we will discuss these ideas in a very simple, elementary way.

Maxwells Equations

The four Maxwell's equations are:

(1) Gauss Law of Electrostatics

According to Gauss law the electric flux ϕ_E through any closed surface is $\frac{q}{\varepsilon_0}$; where q is charge inside the closed surface. Expressed mathematically:

$$\phi_E = \oint_S \mathbf{E}.d\mathbf{S} = \frac{q}{\varepsilon_0} \qquad .....(1)$$

q is charge in side closed surface S

(2) Gauss Law of Magnetostatics

The net magnetic flux ϕ_m through any closed surface is zero. Expressed mathematically:

$$\phi_m = \oint \mathbf{B}.d\mathbf{S} = 0 \qquad .....(2)$$

This is basically a restatement of the fact that magnetic monopoles donot exist.

(3) Ampere is Circuital Law

The law states that the line integral of magetic field over any closed surface equals $\mu_0 I$, where I is current inside the closed loop considered. Expressed mathematically:

$$\oint \mathbf{B}.d\ell = \mu_0 I \qquad .....(3)$$

(4) Faraday's Law of Electromagnetic Induction

The induced emf is numerically equal to the time rate of change of magnetic flux.

$$\varepsilon_{ind} = \oint \mathbf{E}.d\ell = -\frac{d\phi_m}{dt}$$

$$= -\frac{d}{dt}\oint \mathbf{B}.d\mathbf{S} \qquad .....(4)$$

The Displacement Current

Maxwell pointed a fundamental inadequecy of the above four equations. Maxwell argued that if a varying magnetic flux produces an induced electric field and therefore an induced emf, a varying electric flux should also produce an induced magnetic field due to an induced current. In other words Maxwell demanded "Parity" between electric and magnetic fields.

Maxwell also shared that the Ampere's circuital law as expressed in Eqn (3); in general; is not valid. As expressed in Eqn (3); the law is valid for direct currents (d.c). But is is not applicable to alternating currents.

We know that a direct current cannot flow through a capacitor. However an a.c current can flow through a capacitor, though an insulator is present between plates of the capacitor. Obviously conduction electrons ,that move in wires conneting a.c source to capacitor, cannot flow from one plate of the capacitor to the other. How circuit is completed when a.c current flows?

To overcome these difficulties Maxwell proposed that a new current, now known as displacement current I_d is produced due to a varying electric flux. Maxwell showed that:

$$I_d = \varepsilon_0 \frac{d\phi_E}{dt} \qquad .....(5)$$

The modified Ampere's circuital law is

$$\oint \mathbf{B}.d\ell = \mu_0 \left(I_c + I_d\right) \qquad .....(6)$$

where I_c is conduction current and I_d is displacement current.

Though displacement current is not due to movement of free electrons; it has same effects as conduction current. We say displacement current can produce a magnetic field in the same manner as a conduction current.

Existence of Displacement Currnet

We discuss briefly, by using idea of displacement current, how do we get a continuity of current through a capacitor connected across an a.c. Let a capacitor (parallel-plate) of capacity C be connected across an a.c voltage given by

$$V(t) = V_0 \sin \omega t$$

The capacity $C = \frac{A \varepsilon_0}{d}$; where A is aera of cross-section and d is distance between plates of capacitor.
The instantaneous charge q(t) on capacitor is:

$$q(t) = CV_{(t)}$$

$$= \left[\frac{A \varepsilon_0}{d} \right] V_0 \sin \omega t \qquad .....(7)$$

The conduction current I_c; in the wires connecting a.c source to capacitor; by defintion, is

$$I_c = \frac{dq(t)}{dt} = \frac{A \varepsilon_0}{d} V_0 \omega \cos \omega t \qquad .....(8)$$

According to Maxwell, in between plates of capacitor there is a varying electric flux which produces displacement current I_d. The instantaneous electric flux ϕ_E; in between plates of capacitor, is

$$\phi_E = AE(t) = A \frac{V(t)}{d}$$

$$= \left(\frac{A}{d} \right) V_0 \sin \omega t$$

The displacement current, I_d, is

$$I_d = \varepsilon_0 \frac{d\phi_E}{dt} = \left(\frac{A\varepsilon_0}{d} \right) \frac{d}{dt} (V_0 \sin \omega t)$$

$$= \left(\frac{A\varepsilon_0}{d} \right) \omega V_0 \cos \omega t \qquad .....(9)$$

From eqns (8) and (9) we have

$$I_d = I_c$$

Let $I = I_d + I_c$ be the total current in circuit. In conducting wires $I_d \to 0$, because $\phi_E \to 0$ and $I = I_c$. In between plates of capacitor $I_c = 0$ and $I = I_d$. There is a continuity of a.c current in a capacitor.

Electromagnetic Waves

Maxwell, using modified Ampere's law and the other three equations showed that the electric field (**E**) and the magnetic field (**B**) satisfy the equation.

$$\nabla^2\mathbf{E} = \left(\frac{1}{\mu_0\varepsilon_0}\right)\frac{\partial^2\mathbf{E}}{\partial t^2}$$

and $$\nabla^2\mathbf{B} = \left(\frac{1}{\mu_0\varepsilon_0}\right)\frac{\partial^2\mathbf{B}}{\partial t^2}$$

These two equatinos are standard equations of wave motion. The speed of waves V is

$$v = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

For free space

$$\varepsilon_0 = \frac{1}{4\pi\times 9\times 10^9} \text{ SI Unit}$$

and $$\mu_0 = 4\pi\times 10^{-7} \text{ SI Unit}$$

$$\therefore \quad v = \frac{1}{\left[4\pi\times 10^{-7}\times\frac{1}{4\pi\times 9\times 10^9}\right]^{1/2}} = \left[9\times 10^{16}\right]^{1/2}$$

$$= 3\times 10^8 \text{ m/s}$$

v is same as speed of light = c, in free space. Therefore Maxwell postulated that light is electric and magnetic fields propogating in sapce! Since electric and magnetic fields can exist in vaccum, light can propagate in vaccum!

Origin of Electromagnetic Wave

Maxwell explained theoretically, the production of EM waves with the help of the two equations, namely:

$$\oint \mathbf{B}.d\ell = \mu_0\varepsilon_0\frac{d\phi_E}{dt}$$

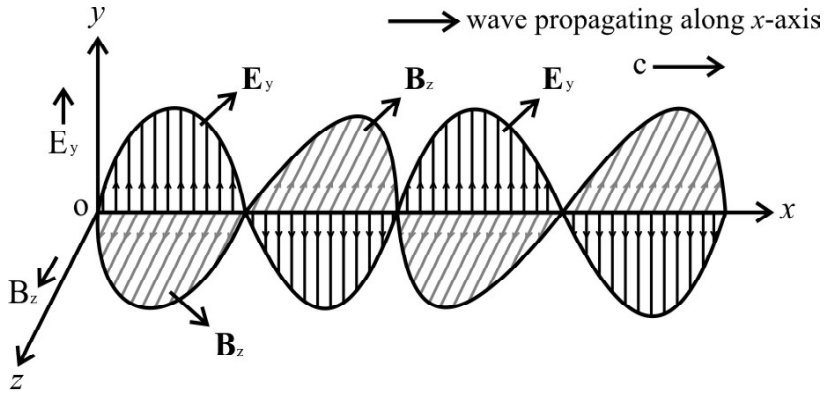
and $$\oint \mathbf{E}.d\ell = -\frac{d\phi_B}{dt}$$

In order to produce a magnetic field, we require a time varying electric field in space and a time-varying electric field can only be produced by an accelerating or oscillating electric change. Hence an oscillating electric change acts as the source of EM waves. As long as the charge continues to oscillate, EM waves are produced from those oscillations.

Nature of Electromagnetic Waves

An electromagnetic wave produced by oscillating charge (and transmitted with the help of a dipole antenna) consists of time-varying E and B fields, the variation in one sustaining the production of the other. As we know, the varition in E-field is the source of displacement current I_d, which actually produces the B-field. Using Bio-savart law, it can be shown that the direction of B is perpendicular to the direction of the displacement current. As the displacement current is always along the direction of E-field, it is clear that these two fields are mutually perpendicular. E and B-fields of EM waves are mutually perpendicular and they are also perpendicular to the direction of propagation of the wave. Thus EM-waves show transverse nature.

Electromagnetic waves propagate, in space with the speed equal to that of light (c). If the electric field **E** is along the y–axis and the magnetic field **B** is along the z–axis, then the direction of wave propagation is along the vector **E** × **B**, that is, along the positive *x*–axis as shown in the figure below. Both **E** and **B** vary sinusoidally with time and space and have same frequency. $\mathbf{E}_y$ and $\mathbf{B}_z$ vary always in same phase.



Since E_y and B_z are progressing along the *x*–axis, with speed equal to v, in space, we can mathematically represent these sinusoidal field variation as under:

$$E_y = E_0 \sin(k x - \omega t)$$

or $$E_y = E_0 \sin\left\{2\pi\left(\frac{x}{\lambda} - \upsilon t\right)\right\}$$

or $$E_y = E_0 \sin\left\{2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right\}$$

and $B_z = B_0 \sin(k x - \omega t)$

or $B_z = B_0 \sin\left\{2\pi\left(\frac{x}{\lambda} - \upsilon t\right)\right\}$

or $B_z = B_0 \sin\left\{2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right\}$

Here, E_0 and B_0 are the respective amplitude or peak values of electric and magnetic fields. It may be recalled here that:

ω is angular frequency of the EM wave

υ is linear frequency

$$\omega = 2\pi\upsilon$$

v is wave velocity (along *x*–axis, here)

$$v = \upsilon\lambda$$

λ is wave length

k is angular wave number

$$k = \frac{2\pi}{\lambda}$$

and $v = \frac{\omega}{k}$

Using Maxwell's equations it can be proved that:

$v = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$, for EM waves, in free space.

or $v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c$

It can also be proved, from Maxwell's equations, that the electric field amplitude (E_0) and the magnetic field amplitude (B_0) in an electromagnetic wave are related as:

$$\frac{E_0}{B_0} = c$$

As we know, both E and B fields in EM wave vary is same phase, equation (15.21) can also be written as:

$$\frac{E}{B} = C$$

[Here E and B are the instantaneous values of electric and magnetic fields, at a particular time instant (t)].

It is well known to all that light, an electromagnetic wave, does propagate through glass and water. We have also seen, in earlier uits, that electric and magnetic field can be established in material medium and that the total electric field inside a medium is described in terms of permittivity (ε) where $\varepsilon = \varepsilon_r \varepsilon_0$ and the magnetic field insdie the medium is described in terms of permeability (μ), where $\mu = \mu_r \mu_0$.

Thus the speed of electromegnetic waves, and therefore light, can be written in terms of μ and ε as:

$$v = \frac{1}{\sqrt{\mu\varepsilon}}$$

Here v is the speed of EM waves in a medium for which μ and ε are respectively permeability and permittivity of the material of the medium. It can also be written as:

$$v = \frac{1}{\sqrt{\mu_r \mu_0 \varepsilon_r \varepsilon_0}}$$

$$= \frac{1}{\sqrt{\mu_r \varepsilon_r} \times \sqrt{\mu_0 \varepsilon_0}}$$

$$v = \frac{c}{\sqrt{\mu_r \varepsilon_r}} \qquad \left(\because \quad \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c \right)$$

or $$v = \frac{c}{\sqrt{\mu_r K}}$$

Here $K = \varepsilon_r$ is the dielectric constant of the material.

Heinrich Rudolf Hertz not only proved that accelerating / oscillating charges produced electromagnetic waves, but also demonstrated these waves, which had wavelength ten million times that of the light waves, could be diffracted, refracted and polarised. Further Hertz produced stationary electromagnetic waves and determined their wavelength, by measuring the distance between two successive nodes. As the frequency of the wave (equal to the frequency of the oscillator) is known, he then calculated the speed of these waves using the equation $v = \upsilon\lambda$ and found that the waves travelled with the speed of light.

The electric and the magnetic fields of an electromagnetic wave are plane polarised. This nature of EM waves can be easily understood in the response of a portable AM radio to a broadcasting station. If the radio has a telescopic antenna it responds to the electric field of the signal. When this antenna is kept horizontal the signal strength will be greatly diminished.

Electromagnetic waves do carry energy and momentum, like other waves. To establish this, consider a plane perpendicular to the direction of propagation of the electromagnetic wave. If there are electric charges (like the free electrons in a conductor) in this plane, they will be set and sustained in motion by the electric and magnetic fields of the EM wave. The charges thus acquire energy and momentum from these waves. We have seen in earlier units, that, in a region of free space,with electric field E, there is an energy density of $\overline{U}_E = \frac{1}{2}\varepsilon_0 E^2$. Similarly, if there is a magnetic field, the associated magnetic energy density is $\overline{U}_B = \frac{1}{2}\frac{B^2}{\mu_0}$.

Since electromagnetic waves contain both electric and magnetic fields, there is non-zero energy density associated with the electromagnetic waves and therefore, they carry energy.

The great technological importance of EM waves stems from the fact that they carry energy from one place to another. The radio, TV and mobile signals carry energy. Light carry energy from the sun to the earth, thus making things visible to human eye, helping leaves to photo-synthesis and enabling solar panels to produce electricity.

Since electromagnetic waves carry momentum, they can also exert pressure, called radiation pressure. If the total energy transferred to a surface, in time t, is U, the toal momentum imparted to the surface, for complete absorption is

$$P = \frac{U}{c}$$

This radiation pressure, exerted by sunlight, is found to be of the order of 7×10^{-6}N-m^{-2}. Thus, on a surface of area 10 cm^2, sunlight exerters a force (radiation force) of around 7×10^{-9}N.

Suppose EM waves are incident normally on the surface of a perfectly black block of surface area A and thickness d. The total energy transferred to this block, over a cycle of these EM waves is equal to:

$$\left[\frac{1}{2}\in_0 E_{rms}^2 + \frac{1}{2}\frac{B_{rms}^2}{\mu_0}\right](Ad)$$

Characterstics and Properties of EM Waves

1. Electromagnetic waves are produced by accelerating or oscillating electric charges.

2. They are self-sustaining oscillations of electric and magnetic fields, which are coupled to each other.

3. The time–varying electric and magnetic fields are mutually perpendicular and are perpendicular to the direction of propagation of the wave.

   Hence electromagnetic waves are transverse is nature.

4. The coupled sinusoidal electric and magnetic field variations are occuring in same phase, so that they attain their maximum or peak values at the same time.

5. Electromagnetic waves do not require any material medium for propagation.

6. They propagate in free space with a speed equal to c, given by

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \qquad \left(c = 3 \times 10^8 \, ms^{-1}\right)$$

7. When electromagnetic wave propagate through a material medium, the speed becomes v, given by

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}} = \frac{c}{\sqrt{\mu_r K}}$$

   (v for any material medium is lesser than c)

8. The speed of electromagnetic waves is also obtained as ratio of the magnitude of electric field to that of the magnetic field at any instant

$$\text{i.e. } c = \frac{E_0}{B_0} = \frac{E}{B}$$

9. EM waves do not carry any charge and so they are not deflected by external static electric or magnetic fields.

10. All the electromagnetic waves, irrespective of their frequency (low or high), travel with the speed equal to c, in space.

11. The speed of EM waves follows the general expression for wave-speed, $v = \upsilon\lambda$.

12. Experiments by Hertz demonstrated that elecromagnetic waves of any frequency (or wavelength) undergo refraction, diffraction, polarisation and can form standing waves.

13. Electromagnetic waves carry energy which is shared equally by the electric field (**E**) and magnetic field (**B**) vectors.

    In a region of space where **E** and **B** fields vary sinusoidally, the average energy density $\left(\overline{U}_E\right)$ associated with the electric field (**E**) and the average energy density $\left(\overline{U}_B\right)$ associated with the

magnetic field (**B**) are given, respectively, as:

$$\overline{U}_E = \frac{1}{2}\varepsilon_0 E^2$$

and $$\overline{U}_B = \frac{1}{2}\frac{B^2}{\mu_0}$$

Here E and B represented, obviously, the rms values of the sinusoidally varying electric and magnetic fields, where $E = E_{rms} = \frac{E_0}{\sqrt{2}}$ and $B = B_{rms} = \frac{E_0}{\sqrt{2}}$.

Therefore, $\overline{U}_E = \frac{1}{4}\varepsilon_0 E_0^2$

and $$\overline{U}_B = \frac{1}{4}\frac{B_0^2}{\mu_0}$$

The total average energy density possed by the electromagnetic waves is therefore,

$$\overline{U} = \overline{U}_E + \overline{U}_B = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4}\frac{B_0^2}{\mu_0}$$

But $\overline{U}_E + \overline{U}_B$, as proved below.

$$\overline{U}_B = \frac{1}{4}\frac{B_0^2}{\mu_0} = \frac{1}{4}\times\frac{1}{\mu_0}\left(\frac{E_0^2}{C^2}\right) \qquad \left(\because \quad \frac{E_0}{B_0} = c\right)$$

$$= \frac{1}{4}\frac{E_0^2}{\mu_0}\times\mu_0\varepsilon_0 \qquad \left(\because \quad c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}\right)$$

$$= \frac{1}{4}\varepsilon_0 E_0^2$$

$$= \overline{U}_E$$

Therefore, total energy density $\left(\overline{U}\right)$ can be written as:

$$\overline{U} = \overline{U}_E + \overline{U}_B$$

or $\overline{U} = 2\overline{U}_E$

or $\overline{U} = \frac{1}{2}\varepsilon_0 E_0^2$

14. Electromagnetic waves carry energy and linear momentum (P). If U is the total energy transferred to a surface (total absorption), is time t, then

$$P = \frac{U}{c}$$

(In case the waves are totally reflected from the surface, backwards, than the momentum transferred to the surface will be zero)

15. Since electromagnetic waves possess momentum, they can exert radiation pressure.

$$\text{Radiation pressure} = \frac{\text{Linear momentum transferred by the EM waves}}{\text{Area of surface} \times \text{time}}$$

16. The intensity of electromagnetic radiation (I), during the propagation of the EM wave, at a point, is defined as the amount of energy passing through unit normal area around that point in unit time.

That is:

$$I = \frac{\text{Energy / Time}}{\text{Area}}$$

or $I = \frac{\text{Power}}{\text{Area}}$

If $\overline{U}$ is the total average energy density carried by the electromagnetic wave, then total energy passing through unit normal area in one second will be:

$I = \overline{U}c$, where $\overline{U} = \frac{1}{2}\varepsilon_0 E_0^2$

$\therefore \quad I = \frac{1}{2}\varepsilon_0 c E_0^2$

Since $\frac{1}{2}\varepsilon_0 c$ is a constant, we can write:

$I \propto E_0^2$

17. If **E** and **B** are the electric and magnetic field vectors, then the velocity vector (**v**) of wave propagation is along **E** X **B**. i.e. **v** is in the direction of **E** X **B**.

It may be noted that a vector called poynting vector (**S**) gives the magnitude and direction of energy transported by the EM waves per second per unit normal area, and **S** is given as:

$$\mathbf{S} = \frac{\mathbf{E}\mathrm{X}\mathbf{B}}{\mu_0}$$

Now, by definition, $I = |\mathbf{S}|$

$$\therefore \quad I = |\mathbf{S}| = \frac{|\mathbf{E}\mathrm{X}\mathbf{B}|}{\mu_0}$$

$$\text{or} \qquad I = S = \frac{EB}{\mu_0} \qquad (\because \ \mathbf{E} \perp \mathbf{B})$$

$$\text{or} \qquad I = S = \frac{1}{2}\frac{E_0 \times B_0}{\mu_0}$$

$$\text{or} \qquad I = S = \frac{1}{2}\frac{E_0}{\mu_0}.\frac{E_0}{C}$$

$$\text{or} \qquad I = S = \frac{1}{2}\varepsilon_0 C E_0^2 \qquad \left(\because \ \mu_0 C = \frac{1}{\varepsilon_0 C}\right)$$

18. The total energy transferred to a black surface, over a cycle, by normally incident EM waves is equal to : $\left[\frac{1}{2}\in_0 E_{rms}^2 + \frac{1}{2}\frac{B_{rms}^2}{\mu_0}\right](Ad)$.

(A= surface area and d = thickness of the body)

Example–1 :

A plane electromagnetic wave of frequency 25 MHz travels in free space along the *x*–diretion. At a particular point in space and time, $\mathbf{E} = 6.3\hat{j}\,\mathrm{Vm}^{-1}$. What is **B** at this point?

Solution :

Given : $\mathbf{v} = c\hat{i} = 3\times10^8\hat{i}\,\mathrm{ms}^{-1}$

$$\mathbf{E} = 6.3\hat{j}\,\mathrm{Vm}^{-1}$$

As we know that (**E**X **B**) is along **v** and only $\left(\hat{j}\times\hat{k}\right)$ gives $\hat{i}$. Hence $\mathbf{B} = B\hat{k}$.

Now, $B = \frac{E}{v} = \frac{E}{c}$

$= \frac{6.3}{3\times10^{8}}\,T$

$= 2.1\times10^{-8}\,T$

$\therefore \quad \mathbf{B} = \left(2.1\times10^{-8}\,\hat{k}\right)$ tesla

Example–2 :

The magnetic field in a plane electromagnetic wave is given as:

$$B_y = 2\times10^{-7}\sin(1.5\times10^{11}t + 500x)T$$

(a) What is the wave length and frequency of the wave?

(b) What is the expression for the electric field?

Solution :

Given $B_y = 2\times10^{-7}\sin(1.5\times10^{11}t + 500x)T$ from this expression we obtain:

(i) $\mathbf{v} = c\left(-\hat{i}\right) = 3\times10^{8}\left(-\hat{i}\right)ms^{-1}$

(ii) $\omega = 1.5\times10^{11}\,rad.s^{-1}$

(iii) $k = 500\,rad - m^{-1}$

(iv) $B_0 = 2\times10^{-7}$ tesla

When we compare the above given equation with: $B_y = B_0\left(\omega t + kx\right)$

(a) $k = \frac{2\pi}{\lambda}$

$\therefore \quad \lambda = \frac{2\pi}{k} = \frac{2\times3.14}{500} = 2\times2\times3.14\times10^{-3}\,m$

$= 12.56\times10^{-3}\,m$

$= 1.256\,cm$

Also, $\omega = 2\pi\upsilon$

$$\therefore \quad \upsilon = \frac{\omega}{2\pi} = \frac{1.5\times10^{11}}{2\times3.14}$$

$$= 23.9\times10^{9}\ \text{Hz}$$

$$= 23.9\ \text{GHz}$$

(b) $E_0 = B_0 c = 2\times10^{-7}\times3\times10^{8}$

$$= 60\ \text{Vm}^{-1}$$

Since the wave velocity (**v**) is along $\left(-\hat{i}\right)$ and **B** is along $\hat{j}$, **E** has to be along the positive z–axis (or along $\hat{k}$)

$$\therefore \quad E_z = E_0 \sin\left(\omega t + k x\right)$$

or $E_z = 60 \sin (1.5\times10^{11}t + 500x)\ \text{Vm}^{-1}$

Example–3 :

Light with an energy flux of 18 Wcm^{-2} falls on a non-reflecting surface, at normal incidence. If the surface has an area of 20cm^2, find the average force exerted on the surface by the incident light.

Solution :

Given : Energy flux = 18Wcm^{-2}

$$= 18\frac{\text{J}}{\text{s}-\text{cm}^2}$$

Therefore, energy falling on 20 cm^2 in one second is

$$\text{U} = \text{Flux}\times\text{Area}\times\text{Time}$$

$$= 18\frac{\text{J}}{\text{s}\times\text{cm}^2}\times20\text{cm}^2\times1\text{s}$$

$$= 360\ \text{J}$$

Hence, total momentum delivered (for complete absorption) on to 20 cm^2 area, in 1 second is

$$\text{P} = \frac{\text{U}}{\text{c}} = \frac{360}{3\times10^{8}} = 1.2\times10^{-6}\ \text{N}-\text{s}$$

Therefore the average force exerted on the surface is:

$$F = \frac{p}{t} = \frac{1.2\times10^{-6}\,N-s}{1s}$$

$$= 1.2\times10^{-6}\ N$$

Example–4 :

Calculate the amplitude of electric and magnetic fields produced by the radiation coming from a 100 watt bulb, at a distance of 3m from the bulb. (Assume that the luminous efficiency of the bulb is 2.5% and that it is a point object).

Solution :

The bulb, as a point source,radiates light in all directions unformly. At a distance of 3m from the bulb, the energy radiated by the bulb, and the energy recieved by a spherical surface, of area $A = 4\pi r^2 = 4\pi(3)^2 = 113 m^2$, will be the same.

Therefore, the intensity of light at 3m is:

$$I = \frac{\text{Power}}{\text{Area}} = \frac{100\,W\times2.5\%}{113\,m^2}$$

$$= 0.022\ Wm^{-2}$$

Half of this intensity is provided by the electric field and half is provided by the magnetic field of the radiation.

Now, we have, $I = \frac{1}{2}\varepsilon_0 c E_0^2$

$$\therefore\quad E_0 = \sqrt{\frac{2I}{c\varepsilon_0}} = \sqrt{\frac{2\times0.022}{8.854\times10^{-12}\times3\times10^{8}}}$$

or $\quad E_0 = 4.07\ Vm^{-1}$

Hence, $B_0 = \frac{E_0}{c} = \frac{4.07}{3\times10^{8}} = 1.4\times10^{-8}\ T$

(Note that, even though the magnetic field amplitude is extremely small, as compared to electric field magnitude, the energy associated with both of then are equal. Further $E_0 = 4.07\ Vm^{-1}$ for a 100 W electric bulb, used by us for reading, at a distance of 3m from it, is much higher than that of the TV or FM waves used by us, which is of the order of $10^{-6}\ Vm^{-1}$).

Example–5 :

Suppose that the electric field amplitude of an electromagnetic wave is $E_0 = 120\ NC^{-1}$ and its frequency is $\upsilon = 50.0\ MHz$

(a) Determine the values of B_0, ω, k and λ.

(b) Obtain the expression for **E** and **B**.

Solution :

Given : $E_0 = 120\ NC^{-1}$ (or Vm^{-1})

$\upsilon = 50.0\ MHz$

$= 50\times10^6\ Hz$

(a) (i) $B_0 = \dfrac{E_0}{c} = \dfrac{120}{3\times10^8} = 4\times10^{-7}\ T$

(ii) $\omega = 2\pi\upsilon = 2\times3.14\times50\times10^6 = 3.14\times10^8\ rad-s^{-1}$

(iii) $k = \dfrac{\omega}{v} = \dfrac{\omega}{c} = \dfrac{3.14\times10^8}{3\times10^8} = 1.05\ rad-m^{-1}$

(iv) $\lambda = \dfrac{c}{\upsilon} = \dfrac{3\times10^8}{5\times10^6} = 6.0\ m$

(b) Suppose the wave propagate in the positive *x*–direction. Then, the expression for **E** is

$$\mathbf{E} = E_0 \sin(\omega t - kx)\hat{j}$$

$$= 120\sin\left(3.14\times10^8 t - 1.05x\right)\hat{j}\ Vm^{-1}$$

Expression for **B** is:

$$\mathbf{B} = B_0 \sin(\omega t - k x)\hat{k}$$

$$= 4\times10^{-7}\sin\left(3.14\times10^8 t - 1.05x\right)\hat{k}\ \text{tesla}$$

Example–6 :

Suppose that the electric field part of an electromagnetic wave in vacuum is :

$$\mathbf{E} = 3.1\cos\left(1.8y + 5.4\times10^8 t\right)\hat{i}\ Vm^{-1}$$

Obtain the:

(i) direction of propagation

(ii) wavelength (λ)

(iii) frequency (υ)

(iv) amplitude of the mangetic field part

(v) expression for the magnetic field part of the wave

Solution :

Given : $\mathbf{E} = 3.1\cos\left(1.8\,y + 5.4\times10^{8}\,t\right)\hat{i}\ Vm^{-1}$

The standard equation for **E** is

$$\mathbf{E} = E_0\cos\left(ky + \omega t\right)\hat{i}$$

Comparing the two we get:

(i) The direction of wave propagation is $\left(-\hat{j}\right)$ or along the negative y–direction.

(ii) $k = 1.8$ rad-m^{-1}

$$\therefore\ \lambda = \frac{2\pi}{k} = \frac{2\times3.14}{1.8} = 3.49\,m$$

(iii) $\omega = 5.4\times10^{8}\ rad - s^{-1}$

$$\therefore\ \upsilon = \frac{\omega}{2\pi} = \frac{5.4\times10^{8}}{2\times3.04} = 85.9\times10^{6}\ Hz$$

(iii) $E_0 = 3.1$ N–C^{-1}

$$\therefore\ B_0 = \frac{E_0}{c} = \frac{3.1}{3\times10^{8}} = 1.03\times10^{-8}\ T$$

(iv) $\mathbf{B} = B_0\cos\left(kj + \omega t\right)\hat{k}$ $\qquad\left(\because\ \mathbf{E}\times\mathbf{B}\text{ is along }\mathbf{v}\text{ or }\hat{i}\times\hat{k} = -\hat{j}\right)$

$$= 1.03\times10^{-8}\cos\left(1.08y + 5.4\times10^{8}t\right)\hat{k}\ T$$

Electromagnetic Spectrum

After Hertz's experimental demonstration regarding electromagnetic waves, establihing Maxwell's theory, several types of electromagnetic waves were discovered, in addition to the familiar and well known visible

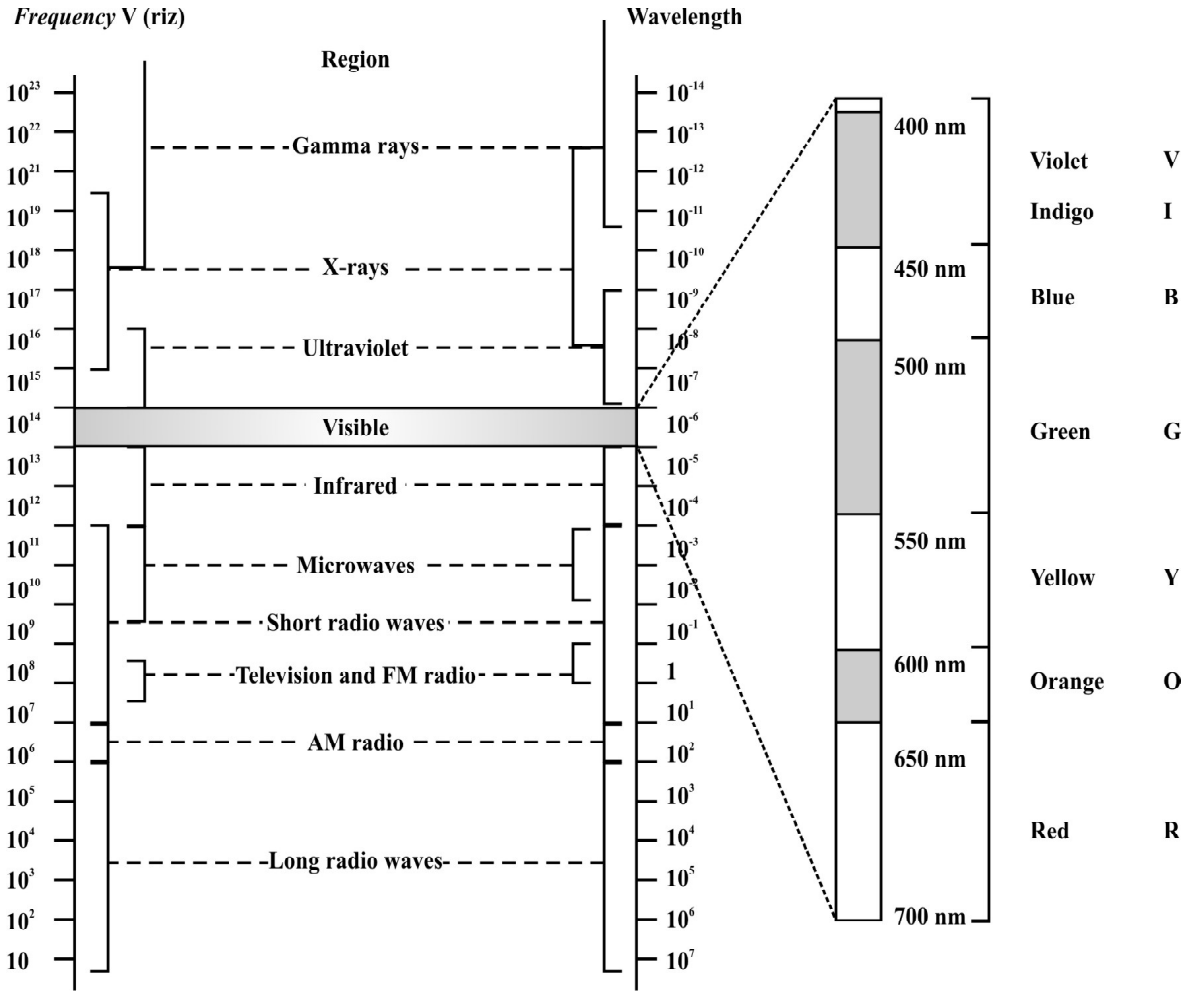
light wave. Now we are all aware that EM waves include visible light, ultra violet radiation,infrared waves, *x*–rays, gamma rays, micro waves and radio waves.

The orderly distribution and classification of electromagnetic waves, according to their frequency or wavelength is known as the electromagnetic spectrum.

The electromagnetic spectrum has a wide range of wavelength, varying from about 10^{-14}m for gamma rays to 6×10^{2}m for radio waves. It may be pointed out that only a small region of the complete electromagnetic spectrum is visible to human eye, since the waves in this 'visible region' can only produce some sensation on the retina of human eye. These radiations in the visible range (starting with $\lambda \approx 400$ nm for violet and ending with $\lambda \approx 700$ nm for red) produce sensation of different colours on the regina.

The following figure, and the table below it, give the details of the various regions of the electromagnetic spectrum with their range of frequency and wavelength, their production as well as the means of their detection. It may be noticed that there is no sharp division between the different regions.

The Electromagnetic Spectrum



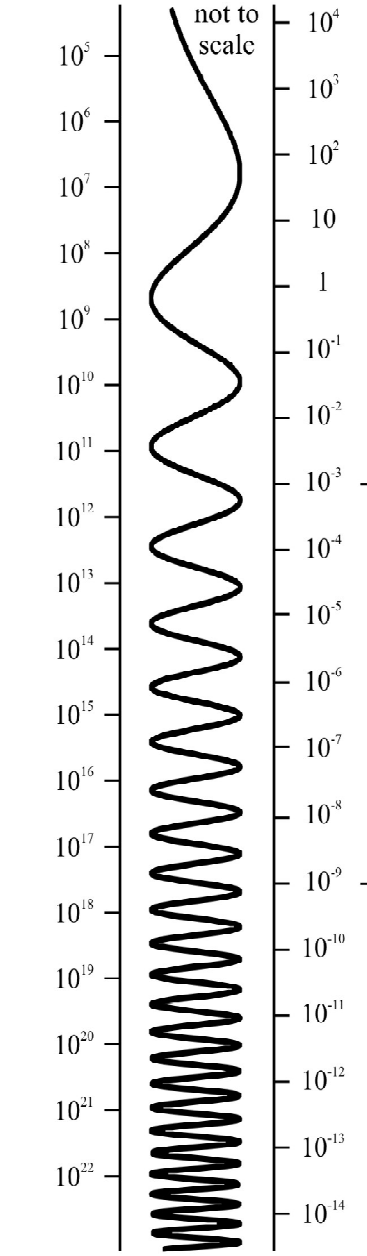
Different types of EM waves and their brief discription

S. No.	Type	Wavelength range	Frequency range, (Hz)	Production	Detection
1.	Radio waves	> 0.1 m (0.1m to 6×10^{2}m)	$3x10^{9}$ to 5×10^{5}	Oscillating circuit or rapid acceleration and decelerations of electrons in aerials	Receiver's aerials
2.	Microwaves	0.1m to 1mm (10^{-3}m to 0.1m)	3×10^{11} to 3×10^{9}	Khystron valve or magnetron valve	Point contact diodes
3.	Infrared waves	1mm to 700nm (7×10^{7}m to 10^{-3}m)	4×10^{14} to 3×10^{11}	Vibration of atoms and molecules	Thermopiles, Bolometer, Infrared photographic films
4.	Visible light waves	800nm to 400nm (4×10^{-7}m to 8×10^{7}m)	8×10^{14} to 4×10^{14}	Excitation of valency electrons in atoms, electrons in atoms emit light when they move from one energy level to a lower energy level	The human eye, Photocells, Photog-raphic film/plate
5.	Ultravoilet waves	400nm to 1nm (1×10^{9}m to 4×10^{-7}m)	1×10^{17} to 8×10^{14}	Excitation of atoms, spark and arc lamps, inner shell electrons in atoms moving from one energy level to a lower level	Photocells, photographic film
6.	*x*–rays	1nm to 10^{-3}nm (10^{-12}m to 10^{-9}m)	3×10^{20} to 1×10^{17}	*x*–rays tubes or excitation of inner shell electrons	Photographic film, Gaiger tubes, Ionisation chamber
7.	Gamma rays	< 10^{3}nm (<10^{-12}m)	5×10^{22} to 3×10^{20}	Radioactive decay of the nucleus, Nuclear origin	Photographic film, Ionisation chamber

The Electromagnetic Spectrum

The electromagnetic spectrum

frequency Hz	wavelength m	type of electromagnetic radiation	examples, uses, and effects



radio waves

- long wave — long-distance AM radio
- medium wave — local AM radio
- short wave — amateur radio
- VHF — FM radio
- UHF — TV boradcasts
- microwaves — mobile phones, TV and communications satellites, telephone links, radar; heating effect used in microwave ovens

infrared

radiant heaters and grills
TV and video remote controllers
security alarms and lamps
'light' pulses in optical fibres

Sun

only type of radiation visible to the eye

causes tanning, skin cancer, and eye damage
causes fluoerscence
(makes some chemicals glow)
kills bacteria

X-rays

used for X-ray photography
causes fluorescence
causes cancer, but can kill cancer cells

gamma rays

emitted by radioactive materials
uses and effects as for X-rays
used for sterilizing medical equipment and food

DANGER
RADIATION

$10^{3} = 1000$

$10^{-3} = \frac{1}{10^{3}} = \frac{1}{1000} = 0.001$

Main Regions of EM Spectrum and uses and Applications of these Region

The entire electromagnetic spectrum is broadly classified in the following regions, in the order of the decreasing wave length (or increasing frequency). We now describe each region, briefly, including some elementary facts about the use of various electromagnetic waves.

(i) Radio Waves

They are produced by the oscillating charges at the ends of a dipole antenna connected to an oscillating voltage source. These radio waves have a frequency range from about 5×10^5 Hz to 10^9 Hz. These waves are used as carrier waves to transport message signals (audio and video) in radio, TV and mobile communication systems.

In early radio-communication, amplitude modulated (AM) waves were used. AM radio were used for this purpose. There are two bands of AM waves (a) Medium wave band (MW bnad), with frequency ranging from 540 KHz to 1710 KHz and are propagated as 'ground waves'. The MW band of AM radios can receive these waves. (b) Short wave band (SW band), with a frequency range from 1710 KHz to 54 MHz. These wave are utilized in sky-wave propagation and are received by the SW bands of AM radios.

TV waves employ EM waves in the frequency range from 54 MHz to 890 MHz. The frequency modulated (FM) radio band extends from 88 MHz to 108 MHz.

The ultra High frequency (UHF) band extends from 300 MHz to 3000 MHz (or 3 GHz) and it is used for cellular or mobile phone communication.

(ii) Microwaves

They are short-wave length radio waves, having frequency range from 1 GHz to 300 GHz. These waves are produced by special vacuum tubes like Klystrons, Mangetrons, Gunndiodes etc.

By virture of their short wave length, microwaves can be transmitted through large distances, without much spreading, like a narrow beam. Thus they are very effectively used in RADAR systems, aircraft navigation and satellite communication.

Microwaves are used in speed guns to measure the speed of fast moving objects and detecting the service speeds of tennis and squash balls, cricket balls, automobile vehicles etc.

Microwaves are also used in Microwave oven for cooking purposes. (Microwave oven works on the principle of resonance between the elcgric field vibrations of the microwaves and electric field variations around the highly polarised water molecules (or dipoles) due to the vibrational and rotational motions of the water molecules. They both have the same or almost same frequencies, producing resonance. It may be noted that at resonance maximum absorption of energy takes place. Hence water moelculas absorbes energy from microwaves due to resonance. In the microwave oven, the energy absored by water molecules heats up these molecules and they then conduct (transfer) the heat to the other food particles, heating the entire food).

(iii) Infrared Waves

These waves have frequency range from 3×10^{11} Hz to 4×10^{14} Hz.

They are produced or emitted by all hot objects and molecules. These waves are easily absorbed by objects in the surroundings, there by cansing heating effect in them. Because of this, infrared (IR) radiations are some times referred to as 'heat waves'.

Infrared waves (or radiations) are used for a variaty of applications like:

- Physical therapy for treating muscular strain (infrared lamps are used for this).
- Solar water heater, solar cooker, solar cells, remote sensing satellites.
- Drying purposes (fruits and vegetables).
- Night vision goggles
- Imprared LEDs, used in remote controles to operate electronic systems in TV sets, video recorders, hi-fi system, ACs, set-top boxes etc.
- Green houses to keep plants warm.

(Infrared waves trapped by green house gases like CO_2 and water vapour, helps keep the earth warm. But excessive green house gases can cause global warming).

(iv) Visible Light

This is the most familiar form of electromagnetic waves. Even though this region forms a significant part of solar spectrum, it is in fact only a very small part of the whole electromagnetic spectrum. Human eye is sensitive only to this part of the EM spectrum.

The visible light waves have a frequency range from 4×10^{14} Hz to 8×16^{14} Hz (or wave length range from 400 nm to 800 nm).

The visible light waves are produced by de-excitation of valance electrons from higher energy to lower energy levels.

(v) Ultra Violet Waves (UV Rays)

This region has a range of frequencies from 8×10^{14} Hz to 5×10^{16} Hz.

Sun is the most aboudant source of U-V rays. They are also produced by special U-V lamps, very hot objects, mercury arcs and welding iron arcs.

Over-exposure to U-V rays from sun can cause very dangerous harmful effects on humans on earth (like skin cancer etc). But the ozone layer in the earth's atmosphere (at around 40-50 km) absorbs most of these radiations from sun, thereby protecting us from harmful effects. However, the ozone depletion caused by chloroflouro carbons (CFCs) emitted by refrigerators, ACs and airosoles is

adversely affecting the absorption of U-V rays by atmosphere and the matter is of international concern for the word community.

Ultravoilet radiations have wide range of uses like:

- in preserving food stuff, by killing germs and bacteria using U-V rays.
- in burglar alarms.
- In the detection of forged documents.
- in finger print analysis.
- in checking and testing mineral samples (through flourescence caused by U-V rays).
- U-V lamps used in water purifiers.
- In LASIK

(It may be noted that UV rays are easily absorbed by ordinary glass and therefore special glass goggles or face masks with glass windows help welders protect their eyes from the UV rays produced by welding arcs).

(vi) x–rays

This part of the EM waves has the range of frequencies from about 10^{16} Hz to 3×10^{21} Hz. The x–rays are produced by bombarding a metal target by high energy electrons. They possess high penatrating power.

x–rays also have wide range of uses and application.

- x–rays are used as diagnosting tool for medical application.
- They are used as treatment tool for certain forms of cancer.
- These rays are used for detection of bone fractures, unauthourised presence of hidden objects like precious metals and stones, bullets etc. inside human body as well as explosive and drugs hidden in the body.
- These rays are used for scientific research in crystal structure etc.
- They are used in engineering, to detect and test any faults, cracks and flows in structures built with metals and concrete.
- They are also used to test the uniformity of insulating materials and to detect leakage of oils in underground pipes.

(It may be noted that over-exposure to x–rays can damage or even destroy living tissues and organisms).

(vii) Gamma Rays (γ –rays)

These rays have a frequency range from about 3×10^{18} Hz to 5×10^{22} Hz. They are highly energetic and are produced or emitted by radio-active nuclei. They have extremely high penetrating power and can cause huge biological damage to tissues and organisms.

They are used in medical field to destroy cancer cells (radio-therapy).

Soft γ –rays are used to kill micro-organisms to help preserve food-stuff for longer period. They are used in the study of the structure of nucleus of atoms and also to detect and test faults in constructed structures.

Example–7 :

Use Wien's formula : $\lambda_m T = 0.29\,\text{cm} - \text{K}$ to find the temperature of a hot body which can possibly emit microwaves of wavelength 1 μm .

Solution :

Given : $\lambda_m T = 0.29\,\text{cm} - \text{K}$

$$\lambda_m = 1\,\mu\text{m} = 1\times10^{-6}\,\text{m} = 1\times10^{-4}\,\text{cm}$$

$$\therefore \quad T = \frac{0.29\,\text{cm} - \text{K}}{10^{-4}\,\text{cm}}$$

$$= 2900\ \text{K}$$

Example–8 :

Given below are some famous numbers associated with electromagnetic radiation in different contexts in physics. State the part of electromagnetic spectrum to which each belongs to:

(i) 21 cm (wavelength emitted by atomic hydrogen in interstellor space).

(ii) 1057 Hz (frequency of radiation arising from two close energy levels in hydrogens known as lamb shift).

(iii) 2.7 K (temperature associated with isotropic radiation filling all space : thought to be a relic of the big bang origin of the universe).

(iv) $5890\,\text{Å}$ and $5896\,\text{Å}$ (double lines of sodium).

(v) 14.4 KeV (energy of a particular transition in Fe^{56} nucleous associated with a famous high resolution spectroscopic method – Mossbauer spectroscopy).

Solution :

(i) Given wave length corresponds to short wavelength radio-waves.

(ii) High frequency end of radio waves (short radio waves).

(iii) $T = 2.7\ K$ gives $\lambda = \frac{0.29\,cm}{T} = 0.11$ cm, which corresponds to microwave region.

(iv) D_1 and D_2 lines in the visible region (yellow).

(v) Given $E = 14.4\ KeV = 14.4\times10^{3}\times1.6\times10^{-19}\ J$

$= 14.4\times1.6\times10^{-16}\ J$

$\therefore\quad \upsilon = \frac{E}{h} = \frac{14.4\times1.6\times10^{-16}}{6.6\times10^{-34}}\ Hz = 3.5\times10^{12}\ MHz$

Which lies high frequency x–rays or soft γ –ray region.

Example–9 :

Electromagnetic waves with wavelenth

(i) λ_1 is used in satellite communication.

(i) λ_2 is used to kill germs in wter purifiers.

(iii) λ_3 is used to detect leakage of oil in undergound pipe lines.

(iv) λ_4 is used to improve visibility in runways during fog and mist condition.

(a) Identify and name the part of EM spectrum to which these radiation belong.

(b) Arrange them in the ascending order of their wavelength magnitude.

(c) Write one more application for each of them.

Solution :

(a) $\lambda_1 \rightarrow$ microwave

$\lambda_2 \rightarrow$ ultraviolet radiation

$\lambda_3 \rightarrow$ x–rays

$\lambda_4 \rightarrow$ Infrared radiation

(b) $\lambda_3 < \lambda_2 < \lambda_4 < \lambda_1$

(c) $\lambda_1 \rightarrow$ microwaves - used in radar (or microwave oven).

$\lambda_2 \rightarrow$ U-V radiation - used in LASIK eye surgery.

$\lambda_3 \rightarrow$ x–rays - used in radio-graphy.

$\lambda_4 \rightarrow$ infrared - used in remote controls.

EXERCISE

1. The magnetic field of a beam emerging from a filter facing a flood light is given as:

   $B = 12\times10^{-8} \sin (1.20\times10^{7}\, z - 3.60\times10^{15}\, t)$ tesla

   What is the average intensity of the bean?

   Ans.: 1.71 $W.m^{-2}$

2. Poyting vector **S** is defined as:

   $$\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E}\times\mathbf{B})$$

   Show graphically the variation of S with t.

3. Show that the radiation pressure exerted by an EM-wave of intensity I on a surface, kept in vaccum, for complete absorption is $\frac{I}{c}$.

4. For frequency $\upsilon = 4\times10^{8}$ Hz, sea water has a permittivity of $\varepsilon = 80\,\varepsilon_0$ and permeability $\mu = \mu_0$. The resitivity of sea water is $\rho = 0.25\,\Omega - m$. Imagine a parallel plate capacitor immersed is sea water and driven by an A.C voltage of $V = V_0 \sin(2\pi\upsilon t)$. What fraction of conduction current density is its conduction current density?

   Ans.: $\frac{4}{9}$

5. If ε_0 and μ_0 are the electric permittivity and magnetic permeability of free space and ε and μ are the corresponding quantities for a given medium, then express the refractive index 'n' of the medium in terms these quantities.

   Ans.: $\sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}$

SOLUTION TO EXERCISE QUESTIONS

1. Given $B = 12\times10^{-8}\sin(1.20\times10^{7}\,z - 3.6\times10^{15}\,t)$ tesla. The standard equation for B of EM-wave is: $B = B_0\sin(kz - \omega t)$.

   Therefore, $B_0 = 12\times10^{-8}$ T

   Now, $I_{av} = \frac{1}{2}c\frac{B_0^2}{\mu_0} = \frac{\frac{1}{2}\times3\times10^{8}\times\left(12\times10^{-8}\right)^2}{4\pi\times10^{-7}}\ \text{W/m}^2$

   $= 1.71\ \text{W–m}^{-2}$

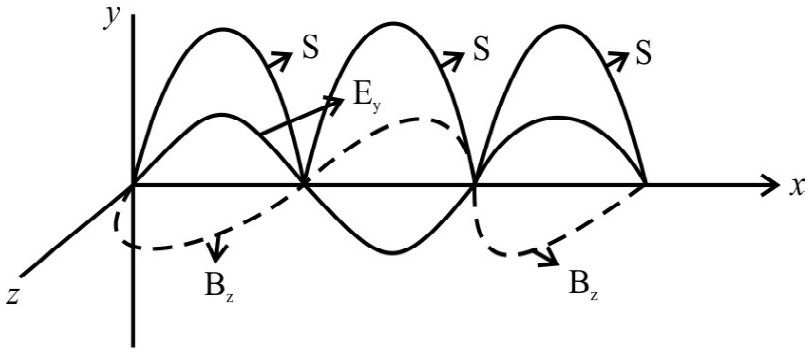
2. For an EM-wave propagating along *x*–direction, the electric field **E** is along the y–direction and the magnetic field B is along the z–direction, such that **E** × **B** is along the direction of wave velocity (i.e. along the *x*-direction).

   Now, $\mathbf{E}_y = E_0\sin(\omega t - kx)$

   and $\mathbf{B}_z = B_0\sin(\omega t - kx)$

   $\therefore\ \mathbf{S} = \frac{\mathbf{E}\times\mathbf{B}}{\mu_0} = \frac{E_0B_0}{\mu_0}\sin^2(\omega t - kx)\hat{i}$

   Hence, the graph of |**S**| verses t varies as square of give function, as shown below.



3. Force $F = \frac{\Delta P}{\Delta t}$ = rate of change of momentum imparted.

   $\therefore$ Pressure $P = \frac{F}{A}$

or $P = \dfrac{\left(\dfrac{\Delta P}{\Delta t}\right)}{A}$

or $P = \dfrac{\left(\dfrac{\Delta P}{\Delta t}\right)}{A} \times \dfrac{c}{c}$

or $P = \dfrac{1}{Ac\Delta t}(\Delta P \times c)$

or $P = \dfrac{1}{Ac\Delta t}(\Delta U)$ ($\Delta U = \Delta P \times c$ is energy imported to the surface)

But $I = \dfrac{\text{Energy imported}}{\text{Area} \times \text{time}}$

$= \dfrac{\Delta U}{A\Delta t}$

Hence, $P = \dfrac{I}{c}$

4. Let d be the distance between the plates of the capacitor. The applied voltage is

$V = V_0 \sin(2\pi\upsilon t)$

Therefore, the field E is

$E = \dfrac{V}{d} = \dfrac{V_0}{d}\sin(2\pi\upsilon t)$

Hence, the conduction current density is

$J_c = \dfrac{E}{\rho} = \dfrac{V_0}{\rho d}\sin(2\pi\upsilon t)$

or $J_c = J_{0_C}\sin(2\pi\upsilon t)$

$\therefore$ The maximum conduction current density $\left(J_{0_C}\right)$ is

$$J_{0_C} = \frac{V_0}{\rho d}$$

Now, the displacement current density is

$$J_d = \frac{i_d}{A} = \frac{1}{A}\ \varepsilon\frac{dQ_E}{dt} = \ \in \frac{dE}{dt} \qquad \left(\because\ Q_E = EA\right)$$

or $$J_d = \varepsilon\frac{d}{dt}\left(\frac{V_0}{d}\sin\left(2\pi\upsilon t\right)\right)$$

or $$J_d = \varepsilon\frac{V_0}{d}.2\pi\upsilon\cos\left(2\pi\upsilon t\right)$$

or $$J_d = J_{0_d}\cos\left(2\pi\upsilon t\right)$$

where $J_{0_d} = \varepsilon\frac{V_0}{d}.2\pi\upsilon$ is the maximum displacement current density.

Hence, $$\frac{J_{0_d}}{J_{0c}} = \frac{\varepsilon\frac{V_0}{d}.2\pi\upsilon}{\left(\frac{V_0}{\rho d}\right)}$$

$$= 2\pi\upsilon\rho\varepsilon$$

or $$\frac{J_{0_d}}{J_{0c}} = 2\pi\varepsilon_r \in_0 .\upsilon\rho$$

$$= \frac{1}{9\times10^9\times2}\times80\times4\times10^8\times0.25$$

$$= \frac{4}{9}$$

Answer: $$\frac{J_{0_d}}{J_{0_C}} = \frac{4}{9}$$

5. Defractive index $n = \dfrac{\text{Speed of light in air}}{\text{Speed of light in medium}}$

$= \dfrac{c}{v}$

Now, $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$v = \dfrac{1}{\sqrt{\mu \varepsilon}}$

$\therefore \quad n = \sqrt{\dfrac{\mu \varepsilon}{\mu_0 \varepsilon_0}}$

(It may be noted that :

$\mu = \mu_r \mu_0$

$\varepsilon = \varepsilon_r \varepsilon_0$

$= K \varepsilon_0$

$\therefore \quad n = \sqrt{\dfrac{\mu_r \mu_0 K \varepsilon_0}{\mu_0 \varepsilon_0}}$

or $n = \sqrt{\mu_0 K}$)

QUESTION BANK

Key Learning Points

1. Maxwell's equation for displacement current (i_d) in space $\left(i.e.\ i_d = \varepsilon_0 \frac{dd_E}{dt} \right)$ gave the first equation of Maxwell's electromagnetic wave thoery i.e. $\oint \mathbf{B}.\mathbf{d}\ell = \mu_0\varepsilon_0 \frac{d\phi_E}{dt}$, which is the modified Amepere's circuital law applicable for an imaginary loop in space where there is a time-varying electrica field existing.

2. The above equation along with Faraday's equation for emf induced in space $\left(\varepsilon = \oint \mathbf{E}.\mathbf{d}\ell\right)$, namely $\oint \mathbf{E}.\mathbf{d}\ell = \frac{-d\phi_B}{dt}$ $\left(\text{read } \varepsilon = \frac{-d\phi_B}{dt} \right)$, predicted the possible production of coupled time-varying electric and magnetic fields, the variation in one field acting as the source for the other field.

   This $\oint \mathbf{B}.\mathbf{d}\ell = \mu_0\varepsilon_0 \frac{d\phi_E}{dt}$ and

   $$\oint \mathbf{E}.\mathbf{d}\ell = \frac{-d\phi_B}{dt}$$

   together, as Maxwell's first two equation in electromagnetic wave theory predicted the production EM waves, which travel with the speed of light.

3. Along with the above two equations, the equations for Gauss's theorem in electrostatics and magnetism (namely $\oint \mathbf{E}.\mathbf{dS} = \frac{1}{\varepsilon_0}(q)$ and $\oint \mathbf{B}.\mathbf{dS} = 0$) were used by Maxwell to develop the complete thoery of EM waves.

4. Hertz experiments to produce and detect EM waves demonstrated the validity of Maxwell's theory.

5. The main properties of Em waves are:

   (i) They are made up of coupled time varying electric and magnetic fields (simusoidally varying).

   (ii) The variation is E-field produces the B-field and vice-versa.

   (iii) E and B fields one mutually perpendicular and both are perpendicular to the direction of propagation of the EM wave, such that: $\mathbf{E} \times \mathbf{B}$ is along **v** (the wave velocity). (This established the transverse nature of the EM-waves.

   (iv) **E** and **B** vary in same phase.

   (v) The equation for the sinusoidal **E** and **B** fields propagating in space along the *x*–direction can be expressed as:

$\mathbf{E} = E_0 \sin(\omega t - kx)\hat{j}$ and

$\mathbf{B} = B_0 \sin(\omega t - kx)\hat{k}$

where, $\omega = 2\pi\nu$ is the angular frequency of the wave and $k = \frac{2\pi}{\lambda}$ is the angular wave number.

Also, $\frac{\omega}{k} = \nu$ is the wave velocity, which is along the *x*–direction, here.

(vi) $v = c$ for free–space, where $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$.

(vii) For any material medium,

$$v = \frac{1}{\sqrt{\mu\varepsilon}} \frac{c}{\sqrt{\mu_r \varepsilon_r}}$$

(viii) The wave velocity is also equal to the ratio of E-field to B-field at any instant of time.

i.e. $v = \frac{E}{B} = \frac{E_0}{B_0}$

(ix) EM-waves carry energy. Where these waves are absorbed, the enery gets transferred to the material which absorbs the waves.

(x) The average energy density carried by the **E** and **B** fields are respectively,

$\overline{\mathbf{U}}_E = \frac{1}{2}\varepsilon_0 E_{rms}^2$ and $\overline{\mathbf{U}}_B = \frac{1}{2}\frac{B_{rms}^2}{\mu_0}$ or

or $\overline{\mathbf{U}}_E = \frac{1}{4}\varepsilon_0 E_0^2$ and $\overline{\mathbf{U}}_B = \frac{1}{4}\frac{B_0^2}{\mu_0}$

(xi) $\mathbf{U}_E = \mathbf{U}_B$ (i.e. both E and B fields carry same amount of energy).

(xii) The total energy density of an EM-wave is

$\overline{\mathbf{U}} = \overline{\mathbf{U}}E + \overline{\mathbf{U}}B$

or $\overline{\mathbf{U}} = 2\overline{\mathbf{U}}_E$

or $\mathbf{U} = \frac{1}{2}\varepsilon_0 E_0^2$

(xiii) The intensity (I) of EM–waves is defined as the total energy flowing through unit normal area in one second.

$$\therefore \quad I = \overline{U} \times c$$

$$\text{or} \quad I = \frac{1}{2}\varepsilon_0 c E_0^2 \qquad \left(\text{or } I \propto E_0^2\right)$$

(xiv) Since EM-waves carry energy and momentum these waves can exert pressure (called radiation pressure) on the incident surface for total absorption,

$$\text{Momentum, } P = \frac{U}{c} = \frac{\text{Total energy absorbed}}{\text{Speed of light in free space}}$$

$$\text{and radiation pressure} = \frac{\text{Change in momentum}}{\text{Surface area}}$$

$$= \frac{P}{A}$$

(xv) Pointing vector **S** is defined as:

$$S = \frac{1}{\mu_0}\left(\mathbf{E} \times \mathbf{B}\right)$$

Also, $I = |\mathbf{S}| = \left|\frac{\mathbf{E} \times \mathbf{B}}{\mu_0}\right|$; the direction of this vector gives the direction of propagation of the EM-wave.

(xiv) To produce a continuous train of EM waves, we need an oscillating / accelerating charge. This is done with the help of a dipole antenna connected to oscillating voltage source. The frequency of EM waves is exactly equal to the frequency of the oscillating charge or oscillating voltage source.

6. The electromagnetic spectrum consists of different regions; namely. Radio waves, micro waves, infrared radiations, visible light, ultraviolet rays, x–rays and gamma rays, in the ascending order of their frequency.

Unit–15 : Electromagnetic Waves

Easy *Source of Electromagnetic Waves*

1. Electromagnetic waves are produced by

(1) charges at rest, having the same linear charge density along two mutually perpendicular directions. (Incorrect)

(2) equal charges, moving with equal uniform velocities, along two mutually perpendicular directinos. (Incorrect)

(3) planer rectangular coils having a steady current flowing through them. (Incorrect)

(4) charges that move with a continuously changing velocity (Correct)

Correct answer: (4)

Solution:

Maxwell's theory, of EM waves, tells us that charges, when accelerated, radiate EM waves. It is only charges, moving with a changing velocity, that can be regarded as accelerated charges. Therefore, it is only such charges that can be a source of EM waves.

Unit–15 : Electromagnetic Waves

Average *Equations for EM Waves*

2. An electromagnetic wave, propagating along the *x*–axis, can be represented by the equations:

(1) $E_y = E_0 \sin(ky - \omega t);\ B_z = B_0 \sin(kz - \omega t)$ (Incorrect)

(2) $E_z = E_0 \sin(ky - \omega t);\ B_y = B_0 \sin(ky - \omega t)$ (Incorrect)

(3) $E_y = E_0 \sin(kx - \omega t);\ B_z = B_0 \sin(k\,x - \omega t)$ (Correct)

(4) $E_z = E_0 \sin(ky - \omega t);\ B_y = B_0 \sin(kz - \omega t)$ (Incorrect)

Correct answer: (3)

Solution:

For an EM wave, propagating along the *x*–direction, the electric and magnetic fields have to be along the y (/z) and z (/y) directions. Further there magnitudes show a simusoidal variation, for different points along their direction of propagation, i.e, along the *x*–axis. This simusoidal dependence, on *x*, corresponds only to the equations given in option (3).

Unit–15 : Electromagnetic Waves

Average *Features of EM Waves*

3. From the following, the only incorrect statement, about EM waves, is the statement:

(1) "EM waves can correspond only to self-sustaining oscillations of electric and magnetic fields in vaccum". (This statement is incorrect)

(2) "We do not need the hypothesis of the (hypothetical) medium 'the ether', to understand the production, and propagation, of EM waves". (It is a correct statement)

(3) "EM waves can propagate through material media". (It is a correct statement)

(4) "The standard, of length, the metre, is defined in terms of the (strictly) constant value of the velocity of EM waves in vaccum". (It is a correct statement)

Correct answer: (1)

Solution:

Statement (1) is incorrect, because of the presence of the word 'only' in it. EM waves can correspond to self sustaining oscillations of electric and magnetic fields, in vaccum, as well as in material media.

Statement (2), (3) and (4) are all (now) well known correct statements about EM waves.

Unit–15 : Electromagnetic Waves

Average *Relating **E** and **B** Vector*

4. The electric field, in a plane EM wave, has a value $\left(9\text{V}\text{m}^{-1}\right)\hat{\text{j}}$, at a point (a, b, c) in space at a time, t_0. The velocity of this wave is given by $\mathbf{c}=\left(3\times10^{8}\,\text{m/s}\right)\hat{\text{i}}$. The magnetic field, in this wave, at the point (a, b, c), and at time t_0, would be given by

(1) $\mathbf{B}=\left(3\times10^{-8}\,\text{T}\right)\left(-\hat{\text{k}}\right)$ (Incorrect)

(2) $\mathbf{B}=\left(27\times10^{8}\,\text{T}\right)\left(\hat{\text{k}}\right)$ (Incorrect)

(3) $\mathbf{B}=\left(3\times10^{-8}\,\text{T}\right)\left(\hat{\text{k}}\right)$ (Correct)

(4) $\mathbf{B}=\left(27\times10^{8}\,\text{T}\right)\left(-\hat{\text{k}}\right)$ (Incorrect)

Correct answer: (3)

Solution:

The magnitude of the magnetic field, in an EM wave, is related to the corresponding magnitude of electric field, through the relation

$$B = \frac{E}{c}$$

Hence the magnitude of the magnetic field (at the same point and at the same time), equals $\left(\frac{9}{3\times10^{8}}\right)$T, i.e. 3×10^{-8}T.

Further, the direction of propagation (i.e., the direction $\left(\hat{n}\right)$ in which energy is transferred, by the EM wave) is related to the directions of **E** and **B**, as $\hat{n} = \frac{(\mathbf{E}\times\mathbf{B})}{|\mathbf{E}\times\mathbf{B}|}$.

Here $\hat{n} = \hat{i}$ (given). We, therefore have

$$\hat{j}\times\frac{(\mathbf{B})}{|\mathbf{B}|} = \hat{i}$$

This gives $\hat{k}$ as the direction of **B**.

Hence $\mathbf{B} = \left(3\times10^{-8}\right)\hat{k}$

Unit–15 : Electromagnetic Waves

Difficult *Relating Directions of **E** and **B***

5. The magnetic field, (at a particular point x_0 **at a time** t_0**) in a plane EM wave, is given by**

$$\mathbf{B} = \mathbf{B}_y = \left[B_0 \sin\left(k x_0 + \omega t_0\right)\right]\hat{j}$$

The electric field, for this EM wave again at the point x_0 and at the time t_0, would be represented by

(1) $\mathbf{E} = \mathbf{E}_z = \left[cB_0 \sin\left(kx_0 + \omega t_0\right)\right]\left(\hat{k}\right)$ (Correct)

(2) $\mathbf{E} = \mathbf{E}_z = \left[cB_0 \sin\left(kx_0 + \omega t_0\right)\right]\left(-\hat{k}\right)$ (Incorrect)

(3) $\mathbf{E} = \mathbf{E}_z = \left[cB_0 \sin\left(kx_0 - \omega t_0\right)\right]\left(\hat{k}\right)$ (Incorrect)

(4) $\mathbf{E} = \mathbf{E}_z = \left[cB_0 \sin\left(kx_0 - \omega t_0\right)\right]\left(-\hat{k}\right)$ (Incorrect)

Correct answer: (1)

Solution:

A close look, at the equation for **B**, (where there is a +sign between the x and t terms) reveals that the EM wave is propagating along the $(-x)$ axis, i.e., has the direction $\left(-\hat{i}\right)$.

The direction of $\mathbf{E} \times \mathbf{B}$, i.e. $\left[\frac{(\mathbf{E})}{|\mathbf{E}|} \times \hat{j}\right]$, must, therefore, be $\left(-\hat{i}\right)$. Hence the direction of **E** has to be along $\left(+\hat{k}\right)$. We also cannot have a representation of **E** (where there is a (–ve) sing between the x and t terms) which implies that the EM wave is propagating along the $(+x)$ axis. The amplitude of $|\mathbf{E}|$ is cB_0. Hence the overall correct representation of **E** is the one given in option (1).

Unit–15 : Electromagnetic Waves

Average *Wavelength Estimation*

6. The electric field (in $\frac{V}{m}$), in a plane EM wave propagating along the x–axis, is given by

$$\mathbf{E} = \mathbf{E}_z = \left[9 \sin\left(10^3 x - 3 \times 10^{11} t\right)\right]\hat{k}$$

The wavelength, λ_0, of this wave, and the amplitude B_0, of its magnetic field, are given by

(1) $\lambda_0 = 1.26 \times 10^{-2}\,m; B_0 = 3 \times 10^{-8}\,T$ (Incorrect)

(2) $\lambda_0 = 0.63 \times 10^{-2}\,m; B_0 = 3 \times 10^{-8}\,T$ (Correct)

(3) $\lambda_0 = 1.26 \times 10^{-2}\,m; B_0 = 27 \times 10^{8}\,T$ (Incorrect)

(4) $\lambda_0 = 0.63 \times 10^{-2}\,m; B_0 = 27 \times 10^{8}\,T$ (Incorrect)

Correct answer: (2)

Solution:

Comparing the given equation with its standard form, namely, $E_z = E_0 \sin(kx - \omega t)$, we find that

$$k = \frac{2\pi}{\lambda} = 10^3 \text{ m}$$

$$\therefore \quad \lambda = \frac{2\pi}{10^3} \text{m} = 6.284 \times 10^{-3} \text{ m}$$

$$\cong 0.63 \text{ cm}$$

Also, the amplitude, B_0, of the magnetic field, equals $\frac{E_0}{c}$. Hence

$$B_0 = \frac{9}{3 \times 10^8} \text{T} = 3 \times 10^{-8} \text{T}$$

Unit–15 : Electromagnetic Waves

Average *Frequency Estimation*

7. The magnetic field, (in tesla) in a plane EM wave, propagating along the *x*–axis, is given by

$$\mathbf{B} = \mathbf{B}_y = 3 \times 10^{-8} \sin(10^3 x - 3 \times 10^{11} t)$$

The frequency of the EM wave, and the part of the EM spectrum, to which this wave could belong, are given, respectively, by

(1) 23.9 GHz; (short radio waves / microwaves) (Incorrect)

(2) 23.9 GHz; (infrared waves) (Incorrect)

(3) 47.7 GHz; (infrared waves) (Incorrect)

(4) 47.7 GHz; (short radio waves / microwaves) (Correct)

Correct answer: (4)

Solution:

Comparing the given equation with its standard form, namely, $B_y = B_0 \sin(kx - \omega t)$, we find that

$$\omega (= 2\pi\upsilon) = 3 \times 10^{11} \text{s}^{-1}$$

$\therefore \quad \upsilon = \dfrac{3\times10^{11}}{2\pi}\,\text{Hz} = 47.7\times10^{9}\ \text{Hz}$

$= 47.7\ \text{GHz}$

This frequency corresponds to the (short radio wave / microwave) part of the EM spectrum. (The infrared waves have frequencies of the order of 10^{12} Hz to 10^{14} Hz).

Unit–15 : Electromagnetic Waves

Average *Estimating the Velocity of EM Waves*

8. The electric field (in SI Units), in a plane EM wave, propagating along the *x*–axis, is given by

$$\mathbf{E} = \mathbf{E}z = 27\sin\left(1.02\times10x - 2.95\times10^{15}\,t\right)$$

The speed of propagation of this wave (as per this data), and the part of the EM spectrum, to which this wave corresponds, are given respectively, by

(1) 3.01×10^{8} m/s; (visible) (Incorrect)

(2) 2.89×10^{8} m/s; (visible) (Correct)

(3) 3.01×10^{8} m/s; (infrared) (Incorrect)

(4) 2.89×10^{8} m/s; (infrared) (Incorrect)

Correct answer: (2)

Solution:

Comparing the given equation, with its standard form, namely $E_z = E_0\sin\left(kx-\omega t\right)$, we find that

$$k\left(=\frac{2\pi}{\lambda}\right) = 1.02\times10^{7}\,\text{m}^{-1} \quad \text{and} \quad \omega\left(=2\pi\lambda\right) = 2.95\times10^{15}\,\text{s}^{-1}$$

$\therefore$ We get $\lambda = \dfrac{2\pi}{1.02}\times10^{-7}\,\text{m} = 6.160\times10^{-7}\,\text{m}$

$= 6160\,\text{Å}$

This corresponds to the green colour in the visible part of EM spectrum.

Also speed of propagation $= \upsilon\lambda = \dfrac{\omega}{k}$

$$= \frac{2.95\times10^{15}}{1.02\times10^{7}}\,\text{m/s}$$

$$\simeq 2.89\times10^{8}\,\text{m/s}$$

Unit–15 : Electromagnetic Waves

Average *Writing the Equation for Electric Field of an EM-Wave*

9. An EM wave has a wavelength 6.284×11^{-7} m and a frequency 4.77×10^{14} Hz. It is propagating along the (+*x*) - axis and its magnet field oscillations are along the (-y) - axis. The electric field, for this wave, can be represented [in its standard form $E = E_0 \sin(kx \mp \omega t)$] as

(1) $E_y = E_0 \sin\left(10^7 x + 2.995\times10^{15} t\right)$ (Incorrect)

(2) $E_z = E_0 \sin\left(2.997\times10^{15} x + 10^7 t\right)$ (Incorrect)

(3) $E_y = E_0 \sin\left(2.997\times10^{15} x - 10^7 t\right)$ (Incorrect)

(4) $E_z = E_0 \sin\left(10^7 x + 2.997 - 10^{15} t\right)$ (Correct)

Correct answer: (4)

Solution:

We have

$$k = \frac{2\pi}{\lambda} \text{ and } \omega = 2\pi\upsilon$$

$\therefore$ In the given case

$$k = \frac{2\pi}{6.284\times10^{-7}}\,\text{m}^{-1} = 10^7\,\text{m}^{-1}$$

and $\omega = 2\pi\upsilon = 2\pi\left(4.77\times10^{14}\right)\text{s}^{-1} = \left(2.997\times10^{15}\right)\text{s}^{-1}$

We, therefore, use these values of k and ω to write the equation in its standard form. We need to use the (–ve) sign between the *x* and t, terms as the wave is propagating along the +*x*–axis. Further, the elctric field has to be along the z–axis, the third of the three perpendicular directions, satisfying the condition that the direction of propagation is along **EXB.**

Unit–15 : Electromagnetic Waves

Difficult *Writing the Equation for the Magnetic Field of an EM-Wave*

10. An EM-wave has a wavelength of $3142\,\text{Å}$ and a frequency of 9.54×10^{14} Hz. Its electrical field oscillations are along the *x*–axis and it is transmitting energy along the (+ z)–axis. The equation, representing its magnetic field oscillations in the standard form (like $B_z = B_0 \sin(kx \mp \omega t)$, would be:

(1) $B_y = B_0 \sin\left[2\times10^7 x - 5.99\times10^{15} t\right]$ (Incorrect)

(2) $B_y = B_0 \sin\left[2\times10^7 z - 5.99\times10^{15} t\right]$ (Correct)

(3) $B_y = B_0 \sin\left[5.99\times10^{15} z - 2\times10^7 t\right]$ (Incorrect)

(4) $B_y = B_0 \sin\left[5.99\times10^{15} x - 2\times10^7 t\right]$ (Incorrect)

Correct answer: (2)

Solution:

The direction of transmission of energy (given to be along the z–axis) is also the direction of the cross product **E** × **B**. Since **E** is (given) oscillating along the *x*–axis and **E** × **B** is along the z–axis, the oscillations of **B** has to be along the y–axis. From the given data

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{3142\times10^{-10}} = 2\times10^7\,m^{-1}$$

and $\omega = 2\pi\upsilon = 2\pi\times9.54\times10^{14}\,s^{-1} \simeq 5.99\times10^{15}\,s^{-1}$

The direction of propagation, being the (+ z)–axis, we would write

$$B = B_0 \sin\left(2\times10^7 z - 5.99\times10^{15} t\right)$$

Unit–15 : Electromagnetic Waves

Average *Electromagnetic Spectrum*

11. The oscillating electric field (E_1) of an EM wave (1), and the oscillating magnetic field (B_2), of an EM wave (2), are represented, respectively, by

$$E_1 = E_0 \sin\left(10^{13} x - 3\times10^{21} t\right)$$

and $B_2 = B_0 \sin\left(6\times10^5 z - 1.8\times10^{14} t\right)$

The parts of the EM spectrum, to which waves (1) and (2), are likely to belong, are

(1) UV rays and microwaves, respectively (Incorrect)

(2) γ –rays and infrared rays, respectively (Correct)

(3) *x*–rays and radio waves, respectively (Incorrect)

(4) UV rays and microwaves, respectively (Incorrect)

Correct answer: (2)

Solution:

For wave (1), $k = \left(10^{13}\right) m^{-1}$

$$\therefore \quad \lambda = \frac{2\pi}{k} = 2\pi\times10^{-13} m$$

This wavelength value corresponds to γ –rays.

For wave (2), $k = \left(6\times10^5\right) m^{-1}$

$$\therefore \quad \lambda = \frac{2\pi}{6\times10^5} \approx 1.02\times10^{-5} m$$

$$= 10200 \ \overset{o}{A}$$

This wavelength value corresponds to infrared rays.

Hence waves (1) and (2) are likely to be γ –rays and infrared rasy, respectively.

Unit–15 : Electromagnetic Waves

Average *EM Spectrum*

12. The equation $E = E_0 \sin\left(k x - \omega t\right)$ could represent the oscillations of the electric field of a microwave if k and ω have the values

(1) 10^{10} m^{-1} and 3×10^{18} s^{-1}, respectively (Incorrect)

(2) 10^7 m^{-1} and 3×10^{15} s^{-1}, respectively (Incorrect)

(3) $6\times10^{2}\ m^{-1}$ and $1.8\times10^{11}\ s^{-1}$, respectively (Correct)

(4) $6\times10^{-2}\ m^{-1}$ and $1.8\times10^{7}\ s^{-1}$, respectively (Incorrect)

Correct answer: (3)

Solution:

For option (1), $\lambda \simeq 2\pi\times10^{-10}\,m$ (x–rays)

For option (2), $\lambda \simeq 2\pi\times10^{-7}\,m$ (visible light)

For option (3), $\lambda \simeq 1.05\times10^{-2}\,m \simeq 1.02\,cm$ (microwaves)

For option (4), $\lambda \simeq 1.05\times10^{2}\,m$ (radiowaves)

(In each case, the value of ω is $(\simeq ck)$, as it should be)

Hence only option (3) could correspond to the representatin of microwaves.

Unit–15 : Electromagnetic Waves

Average *Related Values of ω and k, for an EM Wave*

13. A student writes the following expression for the oscillations of the magnetic field of an em wave: $B = B_0 \sin\left(10^{13}x - 3\times10^{20}t\right)$

Assuming that his choice of the value, of the coefficient of x, is correct, the value, of his quoted coefficient of t, is

(1) less than the correct value by (nearly) 2.7×10^{21} (Correct)

(2) more than the correct value by 2.7×10^{21} (Incorrect)

(3) less than the correct value by (nearly) 3×10^{19} (Incorrect)

(4) more than the correct value by 3×10^{19} (Incorrect)

Correct answer: (1)

Solution:

As per the stand form of representation, namely $B = B_0 \sin\left(10^{13}x - 3\times10^{20}t\right)$

the values of the coefficient of x and t, correspond to 'k' and 'ω', respectively.

Since $\frac{\omega}{k} = c$ (the velocity of em waves), we have $\omega = ck = \left(3\times10^{8}\right)k$

$\therefore$ If k is $10^{13}m^{-1}$, we should have $\omega = 3\times10^{21}s^{-1}$

The given value is $\omega = 3\times10^{20}s^{-1} = 0.3\times10^{21}s^{-1}$

$\therefore$ The given value is less than the correct value by $\left(3-0.3\right)\times10^{21}s^{-1}$ $\left(\text{or } 2.7\times10^{21}s^{-1}\right)$

Unit–15 : Electromagnetic Waves

Average *Related Values of ω and k, for an EM Wave*

14. A student writes the following expression, for the oscillation of the electric field of an em wave: $E_x = E_0 \sin\left(6\times10^{11}z - 1.8\times10^{19}t\right)$

Assuming that she has correctly written the value of the coefficient of t, the value of the coefficient of *x*, given by her, is

(1) less than its correct value by (nearly) $5.4\times10^{11}m^{-1}$ (Incorrect)

(2) more than its correct value by (nearly) $5.4\times10^{11}m^{-1}$ (Correct)

(3) less than its correct value by (nearly) $6.6\times10^{11}m^{-1}$ (Incorrect)

(4) more than its correct value by (nearly) $6.6\times10^{11}m^{-1}$ (Incorrect)

Correct answer: (2)

Solution:

As per the standard representation, namely $E_x = E_0 \sin\left(kz-\omega t\right)$, the value, of the coefficient of z, corresponds to k and that, of the coefficient of t, corresponds to ω. We also have $k = \frac{\omega}{c}$

$\therefore$ For the given (correct) value of $\omega\left(=1.8\times10^{19}\right)$, we should have $k = \frac{1.8\times10^{19}}{3\times10^{8}}m^{-1} = 6\times10^{10}m^{-1}$

The given value is $k = 6\times10^{11}m^{-1} = 60\times10^{10}m^{-1}$

This is more than the correct value by $\left(60-6\right)\times10^{10}m^{-1} = \left(54\times10^{10}\right)m^{-1} = 5.4\times10^{11}m^{-1}$

Unit–15 : Electromagnetic Waves

Average *Identifying the Part of the EM Spectrum*

15. The oscillation, of the magnetic field, of an EM wave, propagating along the *x*–axis, are represented by $\mathbf{B = B_0 \sin(kx - \omega t)}$

This EM wave could be visible light if the values of k and ω are (nearly)

(1) $10^{10}\,m^{-1}$ and $3\times10^{18}\,s^{-1}$, respectively (Incorrect)

(2) $10^{8}\,m^{-1}$ and $3\times10^{16}\,s^{-1}$, respectively (Incorrect)

(3) $10^{7}\,m^{-1}$ and $3\times10^{15}\,s^{-1}$, respectively (Correct)

(4) $10^{3}\,m^{-1}$ and $3\times10^{11}\,s^{-1}$, respectively (Incorrect)

Correct answer: (3)

Solution:

We have $k = \dfrac{2\pi}{\lambda}$

$\therefore \quad \lambda = \dfrac{2\pi}{k}$

For option (1)

$\lambda \simeq 2\pi\times10^{-10}\,m^{-1} \simeq 6.14\ \mathring{A}$ $(x-\text{rays})$

For option (2)

$\lambda \simeq 2\pi\times10^{-8}\,m \simeq 614\ \mathring{A}$ $(uv-\text{rays})$

For option (3)

$\lambda \simeq 2\pi\times10^{-7}\,m \simeq 6140\ \mathring{A}$ (visible light)

For option (4)

$\lambda \simeq 2\pi\times10^{-3}\,m \simeq 6.14\ mm$ (microwaves)

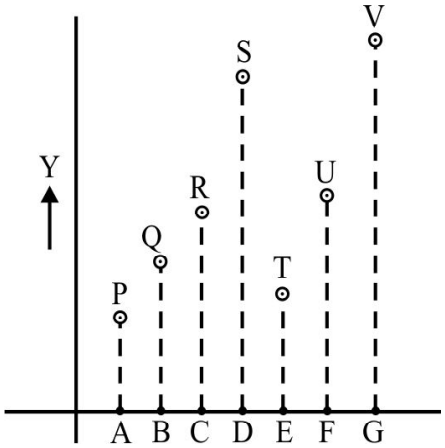
(In each case, the value of $\omega(=ck)$, has been correctly quoted).

Hence only option (3) could correspond to visible light.

Unit–15 : Electromagnetic Waves

Average *Characterstics of EM Waves*

16. In the figure drawn here, the points A, B, C, D, E, F and G, on the *x*–axis, correspond to radiowaves; infrared waves, visible light, *x*–rays; microwaves, uv rays and γ –rays, in that order. The points P, Q, R, S, T, U, V, shown in the figure correspond to certain values, of a varible y, of these waves, on the y–axis. The variable y, could be the



(1) wavelength (λ), or the angular frequency (ω), of these waves (Incorrect)

(2) frequency (ν), or the wave number (k), of these waves (Correct)

(3) angular frequency (ω), or the velocity (v) of these waves in vaccum (Incorrect)

(4) velocity (v), in vaccum, or the wave number (k), of these waves in vaccum (Incorrect)

Correct answer: (2)

Solution:

For the given order of points A, B, C, D, E, F, G; the frequency increases for the first four waves; then decreases (to a value between the first two radius); then increases but to a value between the third and fourth values and finally increases to a value higher than all the other values. The wave number $k\left(=\frac{2\pi}{\lambda}\right)$ showss a similar behaviour (This is in accord with the changes in the values of variable Y, as show in the figure). The wavelength λ behaves is an exactly opposite manner while v has the same value for all the waves.

Hence option (2) is the only correct option.

Unit–15 : Electromagnetic Waves

Average *Characterstics of EM Waves*

17. An em wave is propagating along the positive *x*–axis. The representation, of the oscillations of its electric field, and the direction of oscillations of its E and B vectors, could then be

(1) $E = E_0 \sin(kx - \omega t);\ (\mathbf{E} = \mathbf{E}_y \text{ and } \mathbf{B} = \mathbf{B}_z)$ (Correct)

(2) $E = E_0 \sin(kx - \omega t);\ (\mathbf{E} = \mathbf{E}_z \text{ and } \mathbf{B} = \mathbf{B}_y)$ (Incorrect)

(3) $E = E_0 \sin(kx + \omega t);\ (\mathbf{E} = \mathbf{E}_z \text{ and } \mathbf{B} = \mathbf{B}_y)$ (Incorrect)

(4) $E = E_0 \sin(kx + \omega t);\ (\mathbf{E} = \mathbf{E}_y \text{ and } \mathbf{B} = \mathbf{B}_z)$ (Incorrect)

Correct answer: (1)

Solution:

The wave is propagating along the (+ ve) *x*–axis. Hence we need to have a (– ve) sign between the '*x*' and 't' terms in the representative equation of the oscillations of its electric field.

Further, the cross product ($\mathbf{E} \times \mathbf{B}$) must also have the direction of propagation of the em. Waves, i.e. the (+ ve) *x*–axis. This is possible if we take ($\mathbf{E} = \mathbf{E}_y$) and ($\mathbf{B} = \mathbf{B}z$); the other choice ($\mathbf{E} = \mathbf{E}_z$ and $\mathbf{B} = \mathbf{B}_y$) gives the direction of ($\mathbf{E} \times \mathbf{B}$) as the (– ve) *x*–axis.

Hence option (1), consistent with both these requirements, is the correct option.

Unit–15 : Electromagnetic Waves

Easy *Electromagnetic Spectrum*

18. The angular frequencies, for three em waves E_1, E_2, E_3 have values ω_1, ω_2 and ω_3, respectively. If ω_1 is greater than both ω_2 and ω_3 and ω_2 is less than ω_3, the three em waves could be, respectively,

(1) *x* – rays, visible light, microwaves (Incorrect)

(2) γ – rays, uv – rays, microwaves (Incorrect)

(3) visible light, infrared rays, radiowaves (Incorrect)

(4) uv – rays, radiowaves, visible light (Correct)

Correct answer: (4)

Solution:

We are given that $\omega_1 > (\omega_2 \text{ or } \omega_3)$ and $\omega_2 < \omega_3$

$\therefore$ The angular frequencies are in the order $\omega_1 > \omega_3 > \omega_2$

The three em waves must, therefore, have their frequencies as per this requirement. In all the options given, ω_1 is greater than both ω_2 and ω_3. However, in the first three options, $\omega_2 < \omega_3$. It is only in the fourth option that we have $\omega_2 < \omega_3$. Hence option (4) is the correct choice.

Unit–15 : Electromagnetic Waves

Average *Electromagnetic Spectrum*

19. The wave number (k), for three em waves (x_1, x_2 and x_3) has value k_1, k_2 and k_3, respectively. It is given that k_1 is less than both k_2 and k_3 while k_2 is greater than k_3. The three em waves, (x_1, x_2, x_3) could then be

(1) infrared rays, uv rays, visible light (Correct)

(2) visible light, uv rays, γ – rays (Incorrect)

(3) radiowaves, visible light, uv rays (Incorrect)

(4) microwaves, visible light, γ – rays (Incorrect)

Correct answer: (1)

Solution:

We have $k = \frac{2\pi}{\lambda}$

$\therefore$ We must have λ_1 more than both λ_2 and λ_3.

Also we must have λ_2 less than λ_3. Hence the three em waves must have their wavelenghts in the order.

$$\lambda_1 > \lambda_3 > \lambda_2$$

This ordering of wavelenghts is valid only in option (1). In all the other three options, λ_2 is more than λ_3.

Hence option (1) is the correct choice.

Unit–15 : Electromagnetic Waves

Difficult *Electromagnetic Spectrum*

20. The velocity (v), in a given material medium, has values v_1, v_2 and v_3, respectively for three em waves (y_1, y_2 and y_3). It is given that v_3 is more than both v_1 and v_2 but v_2 is less than v_1. The three em waves (y_1, y_2, y_3) could than be, respectively

(1) γ – rays, uv rays, visible light (Incorrect)

(2) uv rays, visible light, microwaves (Incorrect)

(3) infrared rays, uv rays, radiowaves (Correct)

(4) x – rays, visible light, infrared rays (Incorrect)

Correct answer: (3)

Solution:

We know that the refractive index of a medium decreases with increase in wavelength $\left(\because\ \mu \simeq A + \frac{B}{\lambda^2}\right)$.

Further, since $\mu = \frac{\text{speed of light in vaccum}}{\text{speed of light in medium}}$, a decrease, in the value of μ, implies a higher value of the speed of light in the medium. We, therefore, can conclude, that:

'Higher wavelength implies a higher speed of light in the medium.'

We are given that $v_3 > (v_1 \text{ or } v_2)$ and $(v_2 < v_1)$. Thus $v_3 > v_1 > v_2$. This implies $\lambda_3 > \lambda_1 > \lambda_2$. This sequence of wavelengths values is valid only in option (3).

Hence option (3) is the correct choice.

Unit–15 : Electromagnetic Waves

Difficult *Electromagnetic Spectrum*

21. An em wave is propagating along the (+) *x*–axis. Let there be a point charge initially at rest located in some plane parallel to the y–z plane. At a time instant, just (infinitesimally) greater than the instant at which the em wave reaches this plane, this charge could experience forces, due to the E and B fields, of the em wave, along the

(1) y–axis and *x*–axis, respectively (Correct)

(2) z–axis and y–axis, respectively (Incorrect)

(3) *x*–axis and y–axis, respectively (Incorrect)

(4) y–axis and z–axis, respectively (Incorrect)

Correct answer: (1)

Solution:

When the wave propagates along the (+) *x*–axis, its **E** and **B** fields oscillate in the yz plane; the **E** field along the y–axis and **B** field along the z–axis. (This is because $\mathbf{E} \times \mathbf{B}$ has the same direction as the direction of propagation, i.e. $+x$ – axis, in this case).

The charge, initially at rest, experiences a force due to the **E** field, along the y–axis. If thus acuiqres a velocity along the y–axis. The magnetic field **B** (along the z–axis) would then exert a force along the *x*–axis (the direction of $\mathbf{v} \times \mathbf{B}$). This could happen at a time instant just infinitesimally greater than the time instant at which the em wave reaches the plane containing the charge).

Unit–15 : Electromagnetic Waves

Easy *Transfer of Energy by EM Waves*

22. EM waves can transport energy;

(1) however, it is only the **E** field that contributes to this transfer of energy (Incorrect)

(2) however, it is only the **B** field that contributes to this transfer of energy (Incorrect)

(3) both the **E** and **B** fields contribute to the transfer of energy; their contributions are in the ratio 2 : 1 (Incorrect)

(4) both the **E** and **B** fields contribute to the transfer of energy; their contributions are in the ratio 1 : 1 (Correct)

Correct answer: (4)

Solution:

EM waves, (having energy associated with their electric and magnetic fields) can transport energy from one place to another.

Both the electric and mangetic fields contribute, in this transfer of energy. Further, the energy of an EM wave, is shared equally between its electric and magnetic fields. Hence the ratio of their contributions, in the transfer of energy, equals (1 : 1).

Unit–15 : Electromagnetic Waves

Easy *Transfer of Energy and Momentum by EM Waves*

23. (1) EM waves can transport energy (through their **E** field) but not momentum (Incorrect)

(2) EM waves can transport energy (through their **B** field) but not momentum (Incorrect)

(3) EM waves can transport energy as well as momentum (through their **E** field only) (Incorrect)

(4) EM waves can transport energy as well as momentum (through their **E** and **B** fields) (Correct)

Correct answer: (4)

Solution:

EM waves (having energy, associated with their electric and magnetic fields) can transport energy from one place to another.

It has been shown that they can trasnfer momentum also.

$$\left(\text{Total momentum delivered} = \frac{\left(\text{Total energy transferred}\right)}{c}\right)$$

The electric, and the magnetic fields, of an em wave, contribute equally in this transfer of energy and momentum.

Unit–15 : Electromagnetic Waves

Easy *Energy Density of the **E** and **B** Fields*

24. The (instantaneous) energy density (energy per unit volume) of the electric and magnetic fields, of an em wave, are equal, respectively, to

(1) $\frac{1}{2}\frac{E^2}{\varepsilon_0}$ and $\frac{1}{2}\mu_0 B^2$; with $\frac{1}{2}\frac{E^2}{\varepsilon_0} \neq \frac{1}{2}\frac{B^2}{\mu_0}$ (Incorrect)

(2) $\frac{1}{2}\frac{E^2}{\varepsilon_0}$ and $\frac{1}{2}\mu_0 B^2$; with $\frac{1}{2}\frac{E^2}{\varepsilon_0} = \frac{1}{2}\frac{B^2}{\mu_0}$ (Incorrect)

(3) $\frac{1}{2}\varepsilon_0 E^2$ and $\frac{1}{2}\frac{B^2}{\mu_0}$; with $\frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\frac{B^2}{\mu_0}$ (Correct)

(4) $\frac{1}{2}\varepsilon_0 E^2$ and $\frac{1}{2}\frac{B^2}{\mu_0}$; with $\frac{1}{2}\varepsilon_0 E^2 \neq \frac{1}{2}\frac{B^2}{\mu_0}$ (Incorrect)

Correct answer: (3)

Solution:

The (instantaneous) energy density (energy per unit volume), of the electric and magnetic fields, of an em wave, is given by;

$$\text{(Volume) energy density of electric field} = \frac{1}{2}\varepsilon_0 E^2$$

$$\text{and (volume) energy density of magnetic field} = \frac{1}{2}\frac{B^2}{\mu_0}$$

Here E and B refer to the instantaneous values of the electric and magnetic fields.

The energy, of an em wave, in shared equally between its electric and magnetic fields. We, therefore, also have

$$\frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\frac{B^2}{\mu_0}, \text{ (at all instants)}$$

Unit–15 : Electromagnetic Waves

Easy *Transfer of Energy, Momentum and 'Pressure' by EM Waves*

25. It has been experimentally demonstrated that

(1) EM waves can transport energy but not momentum (Incorrect)

(2) EM waves can transport energy but not momentum; however, they do exert a 'radiation pressure' (Incorrect)

(3) EM waves can transport both energy and momentum; however, they do not exert any 'radiation pressure' (Incorrect)

(4) EM waves can transport both energy and momentum; they also exert a 'radiation pressure' (Correct)

Correct answer: (4)

Solution:

EM waves (having energy associated with their electric and magnetic fields) can transfer energy from one place to another.

It has been shown that they can transfer momentum also (Total momentum delivered in time Δt)

$$= \Delta p = \frac{\text{Total energy transferred in time}(\Delta t)}{c}$$

The average rate of transfer of momentum $\left(= \frac{\Delta p}{\Delta t}\right)$, equals the force exerted on the surface (absorbing the radiant energy). This force, per unit area of the surface, would then equal the 'radiation pressure'.

Unit–15 : Electromagnetic Waves

Average *Energy Transfer by EM Waves*

26. The total energy transfered to a (perfectly black) surface of area A (located normal to the dirction of propagation of the EM wave) and thickness d, by an em wave, over one complete cycle, can be expressd as

(1) $\left[\frac{1}{2}\varepsilon_0\left(\overline{E}\right)^2 + \frac{1}{2}\mu_0\left(\overline{B}\right)^2\right](Ad)$ (Incorrect)

(2) $\left[\frac{1}{2}\varepsilon_0\left(\overline{E}\right)^2 + \frac{1}{2}\frac{\left(\overline{B}\right)^2}{\mu_0}\right](Ad)$ (Incorrect)

(3) $\left[\frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2}\frac{B_{rms}^2}{\mu_0}\right](Ad)$ (Correct)

(4) $\left[\frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2}\mu_0 B_{rms}^2\right](Ad)$ (Incorrect)

Correct answer: (3)

Solution:

We know that

$$\text{(Volume) energy density of electric field} = \frac{1}{2}\varepsilon_0 E^2$$

$$\text{(Volume) energy density of magnetic field} = \frac{1}{2}\left(\frac{B^2}{\mu_0}\right)$$

The energy, of the em wave, is shared equally between its electric and magnetic fields.

Over one complete cycle, we need to use E_{rms} and B_{rms} in the above expressions.

[We cannot use $\overline{E}$ and $\overline{B}$, as these equal zero each over one complete cycle].

The volume absorbing this energy = (Ad)

Hence total energy transferred over one complete cycle

$$= \left[\frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2}\frac{B_{rms}^2}{\mu_0}\right][Ad]$$

Unit–15 : Electromagnetic Waves

Difficult *Force due to EM Wave*

27. An electromagnetic wave (of frequency υ), propagating along the *x*–axis, is incident normally on a perfect black surface of area A and thickness d. The average force, exerted on the surface, over one complete cycle, of this EM wave, is

(1) $\left[\frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2}\frac{B_{rms}^2}{\mu_0}\right]\left(\frac{A\upsilon d}{c}\right)$ (Correct)

(2) $\left[\frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2}\frac{B_{rms}^2}{\mu_0}\right](cA\upsilon d)$ (Incorrect)

(3) $\left[\frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2}\frac{B_{rms}^2}{\mu_0}\right]\left(\frac{c}{A\upsilon d}\right)$ (Incorrect)

(4) $\left[\frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2}\frac{B_{rms}^2}{\mu_0}\right]\left(\frac{cAd}{\upsilon}\right)$ (Incorrect)

Correct answer: (1)

Solution:

We know that

(Volume) energy density of electric field = $\frac{1}{2}\varepsilon_0 E^2$

(Volume) energy density of magnetic field $= \frac{1}{2}\left(\frac{B^2}{\mu_0}\right)$

The energy, of the em wave, is shared equally between its electric and magnetic fields.

Over one complete cycle, we need to use $E_{rms}\left(=\frac{E_0}{\sqrt{2}}\right)$ and $B_{rms}\left(=\frac{B_0}{\sqrt{2}}\right)$ in the above expressions.

$\therefore$ Total energy transferred over one complete cycle

$$= U = \left[\frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2}\frac{B_{rms}^2}{\mu_0}\right][Ad]$$

$\therefore$ Momentum delivered $= p = \frac{U}{c} = \left[\frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2}\frac{B_{rms}^2}{\mu_0}\right]\left[\frac{Ad}{c}\right]$ over one cycle.

Time taken = Time period $= \frac{1}{\upsilon}$

$\therefore$ Average force $= \frac{p}{\left(\frac{1}{\upsilon}\right)} = p\upsilon$

$$= \left[\frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2}\frac{B_{rms}^2}{\mu_0}\right]\left[\frac{\upsilon Ad}{c}\right]$$

Unit–15 : Electromagnetic Waves

Difficult *Energy of an EM Wave*

28. The oscillatory electric field, of an em wave, propagating along the + z–axis, is represented by $E_x = E_0 \sin(kz - \omega t)$.

This em wave is incident on a perfectly black circular disc, of diameter d and height h, located parallel to the *x*–y plane. The total energy absorbed by this surface, over one complete cycle of this em wave, would equal

(1) $\left(\frac{\pi d^2 h E_0^2}{8\mu_0 c^2}\right)$ (Correct)

(2) $\frac{\pi d^2 h E_0^2}{16 c^2 \mu_0}\left[1+\frac{\mu_0 \varepsilon_0}{c^2}\right]$ (Incorrect)

(3) $\frac{\pi d^2 h E_0^2}{4 c^2}\left[\varepsilon_0+\frac{1}{c^2 \mu_0}\right]$ (Incorrect)

(4) $\frac{\pi d^2 h E_0^2}{8 \varepsilon_0 c^2}$ (Incorrect)

Correct answer: (1)

Solution:

We know that

$$\text{(Volume) energy density of electric field} = \frac{1}{2}\varepsilon_0 E^2$$

$$\text{(Volume) energy density of magnetic field} = \frac{1}{2}\left(\frac{B^2}{\mu_0}\right)$$

The energy of the EM wave is shared equally between the electric and the magnetic fields.

Over one complete cycle, we need to use E_{rms} and B_{rms} in the above expressions.

Also $E_{rms} = \frac{E_0}{\sqrt{2}}$ and $B_{rms} = \frac{B_0}{2}$

We also have $B_0 = \left(\frac{E_0}{c}\right)$

$$\text{The volume of the circuluar disc} = \left(\frac{\pi d^2}{4}\right) h$$

$\therefore$ Total energy delivered over one complete cycle

$$= \left[\frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2}\frac{B_{rms}^2}{\mu_0}\right]\left(\frac{\pi d^2}{4}\right) h$$

$$= \frac{1}{16}\left(\pi d^2 h\right)\left[\varepsilon_0 E_0^2 + \frac{E_0^2}{c^2\mu_0}\right] = \frac{\pi d^2 h}{16}\varepsilon_0 E_0^2\left[1 + \frac{1}{c^2\varepsilon_0\mu_0}\right]$$

$$= \frac{\pi d^2 h \varepsilon_0 E_0^2}{8} = \frac{\pi d^2 h E_0^2}{8\mu_0 c^2} \qquad \left(\because \ \mu_0\varepsilon_0 = \frac{1}{c^2}\right)$$

Unit–15 : Electromagnetic Waves

Difficult *Energy Transfer by an EM Wave*

29. The oscillatory magnetic field, of an em wave, propagating along the (–) *x*–axis, is represented by $B_y = B_0 \sin\left(kx + \omega t\right)$

This em wave is incident on a perfectly black circular disc (or radius r and thickness d) kept parallel to the y–z plane. The total energy, transferred to this disc, over one complete cycle, by this em wave, would equal

(1) $\frac{B_0^2}{4}\left[\varepsilon_0 + \frac{1}{\mu_0 c^2}\right]\left(\pi r^2 d\right)$ (Incorrect)

(2) $\frac{B_0^2}{8}\left[\frac{\varepsilon_0}{c^2} + \mu_0\right]\left(\pi r^2 d\right)$ (Incorrect)

(3) $\frac{B_0^2}{8}\left[\varepsilon_0 + \frac{1}{\mu_0 c^2}\right]\left(\pi r^2 d\right)$ (Incorrect)

(4) $\frac{B_0^2}{4}\left[\varepsilon_0 c^2 + \frac{1}{\mu_0}\right]\left(\pi r^2 d\right)$ (Correct)

Correct answer: (4)

Solution:

We know that

(Volume) energy density of the electric field $= \frac{1}{2}\varepsilon_0 E^2$

(Volume) energy density of the magnetic field $= \frac{1}{2}\frac{B^2}{\mu_0}$

The energy, of the em wave, is shared equally between the electric + magnetic fields. Over one complete cycle, we need to use $E_{rms}\left(=\frac{E_0}{\sqrt{2}}\right)$ and $B_{rms}\left(=\frac{B_0}{\sqrt{2}}\right)$ in the above expressions.

Also $E_0 = cB_0$ and $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

$\therefore$ Total energy, transferred to the disc, over one complete cycle

$$= \left[\frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2}\frac{B_{rms}^2}{\mu_0}\right]\left(\pi r^2 d\right)$$

$$= \left[\frac{1}{2}\varepsilon_0 \frac{c^2 B_0^2}{2} + \frac{1}{2}\mu_0 \frac{B_0^2}{2}\right]\pi r^2 d$$

$$= \frac{B_0^2}{4}\left[\varepsilon_0 c^2 + \frac{1}{\mu_0}\right]\left(\pi r^2 d\right)$$

Unit–15 : Electromagnetic Waves

Difficult *Pressure due to EM Waves*

30. The oscillatory electric field, of an em wave propagating along the (+ *x*) direction, is represented by $E_y = E_0 \sin\left(kx - \omega t\right)$

This em wave is incident on a perfectly black surface (of area A and thickness d), located in the y – z plane. The average, rediation pressure, due to this em wave, on this surface, over one complete cycle, would equal

(1) $\frac{1}{2}E_0^2\left[\varepsilon_0 + \frac{1}{\mu_0 c^2}\right]\left(\frac{A\omega d}{2\pi}\right)$ (Incorrect)

(2) $\left[\frac{1}{4c}E_0^2\left(\varepsilon_0 + \frac{c^2}{\mu_0}\right)\right]\left(\frac{\omega d}{2\pi}\right)$ (Incorrect)

(3) $\left(\frac{E_0^2\,\omega\, d}{4\pi\varepsilon_0 c^3}\right)$ (Incorrect)

(4) $\left[\frac{E_0^2 \omega d}{4\pi \mu_0 c^3}\right]$ (Correct)

Correct answer: (4)

Solution:

We know that

Volume) energy density of electric field $= \frac{1}{2}\varepsilon_0 E^2$

(Volume) energy density of magnetic field $= \frac{1}{2}\left(\frac{B^2}{\mu_0}\right)$

The energy of the em wave is shared equally between its electric and magnetic fields.

Over, one complete cycle, we need to use $E_{rms} = \frac{E_0}{\sqrt{2}}$ and $B_{rms} = \frac{B_0}{2}$ is the above expressions.

Also $B_0 = \frac{E_0}{c}$

$\therefore$ Total energy, transferred to the surface, over one complete cycle

$$= U = \left[\frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2}\frac{B_{rms}^2}{\mu_0}\right][Ad] = \frac{1}{4}E_0^2\left[\varepsilon_0 + \frac{1}{\mu_0 c^3}\right](Ad)$$

$\therefore$ Momentum delivered $= \frac{U}{c}$; Time period (for one cycle) $= \frac{2\pi}{\omega}$

$\therefore$ Average force $F = \frac{U}{c} \div \left(\frac{2\pi}{\omega}\right) = \frac{\omega U}{2\pi c}$

Average radiation pressure $= \frac{F}{A} = \frac{\omega U}{2\pi c A}$

$$= \frac{E_0^2}{4c}\left[\varepsilon_0 + \frac{1}{\mu_0 c^2}\right]\left(\frac{\omega d}{2\pi}\right) = \left(\frac{E_0^2 \omega d}{4\pi \mu_0 c^3}\right) \qquad \left(\because \quad c^2 = \frac{1}{\mu_0 \varepsilon_0}\right)$$

Unit–15 : Electromagnetic Waves

Average *Intensity of an EM Wave*

31. The intensity (I) of an em wave (having E_0 and B_0 as the 'peak values' for its electric and magnetic fields), can be expressed in the form:

(1) $\frac{\varepsilon_0 E_0^2}{2c}$ or $\frac{\mu_0 c B_0^2}{2}$ (Incorrect)

(2) $\frac{c\,\varepsilon_0 E_0^2}{2}$ or $\frac{\mu_0 c B_0^2}{2}$ (Incorrect)

(3) $\frac{c}{2}\varepsilon_0 E_0^2$ or $\frac{c B_0^2}{2\mu_0}$ (Correct)

(4) $\frac{\varepsilon_0 E_0^2}{2c}$ or $\frac{c B_0^2}{2\mu_0}$ (Incorrect)

Correct answer: (3)

Solution:

The intensity (at a given point), of an em wave, equals the energy received there per unit area per unit time.

Consider an em wave, propagating along the *x*–axis; consider a cylinder, of unit area of cross section, having a length c (c = velocity of em waves). The axis of this cylinder is assumed to be parallel to the direction of propagation of the em wave.

The volume of this cyilnder is $(1 \times c) = c$

The em wave (covering a distance c in one second) would transfer a total energy.

$$U = \left[\frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2}\frac{B_{rms}^2}{\mu_0}\right][c]$$

Through the cross section (= unity) of this cylinder in one second.

Hence energy transferred per unit area per unit time.

$$= \text{intensity of wave} = \frac{U}{1\times 1} = U$$

$$\therefore \quad I = \left[\frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2}\frac{B_{rms}^2}{\mu_0}\right]c$$

$$= \frac{c}{2}\left[\varepsilon_0 \frac{E_0^2}{2} + \frac{B_0^2}{2\mu_0}\right]$$

$$= \frac{c}{4\mu_0}\left[\mu_0\varepsilon_0 E_0^2 + \frac{E_0^2}{c^2}\right]$$

$$= \frac{c}{4\mu_0 c^2} 2E_0^2 \qquad \left(\because \quad \mu_0\varepsilon_0 = \frac{1}{c^2}\right)$$

$$= \frac{E_0^2}{2\mu_0 c} = \frac{1}{2}\left(\varepsilon_0 E_0^2\right)c = \frac{1}{2}\varepsilon_0 c^3 B_0^2$$

$$= \frac{1}{2}\frac{cB_0^2}{\mu_0}$$

Unit–15 : Electromagnetic Waves

Average *Peak Value of Electric Field in an EM Wave*

32. The peak value of the electric field, in the light radiated by a given (point) source of power P, at a point distance r, from the source equals.

(1) $\left(\frac{\varepsilon_0 P}{4\pi c}\right)^{1/2}\frac{1}{r}$ (Incorrect)

(2) $\left(\frac{P}{2\pi\varepsilon_0 c}\right)^{1/2}\left(\frac{1}{r}\right)$ (Correct)

(3) $\left(\frac{2\pi P}{\varepsilon_0 c}\right)^{1/2}\left(\frac{1}{r}\right)$ (Incorrect)

(4) $\left(2\pi c\,\varepsilon_0 P\right)^{1/2}\left(\frac{1}{r}\right)$ (Incorrect)

Correct answer: (2)

Solution:

The intensity, I, of the light radiated by a (point) source of power P, at a point distance r from the source, is

$$I = \frac{P}{4\pi r^2}$$

The intensity also equals c times the total energy density of the relevant em wave. This implies that

$$I = c\left[\frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2}\frac{B_{rms}^2}{\mu_0}\right]$$

$$\frac{P}{4\pi r^2} = c\left[2\times\left(\frac{1}{2}\varepsilon_0 E_{rms}^2\right)\right] \qquad \left[\because \ \frac{1}{2}\varepsilon_0 E_{rms}^2 = \frac{1}{2}\frac{B_{rms}^2}{\mu_0}\right]$$

$$= c\left[\varepsilon_0 E_{rms}^2\right]$$

$$= c\left[\varepsilon_0 \frac{E_0^2}{2}\right]$$

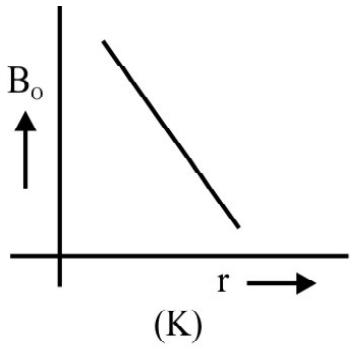
This gives $E_0^2 = \dfrac{P}{2\pi\varepsilon_0 cr^2}$

$$\therefore \quad E_0 = \left(\frac{P}{2\pi\varepsilon_0 c}\right)^{1/2}\left(\frac{1}{r}\right)$$

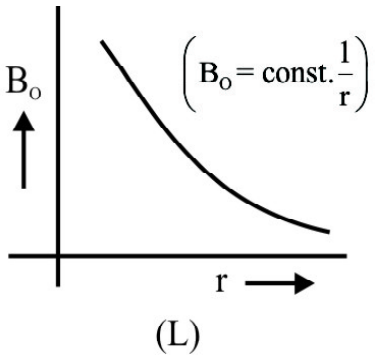
Unit–15 : Electromagnetic Waves

Average *Peak Value of the Magnetic Field in an EM Wave*

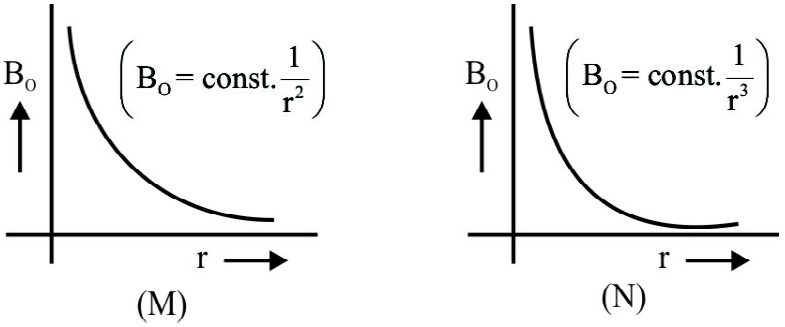
33. A given (point) source, of power P, radiates light, which has a peak value, B_0, for its magnetic field, at a point distant r from the source. The graph, showing the dependence of B_0 on r, would, then, be the graph labelled as graph



(K)



(L)



(1) K (Incorrect)

(2) L (Correct)

(3) M (Incorrect)

(4) N (Incorrect)

Correct answer: (2)

Solution:

The intensity, I, of the light from a (point) source of power P, at a point distant r from it, is

$$I = \frac{P}{4\pi r^2}$$

The intensity I also equals c times the total energy density of the relevant em wave. This implies that

$$I = c\left[\frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2}\frac{B_{rms}^2}{\mu_0}\right]$$

$$= c\left[2\times\frac{1}{2}\frac{B_{rms}^2}{\mu_0}\right] \qquad \left(\because \quad \frac{1}{2}\frac{B_{rms}^2}{\mu_0} = \frac{1}{2}\varepsilon_0 E_{rms}^2\right)$$

$$= \frac{c}{\mu_0}B_{rms}^2 = \frac{c}{2\mu_0}B_0^2 \qquad \left(\because \quad B_{rms} = \frac{B_0}{\sqrt{2}}\right)$$

$$\therefore \quad B_0^2 = \frac{2\mu_0}{c}I = \frac{2\mu_0}{c}\frac{P}{4\pi r^2} = \left(\frac{\mu_0 P}{2\pi c}\right)\frac{1}{r^2}$$

$$\therefore \quad B_0 = \left(\frac{\mu_0 P}{2\pi c}\right)^{1/2}\frac{1}{r} = \text{const.}\frac{1}{2}$$

Thus $B_0 \propto \frac{1}{r}$

This dependance of B_0 on r, is shown in the graph labelled as graph L.

Unit–15 : Electromagnetic Waves

Average *Comparison of Peak Value, of Electric Fields of Two EM Waves*

34. The peak value of the electric field, in the light radiated by a point source of power P_1, at a point, distant r_1 from the source, is E_{01}. The corresponding value, for a point source of power P_2, at a point, distant r_2 from the source, is E_{02}. The ratio $\left(\frac{E_{01}}{E_{02}}\right)$ equals

(1) $\left(\frac{P_1 r_1^2}{P_2 r_2^2}\right)^{1/2}$ (Incorrect)

(2) $\left(\frac{P_2 r_2^2}{P_1 r_1^2}\right)^{1/2}$ (Incorrect)

(3) $\left(\frac{P_1 r_2^2}{P_2 r_1^2}\right)^{1/2}$ (Correct)

(4) $\left(\frac{P_1 r_1}{P_2 r_2}\right)^{\frac{1}{2}}$ (Incorrect)

Correct answer: (3)

Solution:

The peak value, of the electric field (E_0) of an em wave, (at a point distant r from the source) is related to the power (P) of the source as

$$\frac{P}{4\pi r^2} = c\left[2\times\frac{1}{2}\varepsilon_0\left(\frac{E_0}{\sqrt{2}}\right)^2\right] \qquad (= \text{Intensity})$$

$$\therefore \quad E_0^2 = \frac{P}{2\pi r^2 c\varepsilon_0} = \left(\frac{1}{2\pi c\varepsilon_0}\right)\frac{P}{r^2}$$

$\therefore$ The required ratio is

$$\left[\frac{E_{01}}{E_{02}}\right]^2 = \frac{P_1}{P_2}\cdot\left(\frac{r_2}{r_1}\right)^2$$

$$\therefore \quad \frac{E_{01}}{E_{02}} = \left(\frac{P_1 r_2^2}{P_2 r_1^2}\right)^{1/2}$$

Unit–15 : Electromagnetic Waves

Average *Comparison of Peak Value, of Electric Fields of Two EM Waves*

35. The ratio, of the rms values of the magnetic fields in the em waves, radiated by two point sources, at points distance r and R, respectively, from the two sources, is 's'. If the power of one of the sources is P, that of the other, would be

(1) $\left(\frac{PR^2}{r^2 s}\right)$ (Correct)

(2) $\left(\frac{Pr^2}{R^2 s}\right)$ (Incorrect)

(3) $\left(\frac{sR^2}{Pr^2}\right)$ (Incorrect)

(4) $\left(\frac{r^2 s}{PR^2}\right)$ (Incorrect)

Correct answer: (1)

Solution:

The rms value of the magnetic field (B_{rms}), of an em wave (at a point distant r from the source), is related to the power, P, of the source, as

$$\frac{P}{4\pi r^2} = c\left[2\times\frac{1}{2}\frac{B_{rms}^2}{\mu_0}\right] \qquad (= \text{intensity})$$

$$\therefore \quad B_{rms}^2 = \frac{P}{4\pi r^2}\frac{\mu_0}{c}$$

For the given data, we then have

$$s = \left(\frac{P}{4\pi r^2}\frac{\mu_0}{c}\right) \times \left(\frac{c}{\mu_0}\frac{4\pi r^2}{P'}\right)$$

where P' is the power of the other source.

$$\therefore \quad s = \left(\frac{P}{P'}\frac{R^2}{r^2}\right)$$

$$\therefore \quad P' = \left(\frac{P}{r^2}\frac{R^2}{s}\right)$$

Unit–15 : Electromagnetic Waves

Easy *Momentum Transfer due to EM Waves*

36. An em wave, originating from a source, is radiating out 10 J of energy per second. At a distance of 50 cm from the source, a perfectly reflecting surface, of area $8\pi\,cm^2$, has these em waves incident on it. The average force, exerted on the surface, during a 30 seconds span would be

(1) zero (Correct)

(2) $\left(\frac{1}{3750\,c}\right)$ N (Incorrect)

(3) $\left(\frac{1}{125\,c}\right)$ N (Incorrect)

(4) $\left(\frac{1}{2500\,c}\right)$ N (Incorrect)

Correct answer: (1)

Solution:

The surface, being a perfectly reflecting surface, does not absorbe the incident em at all. Hence the energy absorbed, and, therefore, the momentum delivered to the surface, are zero each. The average force (= rate of change of momentum) is, therefore, zero.

Unit–15 : Electromagnetic Waves

Average *Relating between rms Value of Electric Field in an EM Wave + its Intensity*

37. The rms value, of the electric field, in the em waves, produced a point source, at a distance of $\left(\frac{4}{\sqrt{\pi}}\right)$ in from the point source, is 2Vm^{-1}. Taking $\varepsilon_0 = 8.85 \times 10^{-12}$ SI units and C = 3 × 10^8 ms^{-1}, we would find the power of the source to be (nearly)

(1) 2.3 nW (Correct)

(2) 1.15 nW (Incorrect)

(3) 2.3 pW (Incorrect)

(4) 1.15 pW (Incorrect)

Correct answer: (1)

Solution:

Let P be the power of the source. The intensity, at a distance $\left(\frac{4}{\sqrt{\pi}}\right)\text{m}$ is then

$$I = \frac{P}{4\pi\left(\frac{16}{\pi}\right)}\frac{W}{m^2} = \frac{P}{64}\frac{W}{m^2}$$

This equals $\left[2 \times \frac{1}{2}\varepsilon_0 E_{rms}^2\right]$

$\therefore \quad P = \left[64 \times 8.85 \times 10^{-12} \times 4\right]\text{W}$

$= 2265.6 \times 10^{-12}\ \text{W}$

$\approx 2.3\,\text{nW}$

Unit–15 : Electromagnetic Waves

Average *Relation between the Peak Value of the Magnetic Field of an EM Wave and its Intensity*

38. The peak value of the magnetic field, at a certain point, for an em wave is 9 nT. (It is given

that $\mu_0 = 4\pi \times 10^{-7}$ SI units and $c = 3 \times 10^8 ms^{-1}$). The peak value, of its electric field, and its intensity, at the same point, would be (nearly)

(1) $2.7\frac{V}{m}$ and $4.8\times10^{-3}\frac{W}{m^2}$ (Incorrect)

(2) $5.4\frac{V}{m}$ and $9.7\times10^{-3}\frac{W}{m^2}$ (Incorrect)

(3) $5.4\frac{V}{m}$ and $4.8\times10^{-3}\frac{W}{m^2}$ (Incorrect)

(4) $2.7\frac{V}{m}$ and $9.7\times10^{-3}\frac{W}{m^2}$ (Correct)

Correct answer: (4)

Solution:

$E_0 = \text{Peak value of electric field} = cB_0$

$$= 3\times10^8 \times 9\times10^{-9}\frac{V}{m} = 2.7\frac{V}{m}$$

$$\text{Also intensity} = c\left[2\times\frac{1}{2\mu_0}B_{rms}^2\right]$$

$$= c\left[\frac{1}{2\mu_0}B_0^2\right] \qquad \left[\because B_{rms} = \frac{B_0}{\sqrt{2}}\right]$$

$$= \frac{3\times10^8}{8\pi\times10^{-7}}\times81\times10^{-18}\frac{W}{m^2}$$

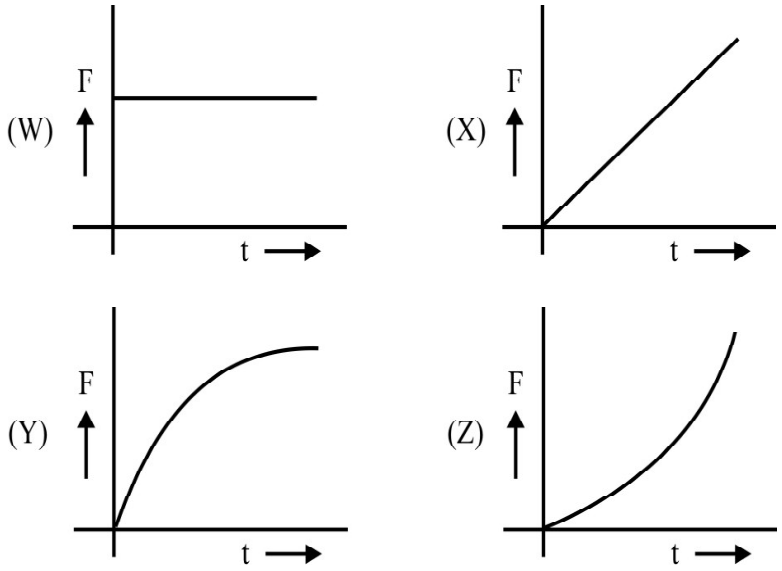
$$= \frac{243}{8\pi}\times10^{-3}\frac{W}{m^2}$$

$$\simeq 9.7\times10^{-3}\frac{W}{m^2}$$

Unit–15 : Electromagnetic Waves

Average *Intensity of EM Wave + Force Exerted on a Surface*

39. EM Waves having a uniform and constant, intensity I, are incident normally on a (perfectly black) surface of area A, for a time t. The average force (F), exerted on the surface by these em waves, depends on the time (t) in the manner shown in graph



(1) W (Correct)

(2) X (Incorrect)

(3) Y (Incorrect)

(4) Z (Incorrect)

Correct answer: (1)

Solution:

When EM Waves, of intensity I, are incident normally, on a surface of area A for a time t, the total energy absorbed by the (black) surface is

$$U = I \times A \times t$$

Therefore, the total momentum, delivered to the surface, is

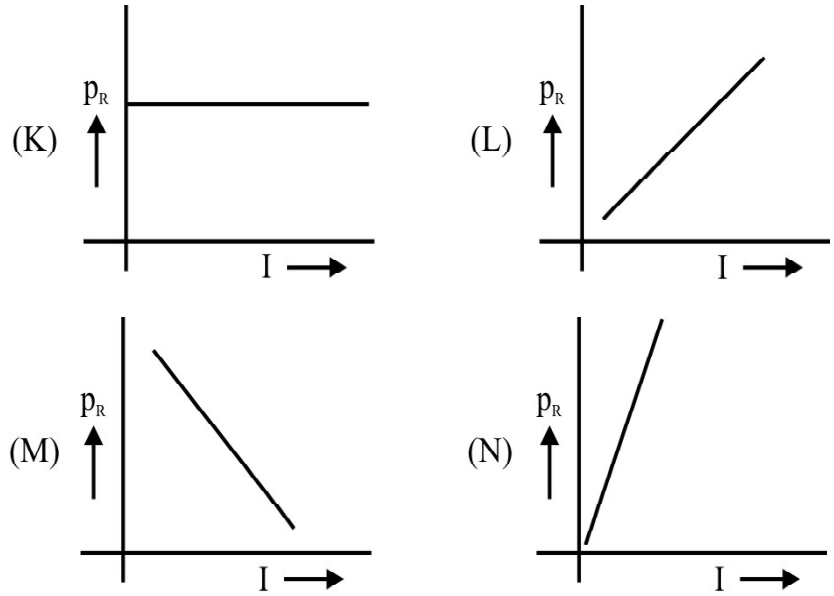
$$F = \frac{p}{t} = \frac{I \times A \times t}{c \times t} = \frac{IA}{c}$$

The average force thus does not depend on the time, t, for which the em waves are incident on. Hence the graph, between F and t, has the form shown in graph W.

Unit–15 : Electromagnetic Waves

Aveage *Radiation Pressure due to an EM Wave*

40. EM waves, of a uniform intensity I $\left(\frac{W}{m^2}\right)$, are incident normally on a perfect black surface, of area $10m^2$, for a time span of 100s. The graph between the 'average radiation pressure' p_R (exerted on the surface by these em waves) and the intensity (I), of the incident radiation, would then be the graph, labelled as graph



(1) K; and its slope would be zero (Incorrect)

(2) L; and its slope would be (nearly) 3.33×10^{-9} SI units (Correct)

(3) M; and its slope would be (nearly) -5.55×10^{-8} SI units (Incorrect)

(4) N; and its slope would be (nearly) + (6.66×10^{-6}) SI units (Incorrect)

Correct answer: (2)

Solution:

When em waves, of intensity I, are incident normally, on a surface of area A, for a time t, the total energy, absorbed by the black surface, is

$U = I \times A \times t = (I \times 10 \times 100)\ J = (1000\ I)\ J$

The take momentum, delivered to the surface, is $\frac{U}{c}$

$$= \frac{(1000\,I)}{c}\,kg\,ms^{-1}$$

$\therefore$ Average force, exerted on the surface, is $\left(\frac{1000\,I}{c \times 100}\right)N$

$$= \left(\frac{1I}{c}\right)N$$

$\therefore$ Average 'radition pressure' $= \left[\left(\frac{10\,I}{c}\right) \times \left(\frac{1}{10}\right)\right]\frac{N}{m^2}$

$\therefore \quad p_R = \frac{I}{c}$

The graph between p_R and I is,therefore, a straight line with a slope $= \frac{1}{c}$

$\therefore$ Slope $= \frac{1}{3 \times 10^8}$ SI units

$\simeq 3.33 \times 10^{-9}$ SI units

Unit–15 : Electromagnetic Waves

Difficult *Radiation Pressure*

41. Electromagnetic waves of intensity I, are incident normally on a material surface. If the reflected, transmitted and absorbed parts of these waves (by the material) are in the same ratio, then the radiation pressure p experienced by the surface equals

(1) $\frac{I}{2\,c}$ (Incorrect)

(2) $\frac{I}{c}$ (Incorrect)

(3) $\frac{I}{3\,c}$ (Correct)

(4) $\frac{2I}{3c}$ (Incorrect)

Correct answer: (2)

Solution:

Let U be the total energy of the EM waves received by the surface of one A in time t.

Therefore, intensity, I = Energy recieved per unit normal area in one second.

$$\text{or } I = \frac{U}{At}$$

Now, momentum imparted for computer absorption (i.e. if the surface is a perfectly block-body surface),

is : $P = \frac{U}{c}$

Pressure (p) exerted on the surface will then be:

$$p = \frac{F}{A} = \frac{\text{Change in momentum / time}}{A}$$

$$\text{or} \quad p = \frac{P/t}{A} = \frac{U}{cAt} = \frac{I}{c}$$

$$\text{or} \quad p = \frac{I}{c} \qquad \text{(for complete absorption)}$$

Now, for complete / total reflection, where the EM waves get bounced back with speed c, the change in momentum is, zero

For complete transmission, p = 0

Now, here, $\frac{1}{3}$ gets reflected, $\frac{1}{3}$ gets transmitted and $\frac{1}{3}$ gets absorbed.

$\therefore$ The pressure exerted by each part, respectively will be 0, 0 and $\frac{I/3}{c}$.

Hence, the total pressure on the surface, here, will be;

$$\text{Pressure} = 0 + 0 + \frac{I}{3c}$$

$= \frac{I}{3c}$

Correct answer is : $p = \frac{I}{3c}$

Unit–15 : Electromagnetic Waves

Difficult *Poynting Vector*

42. An infinitely long thin wire has a uniform charge density of $+\lambda\ Cm^{-1}$. The wire is placed along the z–axis and moved along its length with a uniform velocity $v = v\hat{k}$. The magnitude and direction of poynting vector, at p (0, a, 0) in this situation, equal

(1) $S = \frac{\lambda v^2}{4\pi^2\varepsilon_0 a^2}$, along $\left(+\hat{k}\right)$ (Incorrect)

(2) $S = \frac{\lambda^2 v^2}{4\pi^2\varepsilon_0 a^2}$, along $\left(-\hat{k}\right)$ (Incorrect)

(3) $S = \frac{\lambda^2 v}{4\pi^2\varepsilon_0 a^2}$, along $\left(+\hat{k}\right)$ (Correct)

(4) $S = \frac{\lambda^2 v}{4\pi^2\varepsilon_0 a^2}$, along $\left(-\hat{k}\right)$ (Incorrect)

Correct answer: (3)

Solution:

The figure given aside represents the situation. Due to the uniform **v** along $\hat{k}$, there will be a current $I = \lambda v$, along $\hat{k}$ Now, consider a point P, on the y–axis, at a distance of 'a' from the wire.

The electric field at P, due to the charge on the wire is : $\mathbf{E} = \frac{\lambda}{2\pi\varepsilon_0 a}\hat{j}$

The magnetic field at P, due to the current $I = \lambda v$ along $\hat{k}$, is $\mathbf{B} = \frac{\mu_0 I}{2\pi a}\left(-\hat{i}\right) = \frac{\mu_0 \lambda v}{2\pi a}\left(-\hat{i}\right)$

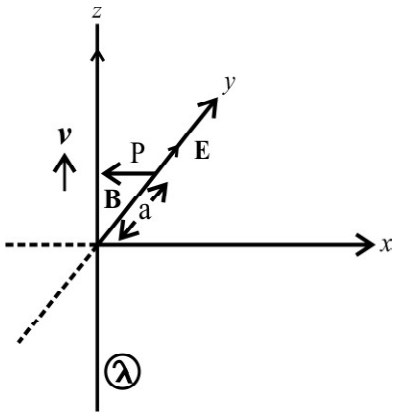
$\therefore \quad \mathbf{S} = \frac{1}{\mu_0}\left(\mathbf{E} \times \mathbf{B}\right)$

or $\mathbf{S} = \frac{1}{\mu_0}\left[\frac{\lambda}{2\pi\varepsilon_0 a} \times \frac{\mu_0 \lambda v}{2\pi a}\right]\left(\hat{j} \times -\hat{i}\right)$

$\mathbf{S} = \frac{\lambda^2 v}{4\pi^2 \varepsilon_0 a^2}\left(\hat{k}\right)$

or $S = \frac{\lambda^2 v}{4\pi^2 \varepsilon_0 a^2}, \text{along } \left(+\hat{k}\right)$

Correct answer is (3)



Unit–15 : Electromagnetic Waves

Average *Intensity of EM Wave / Poynting Vector*

43. The SI unit of the poynting vector (S) defined by the formula: $\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B})$ is

(1) volt – tesla (Incorrect)

(2) volt – metre – ampere^{-1} (Incorrect)

(3) joule – metre^{-2} (Incorrect)

(4) watt – metre^{-2} (Correct)

Correct answer: (4)

Solution:

The magnitude of **S** is same as intensity (I) of EM wave.

$\therefore \quad |\mathbf{S}| = I =$ energy transmitted through unit normal area in one second.

$= \frac{U/\text{Time}}{\text{Area}} \qquad (U = \text{Energy})$

$\therefore$ Unit of $|\mathbf{S}|$ = unit of $I = \frac{J/S}{m^2} = Wm^{-2}$

$=$ watt – metre^{-2}

Answer (4) is correct.

Unit–15 : Electromagnetic Waves

Difficult *Speed EM Waves*

44. An electromagnetic wave of frequency 3 MHz passes from vaccum in to dielectric medium of constant K = 4 and permeability μ_0. The wavelength of this wave in the dielectric is:

(1) 200 m (Incorrect)

(2) 100 m (Inorrect)

(3) 25 m (Correct)

(4) 25 m (Incorrect)

Correct answer: (3)

Solution:

Given, $\upsilon = 3\text{M Hz} = 3\times10^6$ Hz

$c = 3\times10^8\ ms^{-1}$ (is vaccum)

$$\therefore\quad \lambda = \frac{c}{\upsilon} = \frac{3\times10^8}{3\times10^6} = 100\,m$$

Now, $K = \varepsilon_r = 4$

Speed of EM wave in the medium will be $v = \frac{c}{n}$

where n is the refractive index of the medium.

Here $n = \frac{c}{v} = \frac{1}{\sqrt{\mu_0\varepsilon_0}}\Big|\frac{1}{\sqrt{\mu\varepsilon}}$

$$= \frac{\sqrt{\mu\varepsilon}}{\sqrt{\mu_0\varepsilon_0}}$$

$$= \frac{\sqrt{\mu_0 K\varepsilon_0}}{\sqrt{\mu_0\varepsilon_0}} \quad (\because \mu = \mu_0 \text{ and } \in = K \in_0 \text{ for the given medium})$$

$$= \sqrt{K}$$

$= \sqrt{4}$

$= 2$

i.e. refractive index, n = 2

$\therefore$ Speed of EM waves in the dielectric is:

$$v = \frac{c}{2} = 1.5 \times 10^8 \text{ ms}^{-1}$$

But $v = \upsilon\lambda' \qquad (\lambda = 3 \times 10^6 \text{ Hz})$

$$\therefore \quad \lambda' = \frac{v}{\upsilon} = \frac{1.5 \times 10^8}{3 \times 10^6} \text{ m}$$

$= 50$ m

Unit–15 : Electromagnetic Waves

Average *EM Spectrum*

45. A monochromatic source of power 0.66 kW, and efficiency 100%, emits 10^{20} photons is one second. These photons belong to the region of

(1) x – rays (Incorrect)

(2) uv – radiation (Correct)

(3) visible light (Incorrect)

(4) infrarred waves (Incorrect)

Correct answer: (2)

Solution:

Given P = 0.66 kW = 0.66 × 10^3 W

Since efficiency is 100% and source is monochromatic, all the photons have same energy (E = hν) and the entire power is converted to photons energy. Number of photon emitted in 1 second = 10^{20}

$$\therefore \text{ Energy of one photon is } E = \frac{\text{Total energy emitted is one second}}{\text{Number of photons emitted is one second}}$$

$$= \frac{P}{n}$$

$$= \frac{0.66\times10^{3}}{10^{20}} = 0.66\times10^{-17}\ \text{J}$$

Now, $E = h\upsilon = \dfrac{hc}{\lambda}$

$$\therefore\quad \lambda = \frac{h\,c}{E} = \frac{6.6\times10^{-34}\times3\times10^{8}}{0.66\times10^{-17}}\ \text{m}$$

$$= 30\times10^{-9}\ \text{m}$$

$$= 30\ \text{nm} \quad \text{(or uv rays)}$$

Unit–15 : Electromagnetic Waves

Average *Intensity of EM Waves*

46. A point source of electromagnetic radiation has an average power output of 135 W. The maximum value of electric field, at a distance of 3 m from the source, (in Vm^{-1}) is:

(1) $15\sqrt{2}$ (Incorrect)

(2) 30 (Correct)

(3) $30\sqrt{2}$ (Incorrect)

(4) 300 (Incorrect)

Correct answer: (2)

Solution:

Intensity at a distance of r = 3m from the source is:

$$I = \frac{P}{4\pi r^{2}} = \frac{135}{4\pi\times9}\frac{W}{m^{2}}$$

Now, $I = \dfrac{1}{2}\varepsilon_0 c E_0^2 = \dfrac{1}{2}.\dfrac{1}{4\pi9\times10^{9}}\times3\times10^{8}\times E_0^2$

$$\therefore\quad \frac{1}{2}\varepsilon_0 c E_0^2 = \frac{P}{4\pi r^{2}}$$

or $$\frac{1}{2}\frac{1}{4\pi\times9\times10^{9}}\times3\times10^{8}\times E_0^2 = \frac{135}{4\pi\times9}$$

or $E_0^2 = 900$

$E_0 = 30\ Vm^{-1}$

Unit–15 : Electromagnetic Waves

Easy *EM Spectrum*

47. The range of wavelengths allotted for commercial FM radio is

(1) 0.313 m to 0.638 m (Incorrect)

(2) 0.1 m to 1.0 m (Incorrect)

(3) 2.78 m to 3.41 m (Correct)

(4) 277.8 m to 340.9 m (Incorrect)

Correct answer: (3)

Solution:

The range of frequencies alloted for FM radio is: 88 MHz to 108 MHz.

Now this frequency range corresponds to a wavelength range of

$\frac{3\times10^8}{88\times10^6}$ m to $\frac{3\times10^8}{108\times10^6}$ m

or 3.41 m to 2.78 m

Correct answer is (3)

Unit–15 : Electromagnetic Waves

Average *Properties of EM Waves*

48. The electric field vector $\mathbf{E}_{(r,t)}$ and the magnetic field vector $\mathbf{B}_{(r,t)}$ of a plane electromagnetic wave propagating in free-space are given by

$$\mathbf{E}_{(r,t)} = \mathbf{E}_0\hat{\mathbf{i}}\cos(\omega t - kz)$$

and $$\mathbf{B}_{(r,t)} = \mathbf{B}_0\hat{\mathbf{j}}\cos(\omega t - kz)$$

where $B_0 = \frac{E_0}{c}$, $k = \frac{2\pi}{\lambda}$ is angular wave-number and $\omega = 2\pi\upsilon$ is angular frequency. The

average value of the vector $\frac{1}{\mu_0}(\mathbf{E}\times\mathbf{B})$, over a time period of the wave, is

(1) zero (Incorrect)

(2) $\frac{1}{2}\varepsilon_0 E_0^2\,\hat{k}$ (Incorrect)

(3) $\frac{1}{4}\varepsilon_0 c\,E_0^2\,\hat{k}$ (Incorrect)

(4) $\frac{1}{2}\varepsilon_0 c\,E_0^2\,\hat{k}$ (Correct)

Correct answer: (4)

Solution:

The vectors **E** and **B** are perpendicular to each other and are along $\hat{i}$ and $\hat{j}$ respectively, as the wave is progressing along the positive z-axis.

Therefore, $\mathbf{E}\times\mathbf{B} = EB\,\hat{k}$

$E = E_{rms}$ and $B = B_{rms}$ when averaged over a period of the wave. So this can be written as:

$$\frac{1}{\mu_0}(\mathbf{E}\times\mathbf{B}) = \frac{1}{\mu_0}E_{rms}B_{rms}\hat{k}$$

$$= \frac{1}{\mu_0}\frac{E_0}{\sqrt{2}}.\frac{B_0}{\sqrt{2}}.\hat{k}$$

$$= \frac{1}{\mu_0}\frac{1}{2}.E_0.\frac{E_0}{c}.\hat{k}$$

$$= \frac{1}{2}\frac{1}{\mu_0}\frac{E_0^2}{c}.\hat{k}$$

$$= \frac{1}{2}\varepsilon_0 c^2.\frac{E_0^2}{c}.\hat{k} \qquad \left(c^2 = \frac{1}{\mu_0\varepsilon_0} \text{ or } \frac{1}{\mu_0} = c^2\varepsilon_0\right)$$

$$= \frac{1}{2}\varepsilon_0 c E_0^2\,\hat{k}$$

Correct answer is (4)

Unit–15 : Electromagnetic Waves

Average *EM Spectrum*

49. Monochromatic source, of power 4 kW and 100% efficiency produces 10^{20} photons every second. The radiations emitted by it belong to the part of the EM spectrum called:

(1) gamma rays (Incorrect)

(2) *x*–rays (Correct)

(3) microwaves (Incorrect)

(4) infrared waves (Incorrect)

Correct answer: (2)

Solution:

Power P = 4000, W = 4000 J per second.

$n = 10^{20}$ per second

$\therefore \quad n(h\upsilon) = P \qquad (\because$ efficiency is 100%)

or $\upsilon = \dfrac{P}{nh}$

$= \dfrac{4000}{10^{20} \times 6.6 \times 10^{-34}}$ Hz

$= \dfrac{4}{6.6} \times 10^{17}$ Hz

$= 0.61 \times 10^{17}$ Hz

This is in the range of frequencies for x–rays.

Correct option is (2)

Unit–15 : Electromagnetic Waves

Difficult *Intensity EM Radiations*

50. The maximum value of electric field, of the total radiant energy per unit ara (normal to the direction of incidence), recieved at a distance R, from the centre of a star of radius r, (whose outer surface radiates as a black body at temperature TK) can be given, in terms of Stefan's consant σ, as;

(1) $\frac{rT^2}{R}\left(\frac{2\sigma}{\varepsilon_0 c}\right)^{1/2}$ (Correct)

(2) $\frac{r^2T^4}{R^2}\left(\frac{2\sigma}{\varepsilon_0 c}\right)^{1/2}$ (Incorrect)

(3) $\frac{rT^2}{R}\left(\frac{2\sigma}{\varepsilon_0 c}\right)$ (Incorrect)

(4) $\frac{r^2T^4}{cR^2}\left(\frac{2\sigma}{\varepsilon_0}\right)^{1/2}$ (Incorrect)

Correct answer: (1)

Solution:

Enery radiated by the surface of the star per second = $4\pi r^2.\sigma T^4$

Energy recieved per unit normal area per second at a distance $= R = S = \frac{4\pi r^2.\sigma T^4}{4\pi r^2}$

But $S = I = \frac{1}{2}\varepsilon_0 c E_0^2$

$\therefore \quad \frac{1}{2}\varepsilon_0 c E_0^2 = \frac{r^2\sigma T^4}{R^2}$

or $\quad E_0 = \left(\frac{2r^2\sigma T^4}{\varepsilon_0 c R^2}\right)^{1/2}$

$= \frac{rT^2}{R}\left(\frac{2\sigma}{\varepsilon_0 c}\right)^{1/2}$

UNIT–16 : OPTICS

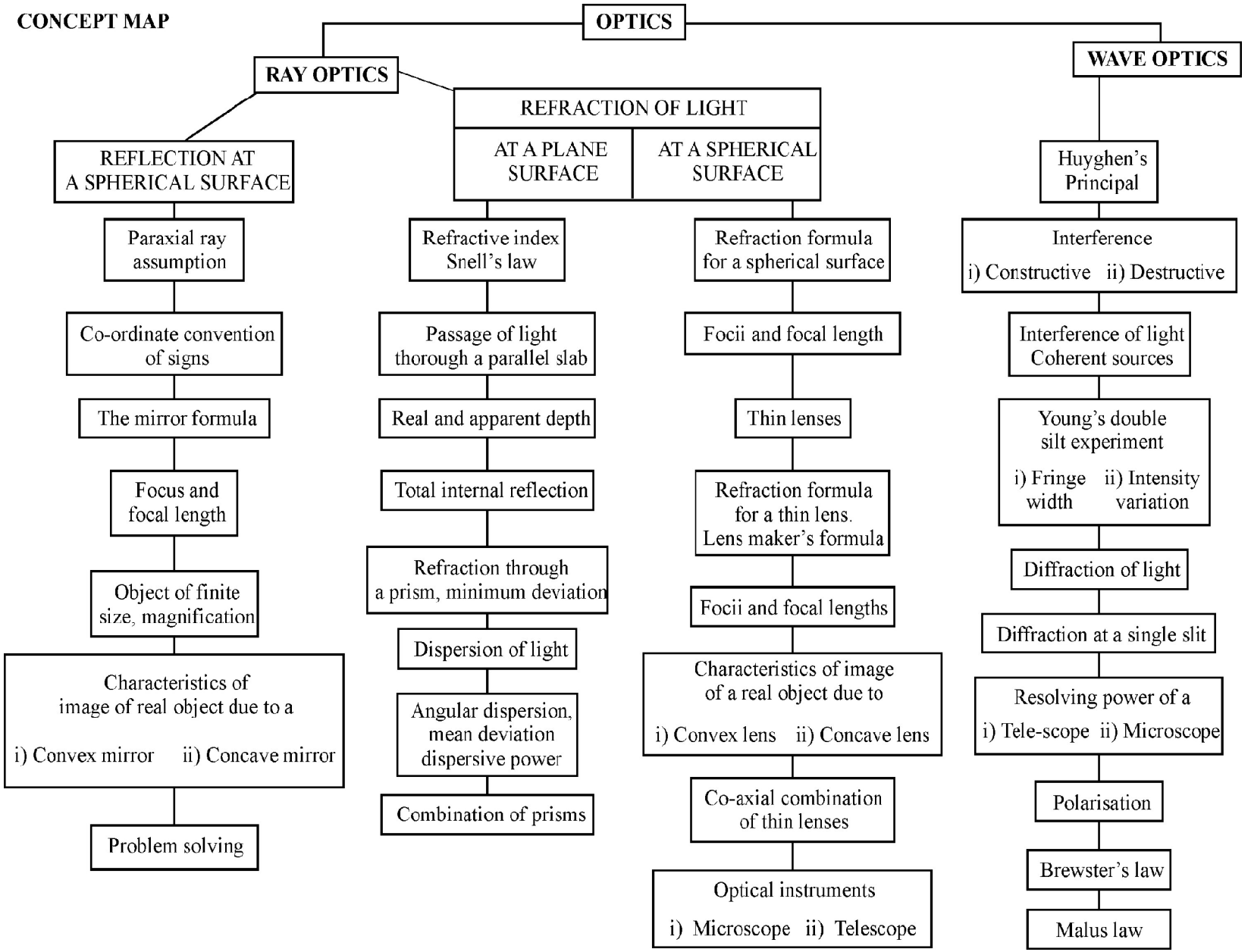
Learning Objectives

After going through this unit, you will be able to understand and use:

- Ray optics; using the rectilinear propagation of light.
- Law of reflection and refraction and their aplications.
- Explain the refraction at a spherical surface.
- Derive the relation between position of object, image and center of curvature – The mirror formula.
- To use the co–ordinate convention of signs.
- Explain the image formation of object of finite size – The transverse magnification.
- Distinguish between the absolute and the relative refractive index and state Snell's law.
- Explain the passage of monochromatic light in a parallel slab – (1) no net deviation (2) only lateral displacement.
- Derive the formula relating the real and the apparent depth.
- Explain total internal reflection (T.I.R.). Applications of T.I.R.
- Explain passage of monochromatic light through a prism. Deviation and minimum deviation.
- Passage of white light through a prism. Dispension of light. Deviation, angular dispersion and dispersive power.
- The refraction of light at a spherical surface. The refraction formula.
- Use the refraction formula at a spherical surface more than once.
- Derive and use the lens maker's formula. The convering or diverging nature of convex and concave lens.
- The lens formula. Transverse magnification.
- Explain the characterstics of image of a real object due to a (1) convex lens (2) concave lens.
- Define the power of a lens.
- Explain the co–axial combination of thin lenses in contact.
- Use the reflection and refraction formula for a combination of lenses, mirrors etc.
- Explain the optical instruments. (1) the microscope and (2) the telescope and their magnifying power.
- Use the concepts of wave motion.

- Explain the Huyghen's principle and its uses.
- Interference of light. Distinguishing between constructive and destructive interference.
- Explain the condition to sustained interference – The coherent sources.
- Explain the Young's double slit experiment. Location of bright and dark fringes. Derive the formula of the fringe width and intensity variation in interference pattern.
- Explain the phenomenon of diffraction of waves and how to observe diffraction of light?
- Derive the relation for the half–angular width and the radius of the central maximum.
- What is polarisation? Explain that only transverse waves can be polarised.
- Distinguish between polariser and analyser. The transverse nature of light.
- Explain the polarisation due to reflection. The Brewster's law.
- State and explain Malu's law and its applications.

CONCEPT MAP


OPTICS
RAY OPTICS
WAVE OPTICS
REFLECTION AT A SPHERICAL SURFACE
REFRACTION OF LIGHT
AT A PLANE SURFACE
AT A SPHERICAL SURFACE
Huyghen's Principal
Paraxial ray assumption
Co-ordinate convention of signs
The mirror formula
Focus and focal length
Object of finite size, magnification
Characteristics of image of real object due to a
i) Convex mirror ii) Concave mirror
Problem solving
Refractive index Snell's law
Passage of light thorough a parallel slab
Real and apparent depth
Total internal reflection
Refraction through a prism, minimum deviation
Dispersion of light
Angular dispersion, mean deviation dispersive power
Combination of prisms
Refraction formula for a spherical surface
Focii and focal length
Thin lenses
Refraction formula for a thin lens. Lens maker's formula
Focii and focal lengths
Characteristics of image of a real object due to
i) Convex lens ii) Concave lens
Co-axial combination of thin lenses
Optical instruments
i) Microscope ii) Telescope
Interference
i) Constructive ii) Destructive
Interference of light Coherent sources
Young's double silt experiment
i) Fringe width ii) Intensity variation
Diffraction of light
Diffraction at a single slit
Resolving power of a
i) Tele-scope ii) Microscope
Polarisation
Brewster's law
Malus law

UDAAN
A program to give wings to girl students

Introduction

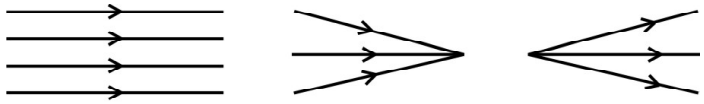
The study of optics is commonly devided into two parts; i.e.

(1) Geometrical or ray optics

and (2) Wave optics

We describe the two heads one by one.

Geometrical or ray optics makes use of rectilinear propagation of light in a homogeneous, isotropic medium. The straight line path light follows is known as a ray. A collection of rays constitue a beam. The beam of light is (1) parallel (2) convergent or (3) divergent as shown in Fig. 1 (a), (b) and (c).



Parallel Beam (a) | Convergent Beam (b) | Divergent Beam (c)

Fig. 1

Reflection of Light

A beam of light incident on a smooth surface is thrown back into the same medium by the surface. This is known as reflection of light. The laws of reflection are:

(a) Incident ray; reflected ray and normal at point of incidence lie in same plane.

(b) The angle of incidence (i) is equal to the angle of reflection (r); i.e. $i = r$.

Characteristics of Reflection by a Plane Mirror

1. The image formed by reflection of a real object is virtual; erect and of same size as object.
2. The image formed is lateraly inverted i.e. left of object appears right in image and vice–versa.
3. The image formed is as far behind the mirror as the object is in front of mirror.
4. When a plane mirror turns through an angle θ; the reflected ray turns through an angle 2θ.
5. The minimum length of a plane mirror, required to see the full size image of a person; by himself; is half the size of the person.
6. For two plane mirrors inclined at an angle θ with one another, the number of images, n; observed due to multiple reflection of a point object placed between the mirrors is

$$n = \left(\frac{360}{\theta} - 1\right) \text{ if } \frac{360}{\theta} \text{ is an even number}$$

$= \left(\frac{360}{\theta}\right)$; if $\frac{360}{\theta}$ is an odd number

7. A ray of light incident at an angle i, undergoes a deviation $\delta = \pi - 2i$, due to reflection at a plane surface.

Example–1:

Two plane mirrors are inclined to one another at an angle θ. A ray of light incident at an angle i is reflected first by one mirror and then by other. The total deviation produced is

(1) 2θ

(2) $(\pi - 2\theta)$

(3) $2\pi - i$

(4) $(2\pi - \theta)$

Solution:

As shown in Fig.2 M_1 and M_2 are the two mirrors inclined to one another at angle θ AB is the incident ray on M_1 at an angle i. The reflected ray from M_1 is incident on the second mirror M_2 at an angle i′. The final reflected ray is CD. Let δ be the total deviation and δ_1 and δ_2 be the deviation due to M_1 and M_2 respectively.

Then $\delta = \delta_1 + \delta_2$, and

$\delta_1 = \pi - 2i$,

$\delta_2 = \pi - 2i'$

$\therefore \quad \delta = 2\pi - 2(i + i')$

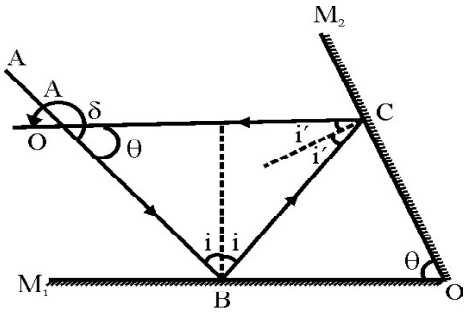


Fig. 2

Now from ΔOBC

$$\left(\frac{\pi}{2} - i\right) + \left(\frac{\pi}{2} - i'\right) + \theta = 2\pi$$

or $i + i' = \theta$

Hence $\delta = 2\pi - 2\theta$

Note that δ is independent of i.

Hence option (4) is correct.

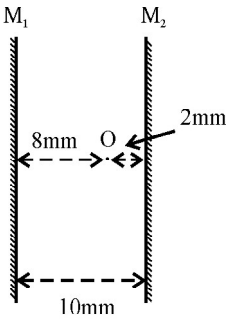


Fig. 3

Example–2:

A point object O is placed between two plane mirrors as shown in Fig. 3. The distance of the first three images formed due to multiple reflections from mirror M_2 are

(1) 2 mm, 8 mm, 18 mm

(2) 2 mm, 18 mm, 28 mm

(3) 2 mm, 18 mm, 38 mm

(4) 2 mm, 18 mm, 58 mm

Solution:

The first image formed is 2 mm behind M_2. Mirror M_1 forms an image of the object 8 mm behind it. This acts as object at a distance of 18 mm from M_2. Second image formed by M_2 is 18 mm behind M_2 and so on.

Hence the option (1) is correct.

SPHERICAL MIRRORS

A spherical mirror is a part of a hollow sphere, one surface of which is silvered. Fig. 4 (a) shows a hollow sphere of center C; radius OC = c. The shpere is cut by plane YY′. AOB is a part of a spherical surface. Fig. 4 (b) and (c) show a convex and a concave mirror respectively. For a spherical mirror.

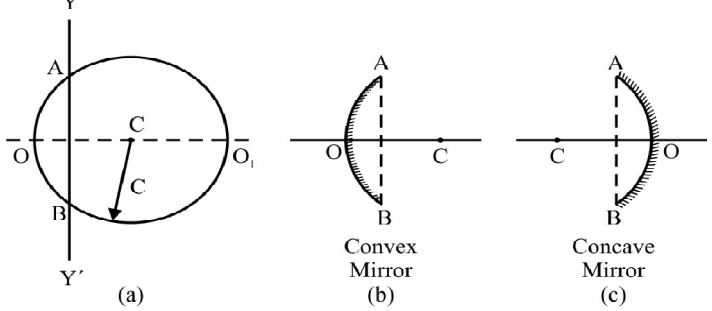


Fig. 4

(1) AB is the apertere of mirror. It measures the size of mirror.
(2) C is the center of curvature and c is the radius of curvature.
(3) Mid point O, is known as the pole of the mirror.
(4) The line joining the pole; O, and center of curvature C, is the *principle axis* of the mirror.

Reflection at a Spherical Mirror

In the FIg. 5 AOA' is a concave mirror. C is center of curvature. P is point object placed on principal axis. PA is one incident paraxial ray at point A of mirror. AC is the normal at point of incidence $\angle PAC = i =$ angle of incidence. In Fig. 5 AB is reflected ray. $\angle CAB = r =$ angle of reflection. In accordance with law of reflection i = r.

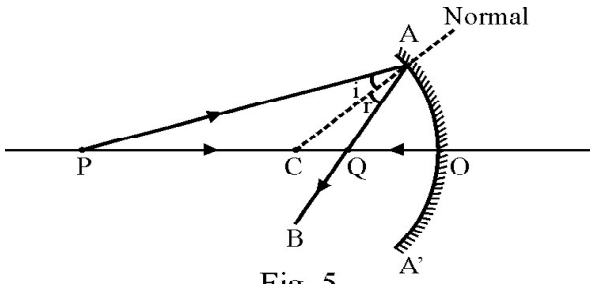


Fig. 5

To locate position of image Q formed by the mirror we consider at least one more incident ray say PO. This ray is incident normally and therefore is reflected back. In Fig. 5 the two reflected rays meet at point Q. We say Q is the real image of P due to reflection by the spherical mirror.

The Co–ordinate Convention of Signs

To obtain a single, convenient relation between the location of object and image we now use co–ordinate convention of signs. The basic features of co–ordinate convention are:

(1) Incident rays are always drawn from left to right.

(2) The pole of spherical mirror or optical center of lens is chosen as origin of co–ordinates.

(3) The principal axis in the direction of incident ray is the direction of positive *x*–axis.

(4) Anticlockwise angles are positive and clockwise angles are negative.

(5) In a direction perpendicular to principle axis, the direction above principal axis is the direction of positive Y axis.

In using co–ordinate convention, note, **distance of object**, image; center of curvature etc. **is a pure number.** There is no + or – sign attached to a distance. However the **position** of an object; image, center of curvature etc., is the co–ordinate of the point considered in the co–ordinate system chosen according to the rules of the co–ordinate convention of sign. **Position** is appropriate *distance with a + or – sign prefixed.*

The Reflection Formula

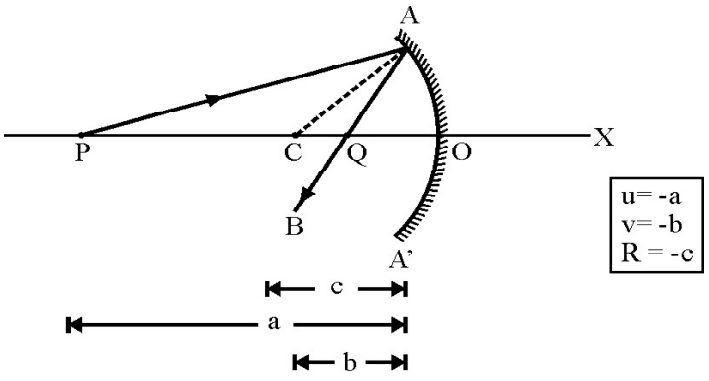


Fig. 6

The mirror formula is the relation between the object position (u); the image position (v) and the radius of curvature, (i.e position of the center of curvature C); R of the mirror. Fig. 6 shows image formation of a point object P due to a concave mirror. OP = a = distance of object P from the mirror. OQ = b = distance of the image formed Q. OC = c = radius of the sphere of which mirror is part. Choosing a co–ordinate system with O as origin and OX as positive X–axis (i.e. in accordance with the co–ordinate convention of signs; we have

u = The object position (i.e. co–ordinate of P) = – a

v = The image position (i.e. co–ordinate of Q) = – b

R = The radius curvature (i.e. co–ordinate of C) = – c

Please note the sign attached before appropriate distance. It can be shown that

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{R} \qquad .....(1)$$

Eqn. (1) is known as the mirror formula.

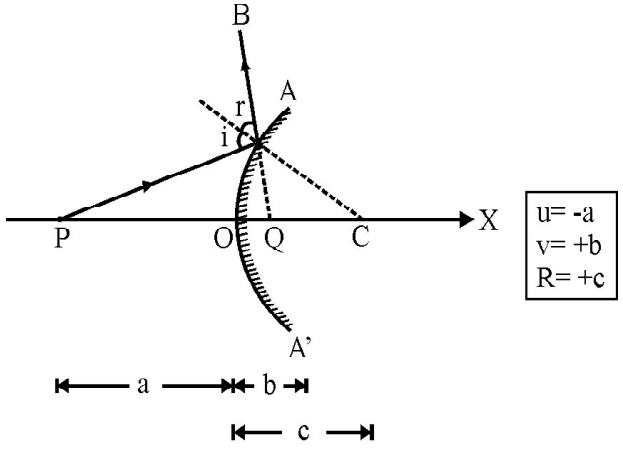


Fig. 7

Fig.7 shows reflection at a convex mirror. The reflected ray AB does not actually meet principal axis, but reflected ray appears to meet principal axis at Q. Q is the *virtual image of the real object P.* Valueof u,v and R in terms of distances a, b and c is shown inside box in Fig. 7. For convex mirror also, the mirror formula is

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{R} \qquad .....(2)$$

This is same as Eqn. (1). In other words same formula is applicable for concave as well as convex mirror. However, u, v, R etc. have to be asinged appropriate + or – sign in accordance with the sign convention.

Principal Focus and Focal Length

Consider an incident beam, parallel to principal axis. The reflected rays

(1) for concave mirror; actually meet principal axis at point F as shown in Fig. 8 (a).

(2) for convex mirror; appear to meet principal axis at point F as shown in Fig. 8 (b).

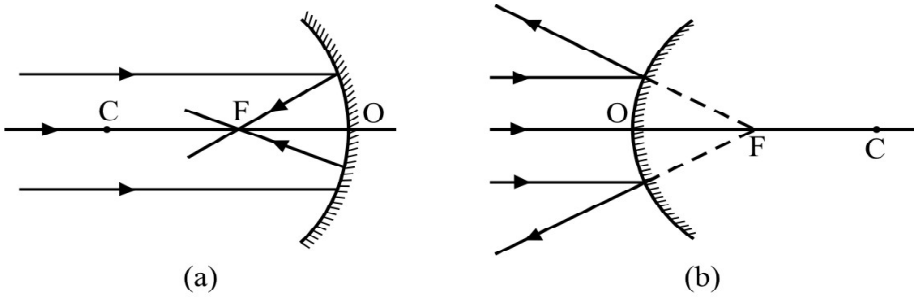


Fig. 8

F is known as the principal focus of the mirror. The position of F gives the focal length (f) of the mirror. It can be shown that F lies in the middle of O and C. Expressed mathematically

$$f = \frac{R}{2} \qquad .....(3)$$

The mirror formula in terms of focal length is

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \qquad .....(4)$$

For an incident ray that passes or appears to pass through F, the reflected ray is parallel to principal axis as shown in Fig. 9 (a) and (b).

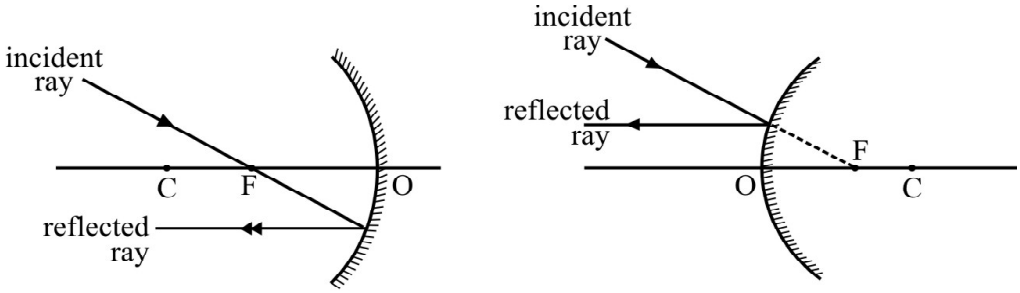


Fig. 9

Images of an Object of Finite Size

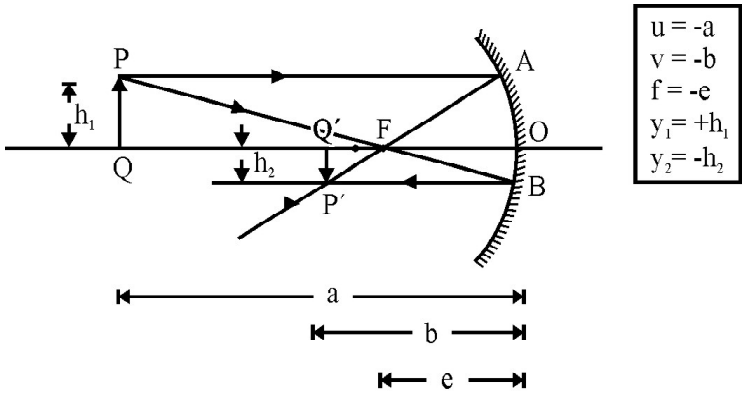


Fig. 10

Fig. 10 shows an object PQ of length h_1 placed perpendicular to principal axis. P'Q' of lengh h_2 is the real, inverted image formed by concave mirror. F is the focus of mirror. The appropriate distances and positions are shown in Fig. 10. The relation between position of object, image and focal length is same as given by Eqn. (4).

The relation between length of object and image formed is expressed by defining *transverse magnification,* m. Let y_1 and y_2 to be the **size** of the object and image. According to co–ordinate convention of signs.

y_1 = Size of object = + h_1

y_2 = Size of image = – h_2

The transverse magnification is ratio of the size of the image and the size of the object, i.e.

$$m = \frac{y_2}{y_1} = -\left(\frac{h_2}{h_1}\right) \qquad .....(5)$$

It can be shown that

$$m = -\left(\frac{v}{u}\right) \qquad .....(6)$$

The magnification (m) has both a sign and magnitude. The *sign of m will give whether image is erect or inverted.* If m is a negative number image formed is inverted w.r.t object and if m is a positive number the image formed is erect w.r.t object. The magnitude of m; gives information regarding length of image. If $|m|>1$, the image formed is enlarged and if $|m|<1$ the image formed is diminished w.r.t the object.

Characteristics of Image of a Real Object due to Reflection by a Concave Mirror

The following table gives position, nature of image as a real object is moved from infinity towards the mirror.

S.No.	Position of Object	Image	
		Position	*Nature*
1.	At infinity	At focus	real, inverted, highly diminished
2.	Between ∞ and 2F	Between F and 2F	real, inverted, diminished
3.	At 2F	At 2F	real, inverted, same size as object
4.	Between 2F and F	Beyond 2F	real, inverted, enlarged
5.	At F	At infinity	real, inverted, highly enlarged
6.	Between F and O	Behind the mirror	virtual, erect and enlarged

Characteristics of Image of a Real Object due to Reflection by a Convex Mirror

A convex mirror forms a virtual, erect and diminished image of a real object; irrespective of the position of the object. Fig. 11 shows image formation by a convex mirror.

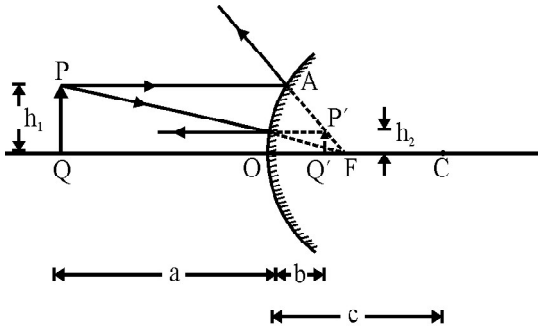


Fig. 11

An Important Note:

For solving problems using mirror formula, the following 'guide lines' are very use full.

(i) The positions of given parameter like object position, image position, focus etc. must be given their appropriate + or – sign in accordance with sign convention.

(ii) The position or size of the unknown parameter will be obtained using Eqn. (4) and (5). The sign and magnitude of unknown parameter must be properly intepretted.

The following table gives signs & their interpretation of commonly occuring parameters.

1.	Mirror	1) Concave	R and f are negative
		2) Convex	R and f are positive
2.	Object	1) Real	u is negative
		2) Virtual	u is positive

3.	Image	1) Real 2) Virtual	v is negative v is positive
4.	Magnification	1) Positive 2) Negative 3) $\|m\|>1$ 4) $\|m\|<1$	Image is erect Image is inverted Image is enlarged Image is diminished

Example–3:

A concave mirror of radius 20 cm has an object of length 2 cm placed perpendicular to principal axis at a distance of (i) 40 cm (ii) 8 cm. In each case what is position, nature and length of image formed by the mirror?

Solution:

R = Radius of curvature of mirror = – 20 cm

Case–I:

Given u = – 40 cm, v = ?

Using mirror formula

$$\frac{1}{v}+\frac{1}{(-40)}=\frac{2}{-20}$$

or $$\frac{1}{v}=-\left(\frac{1}{10}\right)+\frac{1}{40}$$

$$\therefore \quad v=-\left(\frac{40}{3}\right)\text{cm}=-13.33\,\text{cm}$$

Negative sign of v indicates image formed is real at a distance of 13.33 cm from the mirror.

$$m=-\left(\frac{v}{u}\right)=-\left[\frac{\left(-40/3\right)}{(-40)}\right]=-\left(\frac{1}{3}\right)$$

Negative sign of m indicates inverted image $|m|<1$ the image formed is diminished. Also

$$|m|=\frac{h_2}{h_1}\Rightarrow\frac{1}{3}=\frac{h_2}{2}$$

$\therefore \quad h_2 = 0.67\,cm$

Case–II:

Given $u = -8$ cm, $v = ?$

$$\therefore \quad \frac{1}{v}+\frac{1}{(-8)}=\frac{2}{(-20)} \qquad \text{or} \qquad v = +40\,cm$$

The positive sign of v indicates virtual image. The image is 40 cm behind the mirror. Also

$$m = -\left(\frac{40}{-8}\right) = +5$$

The positive sign of m indicates erect image. Also |m|>1 the image formed is enlarged. Also

$$5 = \frac{h_2}{2} \qquad \text{or} \qquad h_2 = 10\,cm$$

Example–4:

A spherical mirror forms an image half the length of the object. The image is at a distance of 25 cm from the mirror. What is nature and focal length of the mirror?

Solution:

We are given |m| = ½. We do not know whether image formed is real or virtual; therefore, we will consider both cases one by one.

Case–I: Image formed is real

Since image is real; it must be inverted; therefore m = (–½). Also v = – 25 cm. We know $m = -\left(\frac{v}{u}\right)$

$$\therefore -\left(\frac{1}{2}\right) = -\left(\frac{-25}{u}\right) \text{ or } u = -50\,cm$$

From mirror formula

$$\frac{1}{(-25)}+\frac{1}{(-50)}=\frac{1}{f}$$

$$\therefore \quad f = -\left(\frac{50}{3}\right) cm = -16.67\,cm$$

The negative sign of f indicates that the mirror is a concave mirror.

Case–II: Image formed is virtual

The virtual image formed is erect, therefore $m = +\frac{1}{2}$, and $v = +25$ cm

$$m = +\frac{1}{2} = -\left(\frac{v}{u}\right)$$

or $\frac{1}{2} = -\left(\frac{25}{u}\right)$ or $u = -50\,\text{cm}$

Using mirror formula

$$\frac{1}{25} + \frac{1}{(-50)} = \frac{1}{f}$$

$\therefore \quad f = +50\,\text{cm}$

The positive sign of f indicates convex nature of the mirror.

Example–5:

A beam of light converges towards a point P as shown in Fig 12(a). A convex mirror is placed in the path of beam at disatnce of 10 cm in front of P. What is position and nature of image formed? Radius of curvature of mirror is 30 cm.

Solution:

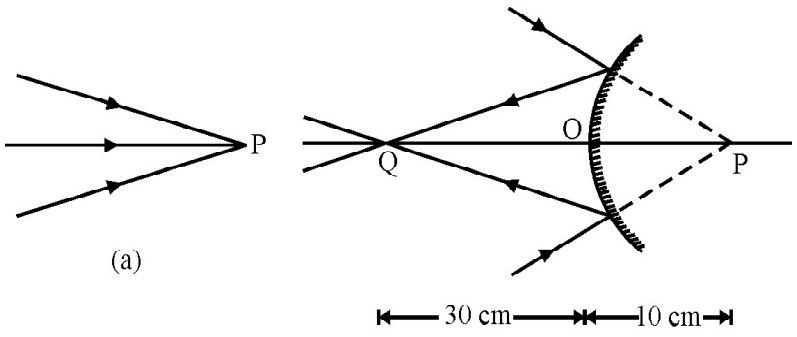


Fig. 12

Fig. 12 (a) shows the incident beam converging at point P. Fig. 12 (b) shows convex mirror placed in the path of incident convergent beam. The incident rays meet convex mirror before reaching point P. Now acts P as a virtual object. Therefore,

u = + 10 cm; R = + 30 cm, v = ?

Using the mirror formula

$$\frac{1}{v}+\frac{1}{(+10)}=\frac{2}{(+30)}$$

or $$\frac{1}{v}=\frac{2}{30}-\frac{1}{10}$$

$\therefore \quad v = -30\,cm$

The negative sign of v indicates a REAL image! This is shown in Fig. 12 (b). The interesting point to be noted specially is that *convex mirror can from a real image of an appropriately placed virtual object.*

Example–6:

A spherical mirror projects image of an object placed in front on a screen. The length of image is twice that of the object. The distance between object and image is 120 cm. What is

(i) position of object and image.

(ii) nature and power of the mirror?

Solution:

Let a and b be the distance of the object and image from the mirror. Since image is projected on screen it is a real and inverted image. Therefore using sign convention

$u = -a,\ v = -b$ and $m = -2$

Also, $m = -\left(\frac{v}{u}\right)$; therefore

$$-2 = -\left[\frac{(-b)}{(-a)}\right] \quad \text{or} \quad b = 2a$$

Given distance between object and image is 120 cm. Therefore

$b - a = 120$

or $2a - a = 120$ or $a = 120$ cm and $b = 240$ cm

Let f be the focal length of the mirror. Using mirror formula

$$\frac{1}{(-240)}+\frac{1}{(-120)}=\frac{1}{f}$$

or $f = -80$ cm

The negative sign of f indicates that the mirror is a concave mirror. The power of mirror

$$P = \frac{1}{(-0.8)} D = -1.25 D$$

The object is placed 120 cm in front of the mirror and image is at a distance of 240 cm from the mirror.

Example–7:

A concave mirror of focal length 10 cm and a convex mirror of focal length 15 cm are placed co-axially facing each other and 40 cm apart. A point object is placed in between the mirrors; on their common axis; 15 cm from the concave mirror. Find the position and nature of image formed due to first reflection at concave mirror and then convex mirror.

Solution:

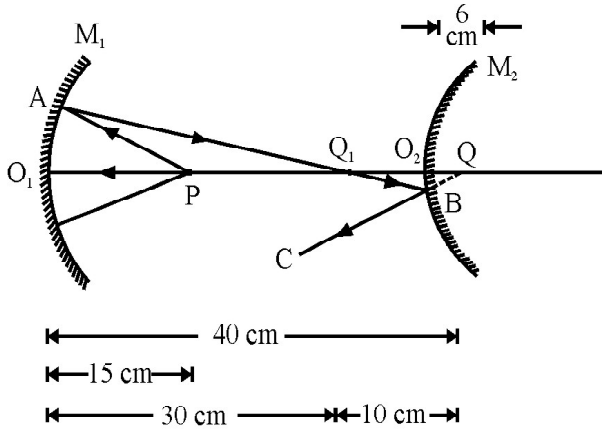


Fig. 13

Fig. 13 shows M_1 and M_2 as the two mirrors. P is object on principal. axis O_1O_2 at a distance of 15 cm from M_1. The incident ray PA on concave mirror is moving from right to left. This is not in accordance with co–ordinate convention of signs. To make consistent use of co–ordinate convention imagine the diagram to be rotated by 180^0 as far as concave mirror is concerned. Then for concave mirror.

$u_1 = -15$ cm, $f_1 = -10$ cm and $v_1 = ?$

From the mirror formula

$$\frac{1}{v_1} + \frac{1}{(-15)} = \frac{1}{-10} \qquad \text{or} \qquad v_1 = -30\,\text{cm}$$

The negative sign of v_1 indicates real nature of image Q_1 formed by concave mirror. The incident ray PA and reflected ray AQ_1, is shown in Fig. 13

Q_1 acts as a real object for convex mirror. The ray Q_1B is incident on convex mirror. In Fig. 13 this is in accordance the co–ordinate convention of signs. For convex mirror.

$u_2 = +10$ cm; $f_2 = 15$ cm; $v_2 = ?$

$$\therefore \quad \frac{1}{v_2}+\frac{1}{(10)}=\frac{1}{15}$$

or $v = +6$ cm

The positive sign of v indicates virtual nature of the final image Q. The incident and reflected rays for convex mirror as shown in Fig. 13.

Example–8:

A thin rod of length $\frac{f}{3}$ is placed along the optic axis of a concave mirror of focal length f such that its image which is enlarged, real and just touches the rod. Calculate magnification.

Solution:

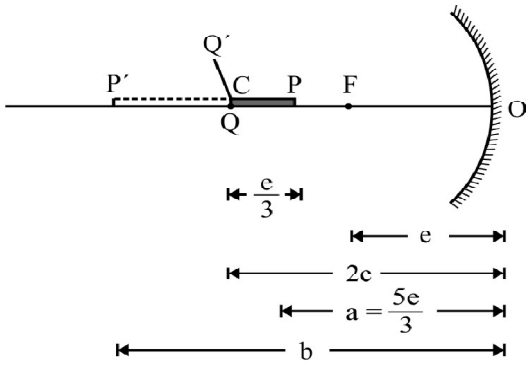


Fig. 14

Let OF = e = magnitude of focal length of concave mirror. OC = 2e = distance of center of curvature from O. PQ = e/3 is the length of object placed along principal axis. The image formed is P'Q'. For end Q of rod its image coincides with itself, therefore, Q is at center of curvature, C, of mirror. For end P

$$a = OP = \text{object disatnce} = OC–PQ = 2e–\left(\frac{e}{3}\right)=\frac{5e}{3}$$

$b = OP' =$ image distance

In accordance with co–ordinate convention of signs

$$u = \frac{-5e}{3},\ v = -b,\ f = -e$$

Using mirror formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ we have

$$\left(\frac{1}{-b}\right)-\left(\frac{3}{5e}\right)=\left(\frac{1}{-e}\right) \quad \text{or} \quad b=2.5e$$

The length of image $= P'Q' = b - 2e = \frac{e}{2}$

The magnitude of axial magnification

$$=\left|m_{axis}\right|=\frac{P'Q'}{PQ}=\frac{e/2}{e/3}=1.5$$

Example–9:

Image of an object approaching a convex mirror of curvature 20 m along its optical axis is observed to move from $\frac{25}{3}$ m to $\frac{50}{7}$ m in 30 s. What is the average speed of object is kmh^{-1}?

Solution:

Let at t = 0; the object be at a distance of a_1 meter from convex mirror. The image formed is at a distance $b_1=\frac{25}{3}$m from the mirror. Obviously

$$u_1=-a_1 \qquad \text{and} \qquad v_1=+\frac{25}{3}\text{m}$$

Given, $f=\frac{R}{2}=+\frac{20}{2}=+10\,\text{m}$

Using mirror formula, we have

$$\frac{3}{25}-\left(\frac{1}{a_1}\right)=\frac{1}{10} \qquad \text{or} \quad a_1=50\,\text{m}$$

Similarly a_2 is distance of object from mirror at t = 30 s. The image distance $b_2=\frac{50}{7}$m . Therefore

$$u_2=a_2; \qquad v_2=+\frac{50}{7}\text{m}$$

Using mirror formula

$$\frac{7}{50}-\left(\frac{1}{a_2}\right)=\frac{1}{10} \qquad \text{or} \qquad a_2 = 25\,\text{m}$$

The distance moved by object in 30 s is (50–25) m = 25 m. Therefore,

$$\text{The average speed of object} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$= \frac{25\,\text{m}}{30\,\text{s}} = \frac{25\times10^{-3}\,\text{km}}{\left({}^{30}\!/_{300}\right)\text{hr}} = 3\ \text{kmh}^{-1}$$

REFRACTION OF LIGHT–SNELL'S LAW

Whenever light goes from one medium to another medium; in general; the light bends either towards or away from the normal at the point of incidence. This is known as refraction of light. Refraction of light is shown in Fig. 15 (a) and (b).

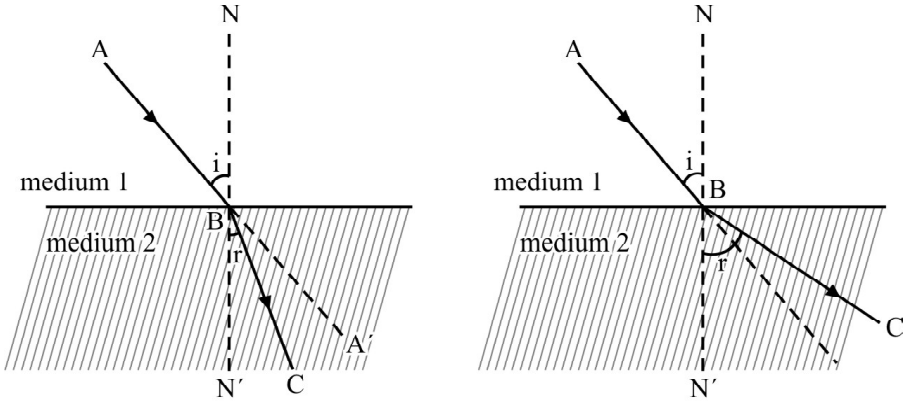


Fig. 15

In Fig. 15 (a) we say medium 2 is optically denser than medium 1; whereas in Fig. 15 (b) medium 2 is optically rarer than medium 1.

The optical density of a medium is measured by the absolute refractive index (n) of the medium. By definition

$$n = \frac{\text{Speed of light in vaccum}}{\text{Speed of light in medium}} = \frac{c}{v}$$

$$= \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}} = \sqrt{\varepsilon_r\mu_r}$$

ε_r and μ_r represent the relative permittivity and the relative permeability of the medium. v is always less than c hence *absolute refractive index can never be less than one.*

Refraction of light obey's *Snell's law*. According to Snell's law.

(1) The incident ray; the refracted ray and normal at point of incidence lie in same plane.

(2) The ratio of sine of angle of incidence (i) and sine of angle of refraction (r) is a constant. Expressed mathematically

$$\frac{\sin i}{\sin r} = \text{constant}$$

Let n_1 be the absolute refractive index of medium 1 known as incident space and n_2 the refractive index of medium 2, known as refracted space. According to Snell's law.

$$n_1 \sin i = n_2 \sin r$$

or $$\frac{\sin i}{\sin r} = \frac{n_2}{n_1} = {}^2_1 n = \frac{v_1}{v_2} \qquad .....(1)$$

${}^2_1 n$ is the relative refractive index of medium 2 with respect to medium 1. In Eqn. (1), v_1 and v_2 is speed of light in medium 1 and medium 2 respectively. *The relative refractive index* ${}^2_1 n$ *can be more or less than one.* Obviously

$${}^1_2 n \times {}^2_1 n = \frac{v_1}{v_2} \times \frac{v_2}{v_1} = 1 \qquad .....(2)$$

In general Eqn. (2) can be written as

$${}^1_2 n \times {}^2_3 n \times {}^3_4 n = {}^1_4 n = \frac{1}{{}^4_1 n} \qquad .....(3)$$

Refraction of Light at a Parallel Slab

In the Fig. 16 PQRS is a parallel slab of thickness t, having refractive index n. AB is a monochromatic incident ray on face PQ at an angle of incidence, i. Fig. 16 shows refracted, ray BC inside slab and the emergent ray CD. i′ is angle of emergence. Applying Snell's law at point B and C, we have

$$1 \sin i = n \sin r \quad \text{and} \quad n \sin r = 1 \times \sin i'$$

$\therefore \quad \sin i = \sin i' \quad \text{or} \quad i = i'$

The angle of emergence equals angle of incidence. In other words emergent ray CD is parallel to incident ray ABA′. The net deviation due to refraction at both faces of parallel slab is zero. However the incident ray and the emegent ray are not coincident. CE is the lateral displacement. From Fig.16 it can be shown that

$$\text{Lateral Displacement} = CE = \frac{t \sin(i - r)}{\cos r}$$

.....(4)

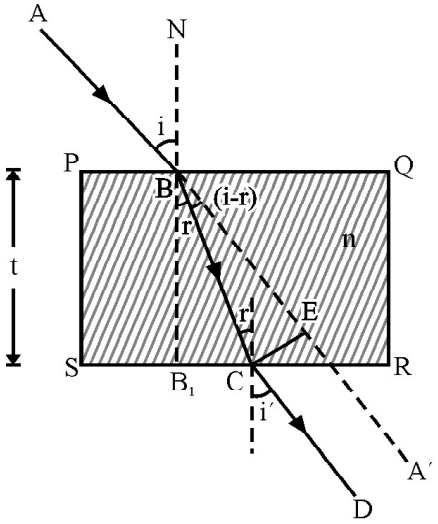


Fig. 16

Refraction Through Multiple Slabs

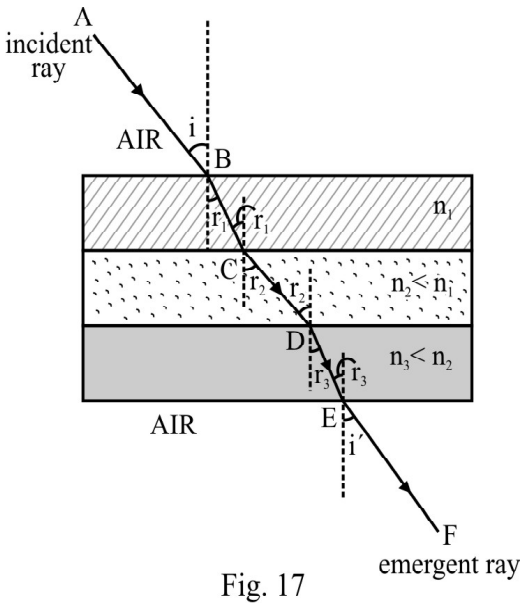


Fig. 17

Fig. 17 shows path of incident ray AB inside parallel slabs of different materials. i is angle of incidence and i′ is angle of emergence. Applying Snell's law repeatedly.

$$i = i'$$

When medium on the upper side and lower side is same [in Fig. 17 it is air]; $i = i'$. If medium above and below is not same, $i \neq i'$.

Real and Apparent Depth

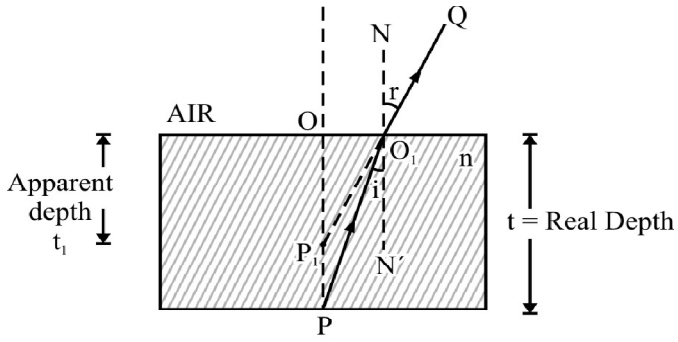


Fig. 18

P is a point object at the bottom of a medium of refractive index n of real thickness OP = t. PO is an incident ray. The refracted ray O_1Q bends away from normal, i.e. r > i. The emergent ray appears to come from point P_1. P_1 is apparent position of P as viewed from outside. $OP_1 = t_1$ = the apparent depth of P. For near normal incidence (i.e. i and r are small angles; sin i ≃ tan i ≃ i)

n = The refractive index of the medium

$$= \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{t}{t_1} \qquad .....(5)$$

$$\text{The apparent shift} = PP_1 = t\left(1 - \frac{1}{n}\right) \qquad .....(6)$$

For Fig. 18 shown light going from a denser medium to a rarer medium (air) apparent depth viewed from air is less than the actual depth.

In general; if instead of air there is some other medium; the apparent depth may be more or less than the real depth depending to value of refractive indices of media involved.

For object at a real depth t, inside a medium of refractive index n_1 when viewed from a medium of refractive index n_2; the apparent depth t_1 is

$$^{2}_{1}n = \frac{n_2}{n_1} = \frac{t}{t_1}$$

For $n_1 > n_2$; $t > t_1$ whereas for $n_1 < n_2$; $t < t_1$.

For a number of media of different thickness and refractive indices as shown in Fig. 19, we have

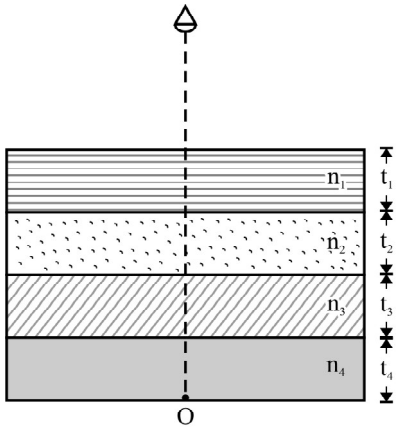


Fig. 19

Total apparent depth $= \frac{t_1}{n_1} + \frac{t_2}{n_2} + ... = \sum \frac{t_i}{n_i}$

and total apparent shift $= \sum t_i \left(1 - \frac{1}{n_i}\right)$

n_i is the refractive index of i^{th} medium of real thickness t_i.

Example–10:

A ray of light in air is incident at angle of 60^0 on the plane boundary seprating it from a medium. The angle of deviation of refracted ray is 15^0. What is refractive index of the medium?

Solution:

Let n be refractive index of medium. Incident light goes from a rarer medium (air) to a denser medium. It bends towards normal i.e. r < i. The angle of deviation $\delta = i - r$. Given

$i = 60^0, \quad \delta = 15^0 \quad \therefore \; r = 45^0$

From Snell's law

$1 \sin 60^0 = n \times \sin 45^0$

or $n = \sqrt{\frac{3}{2}}$

Example–11:

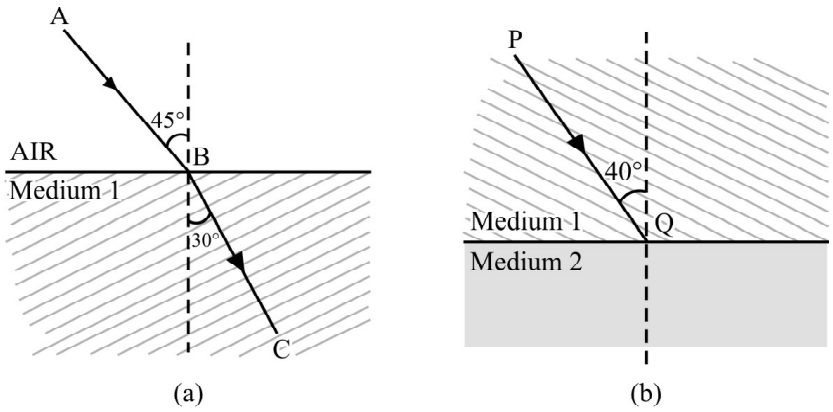


Fig. 20

In Fig. 20 (b) the incident ray PQ bends away from normal; suffering a deviatino of 10^0. What is speed of light in medium 2? Given $c = 3 \times 10^8$ ms^{-1}.

Solution:

Let n_1 be absolute refractive index of medium 1. From Fig. 20 (a); using Snell's law

$$1 \sin 45^0 = n_1 \sin 30^0$$

or $$n_1 = \sqrt{2}$$

In Fig. 20 (b); the angle of refraction $r = i + \delta = 40 + 10 = 50^0$. From Snell's law

$$n_1 \sin 40° = n_2 \sin 50°$$

or $$n_2 = \frac{1.414 \times 0.643}{0.766} \simeq 1.187$$

The speed of light V_2 in medium 2 is

$$V_2 = \frac{c}{n_2} = \frac{3 \times 10^8}{1.187} \text{ ms}^{-1} \simeq 2.527 \times 10^8 \text{ ms}^{-1}$$

Example–12:

Fig. 21 shows a small bubble, P; trapped inside a glass slab ($n_1 = 1.5$). **When viewed from position O_1 the bubble appears to be at a distance of 8 cm from the face through which it is viewed. When viewed from position O_2 the bubble appears to be at a distance of 10 cm from the face of slab nearer the observer. What is n_2?**

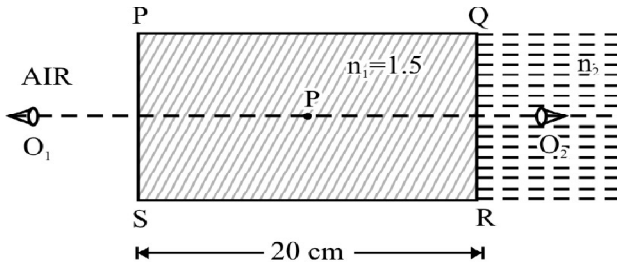


Fig. 21

Solution:

Let *x* can be the actual distance of bubble from face PS. For observer. O_1; the apparent distance from PS is 8 cm. Therefore

$$n_1 = 1.5 = \frac{x}{8}$$

or $x = 12$ cm

The actual distance of bubble from face QR is (20–12) = 8 cm. The apparent distance is 10 cm. Therefore

$$_{1}^{2}n = \frac{n_1}{n_2} = \frac{\text{Real distance}}{\text{Apparent distance}}$$

or $$\frac{1.5}{n_2} = \frac{8}{10}$$

$$\therefore \quad n_2 = \frac{15}{8} = 1.875$$

Example–13:

One face of a parallel rectangular slab of a transparent material is silvered. An object is placed at a perpendicular distance of 8 cm from the unsilvered face of slab. The image of the object is observed at a distance of 12 cm from the silvered face of slab. What is refractive index of material of slab?

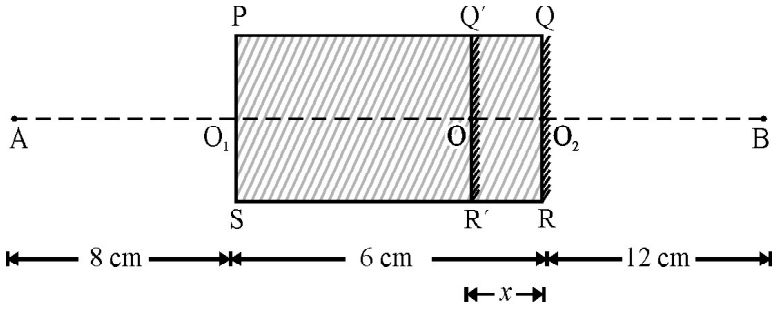


Fig. 22

Solution:

Fig. 22 shows slab PQRS. A is object at a distance $AO_1 = 8$ cm. The image B is formed at a distance $O_2B = 12$ cm. When observed from A; the mirrored face QR appears to be at Q'R'. Let x be the apparent shift in position of QR. Now for reflection from Q′R′;

Object distance = OA = Image distance = OB

$$8 + (6–x) = 12 + x$$

or $$x = 1 \text{ cm}$$

From face PS of slab; the real distance of face QR = 6 cm. The apparent distance = (6–x) = (6–1) = 5 cm. Therefore

$$n = \frac{6}{5} = 1.2$$

Example–14:

An observer can see through a pin–hole at the top end of a thin glass rod of height h placed as shown in Fig. 23. The actual height of beaker is 3h and its radius is h. When the beaker is filled with a liquid upto a height of 2h, the observer can see the lower end of the rod. What is μ of liquid? (I.I.T 2002)

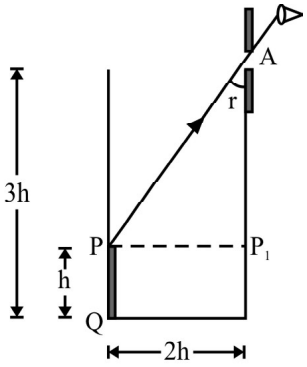


Fig. 23

Solution:

In Fig. 23 QO is incident ray from lower end Q of rod inside liquid. The refracted ray OA appears to meet rod at end P as shown.

$\angle QON_1 = i =$ angle of incidence at O, $\angle NQA = \angle PQN_1 = r =$ angle of refraction at O.

Using Snell's law

$$\mu \sin i = 1 \times \sin r \qquad .....(1)$$

From Fig. 24;

$$\tan \angle PAP_1 = \tan r = 1$$

$$\therefore \quad r = 45^0 \qquad .....(2)$$

From right angled triangle QN_1O

$$\sin i = \frac{h}{\left[(h)^2 + (2h)^2\right]^{1/2}} = \frac{1}{\sqrt{5}} \qquad .....(3)$$

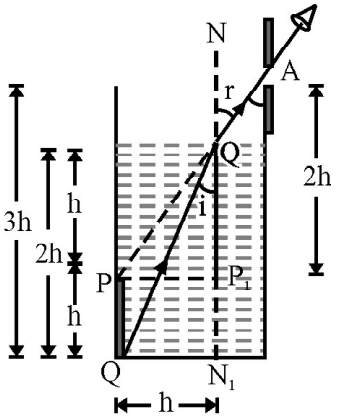


Fig. 24

From Eqns. (1), (2) and (3) we have

$$\mu \times \frac{1}{\sqrt{5}} = \frac{1}{\sqrt{2}}$$

$$\therefore \quad \mu = \sqrt{\frac{5}{2}}$$

Total Internal Reflection

Fig. 25 shows a monochromatic incident ray going from a denser medium (i.e. medium 1, refractive index μ_1) into a rarer medium (i.e. medium 2, refractive index $\mu_2 < \mu_1$). The refracted ray bends away from

normal i.e. r > i as shown in Fig. 25 (a). On gradually increasing i, r also increases. For an angle of incidence (i_c) the angle of refraction $r = 90^0$ as shown in Fig. 25 (b). In Fig. 25 (c) $i > i_c$; there is no refracted ray in medium 2. The entire incident energy is thrown back to medium 1 obeying law of reflection. This is known as *total internal reflection (T.I.R)*. i_c is the critical angle of incidence. Applying Snell's to Fig. 25 (b); we have

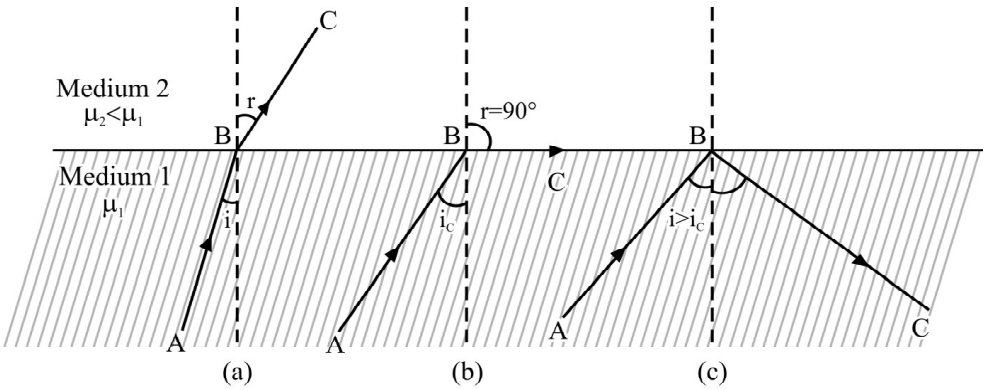


Fig. 25

$$\mu_1 \sin i_c = \mu_2 \sin \frac{\pi}{2}$$

or $$i_c = \sin^{-1}\left(\frac{\mu_2}{\mu_1}\right)$$

For T.I.R to take place light must go from a denser to a rarer medium.

Total Internal Reflection at Glass - Air Surface

Consider an incident ray going from glass ($\mu = 1.5$) to air. The critical angle of incidence, i_c is

$$i_c = \sin^{-1}\left(\frac{1}{1.5}\right) \simeq 41.8^0$$

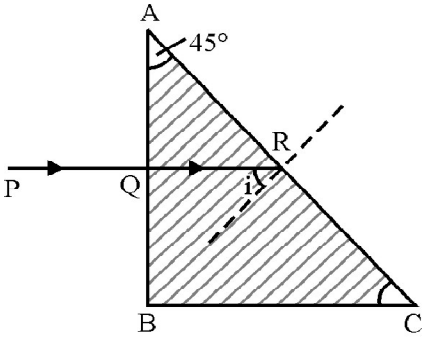


Fig. 26

Fig. 26 shows an issoceless right angled glass (refractive index μ) prism. PQ is a ray incident normally at face AB. It goes straight and is incident on face AC at an angle of incident $i = 45^0$. Total internal reflection can occur at R. For T.I.R to take place at face AC, i.e. no light emerges out of face AC, $i \geq i_c$. The minimum value of i; should be equal to i_c. Therefore for T.I.R to take place at face AC

$$i_{min} = 45^0 = i_c$$

The minimum refractive index μ_m of prism is

$$\mu_m = \frac{1}{(\sin i_c)} = \frac{1}{\sin 45} = \sqrt{2} = 1.414$$

Such a glass $(\mu = 1.5)$ issocless prism is known as a totally reflecting prism $(\mu > \mu_m)$. Fig. 27 (a) and (b) show how such a prism is used to deviate an incident ray by $\frac{\pi}{2}$ or π.

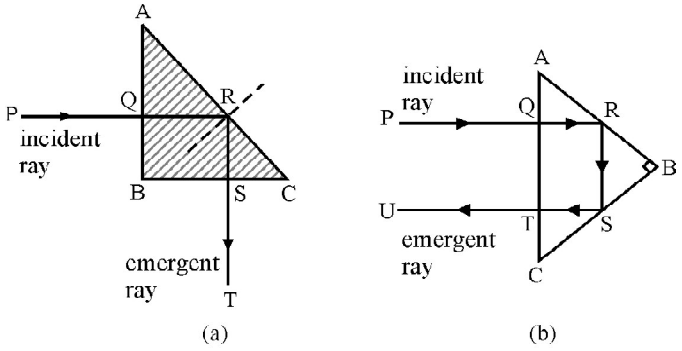


Fig. 27

Total Internal Reflection at Water - Air Surface

Total internal reflection can occur when an incident ray in water ($\mu = 1.33$) goes to air. The critical angle, i_c, is

$$i_c = \sin^{-1}\left(\frac{1}{1.33}\right) \simeq 48.6°$$

A fish under water in a pond sees the entire outside world canfired within a cone of angle= $2i_c \simeq 98^0$.

ILLUSTRATIONS OF T.I.R

The optical illusion known as mirage is observed during day time in a hot desert. A thirsty deer, looks around for water. The deer sees a clear inverted image of a distant tree and thinks that the tree is on the bank of a water pond. The deer runs in that direction and on reaching the tree finds no water! The phenomenon of mirage is due to *variation of refractive index with temperature.* In a 'hot desert' during day time the temperature of air layers decreases as one moves upwards from ground. Therefore refractive index of air layers decreases as one moves towards ground. An incident ray from top of a distant tree undergoes refraction from a denser layer to a rarer layer of air and undergoes T.I.R producing a clear inverted image of the tree.

The shinning of diamonds is also due to repeated T.I.R. A ray inside diamond undergoes repeatd T.I.R inside diamond before it emerges out. Take $\mu = 2.42$ for diamond, the critical angle.

$$i_c = \sin^{-1}\left(\frac{1}{2}.42\right) \simeq 24.4^0$$

Due to small values of i_c; repeated T.I.R occures easily inside diamond.

REFRACTION THROUGH A PRISM

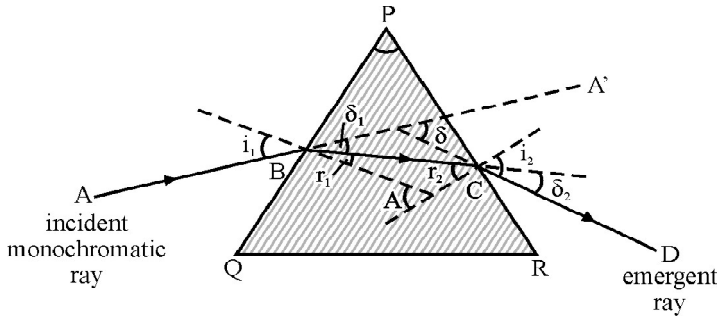


Fig. 28

In Fig. 28 PQR is a prism of a material of refractive index μ. AB is a monochromatic incident ray on face PQ at angle of incidence i_1. Fig. 28 shows refracted ray BC inside prism and the emergent ray CD. $\underline{|QPR} = A$ (i.e. angle between the two faces where refraction occurs) is known as the refracting angle, A; of prism. i_2 is angle of emergence.

Due to refraction at both faces PQ and PR the incident ray is deviated by an angle δ.

δ = The angle of total deviation = $\delta_1 + \delta_2$

$$= i_1 + i_2 - (r_1 + r_2) \qquad .....(1)$$

Also, $r_1 + r_2 = A$ .....(2)

$\therefore \quad \delta = i_1 + i_2 - A$ .....(3)

For a given prism; δ depends on angle of incidence. Fig. 29 shows δ vs i graph. From graph we note that there is a particular angle of incidence i_m for which deviation is minimum (δ_m). It can be shown that for minimum deviated ray.

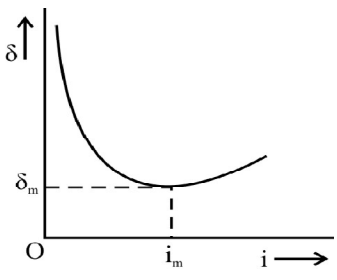


Fig. 29

$$i_1 = i_2, \quad r_1 = r_2 = \frac{A}{2} \qquad .....(4)$$

and $\delta_m = 2i_1 - A$

or $i_1 = (i)_m$ = angle of incident for minimum deviated ray $= \dfrac{A + \delta_m}{2}$ .....(5)

For minimum deviated ray; the path of ray inside prism is parallel to the base QR of prism as shown in Fig. 30. Applying Snell's law to the minimum deviated ray; i.e.

$i_1 = \dfrac{A + \delta_m}{2}$ and $r_1 = \dfrac{A}{2}$; we have

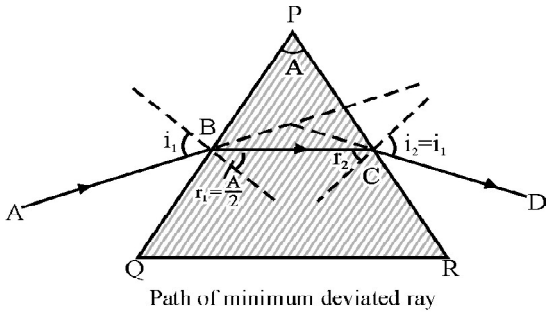


Fig. 30

$$\mu = \frac{\sin\left(\dfrac{A + \delta_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)} \qquad .....(6)$$

For a small angled prism; A; δ and i are small. Taking $\sin\theta \simeq \theta$; Eqn. (6) reduces to

$$\mu \simeq \frac{\left(\dfrac{A + \delta_m}{2}\right)}{\left(\dfrac{A}{2}\right)} = 1 + \frac{\delta}{A}$$

or $\delta = (\mu - 1)A$ .....(7)

DISPERSION OF LIGHT

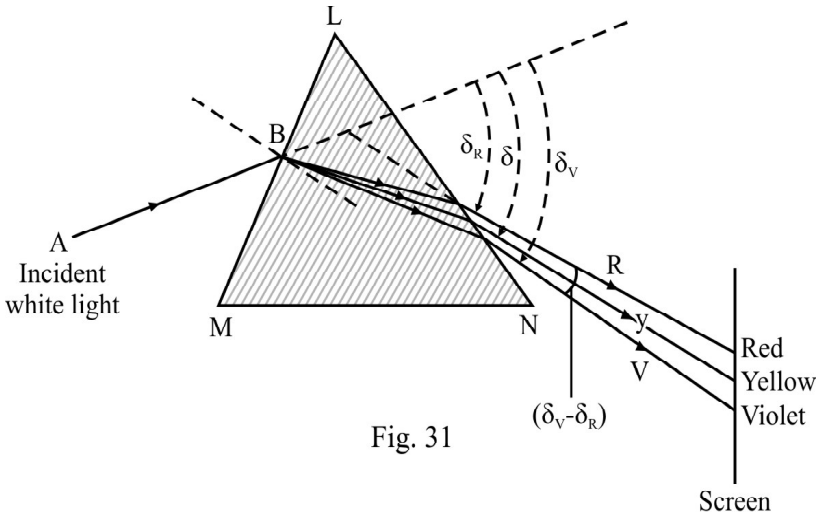


Fig. 31

Fig. 31 shows an incident white ray of light, AB on a prism LMN. In passing through the prism the light breaks up into its constituent colours; VIBGYOR. This is known as dispersion of light. In Fig. 31; δ_V, δ and δ_R are the angle of deviation for violet; yellow and red colour respectively. δ is known as angle of mean deviation. Also $\delta_V > \delta_R$ and $\delta_R - \delta_R = \Delta$, is known as angular dispersion. On the screen we see each constituent colour seperately. We say a spectrum of incident white light is produced on screen.

Cause of Dispersion of Light

The dispersion of a polychromatic (i.e. white) light due to refraction in a medium occurs due to the variation of refractive index (μ) of the material with the wave length (λ) of the light. According to Cauchy

$$\mu = a + \frac{b}{\lambda^2} \qquad .....(8)$$

where a and b are constants and λ is wave length of light. This is known as CAUCHY'S RELATION. According to Eqn (8) μ decreases as λ increases. For $\lambda \to 0$; $\mu \to \infty$ and for $\lambda \to \infty$; $\mu \to a$. The variation of μ with λ is shown graphically in Fig. 32 (a) and Fig. 32 (b) is graph of μ vs $\frac{1}{\lambda^2}$. This is a straight line with slope = b and intercept on μ axis equal to a.

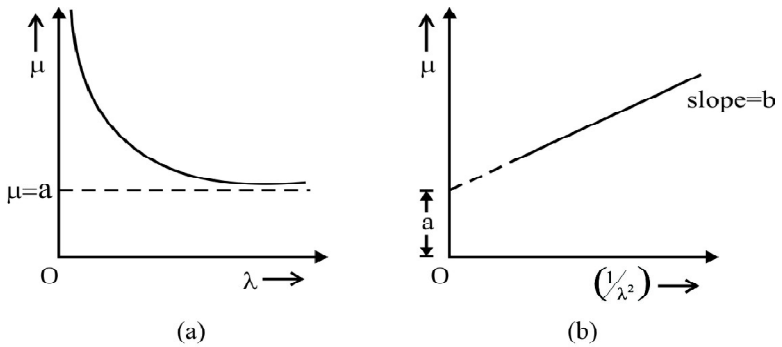


Fig. 32

For incident white light AB on face LM of prism; angle of incidence i for all colours is same. Since μ is different for different colours angle of refraction (r) is different for different colour. Therefore different colours follow different paths and emerge out of prism in different directions. This is dispersion.

For white light we know that $\lambda_{min} = \lambda_V$ and $\lambda_{max} = \lambda_R$. Therefore $\mu_{max} = \mu_V$ and $\mu_{min} = \mu_R$. For small angled prisms, δ = angle of deviation = $(\mu - 1)A$ therefore violet colour undergoes maximum deviation and deviation is minimum for red. The angle, Δ; between violet and red, is the angular dispersion produced by the prism, i.e.

$$\Delta = \text{Angular dispersion} = \delta_A - \delta_R = (\mu_V - \mu_R)A \qquad .....(9)$$

Dispersive Power

The dispersive power, ω, of a refracting medium is angular dispersion per unit mean deviation; i.e.

$$\omega = \text{Dispersive power} = \frac{\text{Angular dispersion}}{\text{Mean deviation}} = \frac{\delta_V - \delta_R}{\delta}$$

For small angled prism, ω is a constant for a given medium. Let α be the refracting angle of a small angled prism then

$$\delta = \text{Mean deviation} = (\mu - 1)\alpha$$

$$\Delta = \text{Angular dispersion} = (\mu_V - \mu_R)\alpha$$

$$\omega = \text{The dispersive power} = \frac{\mu_V - \mu_R}{\mu - 1} \qquad .....(10)$$

$$= \frac{d\mu}{(\mu - 1)}$$

Combination of two small angled prisms

Consider two small angled prisms of angles α_1 and α_2 having dispersive power ω_1 and ω_2 respectively. Let μ_{1V}; μ_{1R} and μ_1 be the refractive index of prism 1 for violet; red and yellow (mean) colour. The corresponding quantities for prism 2 are μ_{2V}; μ_{2R} and μ_2 .Then

$$\omega_1 = \frac{\mu_{1V} - \mu_{1R}}{\mu_1 - 1}, \text{ and } \omega_2 = \frac{\mu_{2V} - \mu_{2R}}{\mu_2 - 1}$$

Let δ_{1V}, δ_{1R} and δ_1 be the angle of deviation of violet, red and mean colour for prism 1. Corresponding quantities for prism 2 are δ_{2V}, δ_{2R} and δ_2 respectively. For the combination of the two prisms

$$\delta = \text{Net mean deviation} = \delta_1 + \delta_2$$

$$= (\mu_1 - 1)\alpha_1 + (\mu_2 - 1)\alpha_2$$

and $\Delta = \text{Net angular dispersion} = \Delta_1 + \Delta_2$

$$= (\mu_{1V} - \mu_{1R})\alpha_1 + (\mu_{2V} - \mu_{2R})\alpha_2$$

By choosing α_1 and α_2 properly we can have an arrangment causing

(i) Dispersion without deviation, i.e.; $\delta = 0;\ \Delta \neq 0$

and (ii) Deviation without dispersion, i.e.; $\Delta = 0$ and $\delta \neq 0$

The two arrangements are shown in Fig.33 (a) & (b) respectively.

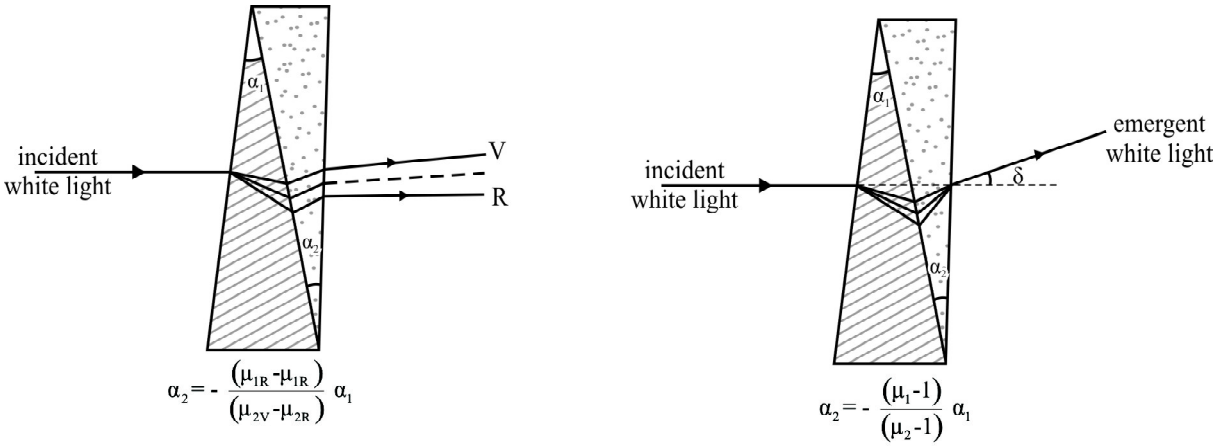


Fig. 33

Example–15:

A rectangular glass slab ABCD of refractive index n_1 is immersed in water of refractive index n_2 ($n_1 > n_2$). A ray of light is incident at surface AB of the slab as shown in Fig. 34. What is maximum value of angle of incidence on AB such that the ray imerges out only from the surface CD?

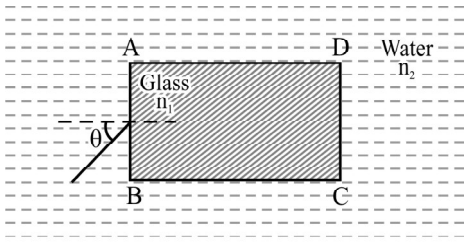


Fig. 34

(I.I.T 2000)

Solution:

In Fig. 35 PQ is incident ray inside medium of refractive index n_1. QR is refracted ray inside slab ABCD. Let θ_1 be angle of refraction. From Snell's law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad .....(1)$$

The ray QR is incident on face BD at angle θ_2. It will not emerge out from BD if it undergoes total internal reflection at R. This will be so it

$$\theta_2 \geq i_c$$

where i_c is critical angle of incidence from glass (n_2) to water (n_1). We know

$$n_2 \sin i_c = n_2 \sin \frac{\pi}{2} = n_1 \qquad .....(2)$$

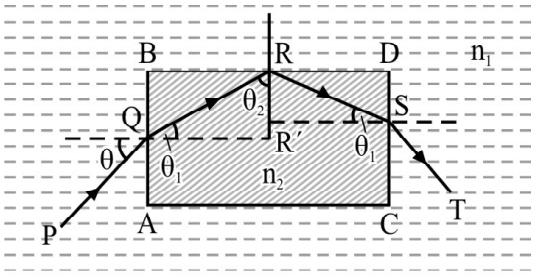


Fig. 35

Also from $\Delta QRR'$;

$$\theta_2 = \frac{\pi}{2} - \theta_1 \qquad .....(3)$$

For T.I.R to occur on face BD;

$$(\theta_2)_{min} = i_c;$$

$$\therefore \quad (\theta_1)_{max} = \frac{\pi}{2} - i_c$$

From Eqn. (1); we have

$$n_1 \sin(\theta)_{max} = n_2 \sin(\theta_1)_{max}$$

or $$\sin(\theta)_{mix} = \frac{n_2 \cos i_c}{n_1}$$

or $$\theta_{max} = \sin^{-1}\left[\frac{n_2}{n_1}\cos i_c\right]$$

$$= \sin^{-1}\left[\frac{n_2}{n_1}\cos\left\{\sin^{-1}\frac{n_1}{n_2}\right\}\right] \qquad \text{[From Eqn. (2)]}$$

Example–16:

ABC is a right angled issoceless prism of a material of $\mu_1 = 1.352$. A plane of refrative index μ_2 $(\mu_2 < \mu_1)$ is cemented to diagonal face AC. PQ is an incident ray. The refracted ray passes undeviated through the diagonal face AC. What is i?

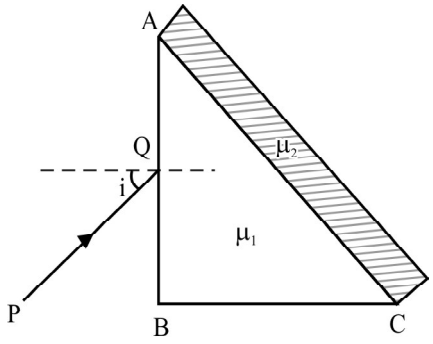


Fig. 36

Solution:

PQ is incident ray on face AB. The refracted ray QR undergoes no deviation on diagonal face AC if its incident normally on AC as shown in Fig. 37. From Fig. 37 r = angle of refraction on face AB = 45^0. From Snell's law

$1 \sin i = 1.352 \sin 45^0$

$$= \frac{1.352}{\sqrt{2}} \simeq 0.956$$

$$\therefore \quad i = \sin^{-1}(0.956) \simeq 73^0$$

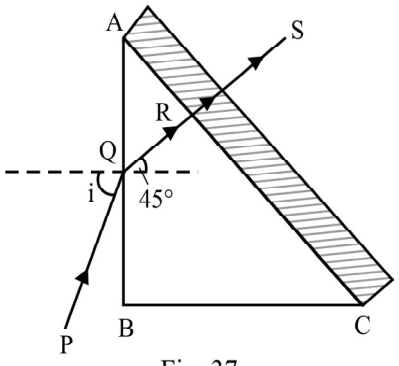


Fig. 37

Example–17:

A ray of light is incident at an angle of 60^0 on one face of a prism of angle 30^0. The ray emerging out of the prism makes an angle of 30^0 with incident ray. Show that the emergent ray is perpendicular to the face through which it emerges and calculate the refractive index of the material of prism.

Solution:

Let i_1 and i_2 be the angle of incidence and angle of emergence. $A = 30^0$ is the refracting angle of prism. The angle of deviation δ is

$$\delta = i_1 + i_2 - A$$

Given $\quad i_1 = 60^0,\ i_2 = ?,\ \delta = 30^0 \quad$ and $\quad A = 30^0$

$$\therefore \quad 30 = 60 + i_2 - 30$$

or $\qquad i_2 = 0$

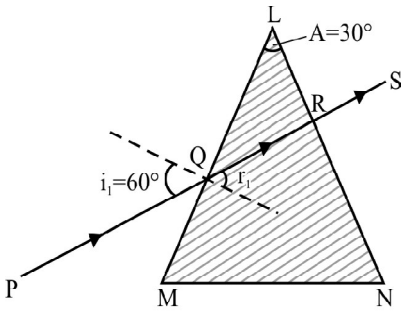


Fig. 38

Since angle of emergence is zero; the light is incident normally on the face LN of prism as shown in Fig. 38. The angle of incidence inside prism at point R = $r_2 = 0$.

Also, $r_1 + r_2 = A$

$\therefore \quad r_1 + 0 = 30$

or $\quad r_1 = 30^0$

Applying Snell's law at point Q

$$1 \sin 60 = \mu \times \sin 30$$

$\therefore \quad \mu_1 = \sqrt{3} = 1.732$

Example–18:

Monochromatic light is incident on a glass prism of angle A. The refractive index of the material of prism is μ. A ray is incident on one face at angle of incidence θ as shown in Fig. 39. Show that light will be transmitted through face AC; if

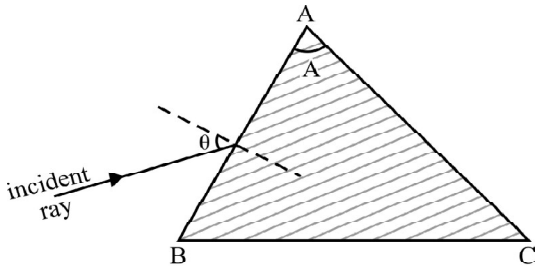


Fig. 39

$$\theta > \sin^{-1}\left[\mu \sin\left\{A - \sin^{-1}\left(\frac{1}{\mu}\right)\right\}\right]$$

Solution:

In Fig. 40 PQ is incident ray on face AB. Let r_1 be angle of refraction. From Snell's law

$$1 \sin\theta = \mu \sin r_1$$

or $$r_1 = \sin^{-1}\left[\frac{\sin\theta}{\mu}\right] \qquad .....(1)$$

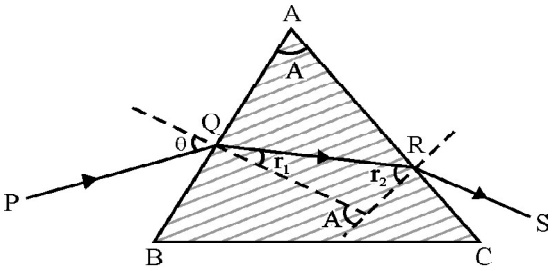


Fig. 40

Let r_2 be angle of incidence for ray QR on face AC of prism. The ray QR will emerge out of face AC if

$$r_2 < i_C \qquad .....(2)$$

where i_C is critical angle of incidence inside glass. Also

$$i_C = \sin^{-1}\left(\frac{1}{\mu}\right) \qquad .....(3)$$

From Fig. 40; we have

$$r_1 + r_2 = A$$

or $r_2 = A - r_1$

From Eqns. (2), (3) and (4) we have

$$r_2 < i_C$$

or $$A - \sin^{-1}\left[\frac{\sin\theta}{\mu}\right] < \theta_C$$

or $$\sin^{-1}\left[\frac{\sin\theta}{\mu}\right] > A - \theta_C$$

or $$\frac{\sin\theta}{\mu} > \sin\left(A - \theta_C\right)$$

$$\therefore \quad \theta > \sin^{-1}\left[\mu \sin\left(A - \theta_C\right)\right]$$

$$> \sin^{-1}\left[\mu \sin\left\{A - \sin^{-1}\left(\frac{1}{\mu}\right)\right\}\right]$$

Example–19:

A right angled prism of refractive index μ_1 is placed inside a rectangular block of refractive index μ_2, which is surrounded by a medium of refractive index μ_3 as shown in Fig. 41. An incident ray *e* enters the rectangular block at normal incidence. Four possible paths (1) e → i (2) e → f (3) e → g and (4) e → h are shown. In each case what is relation between μ_1, μ_2 and μ_3?

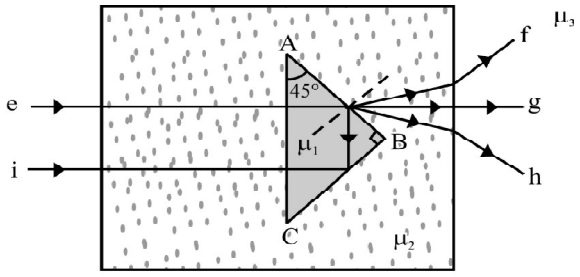


Fig. 41

(I.I.T 2013; Adv)

Solution:

Case–I : The incident ray 'e' emerges out as 'i'. Total internal reflection occurs at face AB of prism; therefore $\mu_1 > \mu_2$. The critical angle of incidence i_C from μ_1 to μ_2 is less than 45^0 ie., $45° > i_C$.

$$\therefore \quad \sin 45 > \sin i_C \qquad \text{or} \qquad \frac{1}{\sqrt{2}} > \sin i_C \qquad \text{.....(1)}$$

Also,

$$\mu_1 \sin i_C = \mu_2 \sin \frac{\pi}{2}$$

$$\therefore \quad \sin i_C = \frac{\mu_2}{\mu_1} \qquad .....(2)$$

Fron Eqn. (1) and (2)

$$\frac{1}{\sqrt{2}} > \frac{\mu_2}{\mu_1} \qquad \text{or} \qquad \mu_1 > \sqrt{2}\,\mu_2$$

Case–II : The ray incident on AB emerges out and bends towards normal, i.e. $\mu_2 > \mu_1$. This ray incident on face of slab emerges out and bends away from normal; i.e. $\mu_3 > \mu_2$. Therefore

$$\mu_2 > \mu_1$$

$$> \mu_3$$

Case–III : The ray incident on face AB of prism as well as the face of rectangular slab emerges out straight i.e. there is no deviation. Therefore,

$$\mu_1 = \mu_2$$

Case–IV : The incident ray 'e' emergs out as ray 'h'; on face AB of prism the ray undergoes refraction and moves away from normal; therefore

$$\mu_2 < \mu_1$$

or $\mu_1 < \sqrt{2}\,\mu_2$

For the ray incident on slab; the refracted ray bends away from normal; therefore $\mu_2 < \mu_3$. Therefore

$$\mu_1 < \sqrt{2}\ \mu_2 \qquad \text{and} \qquad \mu_2 > \mu_3$$

Example–20:

A ray of light is incident at grazing angle of incidence on one face of a prism of refracting angle A and refractive index μ. Show that angle of emergence i_2 is

$$\sin^{-1}\left[\left(\mu^2 - 1\right)^{1/2} \sin A - \cos A\right]$$

Solution:

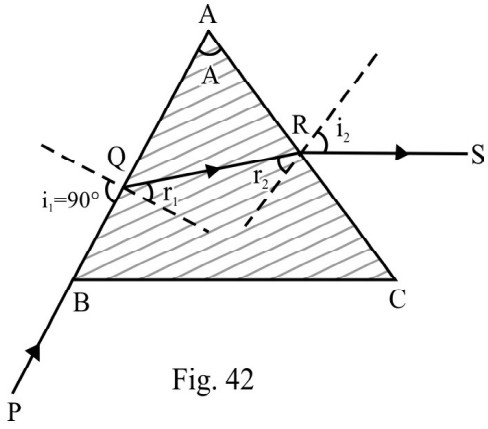


Fig. 42

Fig. 42 shows incident ray PQ grazing face AB of prism ABC. The angle of incidence i_1 at point Q is 90^0. Let r_1 be the angle of refraction inside prism. Let i_C be the critical angle of incidence from prism to air. Obviously $r_1 = i_C$. Also

$$r_1 + r_2 = A$$

$$\therefore \quad r_2 = A - r_1 = A - i_C$$

Let i_2 be angle of imergence at point R from face AC of prism. Applying Snell's law at R

$$\mu \sin r_2 = 1 \times \sin i_2$$

$$\therefore \quad \sin i_2 = \mu \sin\left[A - i_C\right]$$

$$= \mu\left[\sin A \cos i_C - \cos A \sin i_C\right]$$

We also know that $\sin i_C = \frac{1}{\mu}$. Therefore

$$\sin i_2 = \mu\left[\left(1 - \frac{1}{\mu^2}\right)^{1/2} \sin A - \frac{\cos A}{\mu}\right]$$

$$= \left(\mu^2 - 1\right)^{1/2} \sin A - \cos A$$

$$\therefore \qquad i_2 = \sin^{-1}\left[\left(\mu^2 - 1\right)^{1/2} \sin A - \cos A\right]$$

Example–21:

A prism of refractive index n_1 and another prism of refractive index n_2 are stuck together with a glue as shown in Fig. 43. The angles of the prisms are shown in Fig. 43. n_1 and n_2 depend on the wave length of light λ according to the relation

$$\mathbf{n_1 = 1.20 + \frac{10.8 \times 10^4}{\lambda^2}}$$

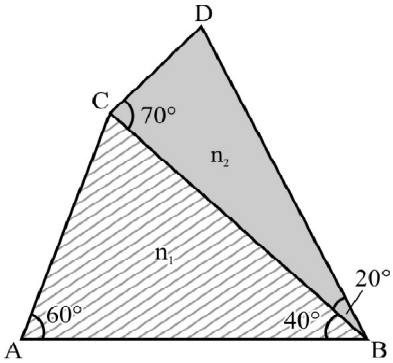


Fig. 43

and $\mathbf{n_2 = 1.45 + \frac{1.80 \times 10^4}{\lambda^2}}$

where λ is in nm.

(a) Calculate the wave length λ_0 for which rays incident at any angle on the interface BC pass without any bending at that interface.

(b) For light of wave length λ_0 find the angle of incidence i on the face AC such that the deviation produced by the combination is minimum. (I.I.T 1998)

Solution:

(a) There will be no deviation at face BC for any angle of incidence if $n_1 = n_2$. Therefore

$$1.20 + \frac{10.8 \times 10^4}{\lambda_0^2} = 1.45 + \frac{1.8 \times 10^4}{\lambda_0^2}$$

or $\lambda_0 = 600\,\text{nm}$

(b) For $n_1 = n_2$; the given arrangement is a part of a prism ABE of angle 60^0. Let i be angle of incidence of ray PQ on face AC undergoing minimum deviation. For minimum deviated ray

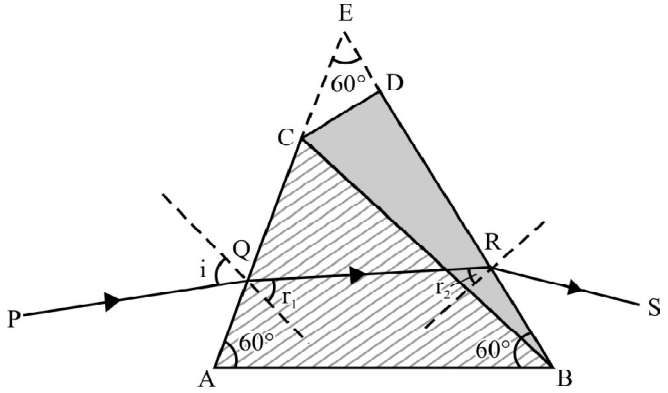


Fig. 44

$$r_1 = r_2 = \frac{60}{2} = 30^0 \qquad .....(1)$$

Applying Snell's law at point Q; we have

$$1 \sin i = n_1 \sin 30^0$$

Also,

$$n_1 = 1.20 + \frac{10.8\times10^4}{\left(6\times10^2\right)} = 1.20 + 0.3 = 1.50$$

From Eqn. (2) and (3) we have

$$i = \sin^{-1}\left[\frac{1.50}{2}\right] = \sin^{-1}\left(\frac{3}{4}\right) \simeq 48.6^0$$

Example–22:

A crown glass prism has a refracting angle of 60^0. A white ray of light is incident on one face at an angle of 45^0. What is

(1) mean deviation

and (2) angular dispersion?

Given $\mu_V = 1.538$ and $\mu_R = 1.500$.

Solution:

For RED incident ray

Let r_1 be angle of refraction at the first face of prism. Given $i_1 = 45^0$ and = 1.500. From Snell's law

$$1\sin 45 = 1.500\sin r_1$$

or $$\sin r_1 = \frac{1}{\sqrt{2}\times1.500} \simeq 0.471$$

$$\therefore \quad r_1 = \sin^{-1}\left(0.471\right) \simeq 28.1^0$$

Let r_2 be angle of incidence on second face of prism. We know

$$r_1 + r_2 = A$$

$$\therefore \quad r_2 = 62 - 28.1 = 31.9^0$$

Let i_2 be the angle of emergence. Applying Snell's law

$$1.5\sin\left(31.9^0\right) = 1\times\sin i_2$$

or $$\sin i_2 = 0.793$$

$$\therefore \quad i_2 = \sin^{-1}\left(0.793\right) \simeq 52.5^0$$

Let δ_R be angle of deviation for red colour. Then

$$\delta_R = i_1 + i_2 - A = 45^0 + 52.5^0 - 60 = 37.5^0$$

For Violet Colour

Proceeding exactly in the same way as above, we have

$$1\sin 45 = 1.538 \sin r_1'$$

$\therefore \quad r_1' = \sin^{-1}(0.460) \simeq 27.3^0$

Also, $r_2' = 60 - 27.3 = 32.7^0$ and

$$1.538 \sin\left(32.7^0\right) = 1 \times \sin\ i_2$$

or $\quad i_2 = \sin^{-1}(0.831) \simeq 56.1^0$

$\therefore \quad \delta_V = 45 + 56.1 - 60 = 41.1^0$

The mean deviation $= \delta = \dfrac{\delta_V + \delta_R}{2} = \dfrac{41.4 + 37.5}{2} = 39.3^0$

The angular dispersion $= \Delta = \delta_V - \delta_R = (41.1 - 37.5) = 3.6^0$

REFRACTION AT A SPHERICAL SURFACE

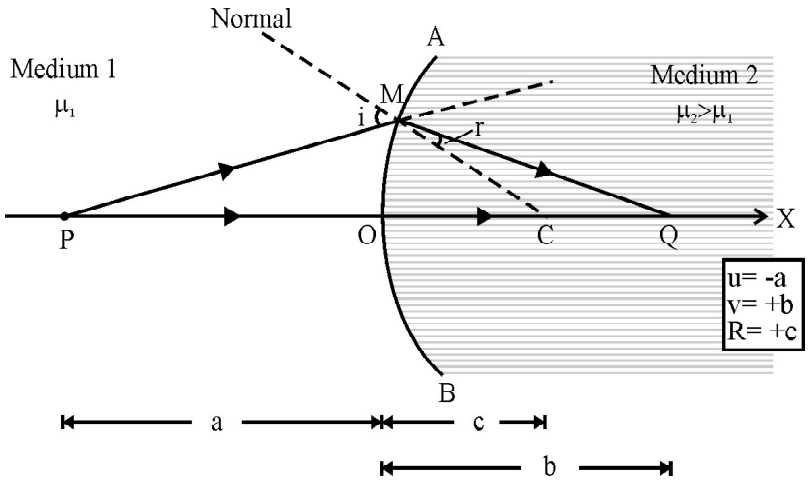


Fig. 45

Fig. 45 shows a convex surface AOB seprating two media of refractive index and μ_1 and $\mu_2 (> \mu_1)$. C is the center of curvature of spherical surface. P is a point object on principal axis at a distance OP = a. OC = c = The radius of spherical surface. Medium 1 of refractive index μ_1 is the *object space.* The refracted ray goes into medium 2 of refractive index μ_2. This is the *image space* Fig. 45 shows incident ray PM and refracted ray MQ. Since we have $\mu_2 > \mu_1$; the refracted ray bends towards normal. The refracted ray meets principal axis at Q. We say Q is real image of real object P. Using co–ordinate convention of signs.

u = The object position = –a

v = The image position = +b

R = The radius of curvature = +c

It can be shown that

$$\frac{\mu_2}{v} - \left(\frac{\mu_1}{u}\right) = \frac{\mu_2 - \mu_1}{R} \qquad .....(1)$$

Eqn. (1) is 'refraction formula for a spherical surface'. The same formula is valid for a concave spherical surface. Fig. 46 shows refraction at a concave surface. Note the image formed Q is virtual.

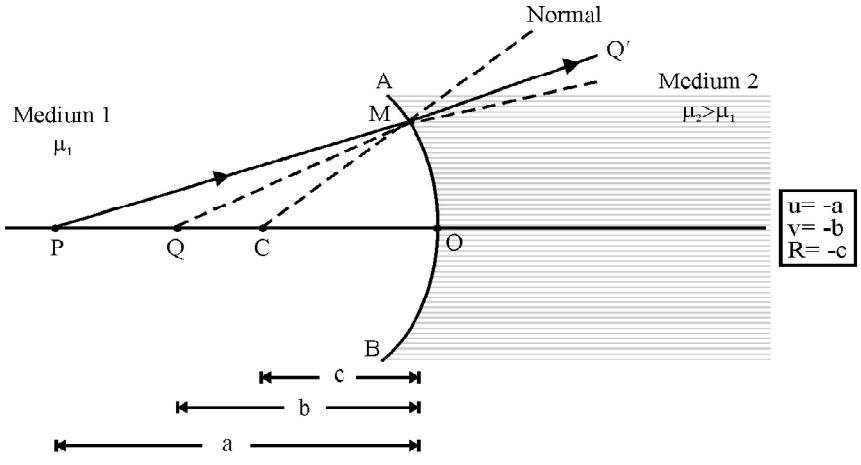


Fig. 46

Note v is positive for a 'real image' and is negative for a 'virtual image'.

Principal Focus; Focal Length and Focal Plane

Let a parallel beam; parallel to principal axis be incident in medium 1. The refracted rays.

(i) meets principal axis at a point F_2 for a convex surface

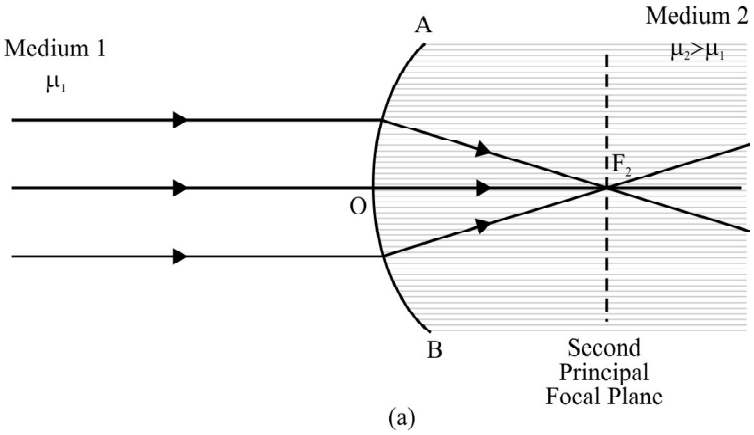
(ii) appear to meet principal axis at a point F_2 for a concave surface

as shown in Fig. 47 (a) and (b); if $\mu_2 > \mu_1$. F_2 is known as second principal focus. Its position given the second principal focal length f_2. Expressed mathematically,

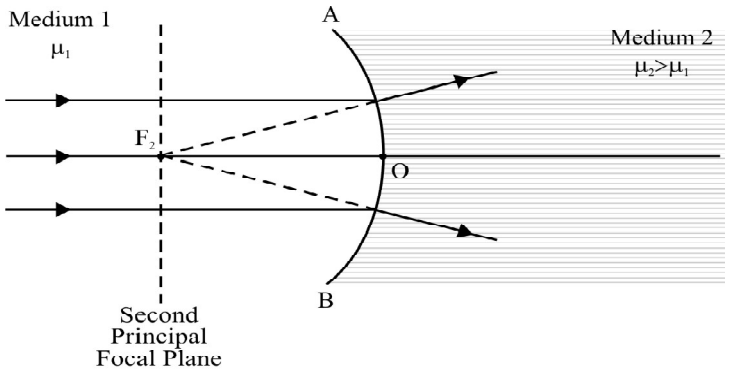
$$u = -\infty, \quad v \equiv f_2$$

From Eqn. (1), we have

$$\frac{\mu_2}{f_2} = \frac{\mu_2 - \mu_1}{R} \qquad .....(2)$$



(a)



(b)

Fig. 47

For $\mu_2 > \mu_1$; the convex surface behaves as a converging surface and the concave surface as a diverging surface. However it $\mu_2 < \mu_1$; the convex surface behaves as a diverging surface and concave surface as converging surface. The converging or diverging nature of a surface depends on value of μ_1 and μ_2.

A plane perpendicular to principal axis through point F_2 is the second principal focal plane. A parallel incident beam inclined to principal axis; after refraction meets or appears to meet at a point on the second principal focal plane.

The 'First Principal Focus'; Focal Length and Focal Plane.

Let $\mu_2 > \mu_1$, then for a

(1) Convex Surface : F_1 is a point on principal axis where if an object is placed the refracted rays are parallel to principal axis as shown in Fig. 48 (a).

(2) Concave Surface : F_1 is a point on principal axis where if an object is imagined to be placed (i.e. virtual object) the refracted rays are parallel to principle axis as shown in Fig. 48 (b).

F_1 is known as the 'first principal focus'. Position of F_1 gives first principal focal length f_1. Expressed mathematically

$$u \equiv f_1;\ v = +\infty$$

From Eqn. (1) we have

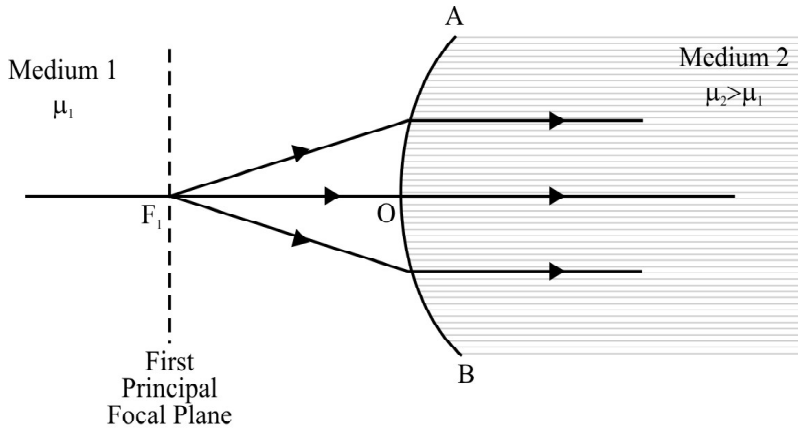
$$-\left(\frac{\mu_1}{f_1}\right) = \frac{\mu_2 - \mu_1}{R} \qquad .....(3)$$

Note $\frac{\mu_2}{f_2} = -\left(\frac{\mu_1}{f_1}\right)$. Since $\mu_1 \neq \mu_2;\ |f_1| \neq |f_2|$. In general, the two focii are not equidistant from the spherical surface.

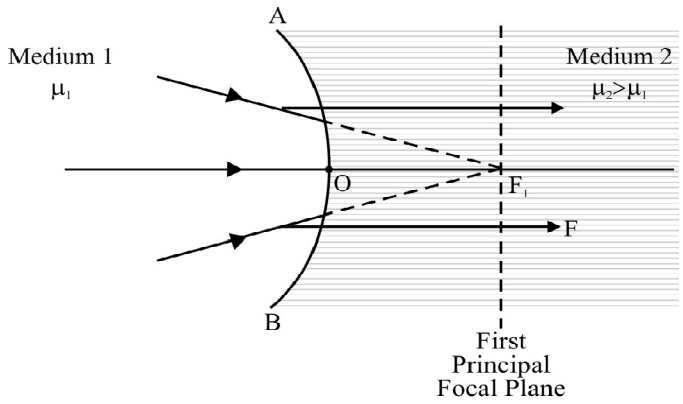
A plane perpendicular to principal axis through F_1 is the "first principal focal plane".

By convention; the focal length f of a spherical surface is the position of F_2; i.e. $f \equiv f_2$. From Eqn. (2);

$$f \equiv f_2 = \frac{\mu_2 R}{(\mu_2 - \mu_1)} \qquad .....(4)$$



(a)



(b)

Fig. 48

In terms of focal length; f; the refraction formula expressed in Eqn. (1) can be rewritten as

$$\frac{\mu_2}{v} - \left(\frac{\mu_1}{u}\right) = \frac{\mu_2}{f} \qquad .....(5)$$

The Power of a Spherical Surface

The reciprocal of focal length f of the refracting surface expressed in meter; is the power P of the surface. SI unit of P is diopter. Expressed mathematically

$$P\,(\text{in diopter}) = \frac{1}{f\,(\text{in meter})} \qquad .....(6)$$

For $\mu_2 > \mu_1$; P is a positive number for a convex refracting surface and is a negative number of a concave surface. However sign of P changes for both surfaces it $\mu_2 < \mu_1$.

Refraction at Spherical Surface for an Object of Finite Size

Fig. 50 shows an object $PQ = h_1$ placed in medium 1. $P'Q'$ is the real, inverted image formed in medium 2. $P'Q' = h_2$ is length of image formed. In terms of co–ordinate convention of sign

$y_1 = \text{SIZE of image} = +h_1$

$y_2 = \text{SIZE of object} = -h_2$

The transverse magnification $= m = \frac{y_2}{y_1}$. It can be shown that

$$m = \frac{\left(v/\mu_2\right)}{\left(u/\mu_1\right)} = \frac{v'}{u'} \qquad .....(7)$$

Where $v' = \frac{v}{\mu_2} =$ reduced image posion and $u' = \frac{u}{\mu_1} =$ reduced object position.

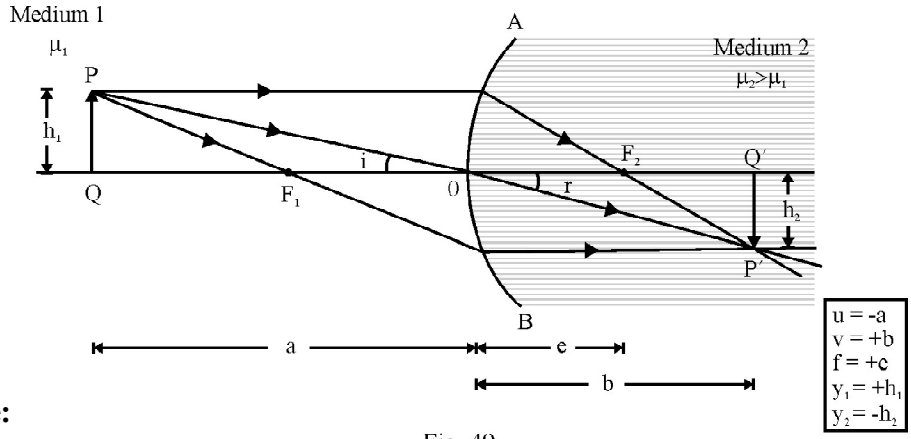


Fig. 49

Important Note:

The following table gives the sign of u, v, f etc. for refraction at a spherical surface.

1.	(i) Convex Surface	f is +ve for $\mu_2 > \mu_1$
		is –ve for $\mu_2 < \mu_1$
	(ii) Concave Surface	f is –ve for $\mu_2 > \mu_1$
		is +ve for $\mu_2 < \mu_1$
2.	Objecct	u is –ve for a real object
		is +ve for a virtual object
3.	Image	v is +ve for a real image
		is –ve for a virtual image
4.	Magnification	m is +ve for erect image
		is –ve for inverted image
		|m|> 1, enlarged image
		< 1, diminished image

Example–23:

A point object is placed at a point mid–way between the center and surface of a glass sphere of radius 10 cm refractive index 1.5. What is distance of the image as seen from air outside from the side of the sphere to which object is nearer?

Solution:

In Fig. 51 P is the point object inside glass. Observer is to the right of point O. We have

$\mu_1 = 1.5;\ \mu_2 = 1.0;\ u = -5$ cm ; $R = -10$ cm ; $v = ?$

From refraction formula at a spherical surface

$$\frac{\mu_2}{v} - \left(\frac{\mu_1}{u}\right) = \frac{\mu_2 - \mu_1}{R}$$

we have

$$\frac{1}{v} - \left(\frac{1.5}{(-5)}\right) = \frac{1.0 - 1.5}{(-10)}$$

or $$\frac{1}{v} = \frac{1}{20} - \left(\frac{3}{10}\right) = -\left(\frac{5}{20}\right)$$

or $v = -4$ cm

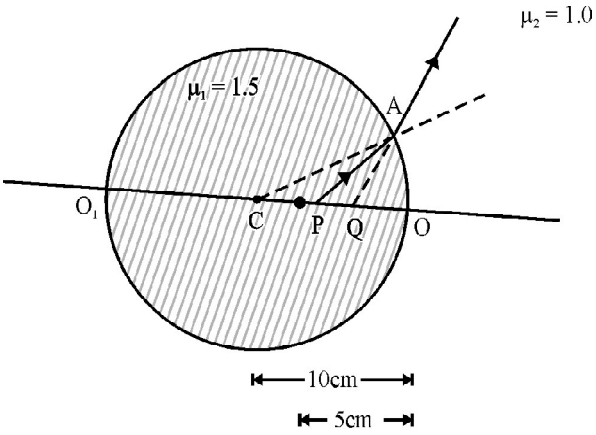


Fig. 50

Since v is negative the image Q is virtual, at a distance of 6 cm from the center of sphere.

Example–24:

A parallel beam of light travelling in water $\left(\mu_W = \frac{4}{3}\right)$ is refracted by a spherical air bubble of radius 2 mm inside water. Find the position of image due to refraction at both surfaces of bubble.

(I.I.T 1992)

Solution:

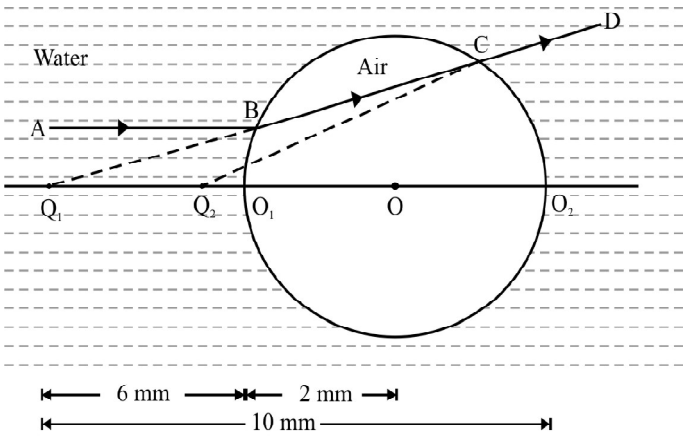


Fig. 51

For refraction at first surface

$\mu_1 = 4/3;\ \mu_2 = 1.0;\ u_1 = -\infty\ ;\ R_1 = +2$mm

Using refraction formula we have

$$\frac{1}{v_1} - \frac{(4/3)}{(-\infty)} = \frac{1-(4/3)}{2}$$

$\therefore \quad v_1 = -6$ mm

For refraction at second surface Q_1 acts as object. The final image is Q_2 as shown in Fig. 52. For second surface

$u_2 = -10$ mm; $v_2 = ?$; $R_2 = -2$ mm; $\mu_1 = 1.0$ and $\mu_2 = 4/3$

Using refraction formula; we have

$$\frac{(4/3)}{v_2} - \left[\frac{1}{(-10)}\right] = \frac{(4/3)-1}{(-2)}$$

$\therefore \; v_2 = -5$ mm

Since v is negative the final image Q_2 is VIRTUAL at a distance of 5 mm from O_2.

Example–25:

PQ is an incident ray on a sphere of radius r and refractive index μ. It passes through the edge Q and R of two radii inclined to one another at an angle of 120^0. The net deviation the ray undergoes in passing through sphere is 30^0. What is μ?

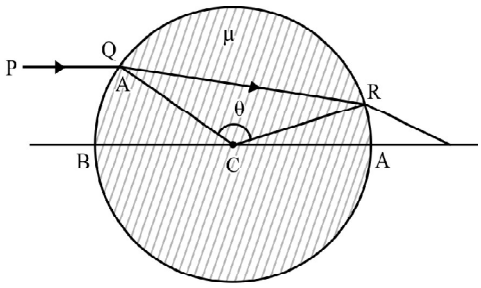


Fig. 52

Solution:

Fig. 54 shows the refraction of ray through the sphere. Let i and r be angle of incidence and refraction at Q. From triangle QCR.

$$\theta + 2r = \pi$$

or $\quad 2r = \pi - \theta \qquad .....(1)$

The total deviation δ; is

$$\delta = \delta_1 + \delta_2 = 2(i - r)$$

or $\quad 2r = 2i - \delta \qquad .....(2)$

From Eqns. (1) and (2) we have

$$\pi - \theta = 2i - \delta$$

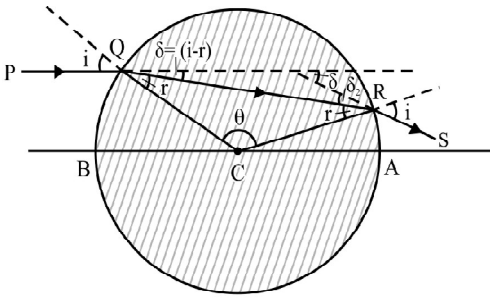


Fig. 53

or $\quad i = \dfrac{\pi + (\delta - \theta)}{2} = \dfrac{180 + (30 - 120)}{2} = 45^0$

From Eqn. (1); we have

$$r = \frac{180 - 120}{2} = 30^0$$

Applying Snell's law at point Q

$$1 \sin i = \mu \sin r$$

or $\quad \mu = \dfrac{\sin(45)}{\sin 30} = \dfrac{2}{\sqrt{2}} = \sqrt{2}$

Example–26:

A spherical surfaces of radius of curvature R seperates air $(\mu_a = 1.0)$ from glass $(\mu_g = 1.5)$. The centre of curvature is in glass. A point object P placed in air is found to have a real image in the glass. The line PQ cuts the surface at a point O and PO =OQ. What is distance PO?

Solution:

Let PO = OQ = a

We have the relation

$$\frac{\mu_2}{v} - \left(\frac{\mu_1}{u}\right) = \frac{\mu_2 - \mu_1}{R} \qquad \text{and} \qquad u = -z,\ v = z$$

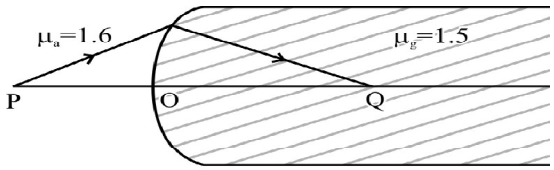


Fig. 54

$$\therefore \quad \left(\frac{1.5}{+a}\right) - \left(\frac{1.0}{-a}\right) = \frac{1.5 - 1.0}{(+R)}$$

or $\quad \dfrac{2.5}{a} = \dfrac{0.5}{R}$

$\therefore \quad a = 5R$

Example–27:

A convex surface of radius 20 cm seperates water ($\mu_1 = 1.32$) and glass ($\mu_2 = 1.5$). An object of length 1 cm is placed perpendicular to principal axis, in water; at a distance of 40 cm. What is position, natural and length of image formed?

Solution:

Given $\mu_1 = 1.32$; $\mu_2 = 1.5$; u = –40 cm; R = +20 cm; v = ?

From refraction formula

$$\frac{\mu_2}{v} - \left(\frac{\mu_1}{u}\right) = \frac{\mu_2 - \mu_1}{R}$$

We have

$$\frac{1.5}{v} - \left[\frac{1.32}{(-40)}\right] = \frac{1.5 - 1.32}{20}$$

or $$\frac{1.5}{v} = \frac{0.18}{20} - \left(\frac{1.32}{40}\right)$$

or $$v = -62.5 \text{ cm}$$

Negative sign of v indicates that image formed is virtual at a distance of 62.5 cm from the convex surface. The magnification, m;

$$m = \frac{v/\mu_2}{u/\mu_2} = \frac{\left(-62.5/1.5\right)}{\left(-40/1.32\right)} + 1.33$$

The positive sign of m indcates erect image. The length, ℓ_2, of image is

$$\ell_2 = |\, m \,| \times \text{length of object}$$

$$= 1.33 \times 1 = 1.33 \text{ cm}$$

The image formed is virtual, erect at a distance of 50 cm and has a length of 1.33 cm

THIN LENSES

A lens is a refracting medium bound by two spherical surfaces. There are two type of lenses (i) convex lenses and (ii) concave lens. The three variation of each type as shown in Fig. 56.

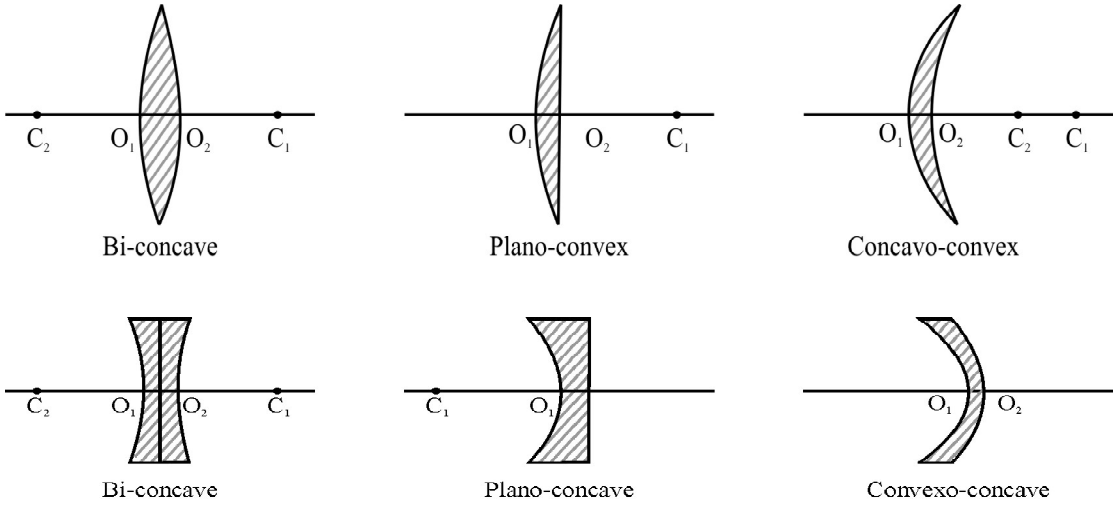


Fig. 55

$O_1O_2 = t$ = The thickness of lens. For a thin lense t is very small as compared to object or image distance or focal length of lens.

Refraction at a Thin Lens

Fig. 57 shows a convex lens of a material of refractive index μ_2. The media on the two sides of the lens have refractive index μ_1 and μ_3 respectively. P is a point object placed on principal axis in medium 1 known as object space. PA is incident ray on the first surface of lens. Had there been only one surface the refracted ray AB would have met principal axis at Q_1. For refraction at first surface

$$u_1 = -a;\ v_1 = +b_1;\ R_1 = +c$$

From refraction formula at a spherical surface

$$\frac{\mu_2}{(+b_1)} - \left[\frac{\mu_2}{(+a)}\right] = \frac{\mu_2 - \mu_1}{(+c)} \qquad .....(1)$$

The refracted ray AB from first surface meets second surface of lens at B and the emergent ray BQ meets principal axis at Q. Q is final image formed by lens due to refraction at both surfaces. For refraction at second surface of lens Q_1 acts as a virtual object forming real image Q. For refraction at second surface

$$u_2 \simeq +b_1;\ v_2 = +b;\ R_2 = -d$$

$$\therefore\ \frac{\mu_3}{(+b)} - \left[\frac{\mu_2}{b_1}\right] = \frac{\mu_3 - \mu_2}{(-d)} \qquad .....(2)$$

Adding Eqn. (1) and (2) we have

$$\frac{\mu_3}{b}-\left[\frac{\mu_1}{(-a)}\right]=\frac{\mu_2-\mu_1}{(+c)}+\frac{\mu_3-\mu_2}{(-d)} \qquad .....(3)$$

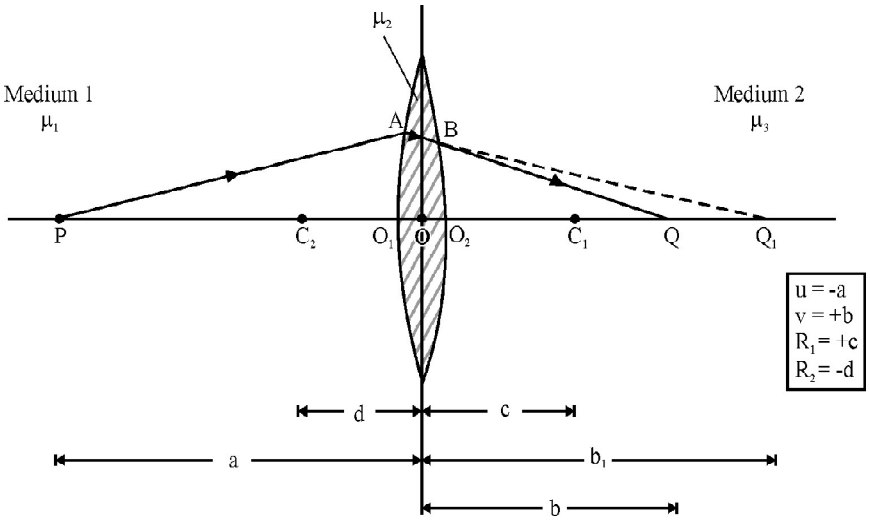


Fig. 56

Let u and v denote the position of object P and final image Q. In terms of co–ordinate convention of signs.

$u = -a;\ v = +b;\ R_1 = +c;\ R_2 = -d$

We rewrite Eqn. (3) as

$$\frac{\mu_3}{v}-\left(\frac{\mu_1}{u}\right)=\frac{\mu_2-\mu_1}{R_1}+\frac{\mu_3-\mu_2}{R_2} \qquad .....(4)$$

Equation (4) in the refraction formula for a thin lens. Commonly the media on both sides of lens is same i.e. $\mu_1 = \mu_3$ Eqn. (4) reduces to

$$\left(\frac{1}{v}\right)-\left(\frac{1}{u}\right)=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad .....(5)$$

This is refraction formula for a thin lens.

Focii and Focal Length of a Lens

(1) Second Principal Focus, F_2

Consider a parallel beam, parallel to principal axis, incident on lens. The refracted rays meets or appears to meet principal axis at f_2. f_2 is the second principal focus of lens. Position of F_2 gives

second principal focal length F_2. Expressed mathematically;

$$u = -\infty,\ v \equiv f_2$$

From Eqn. (4) we have

$$\frac{\mu_3}{f_2} = \frac{\mu_2 - \mu_1}{R_1} + \frac{\mu_3 - \mu_2}{R_2} \qquad .....(6)$$

(2) First Principal Focus, F_1

F_1 is a point on principle axis where if an object is placed or appears to be placed the refracted rays are parallel to principal axis. F_1 is known as the first principal focus. Position of F_1 gives the first principal focal length f_1 of the lens. In language of mathematics

$$u \equiv f_1;\ v = +\infty$$

From Eqn. (4) we have

$$-\left(\frac{\mu_1}{f_1}\right) = \frac{\mu_2 - \mu_1}{R_1} + \frac{\mu_2 - \mu_3}{R_2} \qquad .....(7)$$

Note in general $|f_1| \neq f_2$ In words; in general the two focii are not equidistant from lens.

The focal length f of lens is the position of second principal focus i.e. $f_2 \equiv f$. In terms of focal length, f; Eqn. (4) is rewritten as

$$\frac{\mu_3}{v} - \left(\frac{\mu_3}{u}\right) = \frac{\mu_3}{f} \qquad .....(8)$$

This is the genral refraction formula for a thin lens. However commonly $\mu_1 = \mu_3$; Eqn. (6) and (8) reduces to

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad .....(9)$$

and $$\frac{1}{v} - \left(\frac{1}{u}\right) = \frac{1}{f} \qquad .....(10)$$

Eqn. (9) is known as *Lens Maker's Formula* For a lens placed in air, i.e. $\mu_1 = 1$ and $\mu_2 \equiv \mu$ the lens maker's formula reduces to

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Nature of a Convex and a Concave Lens

Case–I : Convex Lens

For a convex lens; $R_1 = +c$ and $R = -d$. From Eqn. (9);

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{c} + \frac{1}{d}\right) \qquad .....(11)$$

(a) Let $\mu_2 > \mu_1$, i.e. refractive index of lens is more than refractive index of medium in which lens is placed. From Eqn. (11), f is a +ve number. The incident parallel beam on lens, after refraction actually meets principal axis at F_2 as shown Fig. 58 (a). Under these conditions convex lens behaves as a converging lens. Fig. 58 (b) shows position of F_1 since $\mu_1 = \mu_3$; $|f_1| = f_2 = f$; i.e. *the two focci are equidistance from lens if medium on both sides of lens is same.*

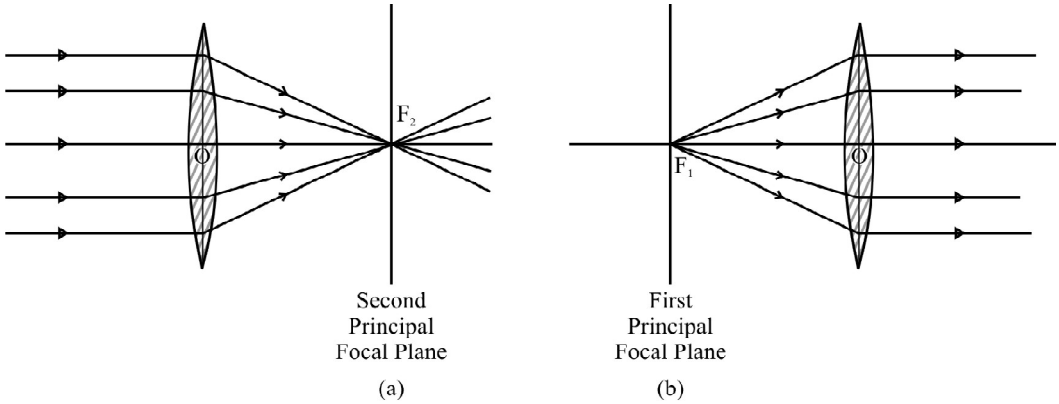


Fig. 57

(b) Let $\mu_2 < \mu_1$. From Eqn. (11) f is a negative number! This means a convex lens behaves as a diverging lens. The converging or diverging nature of lens depends the nature of medium (i.e. μ_1) in which lens is placed.

Case–II : Concave Lens

In accordance with the sign convention, for a concave lens $R_1 = -c$, $R_2 = +d$. From Eqn. (9); the focal length, f, of lens is

$$\frac{1}{f} = -\left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{c} + \frac{1}{d}\right)$$

(a) Let $\mu_2 > \mu_1$; f is a negative number. This means a parallel beam, incident on a concave lens appears to meet principal axis at F_2 as shown in Fig. 59 (a). The concave lens behaves as a diverging lens. Fig. 59 (b) shows position of F_1.

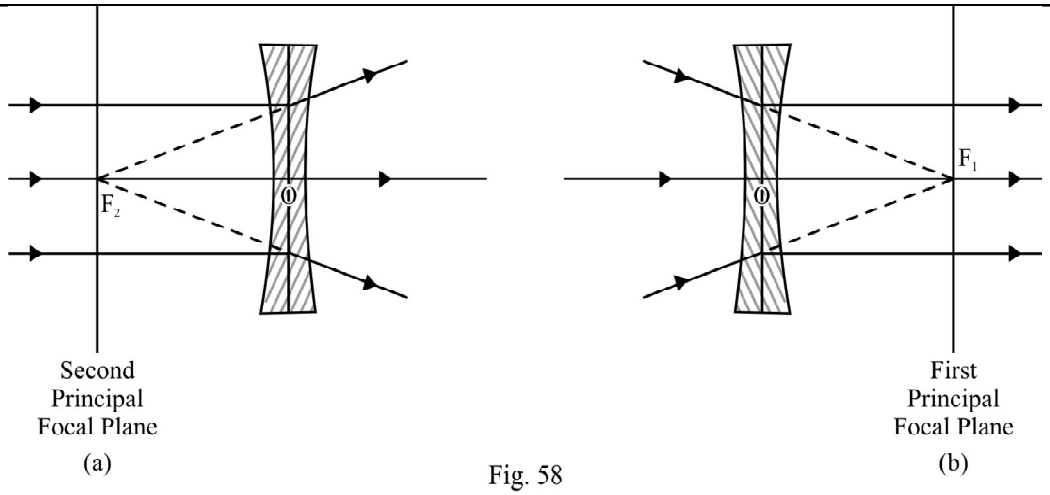


Fig. 58

(b) Let $\mu_2 < \mu_1$; f is a positive number. The concave lens new behaves as converging lens!

The Power of a Lens

The power, P, of a lens describes the ability of the lens to bend incident rays towards or away from principal axis. It is the reciprocal of focal length expressed in meter. S.I unit of power is dropter.

$$P\,(\text{in diopter}) = \frac{1}{f\,(\text{in meter})}$$

A positive value of P indicates converging behaviour and a negative value the diverging behaviour of the lens.

Object of Finite Size

Case–I : Convex Lens

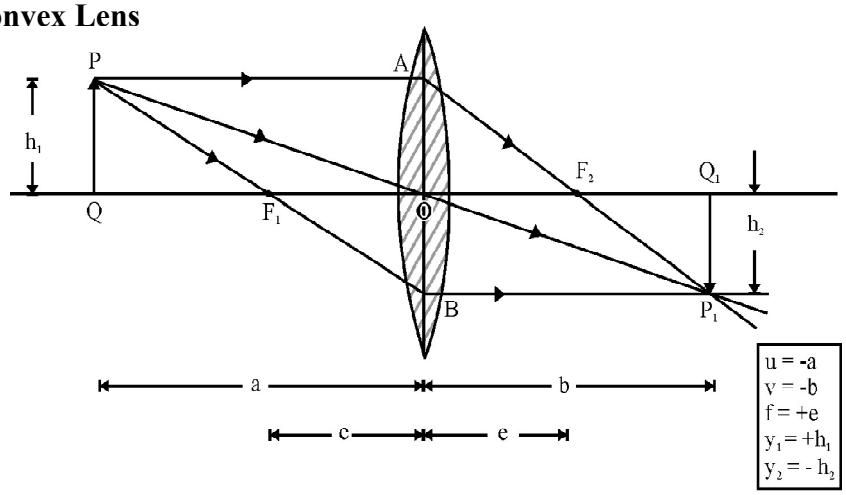


Fig. 59

Fig. 60 shows an object PQ = h_1 placed perpendicular to principal axis at a distance a from the lens. P_1Q_1 = h_2 is the real; inverted image formed. The transverse magnification, m; is

$$m = \frac{\text{size of image}}{\text{size of object}} = \frac{y_2}{y_1}$$

It can be shown that

$$m = \frac{v}{u} \qquad .....(12)$$

Let x_1 be the distance of the object from F_1, and x_2 the distance of image from F_2. Then

$$x_1 x_2 = f^2 \qquad .....(13)$$

This is *Newton's Formula*

The following table gives the nature, position and magnification of a real object as it moves from infinity, towards convex lens.

S.No.	Position of Object	Image	
		Position	*Nature*
1.	At infinity	Focus	real, inverted, highly diminished
2.	In between ∞ and 2F	In between F and 2F	real, inverted, diminished
3.	At 2F	At 2F	real, inverted, same size as object
4.	In between F and 2F	Beyond 2F	real, inverted, enlarged
5.	At F	At infinity	real, inverted, highly enlarged
6.	In between O and F	Beyond lens	virtual, erect and enlarged

Case–II : Concave Lens

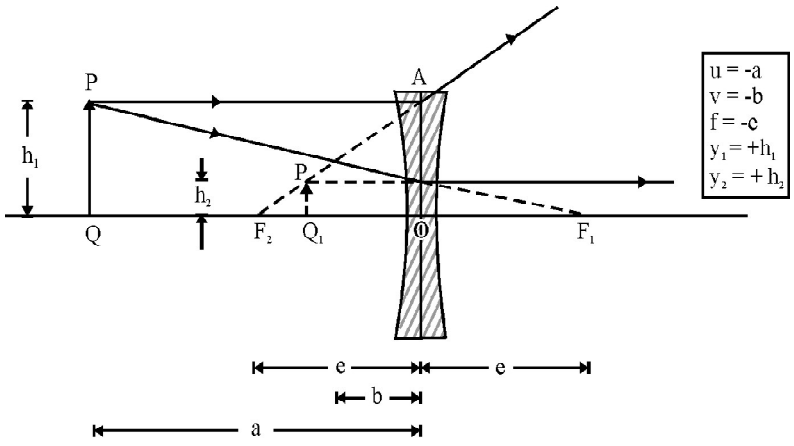


Fig. 60

Fig. 61 shows the virtual, erect and dimished image P_1Q_1 of a real object PQ. A concave lens forms a virtual, erect; dimished image of a real object irrespective of the position of object.

Two Thin Lenses in Contact

Fig. 62 shows two co–axial thin lenses L_1 and L_2 of focal length f_1 and f_2 in contact. Treating the system as a thin lens; the equivalent focal length; f; of the combination is

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

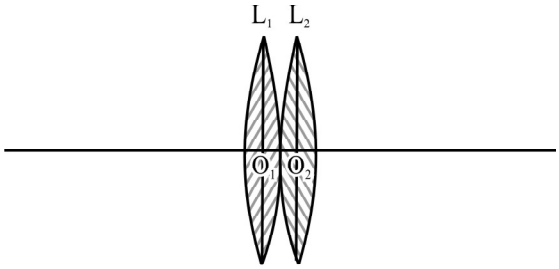


Fig. 61

In terms of power, the above equations is rewritten as

$$P = P_1 + P_2$$

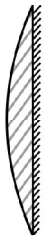
For a number of co–axial thin lenses, in contact, the treating the combinations as a thin lens; we have

$$\frac{1}{f} = \sum \frac{1}{f_j}; \quad P = \sum P_j$$

When one surface of a thin lens is silvered; the rays are reflected back from the silvered surface. The set up behaves like a spherical mirror. The 'thumb rule' for the equivalent focal length f of the spherical mirror is

$$\frac{1}{f} = \sum \frac{1}{f_j}$$

f_j is the focal length of the lens or mirror; to be repeated as many times as reflection or refraction respectively is repeated.



A plano–convex lens silvered at its plane surface as shown in Fig. 63; there are two refractions and one reflection; therefore

$$\frac{1}{f} = \frac{1}{f_l} + \frac{1}{f_m} + \frac{1}{f_l} = \frac{2}{f_l} + \frac{1}{f_m}$$

f_l is focal length of plano convex lens. Obviously; f_m = focal length of mirror = ∞

Fig. 62

$$\therefore \quad \frac{1}{f} = \frac{2}{f_1} = \frac{2(\mu - 1)}{R}$$

Fig. 63

For plano–convex lens silvered at its curved surface, as shown in Fig. 63. We have

$$\frac{1}{f} = \frac{2}{f_1} + \frac{1}{f_m}$$

$$= \frac{2(\mu - 1)}{R} + \frac{2}{R}$$

OPTICAL INSTRUMENTS

Microscope is an optical instrument used to forn an enlarged image of a nearby small sized object.

M.P = The magnifying power of microscope

$$= \frac{\text{angle subtended by image}}{\text{angle subtended by object when placed at distance of distinct vision}} = \frac{\beta}{\alpha}$$

A simple microscope is a short focal length, f; convex lens. The object to be magnified is placed within focus and optical center of lens. The image formed is virtual; erect and enlarged.

The Compound Microscope

A compound microscope consists of two short focal length co–axial convex lenses a finite distance apart. The object to be magnified is placed in front of O–lens of focal length f_0 and is close to but outside the focus F_0. The O–lens forms a real; inverted and enlarged image. This image acts as object for the E–lens of focal length f_e. The final image formed is inverted with respect to the object.

M.P. = The magnifying power = $|m_0| \times |m_e|$

m_0 and m_e denote magnification due to O – and E – lens respectively. For a microscope forming final image at distance of distinct vision;

$$M.P = \left(\frac{b_1}{f_0} - 1\right)\left(1 + \frac{D}{f_e}\right) \qquad .....(1)$$

The length of microscope = Distance between O and E – lens

$$= b_1 + a_1 \qquad .....(2)$$

$a_1 < f_e$; is distance of image formed by O – lens from E – lens.

If microscope is so adjusted that final image is at infinity; the M.P is slightly less than what is given by

Eqn. (1) and the length is more than given by Eqn. (2).

The Telescope

Telescope is an optical instrument used to form an enlarged image of a far off large sized object.

M.P = The magnifying power of telescope

$$= \frac{\text{angle subtended by image}}{\text{angle subtended by object}} = \frac{\beta}{\alpha}$$

An astronomical telescope consists of a large aperture, large focal length (f_0) O – lens. The eye–lens (E – lens) is a co–axial convex lens of small focal length f_0. For an astronomical telescope, in normal, adjustment

$$M.P = \frac{f_0}{f_e}$$

Length of telescope = Distance between O and E – Lens = $f_0 + f_e$

Example–28:

An equi–convex lens of radius 20 cm has a refractive index of 1.5. The difference in the focal length of the lens when surrounded by water $\left(\mu = \frac{4}{3}\right)$ and air is

(a) 60 cm **(b) 20 cm**

(c) 10 cm **(d) 80 cm**

Solution:

$$\frac{1}{f_a} = \left({}_a\mu^g - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.5-1)\left(\frac{1}{20} + \frac{1}{20}\right)$$

$\therefore \quad f_a = 20$ cm

When lens is immersed completely in water

$${}_\omega\mu^g = \frac{\mu^g}{\mu^\omega} = \frac{1.5}{4/3} = \frac{9}{8}$$

$$\frac{1}{f_\omega} = \left({}_\omega\mu^g - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \left(\frac{9}{8} - 1\right)\left(\frac{1}{20} + \frac{1}{20}\right)$$

$\therefore \quad f_\omega = 80\,cm$

The difference in focal length of lens in water and air is

$f_\omega - f_a = 80 - 20 = 60\,cm$

Hence option (a) is correct

Example–29:

A hollow double concave lens is made of very thin transparent material. It can be filled with air or either of two liquids L_1 and L_2 having refractive indices μ_1 and μ_2 respectively ($\mu_2 > \mu_1 > 1$). The lens will diverge a parallel beam of light if it is filled will

(a) air and placed in air **(b) air and immersed in L_1**

(c) L_1 and immersed in L_2 **(d) L_2 and immersed in L_1**

Solution:

Let μ_L denote the refractive index of lens and μ_m the refractive index of medium. From lens maker's formula

$$\frac{1}{f} = \left(\frac{\mu_L}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For a double concave lens R_1 is negative and R_2 is positive. Hence $\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ will be negative number.

The focal length f of the lens has to be negative if it has to behave as a diverging lens. For this to happen $\left(\frac{\mu_L}{\mu_m} - 1\right)$ should be positive implying that $\mu_L > \mu_m$. Also it is given that $\mu_L > \mu_m$ and also $\mu_2 > \mu_1$. considering all this information lens should be filled with L_2 and immersed in L_1.

Hence option (d) is correct.

Example–30:

A square of side 1 cm is placed on principal axis of a lens at a distance of 20 cm. The lens projects image of area 16 cm^2 on a screen.

(a) What is distance between object and screen?

(b) What is nature and power of lens?

Solution:

Since the area of image is 16 times that of object; the magnitude of linear magnification = 4. The image is projected on a screen; therefore it is a real image and therefore inverted, i.e.

$$m = -4$$

Also u = –20 cm; therefore

$$-4 = \frac{v}{(-20)}$$

or $v = +80$ cm

The distance of screen from lens is 80 cm. Obviously distance between screen and object = (80+20) cm = 100 cm. Let f be the focal length of lens. From refraction formula

$$\frac{1}{(+80)} - \left[\frac{1}{(-20)}\right] = \frac{1}{f}$$

$$\therefore \quad f = 16 \text{ cm}$$

The positive sign of f indicates convex lens. The power, P, of lens is

$$P = \frac{1}{0.16} D = 6.25 D$$

Example–31:

A beam of light converges towards a point P as shown in Fig. 64(a). A concave lens is placed 10 cm in front of P. The point where beam now converges moves away by 50% of distance of lens from P. What is focal length of the lens? If lens is made of a material of $\mu = 1.5$; assuming that the two faces of lens have same radius of curvature; calculate the radius of convature of each surface of lens.

Solution:

Fig. 65 (a) shows the incident convergent beam. Fig. 65 (b) shows concave lens placed in front of P. Now P acts as a virtual object forming real image Q. Therefore

$$u = +10 \text{ cm}; \quad v = +15 \text{ cm}; \quad f = ?$$

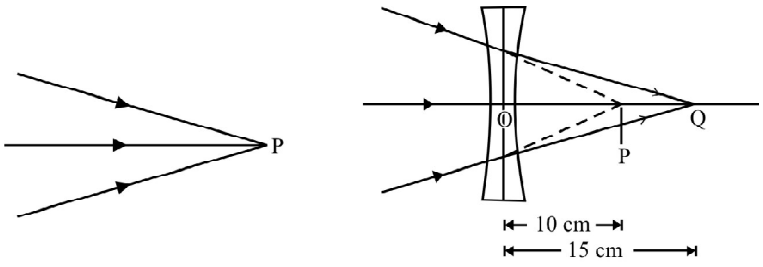
From refraction formula; for a lens

$$\frac{1}{15} - \left(\frac{1}{10}\right) = \frac{1}{f} \qquad \therefore \quad f = -30 \text{ cm}$$

Let c be magnitude of radius of curvature of each surface of lens. According to co–ordinate convention. $R_1 = -c$; $R_2 = +c$. From lens maker's formula

$$\frac{1}{(-30)} = (1.5-1)\left[-\left(\frac{1}{c}\right)-\left(\frac{1}{c}\right)\right] = \left(\frac{1}{c}\right)$$

$$\therefore \quad c = 30 \text{ cm}$$



(a) (b)

Fig. 64

Example–32:

A concave lens has a focal length f. An object is placed at a distance *x* times the focal length of the lens. What is the magnification?

Solution:

Let e be the magnitude of focal length of concave lens. The object is at a distance *x*e in front of lens. Therefore u = –*x*e. Let v be the position of image formed. Using lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ we have

$$\frac{1}{v}-\left[\frac{1}{(-xe)}\right]=\frac{1}{(-e)}$$

$$\therefore \quad v = -\left[\frac{xe}{1+x}\right]$$

The transverse magnification = m = $\frac{v}{u}$

$$= \frac{-\left[xe/1+x\right]}{-(xe)} = \left[\frac{1}{1+x}\right]$$

Since m is positive, image formed is erect. Also |m|<|; the image formed is diminished.

Example–33:

A short linear object of length ℓ lies along the axis of a concave mirror of focal length f at a distance a from the pole of the mirror. The size of the image is approximately equal is

(a) $\ell\left(\frac{u-f}{f}\right)^{1/2}$ (b) $\ell\left(\frac{f}{u-f}\right)^{1/2}$

(c) $\ell\left(\frac{u-f}{f}\right)$ (d) $\ell\left(\frac{f}{u-f}\right)^{2}$

Solution:

In the mirror formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

f is a constant so we have on differentiating

$$-v^{-2}\,dv - u^{-2}\,du - 0 \qquad \text{....(i)}$$

or $$|dv| = \left(\frac{v^2}{u^2}\right)|du|$$

|dv| is representing the size of the image and |du| is size of obejct lying along the axis = ℓ.

From the mirror formula we also have

$$\frac{v^2}{u^2} = \left(\frac{f}{u-f}\right)^2 \qquad \text{....(ii)}$$

From Eqns (i) and (ii) we have

Size of the image = $\ell\left(\frac{f}{u-f}\right)^2$

Hence option (d) is correct.

Example–34:

A thin equiconvex lens is placed on a horizontal plane mirror and a pin held 20 cm above the lens. The system forms an image that coincides in position with the position of the pin. Now the space between the lens and mirror is filled will water $\left(\mu = \frac{4}{3}\right)$ and then to coincide the pin with will its own image, the pin has to be raised until its distance from the lens is 30 cm. What is the radius of curvature of lens?

Solution:

The image of pin coincides with of pin if incident rays from pin after refraction through lens fall normally on plane mirror as shown in Fig. 65. Pin is position of focus of the lens.

$\therefore\ f_1 = 20$ cm

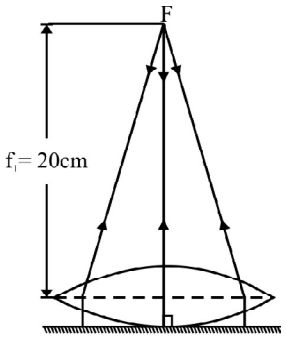


Fig. 65

Let the focal length of water lens be f_2, then for the combination of the two lens, the equivalent focal length F of the combination is 30 cm. We have

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

or $$\frac{1}{30} = \frac{1}{20} + \frac{1}{f_2}$$

$$\frac{1}{f_2} = \frac{1}{30} - \left(\frac{1}{20}\right) = -\left(\frac{1}{60}\right)$$

$\therefore\ f_2 = -60$ cm

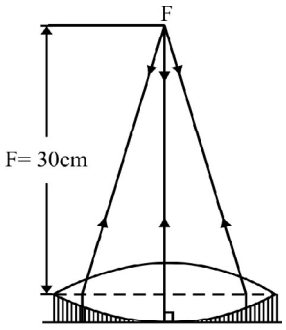


Fig. 66

For the liquid lens

$$\frac{1}{f_2} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where μ is refractive index of water. For plano-convex water lens $R_1 = -c$, $R_2 = \infty$; therefore

$$\frac{1}{(-60)} = \left(\frac{4}{3} - 1\right)\left[\left(\frac{1}{-c}\right) - \frac{1}{\infty}\right] \qquad \therefore\ c = 20 \text{ cm}$$

Example–35:

A lens having focal length f and aperture of diameter d forms an image of intensity I. Aperture of diameter $\frac{d}{2}$ in central region of lens is covered by a black paper. Focal length of lens and intensity of image now are

(a) **f and $\frac{I}{4}$**

(b) **$\frac{3f}{4}$ and $\frac{I}{2}$**

(c) **f and $\frac{3I}{4}$**

(d) **$\frac{f}{2}$ and $\frac{I}{2}$**

Solution:

It is evident that by convering the aperature of diameter $\frac{d}{2}$, focal length of lens does not change. However since the in this process the area is reduced to $\frac{3}{4}$th of its earlier value, the intensity of the image will be reduced by $\frac{I}{4}$.

Hence the focal length will remain f and new intensity of the image will be

$$I-\frac{I}{4}=\frac{3I}{4}$$

Hence option (c) is correct.

Example–36:

The focal length of the objective and eye piece of a telescope is 50 cm and 5 cm respectively. The telescope is focussed for distinct vision on a scale 300 cm away from the objective. The seperation between the objective and eye piece is

(a) 50 – 5 cm **(b) ~ 64 cm**

(c) ~ 56 cm **(d) ~ 60 cm**

Solution:

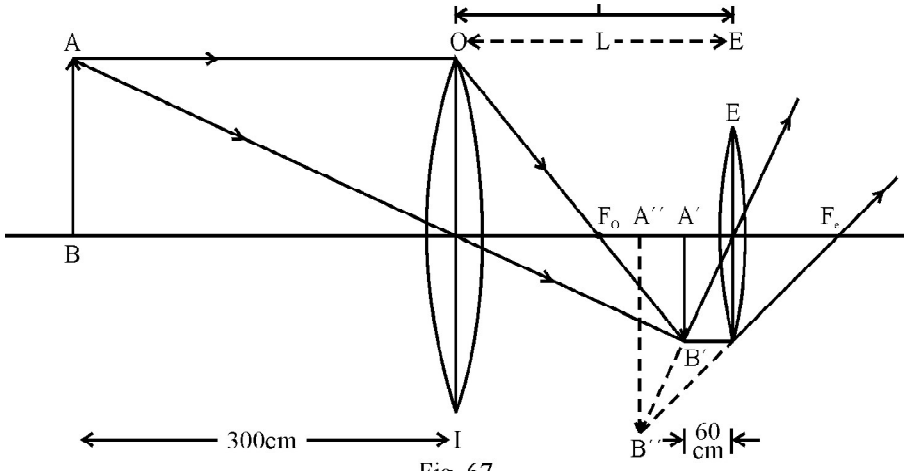


Fig. 67

As shown let AB be the position of the object and let $A'B'$ be the image formed by the objective lens.

Then $v=\frac{uf_0}{u-f_0}=\frac{-300\times 50}{-300+50}=60$ cm. $\quad\left[\because\ u=-300\,\text{cm},\quad f_0=+50\,\text{cm}\right]$

This image $A'B'$ serves as an object for eye piece. The distance between $A'B'$ and the eye piece is

$u' = L - 60$ where L is the seperation between the two lenses. The final image distance is $v' = -25\,cm$ from the eye piece. Now

$$u' = \frac{v'f_e}{v' - f_e} = \frac{-25 \times 5}{-25 - 5} = \frac{25}{6}\,cm \qquad \left[\because f_e = +5cm\right]$$

$$\therefore \quad L = \frac{25}{6} + 60 = \frac{385}{6} \approx 64\,cm$$

Example–37:

A convex lens B of focal length 20 cm is placed at a distance of 30 cm to the right of an identical lens A. A point object is placed at a distance of 30 cm to the left of A. A convex mirror of radius of curvature 6 cm is to be placed on the common axis of the lens at a distance *x* from lens B such that the final image formed by the arrangement coincides with the object. *x* is

(a) 54 cm **(b) 6 cm**

(c) 18 cm **(d) 12 cm**

Solution:

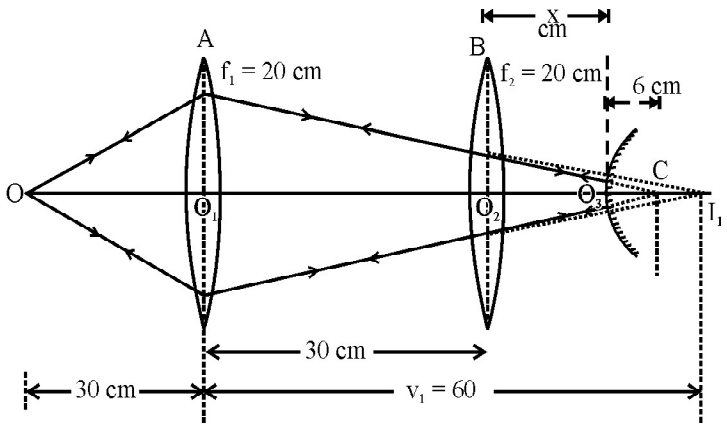


Fig. 68

For the lens A

u_1 = -30 cm, f_1 = 20 cm, v_1 = ?

$$\therefore \quad \frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1} = \left(\frac{1}{20}\right) - \left(\frac{1}{30}\right)$$

or v_1 = 60 cm

Had there been no lens B lens A would have formed a real image of the object at 60 cm shown as I_1 in Fig. 68. For the lens B, I_1 acts as a virtual object. Therefore

u_2 = +30 cm , f_2 = 20 cm

From lens formula we have

$$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = \frac{1}{20} + \frac{1}{30}$$

or $v_2 = 12$ cm

The refracted rays from lens B must fall normally on convex mirror so that rays retrace their path and the final image formed by system coincedes with object O. This will be so if ray incident on convex mirror would have met at center of curvature C of mirror. Obviously from Fig. 68.

$$O_2C = O_2O_3 + O_3C$$

or $12 = 6 + x$ [Given $O_3C = 6$cm]

$\therefore$ $x = 6$m

Example–38:

(a) A thin convex and concave lens of power 5 D and 2.5 D are placed co–axially in contact with one another. An object of length 2 cm is placed 100 cm in front of the combination. What is position and length of image formed?

(b) Keeping object and convex lens in position concave lens is moved away by 10 cm. What is change in position and length of the image formed now as compared to their earlier values?

Solution:

P_1 = The power of convex lens = +5 D, P_2 = The power of concave lens = –2.5 D

$\therefore$ P = The power of the combination = $P_1 + P_2$ = +2.5 D

f = The equivalent focal length of the combination = $\frac{1}{P} = \frac{1}{2.5}$ m = 40 cm

For the combination, given

$u_1 = -100$ cm, $v_1 = ?$ Using the lens formula we have

$$\frac{1}{v_1} - \left[\frac{1}{(-100)}\right] = \frac{1}{40}$$

$\therefore$ $v_1 = \frac{200}{3} = 66.67$ cm

$$\text{The magnification} = m = \frac{v_1}{u_1} = -\left(\frac{2}{3}\right)$$

$$\therefore \text{ Length of image formed} = |m| \times 2 \text{ cm} = \frac{4}{3} = 1.33 \text{ cm}$$

Fig. 69 shows the arrangment when concave lens L_2 is moved away from the convex lens L_1.

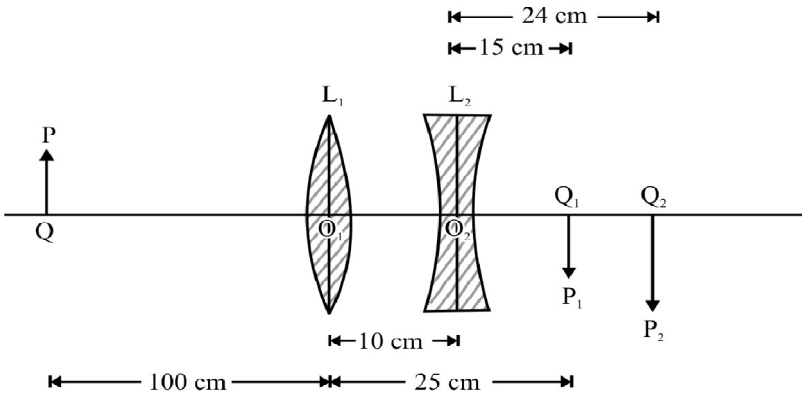


Fig. 69

For Convex Lens

$$u_1' = -100 \text{ cm}; \; f_1 = +20 \text{ cm}, \; v_1' = ?$$

$$\therefore \quad \frac{1}{v_1'} - \left[\frac{1}{(-100)}\right] = \frac{1}{20} \quad \text{or} \quad v_1' = +25 \text{ cm}$$

The magnification m, due to convex lens is

$$m_1 = \frac{v_1'}{u_1'} = \frac{25}{(-100)} = -\left(\frac{1}{4}\right)$$

The image P_1Q_1 formed by convex lens is real, inverted and diminished.

Length of image $P_1Q_1 = |m_1| \times 2 \text{ cm} = 0.5 \text{ cm}$

For Concave Lens

P_1Q_1 acts as a virtual object for concave lens. Therefore

$$u_2 = +(25–10) \text{ cm} = +15 \text{ cm}; \; v_2 = ?; \; f_2 = –40 \text{ cm}$$

$$\therefore \quad \frac{1}{v_2}-\left[\frac{1}{(15)}\right]=\frac{1}{(-40)} \quad \text{or} \quad v_2=+24 \text{ cm}$$

The final image P_2Q_2 is real at a distance of 24 cm from L_2. The magnification m_2 due to concave lens is

$$m_2=\frac{v_2}{u_2}=\frac{24}{15}=1.6$$

The image P_2Q_2 is erect with respect to P_1Q_1 i.e. inverted with respect to object PQ.

The length of image $P_2Q_2 = 1.6\times0.5 = 0.8$ cm

The change in position of image = (24+10) – 66.67 = –32.67 cm

The change in length of image = 0.8 – 1.33 = –0.53 cm

Example–39:

A thin equi–convex lens made of glass of refractive index $\mu_g=\frac{3}{2}$ and focal length 0.3 m in air is sealed into an opening in a tank filled with water $\mu_w=\frac{4}{3}$. On the wall of tank opposite lens a mirror is placed as shown in Fig. 70. The distance between the lens and the mirror is 0.8 m. A small point object P is placed on principal axis at a distance of 0.9 m from the lens. Find the position of the final image with respect to lens, due to the system.

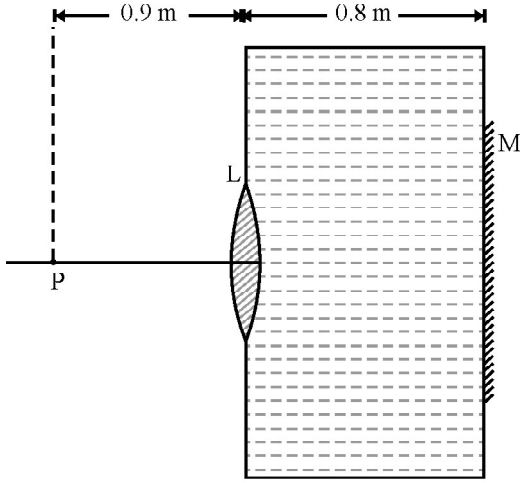


Fig. 70

Solution:

Let c be the magnitude of radius of curvature of each surface of equiconvex lens of f = 0.3 m (given). Using lens maker's formula

$$\frac{1}{0.3}=(1.5-1)\left(\frac{1}{c}+\frac{1}{c}\right) \quad \therefore \quad c=0.3\,\text{m}$$

First image of P is formed due to refraction at lens L. Let μ_1, μ_2 and μ_3 be refractive index of object space, lens material and image space respectively. u and v represent the object and image position. Then

$$\frac{\mu_3}{v}-\left(\frac{\mu_1}{u}\right)=\frac{\mu_2-\mu_1}{R_1}+\frac{\mu_3-\mu_2}{R_2} \qquad .....(1)$$

For first refraction at lens L

$$u_1 = -0.9 \text{ m};\ \mu_1 = 1.0;\ \mu_2 = 3/2;\ \mu_3 = 4/3;\ R_1 = +0.3 \text{ m, and } R_2 = -0.3 \text{ m}$$

$$\therefore \quad \frac{(3/2)}{v_1} - \left[\frac{1}{(-0.9)}\right] = \frac{(3/2)-1}{0.3} + \frac{(4/3)-(3/2)}{(-0.3)}$$

$$\therefore \quad v_1 = 1.2\,\text{m} \qquad \text{.....(2)}$$

Had there been only lens L, the image Q_1 of P would have been real at a distance of 1.2 m from L. However the refracted rays undergo reflection at mirror M on the opposite wall. For mirror M, Q_1 acts as a virtual object at a distance of 0.4 m from M. The mirror forms a real image Q_2; 0.4 m in front of M. Q_2 acts as a real object in water and refraction again takes place at lens L. To apply refraction formula at L, imagine diagram to be rotated by 180^0, so that incident ray on L are moving from left to right, in accordance with sign convention. Now

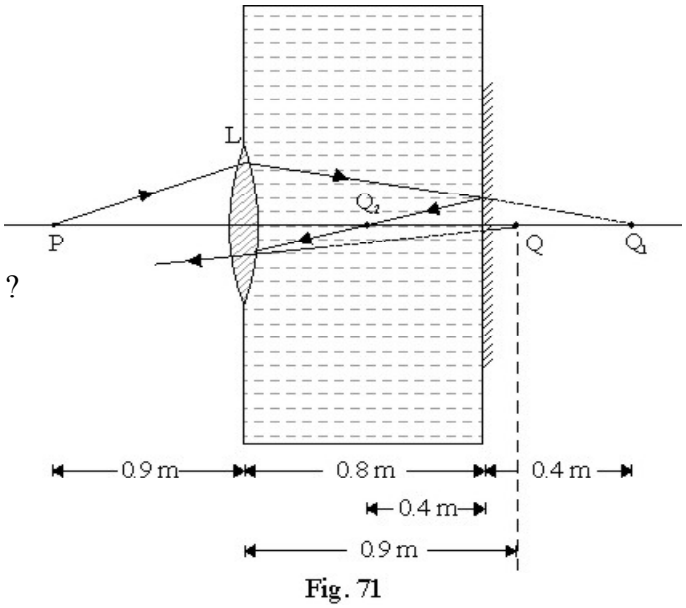


Fig. 71

$$\mu_1 = \frac{4}{3};\ \mu_2 = \frac{3}{2};\ \mu_3 = 1;\ u_2 = -0.4\,\text{m};\ v_2 = ?$$

From Eqn. (1), we have

$$\frac{1}{v_2} - \frac{(4/3)}{(-0.4)} = \frac{(4/3)-(3/2)}{0.3} + \frac{\left(1-\frac{3}{2}\right)}{(-0.3)}$$

$$\therefore \quad v_2 = -0.9\,\text{m}$$

Negative sign of v_2 indicates that final image formed (Q)is virtual at a distance of 0.9 m from lens L.

Fig. 73 shows the ray diagram for image formed by the system.

Fig. 72

Example–40:

Consider a concave mirror and a convex lens (refractive index 1.5) of focal length 10 cm each seperated by a distance of 50 cm in air (refractive index = 1) as shown in Fig. 73. An object is placed at a distance of 15 cm from the mirror. Its erect image formed by combination has a magnification M_1.

When the set up in kept in a medium of refractive index 7/6 the magnification becomes M_2. What is M_1/M_2?

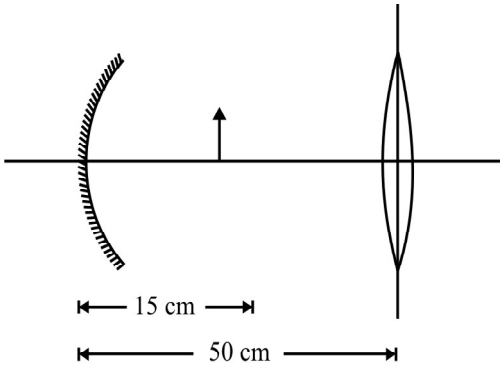


Fig. 73

Solution:

Case–I : Arrangement in Air

(a) For Convave Mirror

$$u_1 = -15 \text{ cm}; v_1 = ? f_1 = -10 \text{ cm}$$

$$\therefore \quad \frac{1}{v_1} + \frac{1}{(-15)} = \frac{1}{-10} \quad \text{or} \quad v_1 = -30\,\text{cm}$$

$$m_1 = \text{magnification due to mirror} = \frac{v_1}{u_1} = -2$$

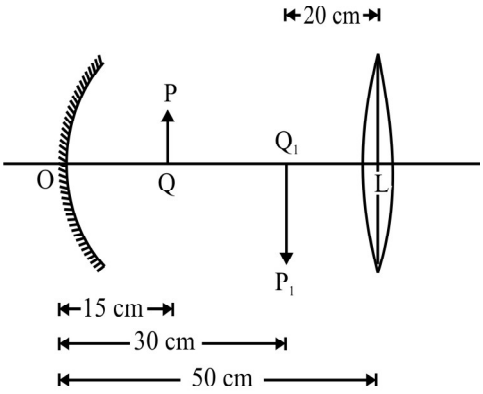


Fig. 74

(b) For Lens

The image P_1Q_1 acts as object for lens. For lens

$$u_2 = -20 \text{ cm}; v_2 + ? f_2 = 10 \text{ cm}$$

$$\therefore \quad \left(\frac{1}{v}\right) - \frac{1}{(-20)} = \frac{1}{10} \quad \text{or} \quad v_2 = 20\,\text{cm}$$

$$m_2 = \frac{v_2}{u_2} = -1$$

$|M_1| = \text{Magnitude of magnification due to combination}$

$$= |m_1| \times |m_2| = 2 \qquad .....(1)$$

Case–II : Arrangement inside medium

For reflection at mirror there is no change. Let f_2' be focal length of convex lens inside medium.

$$\frac{1}{f_2'}=\left(\frac{\frac{3}{2}}{\frac{7}{6}}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \text{ and } \frac{1}{f_2}=\left(\frac{3}{2}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\therefore \quad f_2'=\frac{70}{4}\text{cm}$$

For lens

$$u_2'=-20\,\text{cm},\ v_2'=?\ f_2'=\frac{70}{4}\text{cm}$$

$$\therefore \quad \left(\frac{1}{v_2'}\right)-\frac{1}{(-20)}=\frac{4}{70} \qquad \text{or} \qquad v_2'=140\,\text{cm}$$

$$\therefore \quad m_2'=\frac{140}{(-20)}=-7$$

Now,

$$|M_2|=|m_1|\times|m_2'|=14 \qquad .....(2)$$

$$\therefore \quad \frac{M_1}{M_2}=\frac{2}{14}=\frac{1}{7}$$

Example–41:

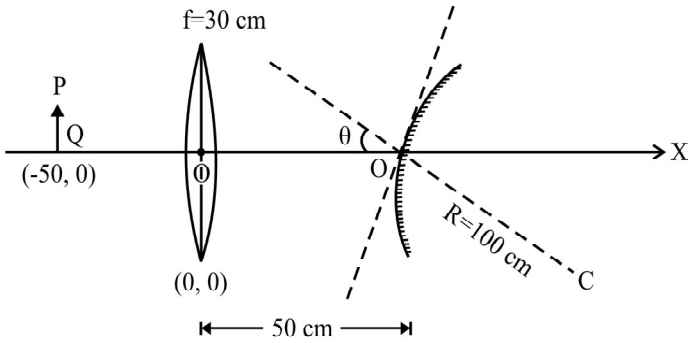


Fig. 75

A small object PQ is placed 50 cm to the left of a thin convex lens of focal length 30 cm. A convex spherical mirror of radius of curvature 100 cm is placed to the right of the lens at a distance of 50 cm. The mirror is tilted such that its axis make an angle of $\theta=30^0$ with axis of lens as shown in Fig. 75 If the origin of co–ordinate system is taken to be the center of the lens, the co–ordinates (in cm) of the point **(*x*, y) at which is formed are:**

(a) $\frac{125}{3}; \frac{25}{\sqrt{3}}$ **(b)** $\left[\left(50 - 25\sqrt{3}\right); 25\right]$

(c) [0, 0] **(d)** $\left[25; 25\sqrt{3}\right]$

(I.I.T Adv. 2016)

Solution:

For lens L; $u_1 = -50$ cm, $v_1 = ?$, $f = +30$ cm. From lens formula

$$\left(\frac{1}{v_1}\right) - \frac{1}{(-50)} = \frac{1}{30} \quad \therefore \quad v_1 = 75 \text{ cm}$$

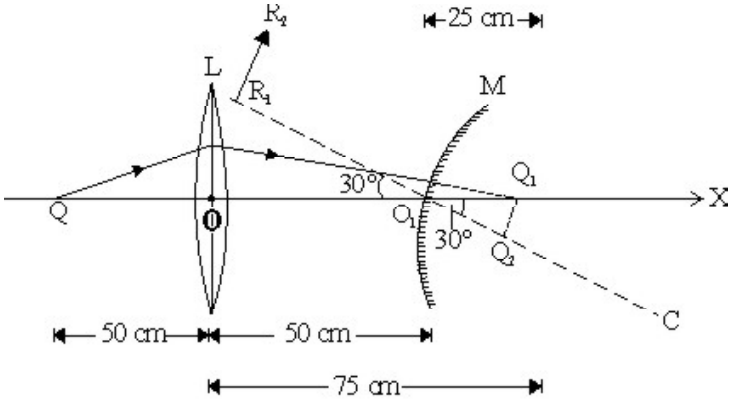


Fig. 76

Had there been only lens L; the image Q_1 would have been at a distance $OQ_1 = 75$ cm as shown in Fig. 76. For mirror M; Q_1Q_2 acts as a virtual object. The distance of point Q_2 from mirror

$$O_1Q_2 = OQ_1 \cos 30 = \frac{25\sqrt{3}}{2} \text{ cm.}$$

$Q_1Q_2 = h_1 =$ Length of virtual object for mirror.

$$= O_1Q_1 \sin 30 = \frac{25}{2} \text{ cm}$$

For mirror

$$u_2 = +\frac{25\sqrt{3}}{2} \text{cm}; \; v_2 = ? \; R = +100 \text{ cm}$$

From mirror formula, we have

$$\frac{1}{v_2}+\frac{1}{\left(\frac{25\sqrt{3}}{2}\right)}=\frac{2}{100}$$

or $$\frac{1}{v_2}=\frac{1}{50}-\left[\frac{2}{25\sqrt{3}}\right]=\frac{\sqrt{3}-4}{50\sqrt{3}}$$

$$\therefore\ v_2=-\left[\frac{50\sqrt{3}}{4-\sqrt{3}}\right]\text{cm}\simeq -38.2\text{ cm}$$

Since v_2 is negative, the image formed is real. Distance $O_1R_1=\frac{50\sqrt{3}}{\left(4-\sqrt{3}\right)}\equiv 38.2\text{ cm}$

The magnification $m=-\left(\frac{v_2}{u_2}\right)=-\frac{\left[\frac{-50\sqrt{3}}{4-\sqrt{3}}\right]}{\left(\frac{25\sqrt{3}}{2}\right)}$

$$\therefore\ R_1R_2=h_2=|m|$$

$$=\frac{25}{3}\times\left[\frac{50\sqrt{3}\times 2}{25\sqrt{3}\left(4-\sqrt{3}\right)}\right]=\frac{50}{\left(4-\sqrt{3}\right)}\text{ cm}\simeq 26.45\text{ cm}$$

In the figure R is final image of point Q. From Fig. 77 the *x*–co–ordinate of point R = 50 – O_1R_1 cos 30 + RR_1 cos 60

$$=50-\left(\frac{50\sqrt{3}}{40-\sqrt{3}}\right)\left(\frac{\sqrt{3}}{2}\right)+\frac{50}{\left(4-\sqrt{3}\right)}\times\frac{1}{2}=25\text{ cm}$$

From Fig. 77 the y–co–ordinate of point R

$= O_1R_1 \sin 30 + h_2 \sin 60$

$$=\left(\frac{50\sqrt{3}}{40-\sqrt{3}}\right)\frac{1}{2}+\left(\frac{50}{4-\sqrt{3}}\right)\left(\frac{\sqrt{3}}{2}\right)$$

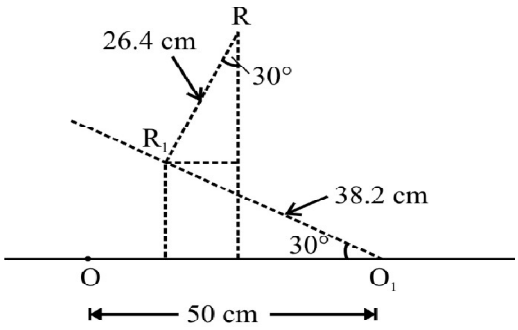


Fig. 77

$= 25\sqrt{3}$ cm

WAVE OPTICS

The study of wave optics involves the study of the phenomenon supporting wave nature of light. The basic phenomenon involved are:

(1) Interfernece

(2) Diffraction

and (3) Polarisation of light.

Wave and Wave–Front

The phenomenon of wave–motion involves the propagation of a disturbance in a medium. The particles of medium execute SHM about their respective mean positions. There is propagation of energy in the direction of wave motion. According to E.M theory light is electromagnetic waves of wave length range 4000–7500 A. This is the visible part of electromagnetic spectrum. In E.M waves the electric and magnetic fields executes SHM in mutually perpendicular direction. Both the directions are also perpendicular to the direction of wave motion.

The concept of wave front is a 'geometrical' idea used to visulise propagation of waves. The wave–front, at any time t = t, is the locus of all those points of medium that are in phase. For a point source S; the wave front is a sphere of radius Vt with S as center shown in Fig 78(a). V is speed of waves in medium. The direction of the ray at any point on wave front is in the direction of normal to wave front at the point considred. Fig. 78 (b) and Fig 79(a) and (b); show wave front of a (i) a parallel beam; (ii) convergent beam and (iii) divergent beam.

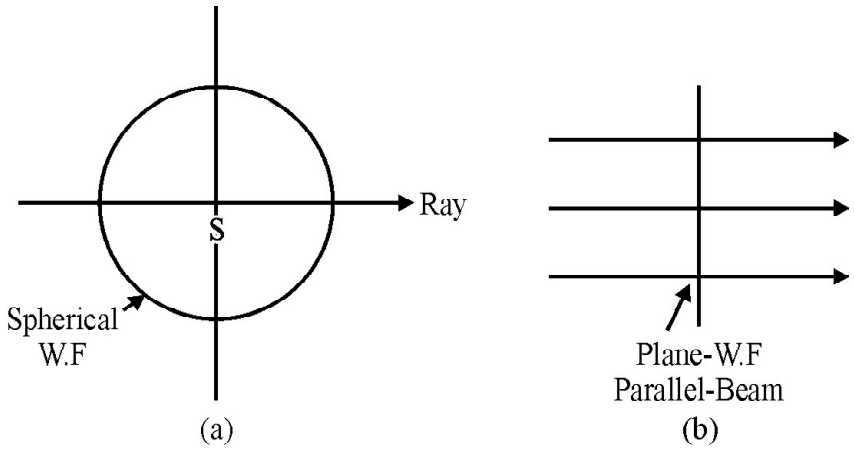


Fig. 78

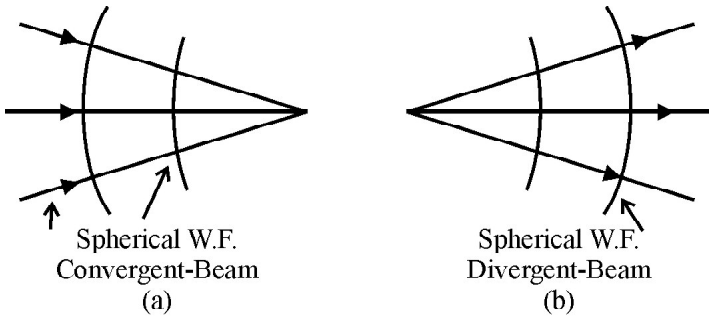


Fig. 79

Huyghen's Principal of Secondary Wave–Lets

Huyghen's principal is a 'geometrical way' to describe propagation of waves via wave front in a medium. Given a wave front say at t = 0; the position and shape of W.F at t = t is obtained using Huyghen's principle. According to this principle:

(1) Regard every point on given W.F as an imaginary point source. The disturbance in medium due to this imaginary point source is known as a *secondary wavelet*.

(2) The speed of secondary wavelets depend on the nature of medium in which the wavelets propagate.

(3) The amplitude a_0 of secondary wavelets in a direction making an angle θ with the forward direction of wave motion, is proportional to $(1 + \cos\theta)$.

(4) The new W.F at t = t; is the tangential plane or envelope of all secondary wavelets.

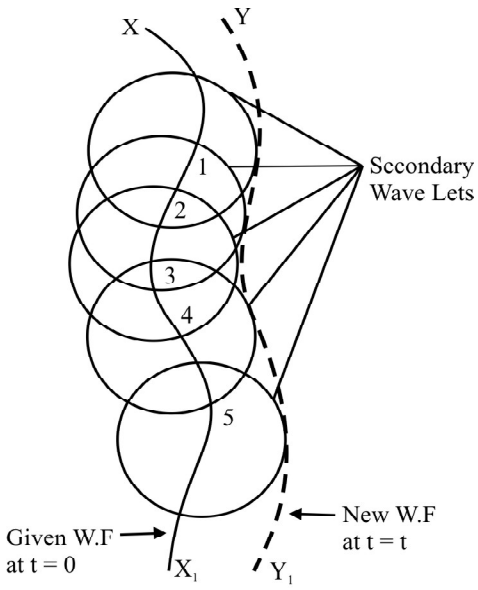


Fig. 80

Fig. 80 shows a given wave front XX_1 at t = 0. The new–wave front YY_1 at t = t is obtained using Huyghen's principal. In a homogeneous, isotropic medium shape of W.F remains same. When light undergoes reflection or refraction in general; the shape and orientation of W.F changes.

INTERFERENCE OF WAVES

Consider two or more than two waves arriving at the same point of a medium simultaneously. The resultant disturbance at this point is the resultant of the disturbance due to each individual wave, in accordance with the principle of superposition of waves. This results in a modification (or redistribution) of energy (or intensity) in the medium. This is known as *interference of waves*. Interference is easily observed with (i) sound (ii) water waves etc.

Fig. 81 shows two identical source of waves S_1 and S_2. The disturbance from the two arrives at a point P on screen travelling path S_1P and S_2P respectively. $S_2P - S_1P = \Delta$ = The "path difference" between the two disturbances at P. If the two disturbances arrive IN PHASE at P; they add up. The resultant disturbance is more. We say *constructive interference* has occured at P. Expressed mathematically

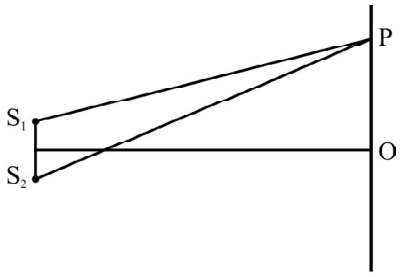


Fig. 81

$$\Delta = S_2P - S_1P = n\lambda; \quad n = 0, 1, 2, 3,..... \quad .....(1)$$

If the two disturbances arrive OUT OF PHASE at P they substract from one another. The resultant disturbance is less than due to each individual wave. We say *destructive interference* has occured. Condition for destructive interference is

$$\Delta = S_2P - S_1P = (2n-1)\frac{\lambda}{2}; \quad n = 1, 2, 3,... \quad .....(2)$$

In terms of phase difference, $\delta = \frac{2\pi}{\lambda}(\Delta)$; the conditions of constructive and destructive interference is

Constructive Interference

$$\delta = 2n\pi, \qquad n = 0, 1, 2, 3,.....$$

Destructive Interference

$$\delta = (2n-1)\pi, \; n = 1, 2, 3,..... \quad .....(3)$$

In writting Eqns. (1), (2) and (3) we have assumed there is no intial path or phase difference between S_1 and S_2. In general if Δ_i is the initial path difference (or δ_i is the initial phase difference) between S_1 and S_2; Eqn. (3) is rewritten as

Constructive Interference

$$\delta_T = \delta_i + \delta = 2n\pi, \qquad n = 0, 1, 2, 3,.....$$

Destructive Interference

$$\delta_T = \delta_i + \delta = (2n-1)\pi, \quad n = 1, 2, 3,..... \quad .....(4)$$

Interference between Two Sources of Light – Coherent Sources

The interference effect at any point is to be observed by the eye. Due to persistence of vision eye cannot detect rapid changes in intensity. To observe interference, we want the effect at any point to remains same at all times. This will be so if δ_T *does not depend on time*. For a given set up; for a particular point P; δ is time independent; therefore δ_T will not depend on time if δ_i remains same at all times. *Any two sources of light having same phase difference at all times are known as coherent sources*. To observe

interference with light we must have coherent sources.

Independent; idential sources of light are not coherent sources. Their phase difference changes 10^8 times in one second. With such sources the interference pattern changes 10^8 times in one second. The eye cannot follow such rapids changes. The interference produced is, therefore, not observed. To observe interference with light we have to produce coherent sources from an incohorent source. This can be achieved by using

(i) reflection

(ii) refraction

(iii) multiple reflection and refraction of light

Young's Double Slit Experiment

Fig. 82 shows standard Y.D.S experimental setup. S_1 and S_2 are two very fine slits distance $S_1S_2 = d$ apart in an opaque screen. S is a monochromatic source emitting light of wave length λ. A screen is placed at a distance D (D>>d) from S_1 and S_2. S is an incoherent source. Since $SS_1=SS_2$; the disturbance from S reaches S_1 and S_2 simultaneously. *Though phase of S_1 and S_2 changes with time; their phase difference is zero at all times.* S_1 and S_2 behave as two coherent sources P is any point on screen at a distance OP = y from centeral point O. It can be shown that

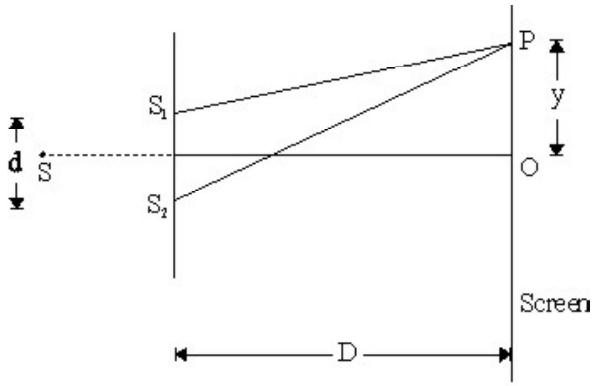


Fig. 82

Δ = The path difference between two disturbances at P

$$= S_2P - S_1P = \frac{y\,d}{D} \qquad .....(1)$$

For constructive interference

$$\Delta = \frac{y_n d}{D} = n\lambda \ , \ n = 0, 1, 2, 3,.....$$

or $$y_n = \left(\frac{nD}{d}\right)\lambda \qquad .....(2)$$

Eqn. (2) gives location of points of constructive interference.

For destructive interference

$$\Delta = \frac{y'_n d}{D} = (2n-1)\frac{\lambda}{2} \quad n = 1, 2, 3,.....$$

or $$y'_n = \left[\frac{(2n-1)D}{2d}\right]\lambda \qquad .....(3)$$

Eqn. (3) gives location of points of distructive interference on screen.

The distance between any two consecutive bright or dark fringes is

$$\beta = y_{n+1} - y_n = y'_{n+1} - y'_n = \frac{\lambda D}{d} \qquad .....(4)$$

β is known as fringe width. β is same for bright or dark fringes. We say fringes, in interference, are equispaced. The angular width, ω; of a fringe is

$$\omega = \frac{\beta}{D} = \frac{\lambda}{d} \qquad .....(5)$$

Note that β is directly proportional to D; however ω is independent of D.

Let I_1 and I_2 be the intensity of interferring sources S_1 and S_2. a_1 and a_2 is amplitude of oscillations due to S_1 and S_2. Then

$$\frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2$$

The resultant intensity; I; at point P where the two disturbances arrive having a phase difference δ is

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta \qquad .....(6)$$

At points of constructive interference; $\delta = 2n\pi$ or $\cos\delta = +1$

$$\therefore \quad I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

$$= (a_1 + a_2)^2 \qquad .....(7)$$

At points of desturctive interference; $\delta = (2n-1)\pi$ or $\cos\delta = -1$

$\therefore \quad I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$

$$= \left(\sqrt{I_1} \sim \sqrt{I_2}\right)^2 = (a_1 \sim a_2)^2 \qquad .....(8)$$

$$\therefore \quad \frac{I_{max}}{I_{min}} = \left(\frac{a_1 + a_2}{a_1 \sim a_2}\right)^2 \qquad .....(9)$$

In standard Y.D.S, experiment; $I_1 = I_2 = I_0$; $a_1 = a_2 = a_0$. Eqn. (6) is reduced to

$$I = 4 I_0 \cos^2\left(\frac{\delta}{2}\right)$$

$$= I_{max} \cos^2\left(\frac{\delta}{2}\right) \qquad .....(10)$$

where $I_{max} = 4 I_0$

There is no violation of law of conservation of energy in interference. There is only redistruction of energy.

Average energy between one max^m and min^m $= \frac{I_{max} + I_{min}}{2}$

$$= \frac{4 I_0 + 0}{2} = 2 I_0$$

Fig. 83 shows I *vs* δ graph. Note all bright fringes are equibright.

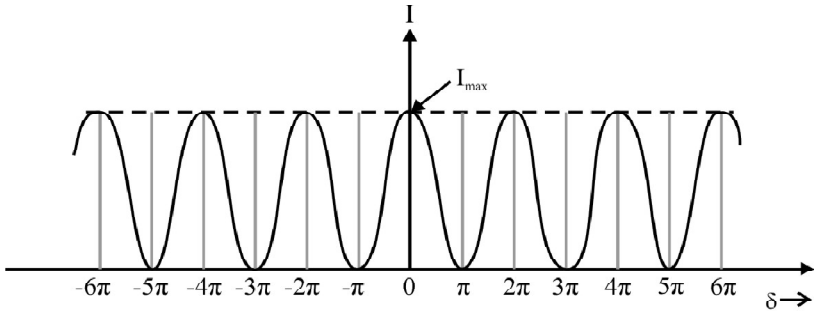


Fig. 83

Distance between mth and nth (n > m)

(i) bright fringes = $(n-m)\lambda$, (ii) dark fringes = $(n-m)\lambda$

Distance between m^{th} dark and n^{th} bright fringe

$$\left(n-m+\frac{1}{2}\right)\lambda \text{ if } n>m$$

$$\left(n-m-\frac{1}{2}\right)\lambda \text{ if } n<m$$

Modified Y.D.S Experiment

Fig. 84 shows a modified Y.D.S experimental set up. Here S is not symmetrically placed with respect to S_1 and S_2. Due to this there is an intial path difference Δ_i between S_1 and S_2.

$$\Delta_i = SS_1 - SS_2$$

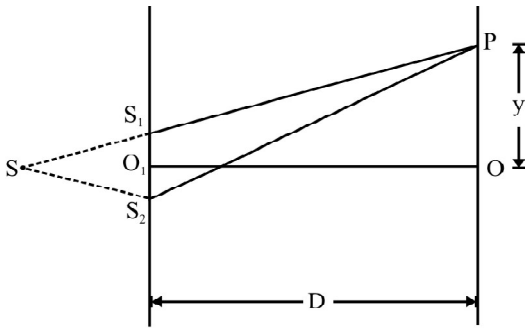


Fig. 84

We can also say there is an initial phase difference δ_i between S_1 and S_2. Obviously $\delta_i = \frac{2\pi}{\lambda}\Delta_i$. The total path difference Δ_T at any point P on screen is

$$\Delta_T = \Delta_i + (S_2P - S_1P) = \Delta_i + \frac{yd}{D}$$

The condition for constructive and distructive interference is

Constructive interference

$$\Delta_T = \Delta_i + \frac{yd}{D} = n\lambda; \quad n = 0, 1, 2, 3,..... \qquad .....(1)$$

Destructive interference

$$\Delta_T = \Delta_i + \frac{y_n' d}{D} = (2n-1)\frac{\lambda}{2} \qquad .....(2)$$

The centeral maximum is not located at O. Let y_0 be the position of central maximum. Putting n = 0; from Eqn. (1) we have

$$\Delta_i + \frac{y_0 d}{D} = 0$$

or $y_0 = -(\Delta_i)\frac{D}{d}$ .....(3)

The negative sign shows that centeral fringe is below point O. If in the modified set up $SS_2 > SS_1$ the centeral fringe shifts above point O. The fringe width β, however remains same.

Let a thin film of thickness t; refractive index μ be introduced is the path of one of the two interferring beams; say S_1P; in standard Young's double slit experiment. The path difference, Δ', between the two disturbances at point P is

$$\Delta' = (S_2P)_{air} - \left[(S_1P - t)_{air} + t_{film}\right]$$

A thickness t of film is equivalent to a path μt in air. Therefore,

$$\Delta' = (S_2P - S_1P) - (\mu - 1)t$$

$$= \Delta - (\mu - 1)t \qquad .....(4)$$

This a change $(\mu - 1)t$ in the path difference. Due to this the fringe pattern shifts but β remains same. For constructive interference

$$\Delta' = \frac{yd}{D} - (\mu - 1)t = n\lambda; \qquad n = 0, 1, 2, 3,..... \qquad .....(5)$$

The centeral fringe shift from O by a distance y_0. y_o is obtained by putting n = 0 in Eqn. (5). Therefore

$$y_0 = (\mu - 1)t\,\frac{D}{d} \qquad .....(6)$$

Let Δn be the fringe shift. Then

$$y_0 = (\Delta n)\beta \quad \text{or} \quad (\mu - 1)t\frac{D}{d} = (\Delta n)\frac{\lambda D}{d}$$

$$\therefore \quad \Delta n = (\mu - 1)\frac{t}{\lambda} \qquad .....(7)$$

White Light Fringes

Replace monochromatic source S by a white source of light in Y.D.S experiment. We observe

(1) Centeral fringe is white.

(2) Coloured fringes of each of the constituent wave lengths of white source are produced simultaneously on screen.

(3) β is minimum for violet and maximum for red colour. Moving away from centeral fringe O for a particular order of interferring (i.e. a particular value of n); the first coloured fringe is violet and the last is red.

(4) Due to superposition of different coloured fringe patterns; the number of fringes (coloured) observed clearly is small (~ 6 to 8) only.

Example–42:

Two coherent sources S_1 and S_2 seperated by a small distance d are shown in Fig. 85. The fringes obtained on screen are:

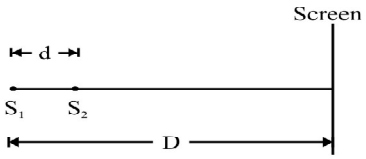


Fig. 85

(a) points **(b) straight lines**

(c) semi–circle **(d) coincenteric circles**

Solution:

P is any point on screen. With P as center draw a circle of radius S_2P meeting S_1P at S_3.

Δ = The path difference at p

$= S_1P - S_2P = (S_1S_3 + S_3P) - S_2P$

$= S_1S_3$

$= d\sin\theta$ [From right angled $\Delta\ S_1S_2S_3$]

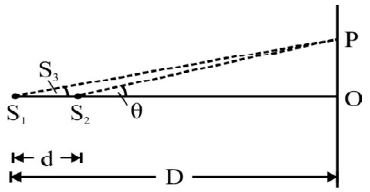


Fig. 86

The condition for say maximum is

$\Delta = d\sin\theta = n\lambda$; $n = 1, 2, 3,.....$

For a particular value of n fringe is locii of all point having same value of θ. The fringes therefore are circular with center O.

Example–43:

Y.D.S experiment is carried out by using green; red and blue light; one colour at a time. The fringe widths recorded are β_G; β_R and β_B respectively. Then

(a) $\beta_G > \beta_B > \beta_R$ **(b)** $\beta_B > \beta_G > \beta_R$

(c) $\beta_R > \beta_B > \beta_G$ **(d)** $\beta_R > \beta_G > \beta_B$

Solution:

In Y.D.S experiment fringe width (β) is directly proportional to λ. We know $\lambda_R > \lambda_G > \lambda_B$. Hence $\beta_R > \beta_B > \beta_B$. The correct choice is (d).

Example–44:

In a Young's double slit experiment 12 fringes are observed to be formed on a certain segment of the screen when light of λ = 600 nm is used. If the wave length of light is changed to 400 nm, number offringes observed in the same segment of the screen is

(a) 12 **(b) 18**

(c) 24 **(d) 30**

(I.I.T 2001)

Solution:

Let y be the length of the segment of screen considered. Let β_1 and β_2 be the fringe width for λ_1 = 600 nm and λ_2 = 400 nm respectively. Let n_1 and n_2 be the number of fringes of λ_1 and λ_2 observed. Obviously

$$y = n_1\beta_1 = n_2\beta_2$$

or $$n_1\frac{\lambda_1 D}{d} = n_2\frac{\lambda_2 D}{d} \quad \text{or} \quad n_1\lambda_1 = n_2\lambda_2$$

$$\therefore \quad n_2 = 12\times\frac{600}{400} = 18$$

Correct choice is (b)

Example–45:

In Young's double slit experiment d = 0.15 mm; D = 1 m. The distance of 10th dark fringe from centeral fringe is 38 mm. What is λ of light used?

Solution:

The distance of 10th dark fringe from the centeral fringe (zeroth order bright fringe)

$$= \left[(10-0)-\frac{1}{2}\right]\beta = \frac{19}{2}\beta$$

where β is fringe width of fringes observed.

$$\therefore \quad \frac{19}{2}\beta = 38\times10^{-3}\ \text{m}$$

or $\beta = 4\times10^{-3}\ \text{m}$

Now $\beta = \dfrac{\lambda D}{d}$ and $d = 0.15\ \text{mm} = 15\times10^{-5}\ \text{m}$; $D = 1\ \text{m}$; therefore

$$\lambda = \frac{\beta d}{D} = \frac{4\times10^{-3}\times15\times10^{-5}}{1}\ \text{m} = 600\ \text{nm}$$

Example–46:

In a Y.D.S experiment d = 0.15 mm, D = 1m. At a point on screen distant 1.64 mm from the centeral point the intensity is 64% of intensity of a bright fringe. What is wave length of light used?

Solution:

In Y.D.S experiment the intensity I at a point distant y from the central point is

$$I = I_{max}\cos^2\left[\frac{\pi yd}{\lambda D}\right]$$

Given $I = 0.64\ I_{max}$; therefore

$$0.64 = \cos^2\left[\frac{\pi yd}{\lambda D}\right]$$

or $$\cos\left[\frac{\pi yd}{\lambda D}\right] = 0.8$$

or $$\frac{\pi yd}{\lambda D} = \cos^{-1}(0.8) \simeq 0.41\pi$$

$$\therefore \quad \lambda = \frac{yd}{0.41D} = \frac{1.64\times10^{-3}\times1.5\times10^{-4}}{0.41\times1}\ \text{m} = 600\ \text{nm}$$

Example–47:

White light is used to illuminate the two slits in Young's double slit experiment. The separation between the two slits is b and the screen is at a distance d (>>b) from the slits. At a point on screen directly opposite one of the slits certain waves lengths are missing. Some of the missing

wave lengths are:

(a) $\lambda = \dfrac{b^2}{d}$ **(b)** $\lambda = \dfrac{2b^2}{d}$

(c) $\lambda = \dfrac{b^2}{3d}$ **(d)** $\lambda = \dfrac{2b^2}{3d}$

Solution:

The wave lengths that are missing at the point P, considered, are the ones undergoing destructive interference. The path difference, Δ; between the two interferring beams at P is

$$\Delta = \frac{\left(\frac{b}{2}\right).b}{d} \qquad \left[\Delta = \frac{yd}{D}. \text{ Here } y = \frac{b}{2}, d = b; D \equiv d\right]$$

$$= \frac{b^2}{2d}$$

For destructive interferrence

$$\Delta = \frac{b^2}{2d} = (2n-1)\frac{\lambda}{2} \qquad n = 1, 2, 3,.....$$

or $$\lambda = \frac{b^2}{(2n-1)d}$$

For different allowed values of n; λ is

$$\frac{b^2}{d}; \frac{b^2}{3d}; \frac{b^2}{5d}.....$$

The missing wave length is given by (a) and (c).

Example–48:

In Y.D.S experiment the two slits act as coherent sources of amplitude A and wave length λ. In an another experiment with the same set up the two slit sources of equal amplitude A and wave length λ, but are incoherent. What is the ratio of intensity of light at mid–point of screen in the first and the second case?

Solution:

In first case the interferring sources are coherent, constructive interference takes place at mid–point O. The resultant intensity.

$$I_1 = 4\,A^2$$

In second the sources are incoherent no interferring is observed at mid–point O. The resultant intensity I_2 is

$$I_2 = 2\,A^2$$

$$\therefore \quad \frac{I_1}{I_2} = 2$$

Example–49:

In a Y.D.S experiment the intensity of each interferring source is I_0. The intensity of one of the two sources is reduced by 19%. Express the change in the intensity of maximum and minimum now observed as a fraction of maximum intensity observed earlier.

Solution:

Let I_{max} and I_{min} be the intensity of maximum and minimum observed initially. Then

$$I_{max} = 4\,I_0 \text{ and } I_{min} = 0$$

In second case. Now

$$I_1 = I_0;\ I_2 = (1–0.19)\,I_0 = 0.81\,I_0$$

Let I'_{max} be the intensity of maximum now observed.

$$I'_{max} = \left(\sqrt{I_0} + \sqrt{0.81 I_0}\right)^2$$

$$= \left(1.9\sqrt{I_0}\right)^2 = 3.61\,I_0$$

$$= \frac{3.61}{4} I_{max} = 0.9025\,I_{max}$$

$$\therefore \quad \frac{I'_{max}}{I_{max}} = 0.9025$$

Let I'_{min} be the intensity of the minimum observed now.

$$I'_{min} = \left[\sqrt{I_0} - \sqrt{0.81 I_0}\right]^2$$

$$= \left[0.1 I_0\right]^2 = 10^{-2} I_0$$

$$= \frac{10^{-2}}{4} I_{max} = 0.25 \times 10^{-2} I_{max}$$

$$\therefore \quad \frac{I'_{min}}{I_{max}} = 0.0025$$

Example–50:

In Y.D.S experimental set up two wave length of 500 nm and 700 nm are used. What is the minimum distance from the centeral maximum where their maximas coincide? Take $\frac{D}{d} = 10^3$.

Solution:

In Fig. 87, P is point where maximas of λ_1 = 500 nm and $\lambda_2 = 700$ nm coincide for first time after centeral maximum. Let β_1 and β_2 be fringe width for wave length λ_1 and λ_2. Obviously $\beta_1 < \beta_2$. Let there be n bright fringes of λ_2 in distance OP = y. There will be (n + m) bright fringes of λ_2. Obviously m = 1, 2, 3,.....

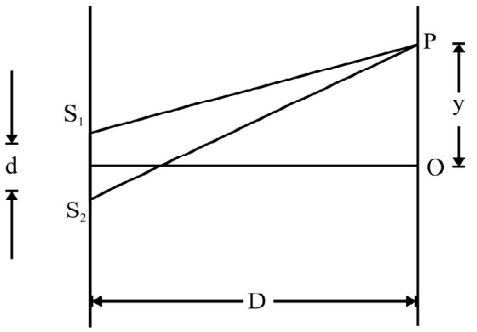


Fig. 87

$$OP = n\beta_2 = (n + m)\beta_1$$

or $$\frac{n + m}{n} = \frac{\beta_2}{\beta_1} = \frac{\lambda_2}{\lambda_1}$$

$$1 + \frac{m}{n} = 1.4$$

or $$\frac{m}{n} = \frac{2}{5}$$

This means 5th bright fringe of λ_2. Coincides with 7th bright fringes of λ_1. Therefore

$$y = 7 \times \beta_1 = 7 \times \frac{\lambda_1 D}{d}$$

$= 7 \times 5 \times 10^{-7} \times 10^{3}$ m

$= 35 \times 10^{-4}$ m $= 3.5$ mm

Example–51:

In Y.D.S experimental set up shown in Fig. 88; the distance between slits S_1 and S_2 is d = λ is wavelength of light. A parallel beam is incident making an angle i with plane of S_1 and S_2. For point P on screen; the condition of (i) constructive (ii) destructive interference is

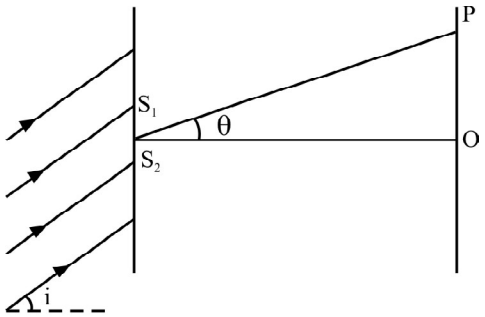


Fig. 88

(a) $\mathbf{d\sin i = n\lambda;}$ $\mathbf{n = 1, 2, 3, ...}$

$$\mathbf{d\sin i = (2n-1)\frac{\lambda}{2}}$$

(b) $\mathbf{d(\sin\theta) = n\lambda}$ $\mathbf{n = 1, 2, 3, ...}$

$$\mathbf{d\sin\theta = (2n-1)\frac{\lambda}{2}}$$

(c) $\mathbf{d[\sin\theta + \sin i] = n\lambda}$ $\mathbf{n = 1, 2, 3, ...}$

$$\mathbf{d[\sin\theta + \sin i] = (2n-1)\frac{\lambda}{2}}$$

(d) $\mathbf{d[\sin i - \sin\theta] = n\lambda}$ $\mathbf{n = 1, 2, 3, ...}$

$$\mathbf{d[\sin i - \sin\theta] = (2n-1)\frac{\lambda}{2}}$$

Solution:

There is on initial path difference $\Delta_i = S_1S_1'$ between S_1 and S_2. From right angled triangle $S_1S_1'S_2$ shown in Fig 89.

$$\frac{S_1S_1'}{S_1S_2} = \sin i$$

or $\Delta_i = d\sin i$ .....(1)

At point P on screen the two disturbances arrived having a path difference, $\Delta = S_2P - S_1P$. From right angled triangle $S_1S_2'S_2$;

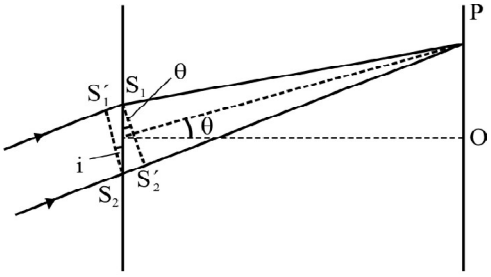


Fig. 89

$$\Delta = d\sin\theta \qquad .....(2)$$

The total path difference Δ_T between the two disturbance at P is

$$\Delta_T = d\sin i - d\sin\theta$$

For constructive interference

$$\Delta_T = d(\sin i - \sin\theta) = n\lambda$$

and for distructive interference

$$d(\sin i - \sin\theta) = (2n-1)\frac{\lambda}{2}$$

The correct choice is (d)

Example–52:

A Young's double slit experiment is performed using monochromatic light of wave length λ. Express intensity on a point where two waves arrive having a path difference $\frac{\lambda}{4}$ as a percentage of maximum intensity.

Solution:

In Y.D.S experiment the intensity I at any point on a screen where the two waves arrive having a phase difference δ is

$$I = I_{max}\cos^2\left(\frac{\delta}{2}\right)$$

Given $\Delta = \frac{\lambda}{4}$; therefore

$$\delta = \frac{2\pi}{\lambda}\left(\frac{\lambda}{4}\right) = \frac{\pi}{2}$$

$$\therefore \quad I = I_{max}\cos^2\left(\frac{\pi}{4}\right)$$

$$= \frac{I_{max}}{2}$$

$$\frac{I}{I_{max}} \times 100 = 50\%$$

Example–53:

A glass plate of refractive index 1.5 is coated with a layer of thickness t and refractive index 1.8. Light of wave length λ travelling in air is incident normally on the layer. It is partly reflected at the upper and the lower surface of the layer and the two reflected rays interfere. Write the condition of constructive interferece. It λ = 648 nm; obtain the least value at t for which rays interfere constructively.

Solution:

In Fig. 90 ABCD is film of thickness t; refractive index $\mu_1 = 1.8$. DCEF is glass plate $\mu_2 = 1.5$. PQ is a ray incident nearly normally on upper face AB. QR is reflected ray. TU is ray produced ray due to reflection at lower face CD of the film. Interference occurs between QR and TU. For nomal incidence path difference $\Delta_1 = 2(t)_{film} = 2(\mu_1 t)_{air}$. The reflection at upper face AB is from boundary of a denser medium. There is an additional path difference $\Delta_2 = \frac{\lambda}{2}$ between the two interferring beams.

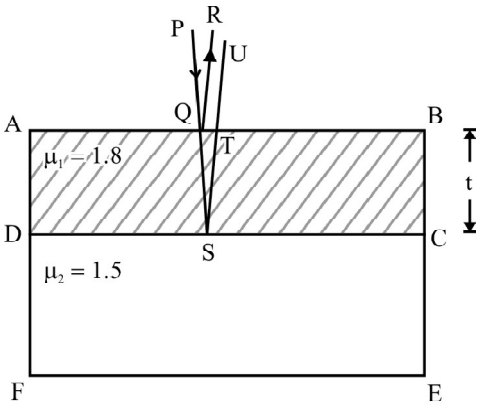


Fig. 90

Δ = The total path differences = $\Delta_1 + \Delta_2$

$$= 2\mu_1 t + \frac{\lambda}{2}$$

For constructive interference

$$2\mu_1 t + \frac{\lambda}{2} = n\lambda$$

or $$2\mu_1 t = \left(n - \frac{1}{2}\right)\lambda \qquad n = 0, 1, 2, 3, ...$$

For minimum value of t; n = 1. Therefore

$$t_m = \frac{\lambda}{4\mu_1} = \frac{648\times10^{-9}}{4\times1.8}\,m$$

$= 90\times10^{-9}$ m = 90 nm

DIFFRACTION

The bending of waves arround the edge of an obstacle in their path and their penetration into the region of 'geometrical shadow' is known as diffraction. All type of waves undergo diffraction. Diffraction is easily observed with

(1) sound waves

(2) waves on surface of water

However apparently light does not exhibet differaction. This is due to very small wave length of visible light. *To observe diffraction size of obstacle must be comparable to wave length of waves used.* For observing differaction of light the size of obstacle used must be very small (comparable to λ). When this is done differaction of light is also easily observed.

Diffraction at a Single Slit

Fig. 91 shows the single slit experimental set up. AB = d (of the order of a fraction of a m.m) is a slit in an opaque screen illuminated with a monochromatic parallel beam. The incident W.F on slit is a plane wave front. The screen is placed at a distance D (>> d) from the slit. If light travels in straight line the part A_1B_1 of screen should be uniformly bright. The screen above A_1 and below B_1 should be completely dark. This is not observed. The intensity on screen varies. The intensity variation is shown in Fig. 91.

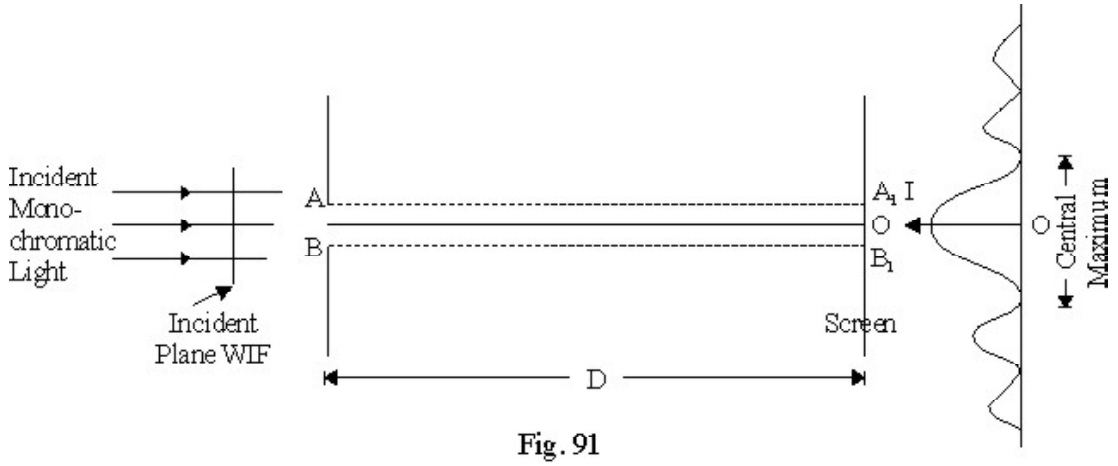


Fig. 91

The central point O is maximum bright. It is sourrounded by alternate maximas and minimas. The intensity of subsidary maximas observed decreases as shown in Fig. 91. The observed intensity pattern is known as

diffraction pattern due to a single slit. The intensity pattern is observed due to bending of light around the edges of the slit i.e. due to diffraction of light.

The observed diffraction pattern is due to interference between secondary wavelets originating from plane of slit. We can say *diffraction is interference between different parts of the same wave front*. It can be shown that for secondary wavelets in a direction θ with the direction of incident light; if

$$d\sin\theta = n\lambda \qquad n \neq 0; \qquad n = 1, 2, 3,.....$$

is condition of minimum; and

$$d\sin\theta = (2n+1)\frac{\lambda}{2}; \qquad n = 1, 2, 3,.....$$

is the condition of subsidary maxima. n = 0 gives the principle maximum.

Let r_1 be the radius of the principle maximum observed i.e. the centeral maximum. It can be shown that

$$r_1 = \frac{\lambda D}{d}$$

The half–angular width θ of principal maximum is; nearly

$$\theta \simeq \frac{\lambda}{d}$$

Diffraction of Light and Rectilinear Propagation of Light

We know light does not travel in straight line but bends around on appropriate obstacle in its path i.e. undergoes diffraction. Does this mean that "ray–otpics" is wrong! Strickly speaking ray optics is an approximation. However it is a very–very good approximation. When size of obstacle is large (of the order of a few cm or more) the diffraction effects are negligible.

Resolving Power

Two nearby objects or spectral lines are resolved if they appear as two images or two spectral lines. Normal human eye can see two objects seprate if they subtend a minimum angle of 1′′ of arc on eye.

The sepration of close objects is termed as resolution and the ability of an optical instrument to produce seperate image of two such objects is known as resolving power of the instrument. Two neighbouring point objects are resolved by the lens system such as telescope or microscope is known as *Geometrical resolution*. When two close spectral lines (i.e. wave lengths) are resolved with the help of a prism; grating etc. we have *spectral resolution*.

Rayleigh's Criterion

When two nearby objects are viewed by any optical instrument; due to wave nature of light (i.e. diffraction) we obtain two nearby diffraction patterns keeping one object fixed the other is moved nearer to the first the two diffraction patterns come nearer and start overlapping.

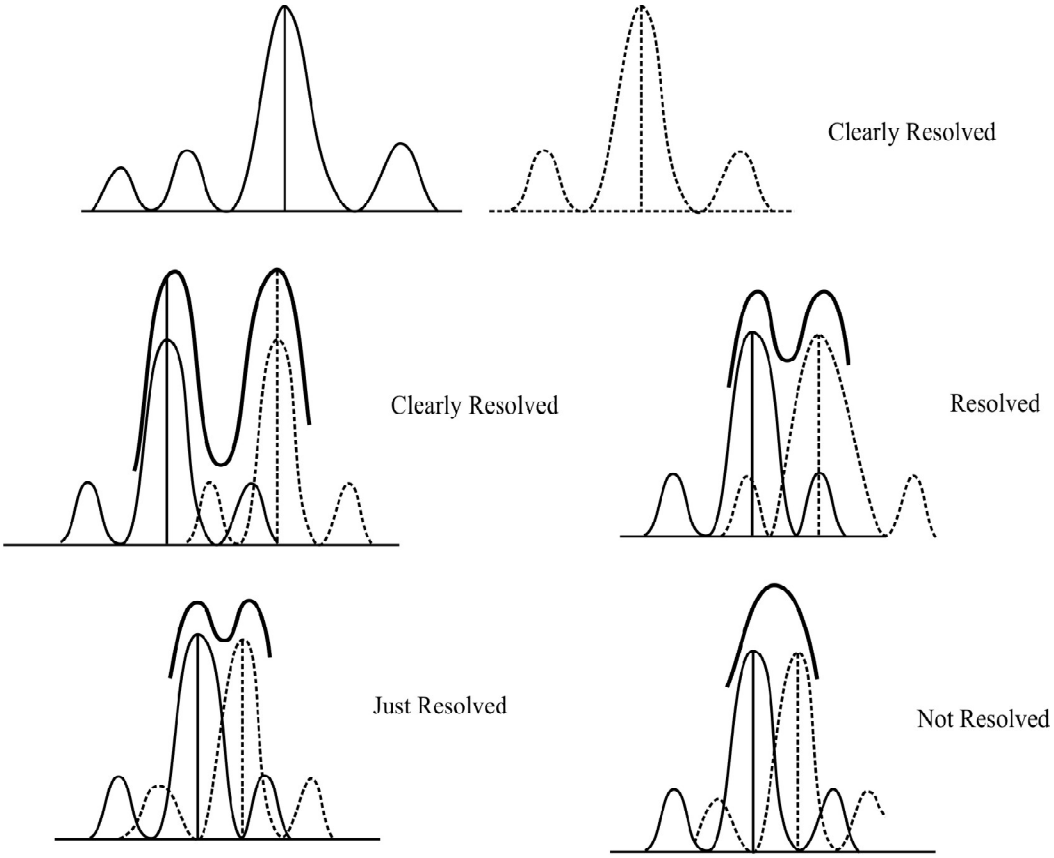


Fig. 92

According to Lord Rayleight two objects or spectral lines are just resolved if the angular sepraction between the principle maxima in the diffraction pattern due to the two equals half the angular width of the principal maximum of the either of the two. Let x be this minimum distance between the two point objects; the geometrical R.P $= \frac{1}{x}$. Let λ and $\lambda + d\lambda$ be two spectral lines just resolved, the spectral resolving power $= \frac{\lambda}{d\lambda}$.

Resolving Power of a Telescope

Let θ_{min} be the angular sepration between two objects just resolved according to Rayleigh's criterion; the

$$\text{R.P. of telescope} = \frac{1}{\theta_{min}}$$

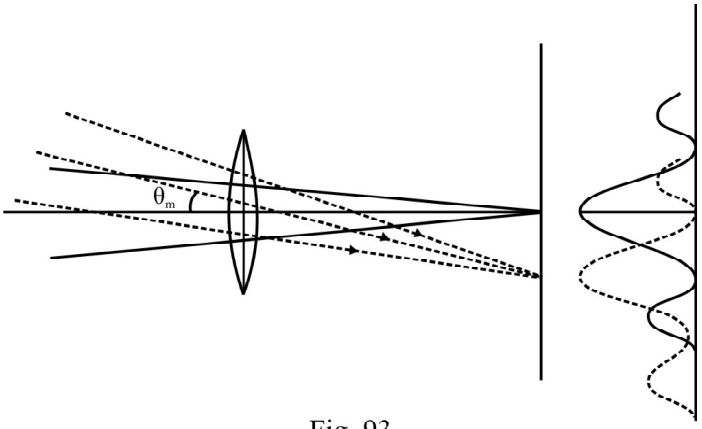


Fig. 93

Let d be the diameter of O–lens of telescope (i.e. aperture of telescope) and λ the wave length of light.

θ = Half angular width of principal maximum of diffraction pattern due to any one star (say the object being viewed)

$$= \frac{\lambda}{d} = \theta_{min}$$

$$\therefore \quad \text{R.P} = \frac{1}{\theta_{min}} = \frac{d}{\lambda}$$

More precise calculation show that

$$\text{R.P} = \frac{d}{1.22\lambda}$$

The magnifying power M of a telescope; the resolving power are related to one another by the relation;

$$\text{M.P of telescope} \times \text{R.P of telescope} = \text{R.P of the eye.}$$

Resolving Power of a Microscope

In Fig. 94 *x* is minimum distance between the two nearby objects forming diffraction pattern shown. The objects are just resolved. It can be shown that

$$\text{PQ} = x = \frac{1.22\lambda}{\sin\theta}$$

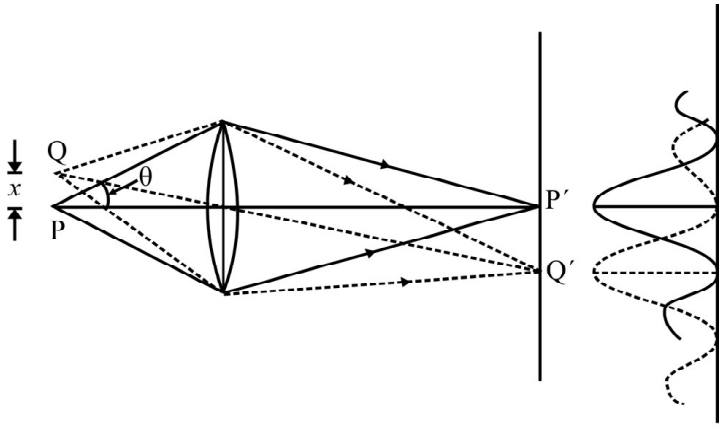


Fig. 94

R.P = $\frac{1}{x}$. Lesser is value of x more is resolving power. If the object and objective lens both are immersed in a liquid of refractive index μ; the minimum values of x for resolution works out to be $\frac{1.22\lambda}{\mu \sin\theta}$. $\mu \sin\theta$ known as the *numerical aperature*. Obviously x is less if μ is more. This is the reason an oil immersion microscope has a higher R.P. Also if λ is less R.P is more. Therefore an ultra–microscope has a higher R.P. It can be shown that

$$\text{Minimum M.P of microscope} = \frac{\text{Limit of resolution of the eye}}{\text{Limit of the resolution of the microscope}}$$

Example–54

A single slit of width 'd' is illuminated with light of λ = 600 nm. The diffraction pattern is observed on a screen at a distance of 120 cm from the slit. The diameter of principal maximum observed is 4 mm. What is

(a) (i) d (ii) angular width of principal maximum.

(b) The wave length of light is increased by 20% and d is reduced by 20%, what is percentage change in angular width of principal maximum.

(c) What is percentage change in distance of screen if angular width is to be restored to its earlier value. What is change in diameter of principal maximum observed now?

Solution:

(a) Given = 600 nm = 6×10^{-7} m

r_1 = radius of principal maximum = 2 mm = 2×10^{-3} m

D = The distance of screen from slit = 120 cm = 1.2 m

We know $r_1 = \frac{\lambda D}{d}$; therefore

$$d = \frac{6\times10^{-7}\times1.2}{2\times10^{-3}} = 3.6\times10^{-4}\,\text{m} = 0.36\,\text{mm}$$

(b) Given $\lambda' = 1.20\lambda$; $d' = 0.8\,d$; $D' = D$

$2\theta'$ = The angular width of principal maximum

$$= \frac{\lambda'}{d'} = \frac{1.2\lambda}{0.8\,d} = 1.5\left(\frac{\lambda}{d}\right) = 1.5(2\theta)$$

where $2\theta = \frac{\lambda}{d}$ = The angular width of principal maximum earlier.

Percentage change in angular width = (1.5–1)×100= +50%

(c) To get same angular width D can change by any amount. The angular width does not depend on D. However diameter of principal max^m is directly proportional to D; therefore

$$r_1' = \frac{D'\lambda}{d} = \left(\frac{D'}{D}\right) r_1$$

Example–55

A single slit diffraction pattern is observed. What is the ratio of the intensity of different maxima's observed? What is ratio of radii of different subsidary maxima's observed.

Solution:

Let I_0, I_1, I_2,..... be the intensity of centeral maximum, first, second,..... subsidary maxima's observed. Very approximately

$$I_0 : I_1 : I_2 : I_3 \ldots\ldots :: 1 : \frac{1}{3} : \frac{1}{5} : \frac{1}{7} \ldots\ldots$$

This because at centeral maximum all secondary wavelets from plane of slit undergo constructive interference.

At first subsidary maximum nearly $\frac{1}{3}^{rd}$ of secondary wavelets donot undergo distructive interference. At second maximum nearly $\frac{1}{5}^{th}$ of secondary wavelets from plane of slit donot undergo destructive interference with one another and so on.

The radii of subsidary maxima's are given by

$$r_n = (2n+1)\frac{D}{2d}; \qquad n = 1, 2, 3, ...$$

$\therefore \quad r_1 : r_2 : r_3 ..... :: 3 : 5 : 7 .....$

Example–56

Angular width of centeral maximum in a Fraunhoffer diffraction pattern of a slit is measured. The slit is illuminated with light of λ = 6000 A. When the slit is illuminated with another wave length the angular width decreases by 30%. Calculate λ of light used. The same decrease in angular width of centeral maximum is obtained when the original apparats is immersed in a liquid. Find the refractive index of the liquid.

Solution:

Let θ_1 be the angular width of centeral maximum when light of wave length λ_1 is used. Then

$$\theta_1 = \frac{\lambda_1}{d}$$

d = width of the slit. Let θ_2 be the angular width when light of $\lambda = \lambda_2$ is used. Then

$$\theta_2 = \frac{\lambda_2}{d}$$

Given $\theta_2 = (1-0.3)\theta_1 = 0.7\,\theta_1$

or $$\frac{\theta_2}{\theta_1} = 0.7$$

$\therefore \quad \frac{\lambda_2}{\lambda_1} = 0.7$ or $\lambda_2 = 0.7 \times 6000 = 4200\,A$

Let μ be refractive index of liquid in which original setup is immersed. The wave length of light inside liquid is $\frac{\lambda_1}{\mu}$. Since change in angular width is same;

$$\frac{\lambda_1}{\mu} = \lambda_2$$

or $$\mu = \frac{\lambda_1}{\lambda_2} = \frac{6000}{4200} = 1.428$$

Example–57

A slit of width d is placed in front of a lens of focal length 0.5 m and is illuminated normally with light of λ = 589 nm. The diameter of centeral maximum observed is 2mm. What is d?

Solution:

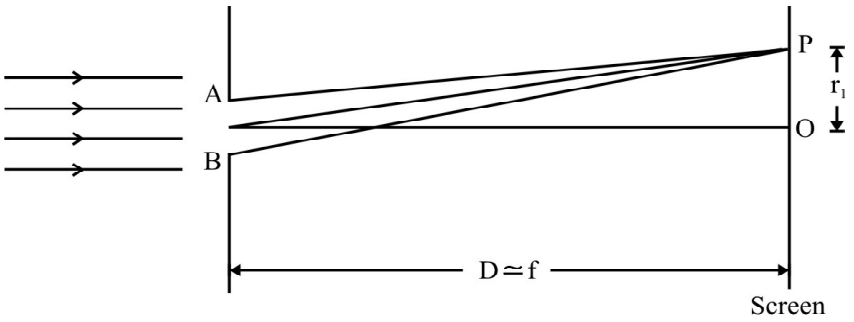


Fig. 95

Fig. 95 shows slit AB = d; illuminated with parallel beam of λ = 589 nm. The convex lens focus diffracted beam on screen. Therefore

D = distance between slit and screen $\simeq$ f = 0.5 m

Let r be radius of centeral maximum. We know $r = \frac{\lambda D}{d}$. Given

$$r = 1 \text{ mm} = 10^{-3} \text{ m}; \ \lambda = 589\times10^{-9} \text{ m}; \ D = 0.5 \text{ m}$$

$$\therefore \quad d = \frac{589\times10^{-9}\times0.5}{10^{-3}} \text{ m}$$

$$\simeq 2.94\times10^{-4} \text{ m} = 2.94 \text{ mm}$$

Example–58

In Y.D.S experimental set up one of the slits is covered with an opaque paper. What change; if any, is observed in the pattern obtained on screen?

Solution:

In Y.D.S experiment interference pattern is observed. When one slit is blocked we obtain diffraction pattern on screen. The difference between the two are:

(1) Earlier fringes observed are equibright and equispaced. Now centeral maximum is most intense. The intensity of other maxima's observed decreases. The fringes observed are not equispaced.

(2) The number of fringes observed in Y.S. D is quite large (~ 20–30). The number of fringes observed clearly in diffraction is small (~ 6–8).

Example–59

A telescope has an objective lens of aperture 4 cm focal length 2 m. λ = 600 nm. What is resolving power of telescope?

Solution:

The aparture of O–lens of telescope is the size of opening, i.e. d = 4 cm = 4×10^{-2} m. Let θ_m be the minimum angular separation between two stars that are just resolved. According to Rayleigh's Criterion; θ_m equals half angular width of centeral maximum due to diffraction at O–lens; i.e.

$$\theta_m = \frac{\lambda}{d} = \frac{6\times10^{-7}}{4\times10^{-2}} = 1.5\times10^{-5} \text{ radian}$$

$$\text{R.P of telescope} = \frac{1}{\theta_m} = \frac{1}{1.5\times10^{-5}} = 6.67\times10^{4}$$

POLARISATION

The phenomenon of polarisation is the preferrential selection of a particular direction of vibration from the various possible directions of vibrations.

In longitudinal waves there is only one direction of vibration, which is the direction of wave propagation, there is no possibility of any perferrential selection of direction of vibration. Therefore longitudinal waves cannot be polarised.

However in transverse waves vibrations can take place in any direction in a plane perpendicular to direction of wave motion. When there is no selection of a preferred direction of vibration i.e. *no selection of a preferred direction of vibration we say waves are unpolarised.* However, if somehow, a particular direction of vibrations is selected and all other direction of vibrations are blocked we have plane polarised waves. The device used for this selection is known as a polariser. *It is only transverse waves that can be polarised.*

In case of light; in accordance with the E.M. theory we discuss about the vibration of electric field. Ordinary light is unpolarised, i.e. the oscillations of **E** take place in all directions perpendicular to direction of wave propagation. A *calcite or a tourmaline crystal,* has the property of allowing oscillations of **E** parallel to its axis to pass through and blocking all other direction of oscillations. It is used to convert ordinary (unpolarised) light from any source to plane polarised light. This crystal acts as a polariser.

In unpolarised light vibrations occur is all allowed directions of vibration. This is equivalent to two equal mutually perpendicular oscillations. A polariser allows all vibrations parallel to its axis to pass through and blocks the perpendicular component. If I_0 is intensity of unpolarised light incident on a polariser, the intensity

of plane polarised light emerging out of polarised is $\frac{I_0}{2}$. It remains same irrespective of orientation of polariser. In other words when a polariser is rotated through are full circle about direction of propagation as axis; there is no change in intensity of light transmitted by the polariser. Fig. 96 shows symbolic representation of unpolarised and plane polarised light.

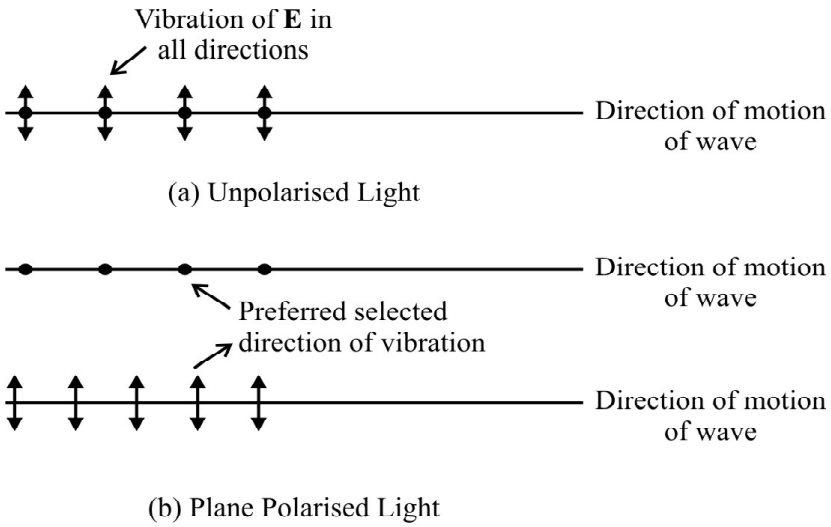
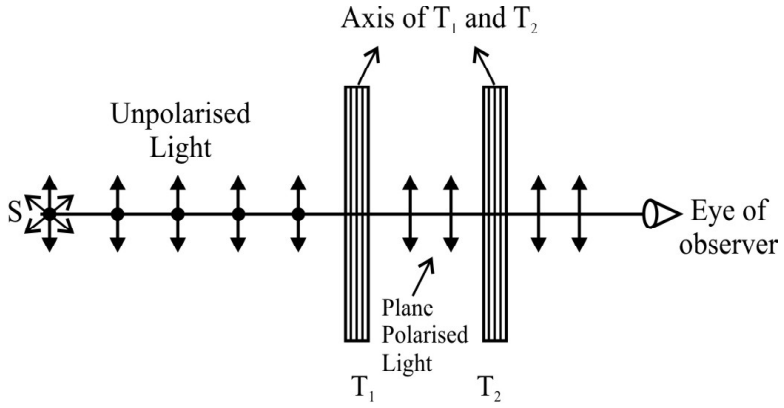


Fig. 96

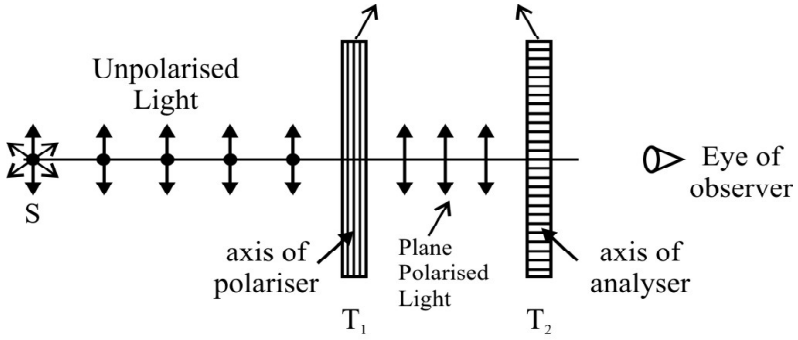
The plane in which vibration of **E** are confined is known as plane of vibration. A plane perpendicular to plane of vibration is known as plane of polarisation.

Detection of Plane Polarised Light the Analyser

Our eye cannot detect whether light used is unpolarised or plane polarised. To do this we use another tourmaline or calcite crystal. This is known as analyser.



(a)



(b)

Fig. 97

In Fig. 97 (a) and (b) S is ordinary source of light emitting unpolarised light. T_1 is polariser and T_2 is analyser. In Fig. 97 (a) axis of polariser (T_1) and analyer (T_2) are parallel; the vibrations passing through T_1 also pass through T_2. The eye observes light passing through T_2. Keeping T_1 fixed T_2 is rotated about direction of propagation of light as axis. The intensity of light passing through T_2 starts decreasing. In Fig. 97 (b); T_2 has accquired an orientation when its axis has rotated by 90^0. The vibrations incident on T_2 are blocked. The intensity of light transmitted is zero. In one full rotation of T_2; there are

(1) two positions when intensity of light transmitted is maximum (i.e. axis of T_1 and T_2 are parallel or antiparallel).

(2) two positions when intensity of light transmitted by T_2 is zero. This is when axis of T_2 make on angle of $\frac{\pi}{2}$ or $\frac{3\pi}{2}$ with axis of T_1.

We summarise by concluding:

(1) When unpolarised light passes through an analyser; intensity of transmitted light is 50% of incident intensity. However on rotating analyser about direction of propagation as axis there is no variation in intensity of light passing through.

(2) When plane polarised light passes through analyser and analyswer is rotated through one full circle; there is a variation in intensity of light passing through and intensity is zero twice in one full rotation.

Brewster's Law

When unpolarised incident light undergoes reflection from a medium of refractive index μ; there is one angle of incidence (i_p) such that the reflected light is plane polarised. *i_p is known as the polarising angle of incidence.* According to Brewster's law.

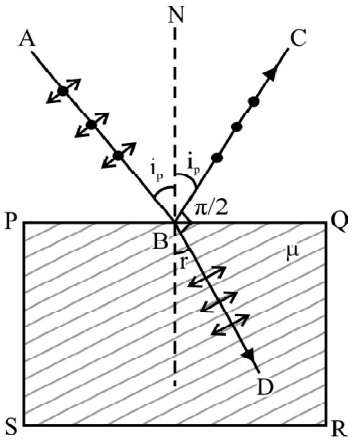


Fig. 98

Fig. 98 shows a block PQRS of a medium of refractive index μ. AB is

an incident unpolarised ray; incident at polarising angle of incidence i_p BC and BD are reflected in mutually perpendicular planes. The reflected BC are refracted ray BD make an angle of 90^0 with one another; i.e.

$\angle CBD = \frac{\pi}{2}$.

The Malus Law

A plane polarised beam of intensity I_0 is incident on an analyser. δ is angle between axis of anglyser and polariser. I is intensity of light transmitted by analyser. According to Malus law

$$I = I_0 \cos^2(\delta)$$

Uses of Polaroids

(1) Sun glasses fitted with polaroid sheets reduce glare of sun.

(2) The glare of head lights of a car; during night driving; is reduced using wind shields made of polariod sheets.

(3) Polariods used in projection and viewing of three dimensional motion pictures.

(4) Polaroids are used to improve colour contrast in old paintings.

(5) Optical stress analyser makes use of polaroids.

Example–60:

Light is reflected from a material in which speed of light is 2×10^8 ms^{-1}. What is polarising angle of incidence? Does this angle depend on wave length of light used?

Solution:

μ = The refractive index of reflector

$$= \frac{c}{v} = \frac{3\times10^8}{2\times10^8} = 1.5$$

Let i_p be the polarising angle of incidence. From Brewster's law

$$\tan i_p = \mu$$

or $$i_p = \tan^{-1}(\mu) = \tan^{-1}(1.5) \simeq 56.3^0$$

The value of μ depends on wave length (λ) of incident light; therefore i_p also depends on λ. For visible light μ is maximum for violet and minimum for red colour. Therefore i_p is also maximum for violet and minimum for red colour.

Example–61:

(a) Fig. 99 shows a source S. P_1 and P_2 are two polaroids. I_0 is intensity of light incident on P_1. Axis of P_1 and P_2 are inclined to one another at $\frac{\pi}{6}$. What is ratio of intensity of light observed by O_1 and O_2.

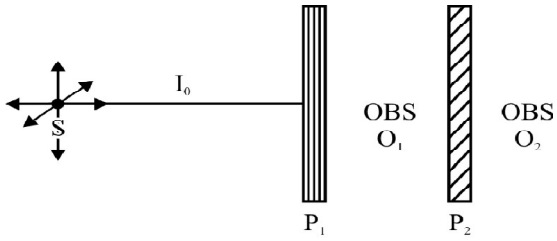


Fig. 99

(b) Keeping P_2 fixed P_1 is rotated through one full circle about incident ray as axis. Draw a graph showing variation of intensity observed by O_1 and O_2.

Solution:

(a) The incident light from S is unpolarised. P_1 acts as a polariser.

I_1 = Intensity of light observed by $O_1 = \frac{I_0}{2}$

P_2 has plane polarised light incident on it. Initially;

δ = Angle between axis of analyser and polariser $= \frac{\pi}{6}$ (Given)

Let I_2 be intensity of light observed by O_2. From Malus law

$$I_2 = \left(\frac{I_0}{2}\right)\cos^2\left(\frac{\pi}{6}\right)$$

$$= \left(\frac{I_0}{2}\right)\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3I_0}{8}$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{I_0/2}{3I_0/8} = \frac{4}{3}$$

(b) On rotating P_1; there is no change in intensity of light observed by O_1. It remains same, having intensity $= \frac{I_0}{2}$ as shown in Fig 100.

Let θ be the angle through which axis of P_1 has been rotated. The instantaneous angle between axis of P_1 and P_2 is

$$\delta' = \left[\theta - \frac{\pi}{6}\right]$$

Therefore I_2'; instantaneous intensity of light, observed by O_2 is

$$I_2' = \left(\frac{I_0}{2}\right)\cos^2\left[\theta - \frac{\pi}{6}\right]$$

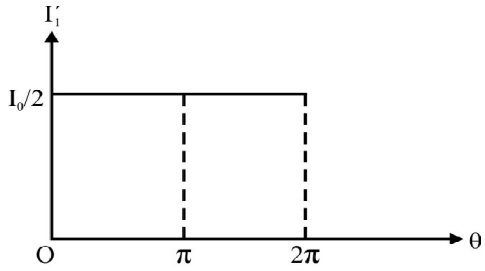


Fig. 100

I_2' is zero if $\cos^2\left(\theta - \frac{\pi}{6}\right) = 0$ or $\cos\left(\theta - \frac{\pi}{6}\right) = 0$

$$\therefore \quad \theta = \frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3} \quad \text{or} \quad \frac{3\pi}{2} + \frac{\pi}{6} = \frac{5\pi}{3}$$

I_2' is maximum if $\cos\left(\theta - \frac{\pi}{6}\right) = \pm 1$. Therefore

$$\theta - \frac{\pi}{6} = 0 \text{ or } \pi$$

or $$\theta = \frac{\pi}{6} \text{ or } \frac{7\pi}{6}$$

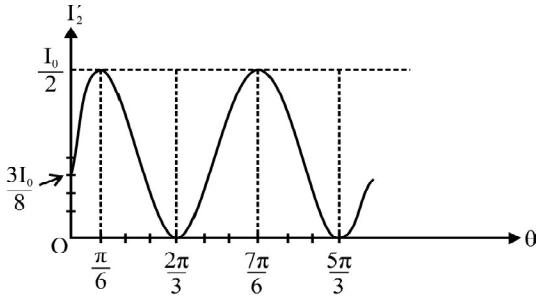


Fig. 101

The variation of I_2' with θ is shown in Fig. 101.

PROBLEMS FOR PRACTICE

1. An object 1 cm high is placed at a distance of 30 cm from a (i) concave mirror (ii) convex mirror of focal lenth 20 cm. In each case what is position, nature and length of image formed?

   [Ans: (i) Real; inverted; 60 cm; 2 cm (ii) Virtual erect; 60 cm; 2 cm]

2. A concave mirror forms an image of length 4 cm of an object of length 1 cm at a distance of 10 cm from the mirror. What is radius of curvature and power of mirror?

   [Ans: 16 cm; –12.5 D]

3. What is distance of an object from a concave mirror of power 2.5 D so as to obtain an image twice the length of the object?

   [Ans: 60 cm or 20 cm]

4. A concave mirror produces an image on a screen of an object at a distance of 22.5 cm. The image is twice the length of object. What is change in position of object so that image formed is three times the length of the object?

   [Ans: 2.5 cm to towards mirror]

5. An object is placed 25 cm in front of a convex mirror. A plane mirror is placed in between convex mirror and object at a distance of 5 cm from the convex mirror. There is no parallex between the image formed by the two mirrors. What is focal length of convex mirror?

   [Ans: 37.5 cm]

6. A plane mirror is placed 22.5 cm in front of a concave mirror of radius of curvature 20 cm. An object is placed in between the two mirros so that the first image formed by both mirror coincide. What is distance of object from plane mirror?

   [Ans: 7.5 cm]

7. Two objects L and M placed one by one in front of a concave mirror of power 13.33 D form image of same size. Length of L is three times that of M. Distance of L from the mirror is 30 cm. What is position of M?

   [Ans: 15 cm]

8. Sun subtends an angle of 30′′ of arc on earth. A concave mirror forms an image of Sun of diameter 2 cm on a screen. What is focal length and power of the mirror?

   [Ans: 2.29 m; –0.4.7 D]

9. A concave mirror of radius of curvature 20 cm and convex mirror of power 2.5 D are placed co–axially facing one another 40 cm apart. A point object is placed on the common axis at a distance of 25 cm from the convex mirror. What is position, nature of the image formed by the system; due to successive reflections first at concave and then convex mirror?

   [Ans: Virtual; 6 cm behind the convex mirror]

10. An object is moved 12 cm towards a concave mirror; the real image formed moves from 25 cm to 75 cm. What is focal length of mirror?

[Ans: 13.2 cm]

11. Speed of light in glass is 2×10^8 m/s. What is absolute refractive index of glass? Is refractive index of glass same for violet and yellow colour?

[Ans: 1.5; No. μ is more for violet than yellow colour]

12. A ray of light is incident on a plane boundary from a medium of refractive index μ_1. The refractive index of second medium is $\mu_2\,(\mu_1 > \mu_2)$. The ray is partially reflected and partially refracted. The reflected and the refracted rays are mutually perpendicular. What is angle of incidence?

$$\left[\text{Ans: } \tan^{-1}\left(\frac{\mu_2}{\mu_1}\right)\right]$$

13. A glass slab of thickness 80 mm contains same number of waves as are there in 10 cm of water for monochromatic light of same λ $\mu_\omega = \frac{4}{3}$. What is μ_g ?

[Ans: 1.67]

14. A ray of light is incident at angle θ with a rectangular plate of thickness d and refractive index n. What is net deviation and lateral displacement as the ray emerges out of slab?

$$\left[\text{Ans: zero; } d\sin\theta\left[1-\frac{\cos\theta}{\left(n^2-\sin^2\theta\right)^{1/2}}\right]\right]$$

15. A beaker is filled with water of refractive index 1.33 up to 10 cm. A layer 15 cm thick of an immisible oil of refractive index 1.4 is formed above water. When viewed from above what is

(i) apparent depth of a point on the inner bottom of beaker.

(ii) equivalent refractive index of the arrangment?

[Ans: (i) 18.23 cm (ii) 1.37]

16. A glass beaker has a base 40 mm thick $(\mu_g = 1.50)$. The beaker has two immisible liquids I and II poured into it upto to a total height of 14 cm. The depth of liquid I is 60 mm and its refractive index is 1.40. A point on the outer bottom of beaker appears raised by 48.8 mm when viewed directly from above. What is refractive index of liquid II?

[Ans: 1.30]

17. A point object is placed 40 cm in front of a plane mirror. A glass slab ($\mu = 1.5$) of thickness 0.2 m is placed in between plane mirror and point object. What is distance between the object and its image formed by the plane mirror?

[Ans: 73.33 cm]

18. A point object is placed 210 mm in front of concave mirror of power 30 D. A glass slab of thickness 30 mm; $\mu_g = 1.5$ is placed with its nearer edge at a distance of 10 mm from concave mirror; in betwwen the mirror and object. What is position of image formed? (I.I.T)

[Ans: 76.7 cm]

19. A rectangular glass slab is placed on a horizontal printed page. What is minimum value of μ of glass so that the letters on the page are not visible from any of the vertical face of slab? (I.I.T)

[Ans: $\sqrt{2}$]

20. A point source S is placed at the bottom of a vessel containing a liquid of refractive index $\frac{5}{3}$. A person is viewing the source from above the surface. There is an opaque disc of radius 1 cm floating on the surface. The center of disc is vertically above S. The liquid in vassel is gradually drained out through a tap. What is maximum height of the liquid for which the source cannot be seen at all from above?

[Ans: 1.33 cm]

21. A prism of a material of $\mu = 1.5$ has a refracting angle A. A ray of light incident on one face emerges out from the other face. What is maximum possible value of A?

[Ans: 83.6^0]

22. A prism produces a minimum deviation of 51^0. The prism produces same deviation of $62^0 48'$ for angle of incidence of either $40^0 6'$ or $82^0 42'$. What is

    (i) refracting angle

    (ii) refractive index of prism

[Ans: (i) 60^0 (ii) 1.648]

23. An equilateral glass ($\mu = 1.5$) prism has a monchromatic ray incident at an angle of 48.6^0. What is the angle of

    (i) emergence

    (ii) deviation?

    (iii) Is the incident ray undergoing minimum deviation?

[Ans: (i) 37.2^0 (ii) 37.2^0 (iii) yes]

24. The refractive index of the material of prism of refracting angle 45^0 is 1.6 for some monochromatic ray. What would be the minimum angle of incidence of this ray on the prism so that no total internal reflection takes place as rays emerge out of the prism?

[Ans: $10^0 8'$]

25. Fig. 102 shows an equilateral prism LMN. The path of an incident ray PQ is as shown in Fig. 102. Angle $\alpha = 40^0$. What is μ of the material of prism?

[Ans: 1.66]

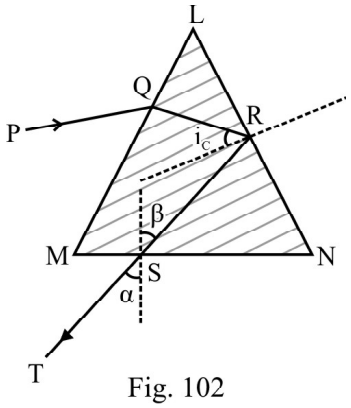


Fig. 102

26. For "crown glass" $\mu_V = 1.538$; $\mu_r = 1.52$. For quartz glass. $\mu_V = 1.557$; $\mu_r = 1.542$.

A quartz prism of angle 4^0 is in contact with a crown glass prism of angle α; such that there is no net deviation. What is α? What is angular dispersion due to the arrangment?

[Ans: 4.16^0, 0.015^0]

27. A very thin glass hollow sphere of radius 20 cm is filled with water ($\mu_\omega = 1.33$). A mark on surface is viewed from the diamaterically opposite end. What is the position and nature of image viewed?

[Ans: 60 cm away from the end of sphere from the side of new; virtual]

28. Two concave glass refracting surfaces each of radius 35 cm; refractive index $\frac{3}{2}$ are facing one another a distance 2R apart. A point Object O is placed on the common axis at a distance of 17.5 cm from on surface. What is distance between the images of O formed by each refracting surface?

[Ans: 4 cm]

29. A concave lens made of glass ($\mu_g = 1.5$) has a power of 5D. The lens is immersed completely in

(i) Water ($\mu_\omega = 1.33$) (ii) carbon–bisuphide $\mu = 1.65$. In each case what is focal length and nature of lens?

[Ans: (i) concave; f = –0.78 m, (ii) convex; f = +1.6 m]

30. A convex lens of glass ($\mu_g = 1.5$) has a focal length of 20 cm. The lens is immersed completely in a liquid. The change in power of lens is –3.722 D. What is refractive index of liquid?

[Ans: 1.33]

31. A lens forms an image of a square of side 1 cm placed at a distance of 80 cm on a screen. The area enclosed by the image formed is 4 cm^2.

(i) What is distance between object and screen?

(ii) What is focal length of lens?

[Ans: (i) 240 cm (ii) 53.3 cm]

32. An object and its real image are at a distance of 25 cm and 40 cm respectively from the two focii of lens. What is

(i) focal length of lens

(ii) magnification?

[Ans: (i) 31.6 cm (ii) –1.27]

33. A lamp and screen are 1 m apart. A convex lens placed in between projects the image of lamp on screen in two positions 40 cm apart.

(i) What is focal length of lens.

(ii) What is magnification in each position of lens?

$\left[\text{Ans: (i) 21 cm (ii) } -\left(\frac{7}{3}\right) \text{ and } -\left(\frac{3}{7}\right)\right]$

34. An equi–convex lens ($\mu = 1.5$) has a power of 10 D. The lens is placed with its lower surface in contact with water ($\mu_\omega = 1.33$). A parallel of beam light parallel to principle.axis of lens is incident in air. What is the distance of image formed in water from the lens? What is change in position of image if the entire system is immersed completely in water?

[Ans: 20 cm, increases by 20 cm]

35. Two thin co–axial lenses A and B are in contact. An object is placed at a distance of 10 cm from the combination. The image formed on a screen has a size three times that of object. B is concave lens of power 3.33 D; what is focal length and power of A?

[Ans: 6 cm; + 16.6 D]

36. A plano–convex lens of focal length 15 cm is fixed on one face of a hollow box with convex surface facing the object. An object 2 cm high is placed 25 cm in front of lens. What is size of image formed when box is (a) empty (b) filled with a liquid of refractive index 1.33? Refractive index of material of lens = 1.5.

[Ans: (a) 3 m (b) 4 cm]

37. A plano–convex lens of radius of curvature 30 cm acts like a concave mirror of radius of curvature 60 cm when its plane surface is silvered. What is μ of the material of lens?

[Ans: 1.5]

38. A convex lens placed in between an object and screen projects image having a size twice that of the object. On moving the lens by 15 cm away again a sharp image is projected on screen. What is focal leng length of lens?

[Ans: 10 cm]

39. A convex lens of power 5 D has an object placed 40 cm in front. The lens projects image on a screen. A concave lens is placed in between convex lens and screen at a distance of 15 cm from convex lens. The screen has to be moved away by 25 cm so that again a sharp image is projected on it. What is focal length of concave lens?

[Ans: 50 cm]

40. A convex lens of focal length 15 cm is placed in front of a convex mirror, co–axially at a distance of 5 cm. An object at a distance of 20 cm fron the lens, has no parallex with its image formed by the system. What is radius of curvature of convex mirror?

[Ans: 55 cm]

41. In Y.D.S experiment the two slits are a distance 0.3 mm apart. Interference fringes are observed on a screen 75 cm away from interferring sources. 20 fringes are observed in a distance of 3 cm from centeral fringe. What is λ? What is percentage change in fringe width if distance of the screen is doubled and seperation between slits is halved?

[Ans: 600 nm, 300% (increases)]

42. Monochromatic light of wave length λ is used in Y.D.S experiment. d = 0.15 mm; D = 120 cm.

    (a) Distance between 8th bright fringe and centeral fringe is 3.2 cm.

    (b) Distance of 7^{th} dark fringe from centeral fringe is 3.9 cm.

    In each case what is λ?

[Ans: (a) 450 nm (b) 750 nm]

43. A beam of light has two wave lengths $\lambda_1 = 650$ nm and $\lambda_2 = 520$ nm is used to observe interference inYoung's double slit experiment d = 2 mm and D = 120 cm. What is minimum distance on screen from centeral fringe where bright fringes of the two wave lengths coincide?

[Ans: 1.56 mm]

44. In an interference experiment the two interferring sources have intensities $I_1 = 5\, I_0$, $I_2 = I_0$ respectively. At a point P the two disturbances arrive having a path difference of $\frac{\lambda}{8}$; whereas at point Q, the path

difference is $\frac{\lambda}{4}$. What is ratio of resultant intensity at P and Q?

[Ans: 1.527]

45. In a Y.D.S. experiment interference is observed using monochromatic light of λ = 600 nm. d = 0.2 mm; D = 1.0 m. Express intensity at a point distance 2.4 mm from centeral point as a percentage of intensity of centeral fringe.

[Ans: 34.5%]

46. In an interference experiment a sheet of thickness of a material in which speed of light is 2×10^8 ms^{-1}; is introduced in the path of one of the two inteferring sources. The centeral fringe shifts to a point where originally 5th bright fringe was located. λ = 589 nm. What is t?

[Ans: 5.66 μm]

47. In a Y.D.S experiment using monochromatic light, the fringe pattern shifts by a certain distance when a mica sheet of thickness 1.964 microns; μ = 1.6 is placed in the path of one of the two waves. The mica sheet is removed and the distance between the slits and screen is doubled. It is observed that the distance between successive maxima and minima now observed is same as the fringe shift observed upon introduction of mica sheet. Calculate λ of light used. (I.I.T)

[Ans: 589.2 nm]

48. Two coherent sources of intensity ratio 25:4 are used in an interference experiment. What is $I_{max} : I_{min}$ in the interference pattern observed?

[Ans: 49:9]

49. Two glass plates A and B are kept near one another as shown in Fig. 103. A monochromatic beam of intensity I is incident on A as shown. Each plate reflects 25% of light incident on it. What is $I_{max} : I_{min}$ due to interference between incident light reflected by A and B?

[Ans: 49.1]

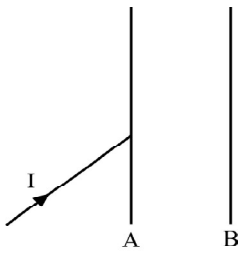


Fig. 103

50. Light of λ = 500 nm is incident normally on a single slit. The distance between the first minimum on either side of centeral maximum is 1 cm. The screen is at a distance of 2 m from slit.

    (i) What is width of slit?

    (ii) What is half–angular width of centeral maximum?

    (iii) What is ratio of radii of centeral maximum and second subsidary maximum?

[Ans: (i) 2.2 mm (ii) ~ 2.27×10^{-4} radian (iii) 5:1]

51. A plane wave–front ($\lambda = 6\times10^{-7}$ m) falls on a slit 0.4 mm wide. A convex lens of focal length 80 cm placed behind the slit focuses diffracted beam on a screen. What is linear distance of

    (i) first minima

    (ii) second maxima from the centeral maxima?

    [Ans: (i) 1.2 mm (ii) 1.8 mm]

52. A telescope has an aperture of 15 cm and focal length of O–lens is 2.5 m. Light of $\lambda = 600$ nm from two distant stars enters the telescope. What is seperation (angular) between the two stars if they just appear resolved?

    [Ans: 4×10^{-6} radian]

53. Two polarising sheets are placed with their axis parallel. The intensity of transmitted light is I. Through what angle one of the two sheets be rotatted so that intensity of light transmitted is 50% of I?

    [Ans: 45^0 or 135^0]

54. An unpolarised beam of intensity I_0 is incident on a polariser. The axis of analyser is at an angle of 60^0 with axis of polariser. What is intensity of light transmitted by the analyser?

    [Ans: $\frac{I_0}{8}$]

55. Unpolarised beam of light is incident on a reflector such that the reflected beam is plane polarised. The speed of light in reflected is 1.875×10^8 ms^{-1}. What is angle of incidence?

    [Ans: $\sim 58^0$]

QUESTION BANK

Key Learning Points

1. Ray optics is study of light using rectilinear propagation of light in an isotropic homogeneous medium.

2. A smooth surface throws back, in same medium light incident on it. This is reflection of light. The law of reflection states:

   Angle of incidence (i) = angle of reflection (r)

3. A spherical mirror is a part of a hollow sphere. The mirror is convex or concave denpending on the polished side of mirror. The principal axis of mirror is a line joining the pole and the center of curvature.

4. For simplicity of analysis we consider paraxial incident rays. These are rays making small angle (θ) with principal axis, such that $\sin\theta \simeq \tan\theta \simeq \theta$.

5. The distance of an object; image center of curvature etc. is a pure number. However position of object; image; center of curvature etc. is an appropriate distance with a + or – sign prefixed according to co–ordinate convention of signs.

6. In co–ordinate convention, (i) the pole of reflecting surface or optical center of refracting surface is origin of co–ordinates (ii) incident rays proceed from left to right (iii) principal axis, in direction of incident ray is direction of positive *x*–axis. (iv) y axis above principal axis is the positive direction (v) clockwise angles are positive.

   u, v, R etc. are co–ordinates of object, image, center of curvature etc.

7. The reflection formula for a spherical mirror is

   $$\frac{1}{v}+\frac{1}{u}=\frac{R}{2}=\frac{1}{f} \quad \text{or} \quad f=\frac{R}{2}$$

   u = object position, v = image position; R = position of center of curvature etc. When using reflection formula proper sign of known parameters must be prefixed before the appropriate distance.

8. Incident parallel beam, parallel to principle.axis after reflection meets or appears to meet principle. axis at a point F. F is the focus of spherical mirror. Position of F gives focal length (f). Also $f=\frac{R}{2}$.

9. m = The transverse magnification $= \frac{\text{Size of image}}{\text{Size of object}} = \frac{y_2}{y_1} = -\left(\frac{v}{u}\right)$

10. A concave mirror is a converging mirror and convex mirror is a diverging mirror. The power P of the mirror is

    $$P\,(\text{in dropter}) = \frac{1}{f\,(\text{in meter})}$$

11. The signs and their interpretation of u, v, R, f etc. are summarised below.

1.	Mirror	1) Concave 2) Convex	R and f are negative R and f are positive
2.	Object	1) Real 2) Virtual	u is negative u is positive
3.	Image	1) Real 2) Virtual	v is negative v is positive
4.	Magnification	1) Positive 2) Negative 3) $\|m\|>1$ 4) $\|m\|<1$	Image is erect Image is inverted Image is enlarged Image is diminished

12. For a real object a concave mirror;
    (i) can form virtual or real image depending on position of object.
    (ii) real image formed is always inverted.
    (iii) real image formed can be diminished or enlarged depending on position of object.
    (iv) virtual image formed is erect and enlarged.
    (v) when object moves towards mirror the image moves away from the mirror.
13. For a convex mirror, the image of a real object is always virtual, erect and diminished irrespective of position of object.
14. Whenever light goes from one medium to another; in general, it either bends towards or away from normal at point of incidence. This is refraction of light.
15. The absolute refractive index n of a medium is

$$n = \frac{\text{Speed of light in vaccum}}{\text{Speed of light in medium}} = \frac{c}{v} = \sqrt{\varepsilon_r \mu_r}$$

ε_r and μ_r denote relative permittivity and relative permeability of medium. n can never be less than one.

16. The relative refractive index ${}_2^1n$ of medium 1 with respect to medium 2 is

$${}_2^1n = \frac{n_1}{n_2} = \frac{v_2}{v_1}$$

The relative refractive index may be greater or less than one. Also

$$^{1}_{2}n \times ^{2}_{3}n \times ^{3}_{4}n = ^{1}_{4}n = \frac{1}{^{4}_{1}n}$$

17. Snell's law is the law of refraction of light. Let i be angle of incidence of light in a medium of refractive index n_1. Let r be angle of refraction in second medium of refractive index n_2. According to Snell's law

$$n_1 \sin i = n_2 \sin r$$

or $$\frac{\sin i}{\sin r} = \text{constant} = \frac{n_2}{n_1} = {}^{2}_{1}n\,\frac{v_1}{v_2}$$

18. When a monochromatic incident ray undergoes refraction through a parallel slab of thickness, t; refractive index n;

(i) The emergent ray is parallel to incident ray i.e. net deviation is zero.

(ii) The lateral displacement, ℓ; depends on (1) angle of incidence (i), (2) thickness (t) and (3) refractive index (n) of the slab.

19. An object in a medium of refractive index n is viewed, nearly normally, from above, say in air; the apparent depth (t_1) is less than real depth (t). Also

$$n = \frac{t}{t_1} = \frac{\text{Real depth}}{\text{Apparent depth}}$$

$$\text{The apparent shift} = t\left(1 - \frac{1}{n}\right)$$

20. For a number of media of different thickness and refractive index

$$\text{Total apparent depth} = \sum_j \frac{t_j}{n_j}$$

t_j is real thickness and n_j refractive index of j^{th} slab.

21. Whenever light goes from a denser to a rarer medium; it bends away from normal i.e. r > i. There is particular angle of incidence i_c for which angle of refraction is $\frac{\pi}{2}$. For $i > i_c$ there is no refracted ray in second medium. The entire incident energy is reflected back obeying law of reflection. This is known as total internal reflection. i_c is known as critical angle of incidence. For light going from a medium of refractive index μ_1 to a medium of refractive index $\mu_2\,(\mu_2 < \mu_1)$; the critical angle of

incidence, i_c, is

$$i_c = \sin^{-1}\left(\frac{\mu_2}{\mu_1}\right)$$

22. A ray of light incident in water ($\mu = 1.33$) at air–water interface undergoes T.I.R if angle of incidence is more than $\simeq 49^0$.

23. For an incident ray going from glass ($\mu = 1.5$) to air, the critical angle of incidence is $i_c \simeq 42^0$. A right angled issoceless prism of $\mu > \sqrt{2}$ is known as totally reflecting prism.

24. The phenomenon of mirage is due to T.I.R. Diamonds shine because a ray inside diamond undergoes repeated T.I.R before at can emerge out.

25. A monochromatic incident ray in passing through a prism; undergoes a net deviation (δ). The angle of deviation, $\delta = i_1 + i_2 - A$; where i_1 and i_2 denote the angle of incidence and angle of emergence. A is refracting angle of prism.

26. For a given prism, (i.e. μ and A are constant) the angle of deviation (δ) varies with angle of incidence (i). For a particular angle of incidence (i_m) the deviation is minimum. For minimum deviated ray; its path inside prism is parallel to the base of prism. Also

$$i_m = \frac{A + \delta_m}{2}; r_m = \frac{A}{2}$$

Therefore, refractive index of prism is

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

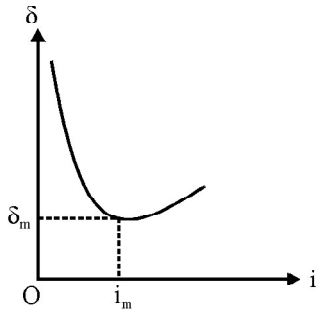


Fig. 1

27. A white incident ray in passing through a prism breaks up into its consituent colours. This is known as despersion of light.

28. Dispersion takes place in a prism due to the variation of refractive index (μ) of the prism with wave length (λ) of incident light. According to Cauchy

$$\mu = a + \frac{b}{\lambda^2}$$

a and b are constants, known as Cauchy's constants.

29. When white light passes through a prism in undergoes:

(i) mean deviation δ

(ii) angular dispersion $(\delta_V - \delta_R)$

δ_V and δ_R is deviation of violet and red colours respectively.

30. For a small angled prism of angle α; let μ_V, μ and μ_R be refractive index for violet, mean (yellow) and red colour respectively. Then

Mean deviation = $\delta = (\mu - 1)\alpha$

Angular dispersion = $\Delta = \delta_V - \delta_R = (\mu_V - \mu_R)\alpha$

31. For a prism, angular dispersion per unit mean deviation is known as dispersive power (ω) of material of prism. For small angled prisms ω is a constant for a particular material.

$$\text{Dispersive power} = \omega = \frac{\delta_V - \delta_R}{\delta} = \frac{\mu_V - \mu_R}{(\mu - 1)} = \frac{d\mu}{(\mu - 1)}$$

32. Consider a spherical surface seperating two media of refractive index μ_1 and μ_2. For incident rays in medium μ_1, the relation between the object position u (in medium μ_1), the image position v (in medium μ_2) and radius of curvature (R) of spherical surface is

$$\frac{\mu_2}{v} - \left(\frac{\mu_1}{u}\right) = \frac{(\mu_2 - \mu_1)}{R}$$

This is known as refraction formula for a spherical surface.

33. Incident parallel beam, parallel to principal axis in medium μ_1; after refraction meets or appears to meet principal axis at a point F_2. F_2 is the second principle focus. Position of F_2 gives second principal focal length f_2. Also $f_2 \equiv f$= the focal length of spherical surface.

$$\frac{\mu_2}{f_2} \equiv \frac{\mu_2}{f} = \frac{\mu_2 - \mu_1}{R}$$

34. A point F_1 on principal axis where an object is placed or appears to be placed so that the refracted rays are parallel to principal axis. F_1 is known as a first principal focus. Position of F_1 gives the first principal focal length f_1 of spherical surface. Also

$$-\left(\frac{\mu_1}{f_1}\right) = \frac{\mu_2 - \mu_1}{R}$$

35. In terms of focal length, f; the refraction formula, at a spherical surface is

$$\frac{\mu_2}{v}-\left(\frac{\mu_1}{u}\right)=\frac{\mu_2}{f}$$

The magnification, m; is

$$m=\frac{\text{size of image}}{\text{size of object}}=\frac{v/\mu_2}{u/\mu_1}$$

36. A thin lens is a refracting medium, in general, bound by two spherical surfaces. The lens is either concave or convex. Each type has three varities.

37. Let a thin lens have refractive index μ_2 and surfaces of radii of curvature R_1 and R_2. μ_1 is refractive index of medium having object placed in it (i.e. incident rays are in medium 1). μ_3 is the refractive index of the medium on the other side of lens. μ_3 is the medium having refracted rays. For a thin lens;

$$\frac{\mu_3}{v}-\left(\frac{\mu_1}{u}\right)=\frac{\mu_2-\mu_1}{R_1}+\frac{\mu_3-\mu_2}{R_2}$$

u and v denote the object position and the image position respectively. This is refraction formula for a thin lens. Commonly $\mu_1=\mu_3$. The above equation reduces to

$$\frac{1}{v}-\left(\frac{1}{u}\right)=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

38. For a lens; the second principal focus (F_2) is a piont on principal axis where incident rays parallel to principal axis, meet or appear to meet after refraction. Position of F_2 gives second principal focal length f_2.

The first principal focus (F_1) is a point on principal axis where if an object is placed or appears to be placed; the refracted rays are parallel to principal axis. Position of F_1 gives first principal focal length f_1 of lens. In general

$$\frac{\mu_3}{f_2}=-\left(\frac{\mu_1}{f_1}\right)$$

i.e. $|f_1|\neq f_2$. The focal length f of a lens is f_2.

39. The focal length, f; of a lens if medium on the two sides of the lens is same and has a refractive index μ_1; is

$$\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

This is known as Lens Maker's Formula.

40. *A convex lens:* If $\mu_2 > \mu_1$; behaves as a converging lens. However if $\mu_2 < \mu_1$ convex lens behaves as a diverging lens.

41. *A concave lens:* If $\mu_2 > \mu_1$ behaves as a diverging lens. However if $\mu_2 < \mu_1$, concave lens behaves as a converging lens.

42. The power, P, of a lens is

$$P\,(\text{in diopter}) = \frac{1}{f\,(\text{in meter})}$$

43. For $\mu_1 = \mu_2$; the refraction formula, for a thin lens reduces to

$$\frac{1}{v} - \left(\frac{1}{u}\right) = \frac{1}{f}$$

The transverse magnification, m; is

$$m = \frac{\text{size of image}}{\text{size of object}} = \frac{v}{u}$$

44. Let x_1 denote the distance of object from F_1 and x_2 the distance of image from F_2 respectively. Then

$$x_1 x_2 = f^2$$

This is Newton's formula

45. A convex lens can form a real or virtual image of a real object depending on position of object. The real image formed is always inverted. The virtual image of real object is erect and enlarged.

46. A concave lens always forms a virtual, erect and diminished image of a real object.

47. Two co–axial thin lens in contact; of focal length f_1 and f_2 respectively; behaves like a thin lens of equivalent focal lenth f; such that

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

In terms of powers

$$P = P_1 + P_2$$

48. Microscope is an optical instrument used to form an enlarged image of a nearby small sized object.

$$\text{Magnifying power} = \text{M.P.} = \frac{\text{Angle subtended by image}}{\text{Angle subtended by object when placed at distance of distinct vision}}$$

49. A compound microscope consists of two co–axial, short focal length convex lens a finite distance apart. The object to be magnified is placed at a distance slightly more than the focal length f_0 of O–lens. f_e is focal length of E–lens. For microscope forming final image at distance of distinct vision (D);

$$\text{M.P} = |m_0| \times |m_e|$$

$$= \left(\frac{b_1}{f_0} - 1\right)\left(1 + \frac{D}{f_e}\right)$$

b_1 is distance of image formed by O-lens from the O-lens.

50. A telescope is an instrument used to form an enlarged image of a large sized, far–off object.

$$\text{Magnifying Power} = \text{M.P.} = \frac{\text{Angle subted by image}}{\text{Angle subtended by object}}$$

51. An astronomical telescope is made up of an O–lens of large aperture and large focal length f_0. The E–lens is a convex lens of small focal length f_e. In normal adjustment

$$\text{M.P} = \frac{f_0}{f_e}$$

Length of telescope = $f_0 + f_e$.

52. Wave–optics is that branch of physics involving wave nature of light. We use concept of wave–front in describing waves. Huyghen's principle of secondary wavelets is a geometrical way of describing propagation of waves in a medium.

53. Whenever there are two exactly identical source of waves producing waves simultaneously; there is a redistribution of energy in the medium. This is known as interference of waves.

At those points of the medium where the two disturbances arrive in phase, the resultant disturbance is more; we say *constructive interference* has occured. Let Δ and δ denote the path difference and phase difference of the two disturbances at the point considered. *For constructive interference.*

$\Delta = n\lambda \qquad n = 0, 1, 2, 3, ...$

or $\delta = 2n\pi$

At those points of the medium where the two disturbances arrive out of phase; the resultant is minimum. We say *destructive* interference has occured at these points. For *destructive interference*

$$\Delta = (2n-1)\frac{\lambda}{2} \qquad n = 1, 2, 3, ...$$

or $\delta = (2n-1)\pi$

54. To observe interference of light; the two interferring sources must be coherent. Any two sources having same phase difference at all times are known as coherent sources.

55. Two exactly identical, independent point sources of light are not coherent sources. No interference pattern is observed due to them. To observe interference coherent sources are derived from an incoherent source.

56.

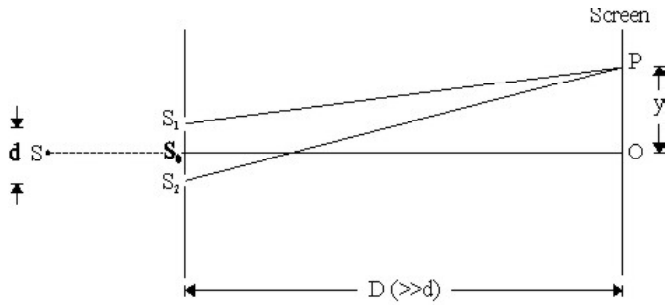


Fig. 2

Fig. 2 shows Y.D.S experiment. S is an incoherent monochromatic source. S_1 and S_2 are two identical slits placed symmetrically w.r.t S. $S_1S_2 = d$. The screen is at a distance $S_0O = D$ ($D >> d$). For a point P on screen.

$$\Delta = S_2P - S_1P = \text{The path of difference between the two waves at point P} = \frac{yd}{D}$$

For constructive interference

$$\frac{y_n d}{D} = n\lambda \qquad \text{or} \qquad y_n = \frac{n\lambda D}{d}, \qquad n = 0, 1, 2, 3, ...$$

For destructive interference

$$\frac{y'_n d}{D} = (2n-1)\frac{\lambda}{2} \qquad \text{or} \qquad y'_n = (2n-1)\frac{\lambda D}{2d}, \qquad n = 0, 1, 2, 3, ...$$

The fringe width β ; is distance between any two consecutive bright or dark fringes.

$$\beta = \frac{\lambda D}{d}$$

β is same for bright or dark fringes. β does not depend on value of n. All fringes are equispaced.

The resultant intensity, I; at point P is

$$I = I_{max} \cos^2\left(\frac{\delta}{2}\right)$$

$$I_{max} = 4\, I_0;\ \ \delta = \frac{2\pi}{\lambda}(\Delta)$$

I_0 is intensity of either S_1 or S_2. I_{max} is the intensity of any bright fringe. All bright fringes are equi–bright and all dark fringes are equidark. There is no violation of law of conservation in interference. Energy is only redistributed.

57. In a modified Y.D.S experiment $SS_1 \neq SS_2$. Let $\Delta_i = SS_2 - SS_1$ be the initial path difference between the two interferring beams. For constructive interference

$$\Delta_T = \Delta_i + \Delta = \Delta_i + \frac{y_n d}{D} = n\lambda \quad n = 0,1,2,3,.....$$

For destructive interference

$$\Delta_T = \Delta_i + \Delta = \Delta_i + \frac{y'_n d}{D} = (2n-1)\frac{\lambda}{2} \quad n = 1,2,3,.....$$

The centeral fringe is not obtained at O. The centeral fringe moves above or below O depending on value of Δ_i. However fringe width, β, remains same.

58. When a thin film of thickness t; refractive index μ is introduced in the path of one of the two interferring beams; the extra path difference introduced = $(\mu - 1)t$. Let Δn be the observed fringe shift.Then

$$\Delta n = (\mu - 1)\frac{t}{\lambda}$$

59. Diffraction is phemomenon of bending of waves around an obstacle in their path and the penetration of waves into the region of geometrical shadow. Diffraction is easily observed with sound or water waves. Since λ of light is very small; to observe diffraction of light the size of obstacle should be also very small i.e. *comparable to wave length of waves. All type of waves undergo diffraction.*

60. In a single–slit diffraction experiment a parallel monochromatic beam is incident on a slit of width d (d $\simeq$ a fraction of a mm). The diffraction pattern is observed on a screen at a large distance D (of the order of a meter) from the slit. The half angular width of centeral maxium, θ, is

$$\theta \simeq \frac{\lambda}{d}$$

The radius r_1 of centeral maximum is

$$r_1 = \frac{\lambda D}{d}$$

61. In a diffraction pattern centeral maximum is most intense. Intensity of subsidary maxima's decreases rapidly. The fringes observed are not equi–spaced. The number of fringes observed is small (~ 8 to 10).

62. Polarisation is the process of selecting a preferred direction of vibration and blocking all other vibrations. Only transverse waves can be polarised. Longitudinal waves cannot be polarised.

63. Light coming of a source has **E** oscillating in all directions perpendicular to direction of wave propagation. Such waves are unpolarised. Light from sun, sodium lamp; fluroscent tube etc. is unpolarised.

64. A *calcite* or *tourmaline crystal* has the property of allowing oscillation of **E** parallel to its axis to pass through and stop vibration of **E** in all other directions. It is used to polariser an unpolarised beam. It is known as a polarsier. The vibration of **E** of light emerging out of polariser are only in a direction parallel to axis of polariser. Such a beam is known as plane polarised.

65. An analyser is a quartz crystal used to find out whether light incident on it is unpolarised or plane polarised.

66. For unpolarised incident light, there is no variation in intensity of light transmitted by analyser when it (analyser) is rotated through are full circle about direction of propagation of light as axis.

67. For an incident plane polarised light; when analyser is rotated through one full circle about direction of propagation of light as axis; the intensity of transmitted light varies. In one full rotation there are two position in which intensity of transmitted light is zero.

68. An unpolarised ray of light can be polarised due to reflection. If angle of incidence equals a particular value, i_p, the reflected beam is plane polarised. i_p is known as polarising angle of incident. According to Brewster's law

$$\tan i_p = \mu$$

μ is refractive index of reflector.

69. Consider a plane polarised beam of intensity I_0 incident on an analyser. Let δ be the angle between the axis of analyser and polariser.

I = The intensity of light transmitted by the analyser

$$= I_0 \cos^2(\delta)$$

This is Malus law.

Unit–16 : Optics

Average *Reflection of Light*

1. A ray of light propagating along the direction $\frac{1}{2}\left(\hat{i}+\sqrt{3}\hat{j}\right)$ is incident on a plane mirror. After reflection it moves along the direction $\frac{1}{2}\left(\hat{i}-\sqrt{3}\hat{j}\right)$. The angle of incident is

(1) 30^0 (Correct)

(2) 45^0 (Incorrect)

(3) 60^0 (Incorrect)

(4) 75^0 (Incorrect)

Correct answer: (1)

Solution:

Let i and r be angle of incidence and reflection in x–y plane. From law of reflection, i = r. For incident ray

$$\tan i = \frac{\sqrt{3}}{1} \Rightarrow \therefore\ i = 30^0$$

For reflected ray

$$\tan r = \frac{\sqrt{3}}{1} \Rightarrow\ r = 30^0$$

Unit–16 : Optics

Difficult *Reflection of Light*

2. In an experiment to determine the focal length (f) of a concave mirror, "by the u–v method", a student places an object pin on the principal axis at a distance *x* from the pole P of the mirror. The student looks at the pin and its inverted image from a distance keeping his / her eye in line with PA. When student shifts his/her eye towards left the image appears to move to right of the object

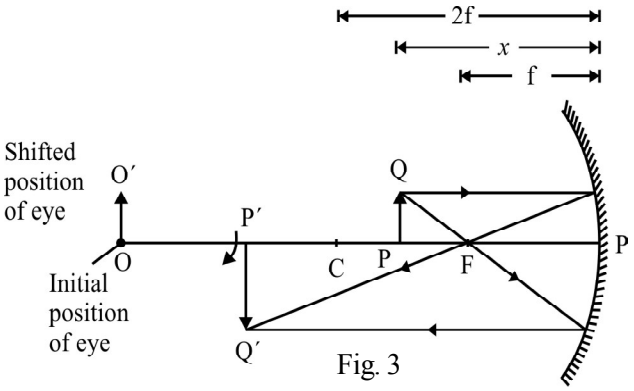


Fig. 3

pins. This implies that

(1) $x < f$ **(Incorrect)**

(2) $f < x < 2f$ **(Correct)**

(3) $x = 2f$ **(Incorrect)**

(4) $x > 2f$ **(Incorrect)**

Correct answer: (2)

Solution:

Fig. 3 show image P'Q' of object PQ lying in between F and C. O is initial position of eye of student. O' is shifted position of eye. The arrow P'Q', shows the apparent direction of motion of image. These movements are in accordance with the statement of question. Hence obejct lies between F and C, i.e.

$f < x < 2f$

Unit–16 : Optics

Average *Spherical Mirror*

3. For the mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

Which of the following statements is correct?

(1) Formula is valid for mirros of large aperature for all position of object (Incorrect)

(2) Formula is valid for mirrors of large aperature and small focal length (Incorrect)

(3) Formula is valid for mirror of small aperature and objects placed at large distances in comparison to focal length (Correct)

(4) Formula is valid for mirror of small aperature and object placed close to mirror (Incorrect)

Correct answer: (2)

Solution:

Statement (3) is restatment of "paraxial ray assumption" used is deriving the mirror formula.

Unit–16 : Optics

Difficult *Reflection*

4. A real object is at a distance a from the focus of a convex mirror of power P. The distance of the image from the mirror is

(1) $\frac{1}{aP^2}$ **(Incorrect)**

(2) $\frac{1}{a}\left[\frac{1+P}{P^2}\right]$ **(Incorrect)**

(3) $\frac{a}{P^2}$ **(Incorrect)**

(4) $\frac{1}{a}\left[\frac{P-1}{P^2}\right]$ **(Correct)**

Correct answer: (4)

Solution:

Fig. 4 shows object PQ, and its virtual image, $P'Q'$, formed by the convex mirror. The image lies between O and F. The magnitude of focal length of mirror, e; is

$$e = OF = \frac{1}{P}$$

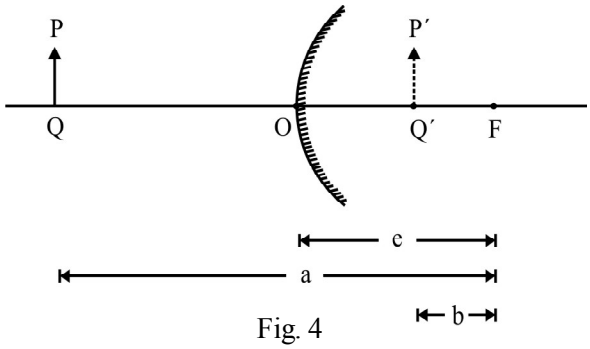


Fig. 4

Let a and b the distance of object (PQ) and image ($P'Q'$) from focus F of the mirror. From Newton's formula

$$a \times b = \frac{1}{P^2} \qquad \therefore b = \frac{1}{aP^2}$$

The distance of image ($P'Q'$) from mirror

$$OQ' = e - b = \frac{1}{P} - \left(\frac{1}{aP^2}\right) = \left[\frac{P-1}{P^2}\right]\frac{1}{a}$$

Unit–16 : Optics

Average *Reflection and Refraction*

5. A concave mirror is placed on a horizontal table with its axis directed vertically upwards. Let O be the pole and C the center of curvature of mirror. A point object is placed at C. The mirror forms its real image, also located at C. The mirror is now filled completely with water. The image, now, would be

(1) real and remains at C (Incorrect)

(2) real and located between C and ∞ (Incorrect)

(3) virtual and located at a point between C and O (Incorrect)

(4) real and located between C and O (Correct)

(I.I.T)

Correct answer: (4)

Solution:

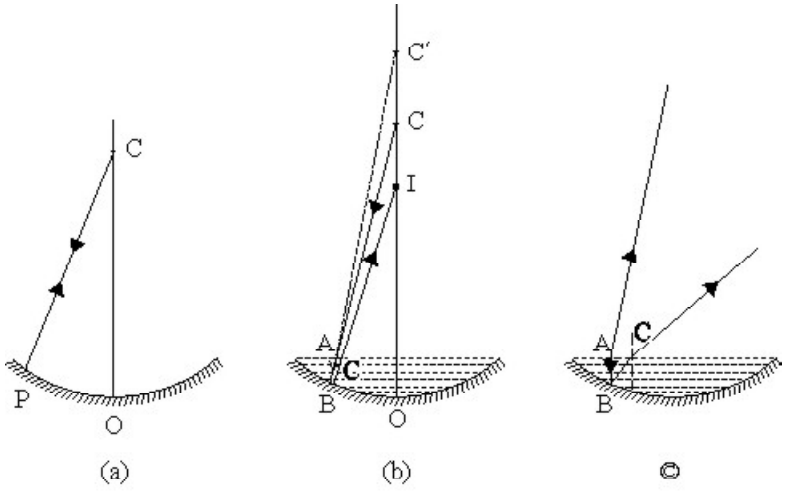


Fig. 5

Fig. 5 (a) shows image at C due to reflection from concave mirror. The ray CP, is, therefore, incident normally on mirror.

When mirror is filled with water; the incident ray CA first undergoes refraction at water. The refracted ray AB is incident on mirror. This ray is not incident normally. It undergoes reflection obeying law of reflection. The reflected ray again undergoes refraction at water surface forming its real image I that lies between C and O.

Unit–16 : Optics

Difficult *Refraction of Light*

6. The *x*–y plane is the boundary between two transparent media. Medium 1, with z > 0, has a

refractive index $\sqrt{2}$ and medium 2, with z < 0, has a refractive index $\sqrt{3}$. A ray of light, incident in medium 1, given by

$$\mathbf{A} = 6\sqrt{3}\,\hat{\mathbf{i}} + 8\sqrt{3}\,\hat{\mathbf{j}} - 10\,\hat{\mathbf{k}}$$

is incident on the plane of seperation. Its angle of incidence (i) and the angle of refraction (r), would be

(1) $i = 60^0;\ r = 45^0$ **(Correct)**

(2) $i = 45^0;\ r = 60^0$ **(Incorrect)**

(3) $i = \sin^{-1}\left(\frac{2\sqrt{3}}{5}\right);\ r = \sin^{-1}\left(\frac{3\sqrt{2}}{5}\right)$ **(Incorrect)**

(4) $i = \sin^{-1}\left(\frac{3\sqrt{3}}{10}\right);\ r = \sin^{-1}\left(\frac{9}{10\sqrt{2}}\right)$ **(Incorrect)**

Correct answer: (1)

Solution:

The angle of incidence (i) is given by "the direction cosine" of **A** with the z–axis; the normal to the plane of incidence. Hence i is given by

$$\cos i = \frac{10}{\left[(10)^2 + \left(6\sqrt{3}\right)^2 + \left(8\sqrt{3}\right)^2\right]^{1/2}} = \frac{1}{2}$$

$\therefore\ i = 60^0$

Applying Snell's law,

$$\sqrt{2}\sin i = \sqrt{3}\sin r$$

$$\sqrt{2} \times \frac{\sqrt{3}}{2} = \sqrt{3}\sin r$$

$\therefore\ \sin r = \frac{1}{\sqrt{2}}$

or $r = 45^0$

Unit–16 : Optics

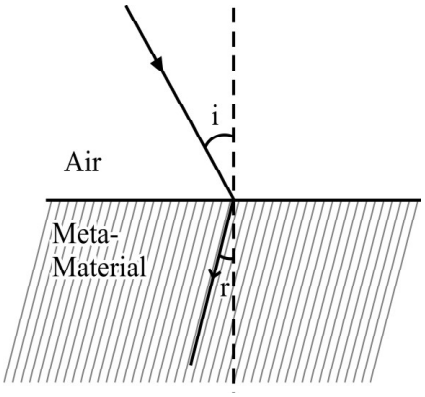
Difficult *Refraction*

7. Speed of light V, in a medium; in accordance with e.m. theory, is given by

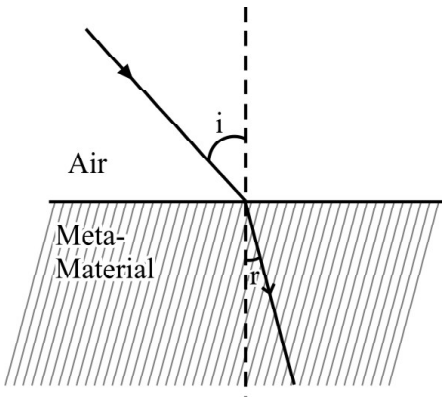
$$V = \frac{c}{(\varepsilon_r \mu_r)^{1/2}} \left(= \frac{c}{n} \right)$$

Here ε_r and μ_r represent the relative permittivity and relative permeability of the medium. For commonly occuring materials, ε_r and μ_r are positive numbers. Aritifically prepared materials, known as 'meta materials', have a negative, value of refractive index n. For a 'meta material', the correct version of the above statement and the fig. showing refraction, is the one given in option

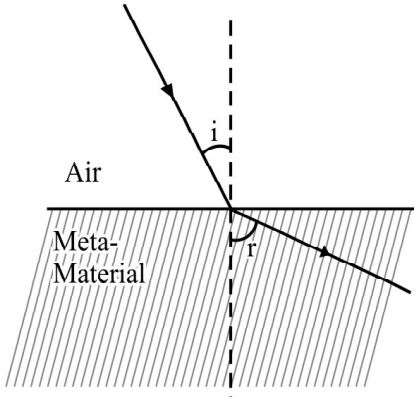
(1) V = speed of light in meta material = $\frac{c}{|n|}$ **(Correct)**



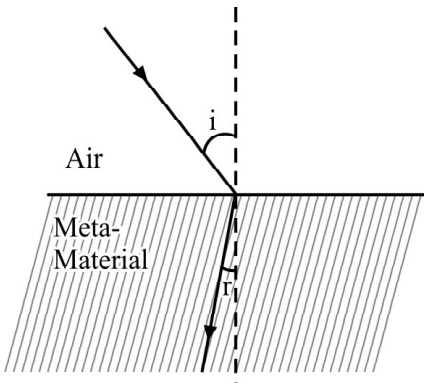
(2) V = speed of light in meta material = c |n| **(Incorrect)**



(3) V speed of light in meta material = c |n| **(Incorrect)**



(4) V = speed of light in meta material = c |n| **(Incorrect)**



(Based on I.I.T, 2012)

Correct answer: (1)

Solution:

For normal materials

$$n = \frac{c}{V}$$

Here V = speed of light in medium. For a 'meta–material' since in is a negative number, V would become a negative number. Therefore, we take

$$V = \frac{c}{|n|}$$

For a 'meta material', the refracted ray is on the same side of normal as the incident–ray is. Hence the figure, given in option (1) correctly shows the refraction of an incident ray, in such a material.

Unit–16 : Optics

Easy *Refraction of Light*

8. A divergent beam of light from a point source S, (having a divergence angle α), falls symmetrically on a glass slab as shown in Fig. 6. The angle of incidence of the two extreme rays are equal. It the thickness of glass slab is t and its refractive index is n, the divergence angle of emergent beam,would be

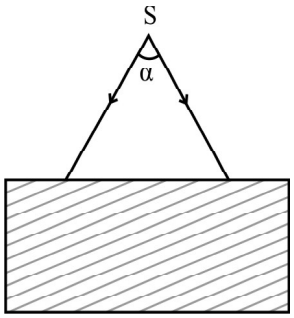


Fig. 6

(1) zero **(Incorrect)**

(2) α **(Correct)**

(3) $\sin^{-1}\left(\frac{1}{n}\right)$ **(Incorrect)**

(4) $2\sin^{-1}\left(\frac{1}{n}\right)$ **(Incorrect)**

Correct answer: (2)

Solution:

Fig. 7 show refraction, through the glass slab, for incident rays SA and SB. The emergent ray CE is parallel to SA and the emergent ray DF is parallel to SB. The emergent rays appear to diverge from point S'. However the angle of divergence $\angle ES'F$ also equals α.

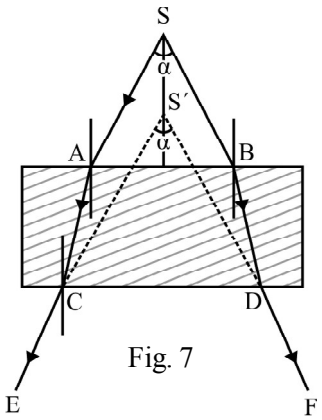


Fig. 7

Unit–16 : Optics

Difficult *Refraction of Light*

9. A parallel glass slab, of thickness 4 cm, has been made from a glass of refractive index $\sqrt{3}$. A ray of light is incident on one face at an angle of incidence of 60^0. The lateral shift of the ray, due to refraction, through the slab, is (nearly)

(1) 2.0 cm **(Incorrect)**

(2) 2.3 cm **(Correct)**

(3) 2.6 cm **(Incorrect)**

(4) 2.9 cm **(Incorrect)**

Correct answer: (2)

Solution:

Fig. 8 shows incident ray PQ undergoing refraction through glass slab. RS is the emergent ray. The lateral displacement = RT = x (say).

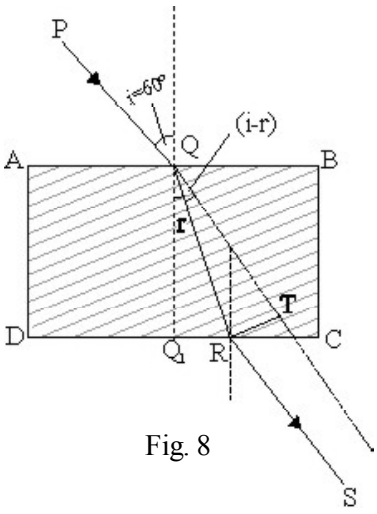


Fig. 8

From right angled ΔQRT

$$\frac{RT}{QR} = \sin(i-r)$$

From right angled ΔQQ_1R

$$\frac{QQ_1}{QR} = \cos r \quad \text{or} \quad QR = \frac{t}{\cos r}$$

$$\therefore \quad RT = x = \frac{t\sin(i-r)}{\cos r} \qquad .....(1)$$

Applying Snell's law at point Q,

$$(1)\sin 60° = \left(\sqrt{3}\right)\sin r$$

or $\sin r = \frac{1}{2} \quad \therefore \; r = 30^0$

Given t = 4 cm. From Eqn. (1) we have

$$x = 4\,\frac{\sin(60°-30°)}{\cos 30^0}\ \text{cm} = 4\tan 30° = \frac{4}{\sqrt{3}}\ \text{cm}$$

$$\simeq 2.3\ \text{cm}$$

Unit–16 : Optics

Average *Refraction of Light*

10. A ray of light is incident, from the 'glass side', at the glass–water interface, at an angle i. It emerges out parallel tothe surface of water as shown in the Fig. 9. The refractive index, μ_g, of glass, is

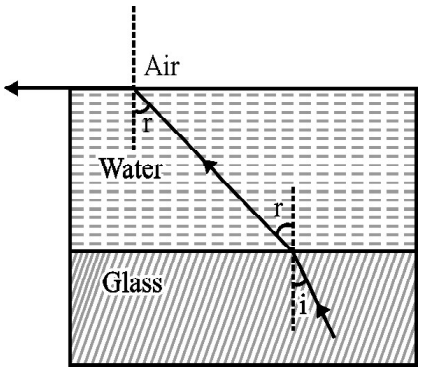


Fig. 9

(1) $\left(\frac{4}{3}\right)\sin i$ **(Incorrect)**

(2) $\frac{1}{\sin i}$ **(Correct)**

(3) $\frac{4}{3}$ **(Incorrect)**

(4) **1** **(Incorrect)**

Correct answer: (2)

Solution:

Let μ_g and μ_ω denote the refractive indices of glass and water, respectivly. Let r be angle of refraction in water, for ray incident in glass at angle i. From Snell's law,

$$\mu_g \sin i = \mu_\omega \sin r \qquad .....(1)$$

For refraction at water–air interface; using Snell's law, we get

$$\mu_\omega \sin r = 1 \times \sin\left(\frac{\pi}{2}\right)$$

$$= 1 \qquad .....(2)$$

From Eqn. (1) and (2) we have

$$\mu_g \sin i = 1$$

$$\therefore \quad \mu_g = \frac{1}{(\sin i)}$$

Unit–16 : Optics

Average *Refraction of Light*

11. A light beam is propagating, from region I to region IV [refer to Fig.10. The refractive indices of region I, II, III and IV are (n_0), $\left(\frac{n_0}{2}\right)$, $\left(\frac{n_0}{6}\right)$ and $\left(\frac{n_0}{8}\right)$ respectively. The angle of incidence, θ, in region I, for which the beam would just miss entering region IV, is

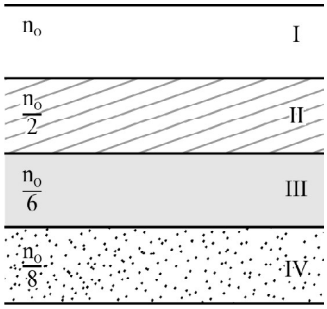


Fig. 10

(1) $\sin^{-1}\left(\frac{3}{4}\right)$ **(Incorrect)**

(2) $\sin^{-1}\left(\frac{1}{8}\right)$ **(Correct)**

(3) $\sin^{-1}\left(\frac{1}{4}\right)$ **(Incorrect)**

(4) $\sin^{-1}\left(\frac{1}{3}\right)$ **(Incorrect)**

(I.I.T 2008 Main)

Correct answer: (2)

Solution:

Fig. 11 shows the ray OA incident, from region I, at an angle of incident θ. Since the incident ray at C, i.e, region III just fails to enter region IV, the angle of incidence at C, must equal the critical angle of incidence between regions III and IV. Hence

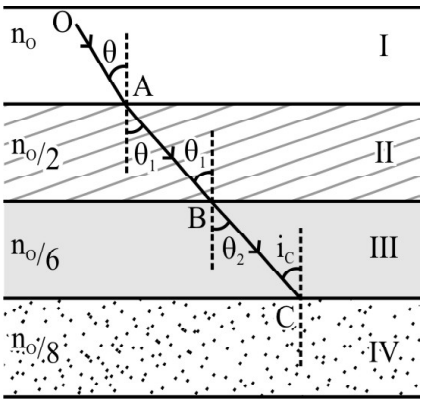


Fig. 11

$$\left(\frac{n_0}{6}\right)\sin i_c = \left(\frac{n_0}{8}\right)\sin\frac{\pi}{2}$$

$$\therefore \quad \sin i_c = \left(\frac{3}{4}\right)$$

Using Snell's law at point A and B, we have

$$n_0 \sin\theta = \left(\frac{n_0}{2}\right)\sin\theta_1$$

and $\left(\frac{n_0}{2}\right)\sin\theta_1 = \left(\frac{n_0}{6}\right)\sin i_c$

$\therefore \quad n_0(\sin\theta) = \frac{n_0}{6}(\sin i_c) \qquad \therefore \quad \sin\theta = \frac{1}{6}\times\frac{3}{4} = \frac{1}{8}$

$\theta = \sin^{-1}\left(\frac{1}{8}\right)$

Unit–16 : Optics

Average *Refraction*

12. A ray of light passes through four transparent media (with refractive indices μ_1, μ_2, μ_3 and μ_4) as shown in the Fig. 12. The surfaces of all media are parallel. If the emergent ray CD is parallel to the incident ray AB; we must have

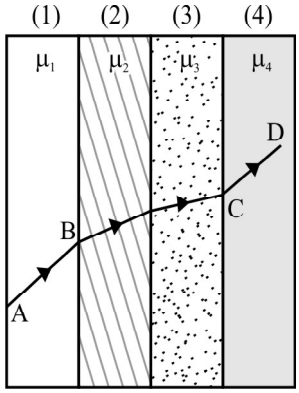


Fig. 12

(1) $\mu_1 = \mu_2$ **(Incorrect)**

(2) $\mu_2 = \mu_3$ **(Incorrect)**

(3) $\mu_3 = \mu_4$ **(Incorrect)**

(4) $\mu_1 = \mu_4$ **(Correct)**

Correct answer: (4)

Solution:

Let i_1 be angle of incidence of ray AB in medium of refractive index μ_1. Let, r_1 be angle of refraction in medium 2. From Snell's law

$\mu_1 \sin i_1 = \mu_2 \sin r_1$ .....(1)

The angle of incidence, in medium μ_2 is r_1. Let angle of refraction, in medium 3 be r_2. Then

$\mu_2 \sin i_1 = \mu_3 \sin r_2$ .....(2)

The angle of incidence in μ_3 is r_2 and angle of emergence of ray CD in medium μ_4 is i_1 (because CD is parallel to AB). Therefore

$\mu_3 \sin i_2 = \mu_4 \sin r_1$

From Eqns. (1), (2) and (3) we have

$\mu_1 \sin i_1 = \mu_4 \sin i_1$

$\therefore \quad \mu_1 = \mu_4$

Unit–16 : Optics

Difficult *Refraction of Light*

13. A parallel glass slab, of thickness t, has a uniformly varynig refractive index. The media, on the two sides of slab, have refractive indices n_1 and n_2 ($n_1 > n_2$), respectively. A ray, in medium of refractive index n_1, enters the slab, at an angle of incidence θ_1. After passing through the slab, it emerges out in the medium of refractive index n_2. The angle of emergence is θ_2. Which of the following statements is incorrect?

(1) $\theta_1 = \theta_2$ **; if** $n_1 = n_2$**; irrespective of the nature of variation of the refractive index of the slab.** **(Correct)**

(2) θ_2 **depends on the nature of variation of refractive index of slab** **(Incorrect)**

(3) $n_1 \sin\theta_1 = n_2 \sin\theta_2$ **(Correct)**

(4) θ_2 **is more than** θ_1 **(if** $n_1 > n_2$**)** **(Correct)**

Correct answer: (2)

Solution:

Let n_A and n_D be the refractive index of the glass layers, AB and CD, respectively. We can put

$n_D = n_A + \alpha t$

(α = constant rate of change of refractive index)

For refraction at layer AB

$$n_1 \sin\theta_1 = n_A \sin\theta_A$$

For adjacent layers, we would have

$$n_A \sin\theta_A = n'_A \sin\theta'_A$$

$$n_x \sin\theta_x = n_{x+dx} \sin\theta_{x+dx}$$

$$n'_D \sin\theta'_D = n_D \sin\theta_D$$

For refraction, at the layer CD, into medium of refractive index n_2, we have

$$n_D \sin\theta_D = n_2 \sin\theta_2$$

From these equations, we get

$$n_A \sin\theta_A = n_D \sin\theta_D$$

and, therefore,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

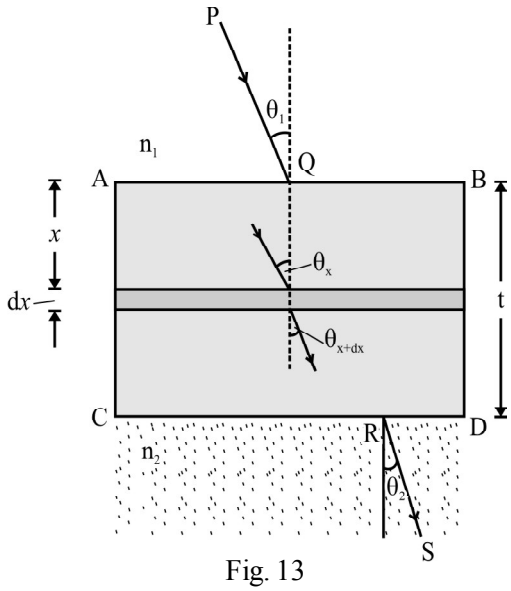


Fig. 13

Hence θ_1 and θ_2 depend on n_1, n_2, and not on the (uniform) nature of variation of the refractive index of the slab.

Thus, out of the given statements, only statement (2) is incorrect.

Unit–16 : Optics

Difficult *Refraction*

14. A transparent slab of thickness d has a refractive index n(z) that increases with z. Here z is the vertical (depth) distance inside slab; measured from top. The slab is placed between two media having uniform refractive index, n_1 and n_2 ($> n_1$) respectively. A ray of light is incident, at an angle θ_i, from medium 1, and emerges out, in medium 2; with refraction angle θ_f, and with a later displacement ℓ. Which of the following statements is incorrect?

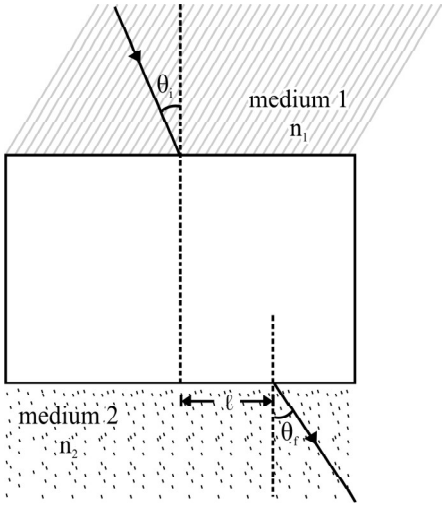


Fig. 14

(1) ℓ **independent of n (z)** **(Correct)**

(2) $n_1 \sin\theta_i = (n_2 - n_1)\sin\theta_f$ **(Incorrect)**

(3) $n_1 \sin\theta_i = n_2 \sin\theta_f$ **(Correct)**

(4) ℓ **depends on** n_2 **(Correct)**

(I.I.T 2016 Adv.)

Correct answer: (2)

Solution:

The lateral displacement depends on (i) n_1 and (ii) n_2 but is indepdent of n(z). The only incorrect statement is

$$n_1 \sin\theta_i = (n_2 - n_1)\sin\theta_f$$

Unit–16 : Optics

Difficult *Total Internal Reflection*

15. A glass slab, of thickness t, has a refractive index $\frac{3}{2}$. A point source is kept at the bottom of the slab. The fraction of light, emitted by S, that emerges out of the upper face of slab, is

(1) **0.5** **(Incorrect)**

(2) $\left[1 - \frac{\sqrt{3}}{2}\right]\left(\frac{1}{2}\right)$ **(Incorrect)**

(3) $\left[1 - \frac{\sqrt{5}}{3}\right]\left(\frac{1}{2}\right)$ **(Correct)**

(4) **0.25** **(Incorrect)**

Correct answer: (3)

Solution:

Fig. 15 shows that only those incident rays from S, that lie within the cone OAA′, emerge out of the upper surface of glass. Also

$$\sin i_c = \frac{1}{\mu} = \frac{2}{3}$$

The area S; from which light emerges out is

$$S = \int_0^{i_c} (2\pi R \sin\theta)(R d\theta) = (2\pi R^2)\left|-\cos\theta\right|_0^{i_c}$$

$$= 2\pi R^2 [1 - \cos i_c]$$

$$= 2\pi R^2 \left[1 - \left(1 - \frac{4}{9}\right)^{1/2}\right]$$

$$= 2\pi R^2 \left[1 - \frac{\sqrt{5}}{3}\right]$$

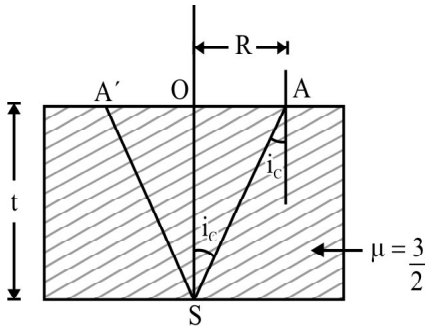


Fig. 15

The total light, emitted by S, gets spread over the surface of a sphere of radius R, i.e., $4\pi R^2$. Therefore Fraction of total light (emitted by the source S) that passes out of the slab, is

$$= \frac{2\pi R^2 \left[1 - \frac{\sqrt{5}}{3}\right]}{4\pi R^2}$$

$$= \left[1 - \frac{\sqrt{5}}{3}\right]\frac{1}{2}$$

Unit–16 : Optics

Average *Real and Apparent Depth / Height*

16. A ball is dropped, from a height of 20 m , above the surface of water in a lake. The refractive index of water is $\frac{4}{3}$. A fish inside the lake; in the line of fall of the ball, is looking at the ball. At the instant when the ball is 12.8 m above the surface, the fish would observe the speed of the ball as

(1) 9 ms^{-1} (Incorrect)

(2) 12 ms^{-1} (Incorrect)

(3) 16 ms^{-1} (Correct)

(4) 21.33 ms^{-1} (Incorrect)

(I.I.T 2009)

Correct answer: (3)

Solution:

In the Fig. 16, B is the actual instantaneous position of the ball when it is at a height y above the surface of water. B_1 is the apparent position of the ball, as seen by the fish. The instantaneous apparent height $OB_1 = y_1$. Let μ be the refractive index of water. We know that

$$y_1 = \mu y$$

$$\therefore \quad \frac{dy_1}{dt} = \mu\left(\frac{dy}{dt}\right)$$

$$\therefore \quad \frac{dy}{dt} = v = \text{Instantaneous speed of the falling ball} = \sqrt{2gh}$$

$$= \sqrt{2\times 9.8(20-12.8)} \simeq 12\,\text{ms}^{-1}$$

$$\text{The apparent instantaneous speed of ball} = \frac{dy_1}{dt}$$

$$= \left(\frac{4}{3}\times 12\right)\text{ms}^{-1} = 16\,\text{ms}^{-1}$$

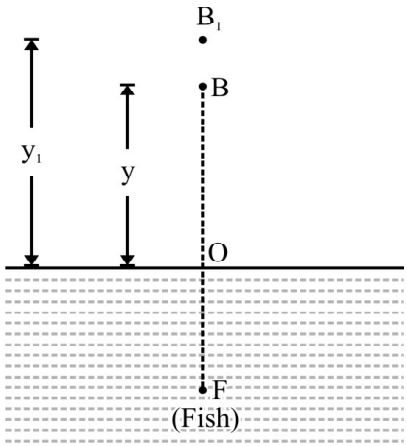


Fig. 16

Unit–16 : Optics

Difficult *Reflection and Refraction*

17. A container is filled with water (μ = 1.33) up to a height of 33.25 cm. A concave mirror is placed 15 cm above the surface of water, as shown in the Fig. 17. The image, of an object O, at the bottom gets formed at I, 25 cm below the water level. The focal length of the mirror is (approximately)

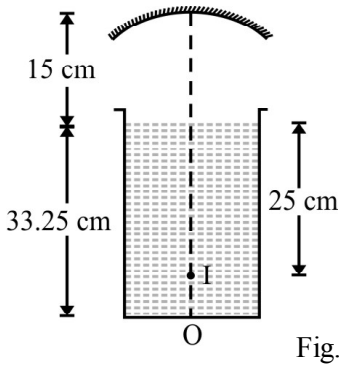


Fig. 17

(1) 10 cm **(Incorrect)**

(2) 15 cm **(Incorrect)**

(3) 20 cm **(Correct)**

(4) 25 cm **(Incorrect)**

(I.I.T 2005)

Correct answer: (3)

Solution:

Due to refraction at water, the object, at O, appears to be at a distance of $\frac{33.25}{1.33}\text{cm} \simeq 25\,\text{cm}$, below the top level of water. Hence the object distance for mirror = (25+15) cm = 40 cm

For concave mirror

u = –40 cm, v = –b (say), f = ?

However, the reflected rays again undergoes refraction at surface of water. The apparent depth of the image, below the water level in tank, is μb′ and is given to be 25 cm. Therefore,

$$b = \left(\frac{25}{1.33} + 15\right)\text{cm} = 33.8\,\text{cm}$$

Using mirror formula, we now get

$$\frac{1}{(-33.8)} + \frac{1}{(-40)} = \frac{1}{f}$$

$\therefore\ f = -18.3\,\text{cm} \simeq (-20)\,\text{cm}$

$\therefore$ Focal length of the mirror $\simeq$ 20 cm

Unit–16 : Optics

East *Refraction*

18. White light is incident on the interface of glass–air, as shown in the Fig. 18. If green light is just totally internally reflected, then the emerging rays, in air would have in them, the colours

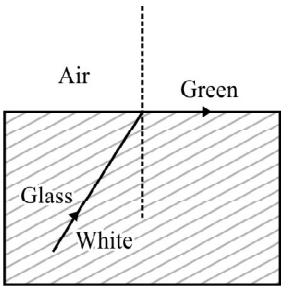


Fig. 18

(1) yellow, orange, red **(Correct)**

(2) violet, indigo, blue **(Incorrect)**

(3) all seven colours of white light **(Incorrect)**

(4) all the other six colours of white light (exept green) **(Incorrect)**

(I.I.T 2004)

Correct answer: (1)

Solution:

Let μ be the refractive index of glass for light of wave length λ. We know $\mu \alpha \frac{1}{\lambda^2}$. Also,

$$i_c = \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$= k \sin^{-1}\left(\frac{1}{\lambda^2}\right)$$

where k is a constant

The angle of incidence, for green colour, equals the critical angle. For light of λ more than that of the green colour, the critical angle would be more than that for green. Only these colours would not undergo T.I.R and would emerge out. Therefore, yellow, orange and red (having $\lambda > \lambda_g$) would emerge out.

Unit–16 : Optics

Average *Refraction of Light*

19. A ray of light, travelling in water, is incident on its surface open to air. The angle of incidence θ is less than the critical angle of incidence. Then there will be

(1) only a reflected ray and no refracted ray **(Incorrect)**

(2) only a refracted ray and no reflected ray (Incorrect)

(3) a reflected and a refracted ray and the angle, between them, is less than $(180^0 - 2\theta)$ (Correct)

(4) a reflected and a refracted ray and the angle, between them, is greater than $(180^0 - 2\theta)$ (Incorrect)

(I.I.T 2007)

Correct answer: (3)

Solution:

Since angle of incidence is less than critical angle, we will have both reflected and refracted rays. In Fig. 19, AO is the incident ray. OB and OC are the reflected and refracted rays. The angle of refraction $\angle NOC = \theta_1 > \theta$.

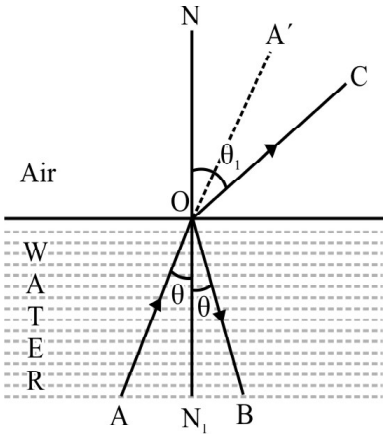


Fig. 19

[Because light is going from a denser to a rarer medium]. Also

$$\angle AON_1 = \angle BON_1 = \theta = \angle A'ON$$

The angle between reflected and refracted ray

$$= \angle BOC = \pi - (\theta + \theta_1)$$

As $\theta + \theta_1 > 2\theta$, $\angle BOC$ is less than $(\pi - 2\theta)$.

Unit–16 : Optics

Average *Refraction Through a Prism*

20. In an experiment to determine the refractive index of glass of a prism; the (i) vs. (δ) graph is plotted. It was found that a ray incident, at an angle of 35^0, suffers a deviation of 40^0 and it emerges at an angle of 79^0. Which is the closest possible value of μ ?

(1) 1.5 (Correct)

(2) 1.6 (Incorrect)

(3) 1.7 (Incorrect)

(4) 1.8 (Incorrect)

(I.I.T 2016 Main)

Correct answer: (1)

Solution:

For a prism of refracting angle A; a ray incident at an angle i_1; emerges out at angle i_2. The deviation, δ, is

$$\delta = i_1 + i_2 - A$$

Given $\delta = 40^0$; $i_1 = 35^0$, $i_2 = 79^0$

$\therefore \quad 40 = (35+79) - A$

or $\quad A = 74^0$

The refractive index μ of prism is given by

$$\mu = \frac{\sin\left(\frac{A+\delta}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Assuming that incident ray has undergone minimum deviation we have

$$\mu = \frac{\sin\left(\frac{74+40}{2}\right)}{\sin 37^0} \simeq 1.51$$

The deviation, for the given incident ray, may not be the minimum value of deviation. We can, therefore, only say that μ is close to 1.5.

Unit–16 : Optics

Easy *Refraction Through a Prism*

21. A ray of light is incident normally on one face of a prism of refracting angle 30^0 and refractive index $\sqrt{2}$. The angle of deviation is

(1) 45^0 **(Incorrect)**

(2) 30^0 **(Incorrect)**

(3) 15^0 **(Correct)**

(4) zero **(Incorrect)**

(I.I.T 1997)

Correct answer: (3)

Solution:

Fig. 20 shows prism PQR. Ray AB is incident normally on its face PQ. It passes straight into the prism. The ray BC, inside the prism, is incident on face PR at an angle of incidence $i = 30^0$. Let r be the angle of refraction. From Snell's law:

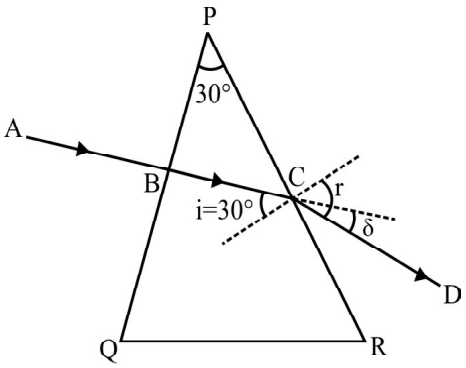


Fig. 20

$$\sqrt{2}\sin 30 = 1 \times \sin r$$

or $$\frac{1}{\sqrt{2}} = \sin r$$

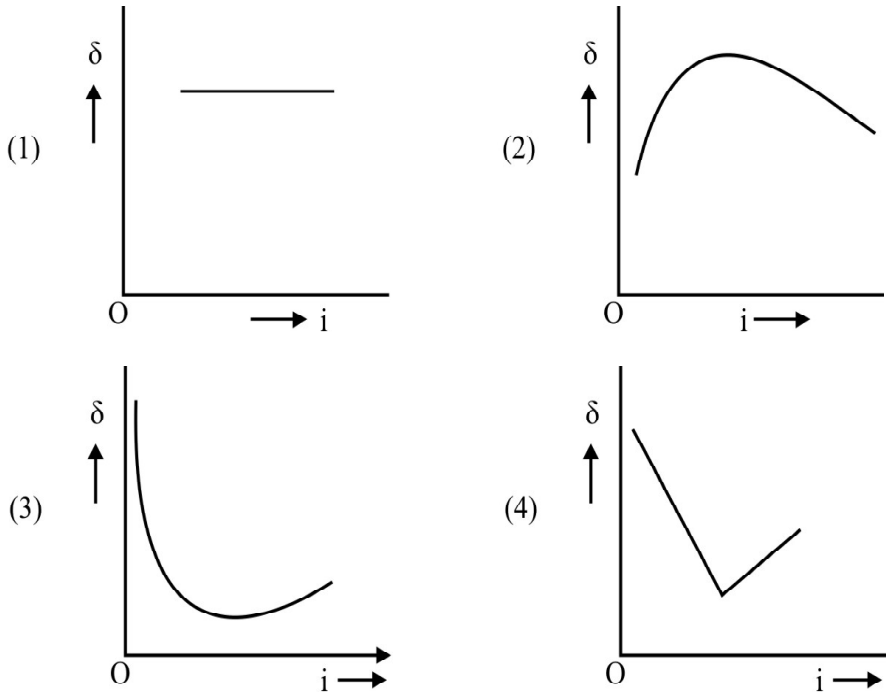
$\therefore \quad r = 45^0$

The angle of deviation = $\delta = r - i = 15^0$

Unit–16 : Optics

Easy *Refraction Through a Prism*

22. The graph between angle of deviation (δ) an angle of incidence (i) for a triangular prism is



Correct answer: (3)

Solution:

Angle of deviation (δ) for a given prism depends on angle of incidence (i). Starting from small values of i, δ decreases as i increases. It accquires a minimum value for a particular angle of incidence; after that it starts increasing as i increases further. This is shown correctly in option 3.

Unit–16 : Optics

Average *Refraction Through a Prism*

23. Beams of red and violet colour, are made to pass through a prism of angle 60^0, one by one r_1 and r_2 are the angles of refraction, when δ_1 and δ_2 are the angles of minimum deviation, in the two caess. Then

(1) $r_1 > r_2;\ \delta_1 > \delta_2$ **(Incorrect)**

(2) $r_1 = r_2 = 30^0\ \ \delta_1 = \delta_2$ **(Incorrect)**

(3) $r_1 < r_2;\ \delta_1 < \delta_2$ **(Incorrect)**

(4) $r_1 = r_2 = 30^0;\ \delta_1 < \delta_2$ **(Correct)**

Correct answer: (4)

Solution:

Under minimum deviation conditions

$$r_1 = r_2 = \frac{A}{2} = 30^0$$

Therefore, $r_1 = r_2 = 30^0$ for both red and violet rays when they are undergoing minimum deviation.

The refractive index μ of prism is

$$\mu = \frac{\sin\left(\frac{A+\delta m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

We know that $\mu_V > \mu_R$; therefore angle of minimum deviotion for violet colour is more than for red colour i.e., $\delta_1 < \delta_2$.

Unit–16 : Optics

Average *Total Internal Reflection*

24. A ray of green light is incident from water to air; (at the air–water interface) at the critical angle of incidence. Select the correct statement:

(1) The entire spectrum of visible light will come out at an angle of 90^0 to the normal (Incorrect)

(2) The part of the spectrum of visible light, for which the frequency in less that of green light, will come out in air (Correct)

(3) The spectrum of visible light, whose frequency is more than that of green light will come out in air. (Incorrect)

(4) The entire spectrum of visible light will come out of water at different angles to the normal (Incorrect)

(I.I.T Main 2014)

Correct answer: (2)

Solution:

Let $(i_c)_g$ be critical angle of incidence for green light at the water air interface. Then

$$(i_c)_g = \sin^{-1}\left(\frac{1}{\mu_g}\right) \qquad .....(1)$$

For $\nu < \nu_g ; \lambda > \lambda_g$. Refractive index decreases as λ increases. From for $\nu < \nu_g$; critical angle of incidence increases. These rays would not undergo total internal reflection at $i = (i_c)_g$. These colours would, therefore, emerge out of the water–air, interface, into the air.

Unit–16 : Optics

Average *Refraction Through a Prism*

25. PQR is an isoceless prism of angle 120^0 and has a refractive index of 1.44. Two parallel rays AB and CD undergo refraction through this prism. The rays, emerging from the opposite face

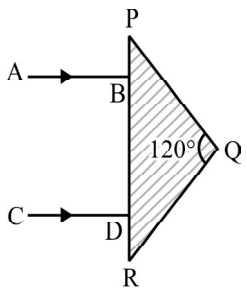


Fig. 22

(1) are parallel to one another **(Incorrect)**

(2) are diverging **(Incorrect)**

(3) make an angle of $2[\sin^{-1}(0.72)-30^0]$ with each other **(Correct)**

(4) make as angle of $2\sin^{-1}(0.72)$ with each other **(Incorrect)**

(I.I.T 1995)

Correct answer: (3)

Solution:

Fig. 23 shows the refraction of rays AB and CD through prism.
Applying Snell's law at point E.

$$1.44 \sin 30 = 1 \times \sin r$$

$$\therefore \quad r = \sin^{-1}(0.72)$$

The angle of deviation δ_1 at E is

$$\delta_1 = r - 30^0 = \sin^{-1}(0.72) - 30^0$$

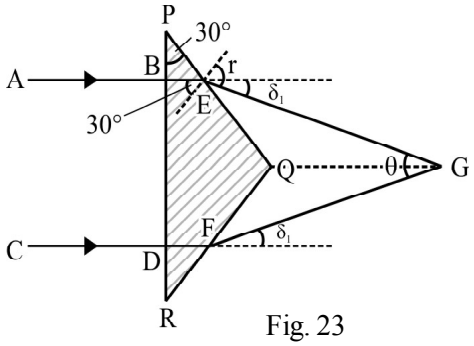


Fig. 23

Similarly, the ray CD, also gets deviated through an angle δ_1, at the point F

The angle θ between the two emergent rays is

$$\theta = 2\delta_1 = 2\left[\sin^{-1}(0.72) - 30^0\right]$$

Unit–16 : Optics

Average *Total Internal Reflection*

26. A parallel beam of light is incident, from air, at an angle α, on the side PQ of a right angled prism of refractive index $\sqrt{2}$. Light undergoes total internal reflection in the prism, at the face PR, when α has a minimum value of 45^0. The angle, θ, of the prism is

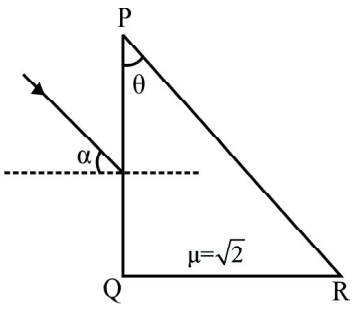


Fig. 24

(1) 15^0 **(Correct)**

(2) 22.5^0 **(Incorrect)**

(3) 30^0 **(Incorrect)**

(4) 45^0 **(Incorrect)**

(I.I.T Adv. 2016)

Correct answer: (1)

Solution:

Fig. 25 shows total internal reflection occuring at the face PR. Applying Snell's law at B

$$1\times\sin\alpha=\sqrt{2}\sin r_1$$

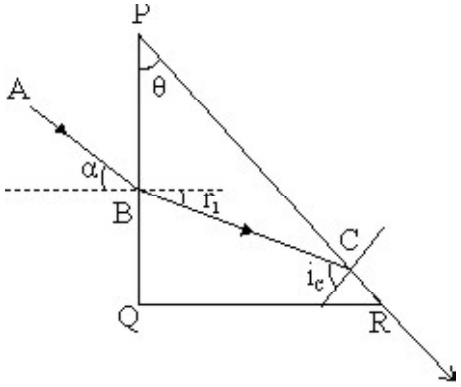


Fig. 25

Given $\alpha=45^0$; therefore

$$\frac{1}{\sqrt{2}}=\sqrt{2}\sin r_1$$

or $r_1=30^0$

Let i_c be the critical angle of incidence inside glass. Then

$$\sqrt{2}\sin i_c=1\times\sin\frac{\pi}{2}$$

$\therefore\ i_c=45^0$

From ΔPBC; we have

$$\theta+\left(\frac{\pi}{2}+r_1\right)+\left(\frac{\pi}{2}-i_c\right)=180^0$$

$\therefore\quad \theta=i_c-r_1=(45-30)^0=15^0$

Unit–16 : Optics

Difficult *Refraction Through a Prism*

27. ABC is a prims of refracting angle 30^0 and refractive index $\sqrt{3}$. The face AC is coated with a film of a transparent material of refractive index 2.2. An incident ray undergoes refraction inside the prism, but does not undergo any deviation at the film. If the net deviation is 30^0, the angle of incidence, i, is

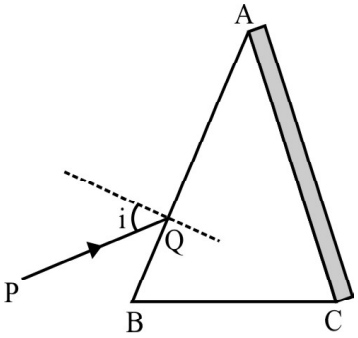


Fig. 26

(1) 30^0 **(Incorrect)**

(2) 45^0 **(Incorrect)**

(3) 60^0 **(Correct)**

(4) 90^0 **(Incorrect)**

Correct answer: (3)

Solution:

Fig. 27 shows the path of the incident ray PQ. QR falls normally on the film on face AC. There is no deviation due to film. The deviation is only due to refraction on face AB of prism ABC. Given

$$\delta = (i - r) = 30^0$$

Applying Snell's law, at point Q, we get

$$(i)\sin i = (\sqrt{3})\sin(i - 30)$$

This relation is satisfied if $i = 60^0$.

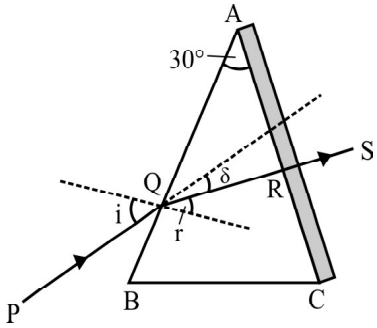


Fig. 27

Unit–16 : Optics

Average *Total Internal Reflection*

28. A right angled prism is to be made by selecting an appropriate material and angles A and B (B < A) as shown in the Fig. 28. A ray PQ, incident normally on face AB, is to emerge parallel to itself, after two internal reflections, as ray RS. The minimum value of μ of material, for this requirement, is

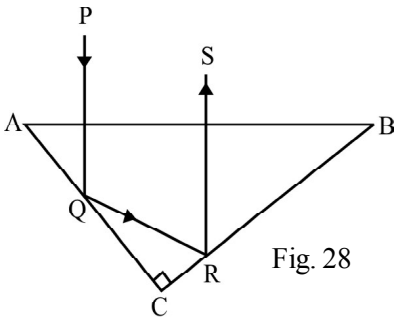


Fig. 28

(1) $\sqrt{2}$ **(Correct)**

(2) $\sqrt{3}$ **(Incorrect)**

(3) $\frac{3}{\sqrt{2}}$ **(Incorrect)**

(4) $\frac{2}{\sqrt{3}}$ **(Incorrect)**

Correct answer: (1)

Solution:

Total internal reflection must take place at points Q and R as shown in Fig. 29. Angle of incidence, at Q, equals angle A and angle of incidence, at R, equals angle B. Let i_c be the critical angle of incidence. Then

$$A \geq B \geq i_c$$

Also, $A+B = 90^0$ (given)

It follows that, μ, for the material of prism is minimum if

$A = B = 45^0$ and this is also the value of the critical angle. Hence

$$\mu_{min} \sin 45^0 = 1 \times \sin \frac{\pi}{2}$$

or $\mu_{min} = \sqrt{2}$

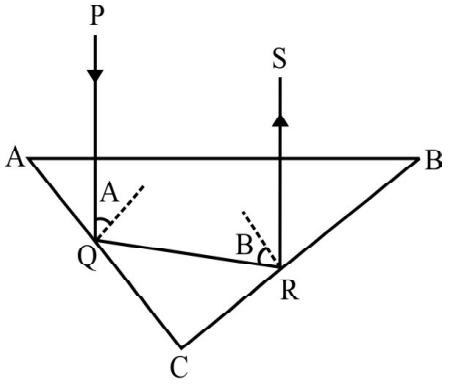


Fig. 29

Unit–16 : Optics

Average *Total Internal Reflection*

29. A right angle prism PQR is made of a material of refractive index 1.67. AB is ray incident normally on face PQ as shown in Fig. 30. Which of the following statements is correct?

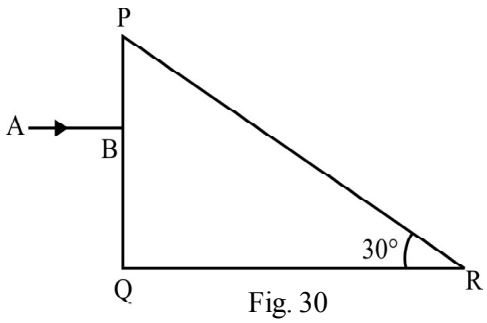


Fig. 30

(1) AB emerges out of face PR of prism (Incorrect)

(2) AB undergoes a deviation of 60^0 on face PR of prism (Correct)

(3) Ray AB, emerges out of face QR at an angle of emergence of 45^0 (Incorrect)

(4) Ray AB emerges out of face QR normally (Incorrect)

Correct answer: (2)

Solution:

The critical angle of incidence, i_c, inside the prism is given by

$$1.67 \sin i_c = 1 \times \sin \frac{\pi}{2}$$

$$i_c = \sin^{-1}\left(\frac{1}{1.67}\right) = 36.8^0$$

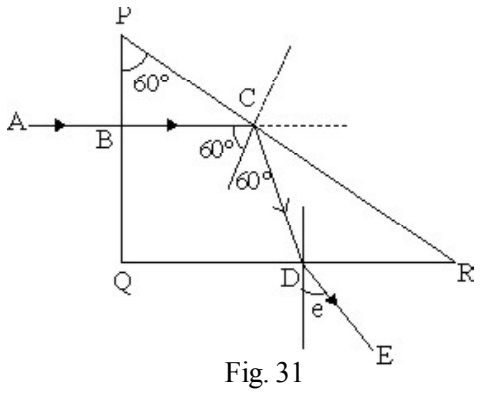


Fig. 31

The angle of incidence on face PR is 60^0. This is more than i_c; hence T.I.R occurs at face PR.

The angle of deviation = $180^0 - 120^0 = 60^0$

The totally reflected ray CD, is incident at 30^0, at point D, on face QR. The angle emergence, e, is given by is

$$1.67 \sin 30^0 = 1 \times \sin e$$

$$e = \sin^{-1}(0.835) \simeq 56.6^0$$

Hence statements (3) and (4) are seen to be incorrect. It is only statement (2) that is correct.

Unit–16 : Optics

Difficult *Refraction Through a Prism*

30. A monochromatic light is incident on a glass prism of angle A. The refractive index of the material of the prism is μ. A ray is incident at an angle θ on the face AB of the prism. It can emerges out of the face AC of prism provided

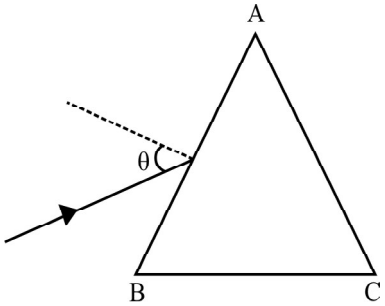


Fig. 32

(1) $\theta < \cos^{-1}\left[\mu\sin\left\{A + \sin^{-1}\left(\frac{1}{\mu}\right)\right\}\right]$ **(Incorrect)**

(2) $\theta < \sin^{-1}\left[\mu\sin\left\{A - \sin^{-1}\left(\frac{1}{\mu}\right)\right\}\right]$ **(Incorrect)**

(3) $\theta > \cos^{-1}\left[\mu\sin\left\{A + \sin^{-1}\left(\frac{1}{\mu}\right)\right\}\right]$ **(Incorrect)**

(4) $\theta > \sin^{-1}\left[\mu\sin\left\{A + \sin^{-1}\left(\frac{1}{\mu}\right)\right\}\right]$ **(Correct)**

(I.I.T Main 2015)

Correct answer: (4)

Solution:

Fig. 33 shows the ray PQ incident on face AB of the prism. Let r_1 be angle of refraction at Q. From Snell's law

$$1 \times \sin\theta = \mu \sin r_1$$

$$\therefore \quad r_1 = \sin^{-1}\left(\frac{\sin\theta}{\mu}\right) \qquad .....(1)$$

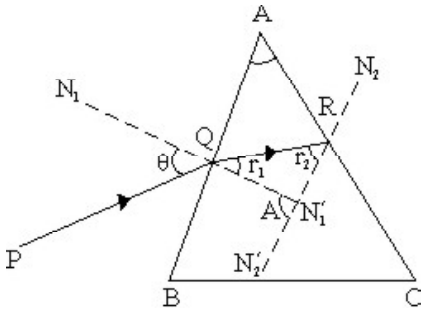


Fig. 33

The refracted ray QR is incident on face AC at an angle r_2. The ray would emerge out of face AC if $r_2 < i_c$ where i_c is the critical angle of incidence (from glass to air). Also

$$i_c = \sin^{-1}\left(\frac{1}{\mu}\right) \qquad .....(2)$$

Now.

$$r_1 + r_2 = A \qquad .....(3)$$

For ray QR to emerge out, we must have $r_2 < i_c$

$$\therefore \quad r_1 = (A - r_2) > (A - i_c)$$

or $\sin\left[\sin^{-1}\left(\frac{\sin\theta}{\mu}\right)\right] > \sin\left(A - i_c\right)$ [Clf Eqn (1)]

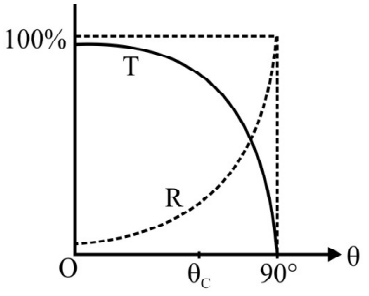
$\therefore \quad \theta > \sin^{-1}\left[\mu \sin\left(A - i_c\right)\right]$

$> \sin^{-1}\left[\mu \sin\left\{A - \sin^{-1}\left(\frac{1}{\mu}\right)\right\}\right]$

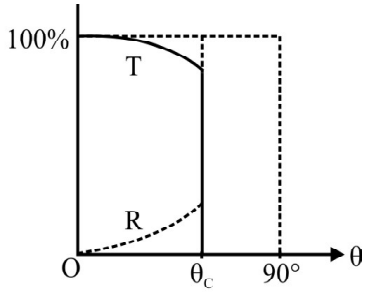
Unit–16 : Optics

Difficult *Total Internal Reflection*

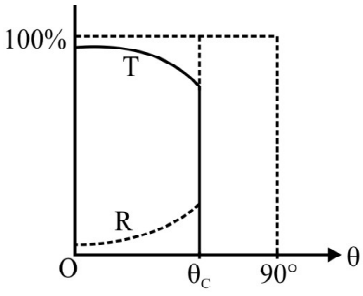
31. A light ray, travelling in glass medium, is incident on the glass–air interface at an angle θ. The reflected (R) and the transmitted (T) intensities, are both plotted, as functions of θ. The correct graph is the graph labelled as



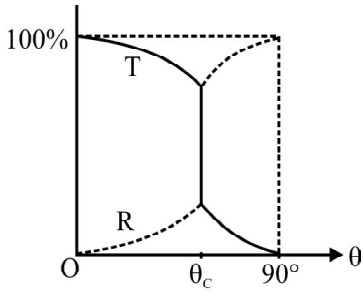
(1)



(2)



(3)



(4)

(I.I.T 2011)

Correct answer: (3)

Solution:

At $\theta = 0^0$; R + T = 1. However, R is not zero and T is not 1. As θ increases, from 0° to the critical angle θ_C; T decreases and R increases. For $\theta > \theta_C$, upto 90^0; there is no transmitted beam; therefore, T = 0. Over this range, due to T.I.R, 100% of incident light is reflected back, i.e., R = 1.

These variation are correctly shown only in graph (3).

Unit–16 : Optics

Average *Refraction Through a Prism*

32. ABC and BCD are two prisms joined together as shown in Fig. 35 μ_1 and μ_2 are the refractive indices of the material of the two prisms. Given:

$$\mu_1 = 1.20 + \frac{\left(10.8 \times 10^4\right)}{\lambda^2} \text{ and}$$

$$\mu_2 = 1.45 + \frac{\left(1.80 \times 10^4\right)}{\lambda^2}$$

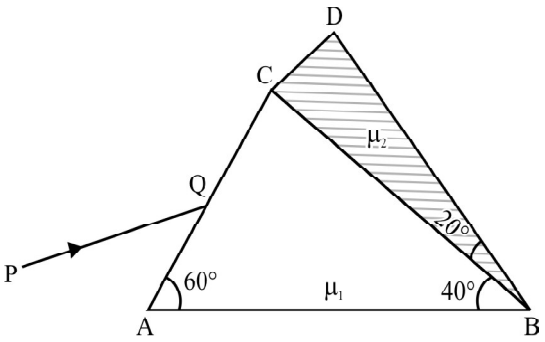


Fig. 35

where λ is in nm. PQ is an incident ray, incident on face AC.There is no bending of ray inside the prisms, on the face BC, the common face of the two prisms. The wavlength, λ, of the incident ray, PQ, must be

(1) 400 nm (Incorrect)

(2) 450 nm (Incorrect)

(3) 600 nm (Correct)

(4) 650 nm (Incorrect)

Correct answer: (3)

Solution:

For no deviation of ray incident on BC; (irrespective of the angle of incidence on BC), we must have $\mu_1 = \mu_2$. Therefore

$$1.20 + \frac{10.8 \times 10^4}{\lambda^2} = 1.45 + \frac{1.80 \times 10^4}{\lambda^2}$$

This gives

$\lambda = 600\,\text{nm}$

Unit–16 : Optics

Average *Refraction Through a Prism*

33. Three prisms, made from materials of, refractive indices μ_1, μ_2 and μ_3, are arranged as shown in Fig. 36. An incident ray, AB, emerges out as EF, following the path shown in the figure. Then the rays undergo minimum deviation in

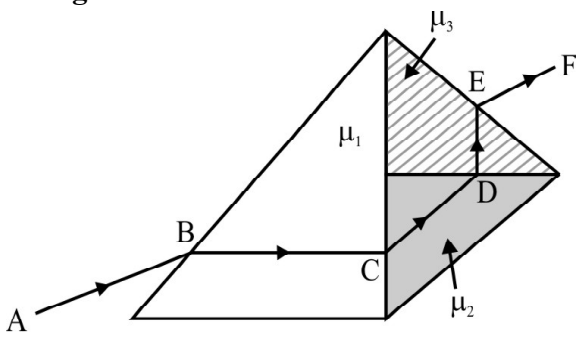


Fig. 36

(1) prism, μ_1, only **(Incorrect)**

(2) prism μ_1 and prism μ_2 but not in prism μ_3 **(Incorrect)**

(3) prism μ_3 only **(Incorrect)**

(4) in all the three prims **(Correct)**

Correct answer: (4)

Solution:

The path of rays BC; CD and DE, inside the respective prisms, are parallel to the base of prism considered. Hence minimum deviation occurs inside all the three prisms.

Unit–16 : Optics

Average *Refraction and T.I.R*

34. PQR is a right angled issoceless prism of a material of refractive index μ_1. It is placed inside a parallel slab, made of two parts, having refractive indices μ_2 and μ_3, as shown in Fig. 37. An incident ray AB follows the path shown. We must then have

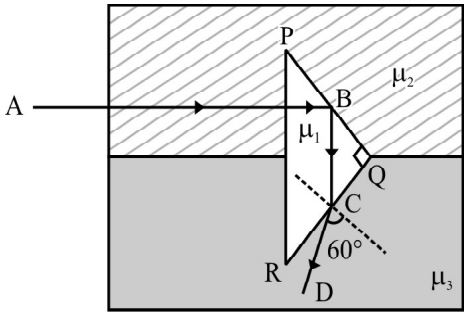


Fig. 37

(1) $\mu_2 \leq \frac{\mu_1}{\sqrt{2}} \leq \mu_1 = \sqrt{\frac{3}{2}}\,\mu_3$ **(Correct)**

(2) $\mu_2 \geq \frac{\mu_1}{\sqrt{2}};\ \mu_1 = \sqrt{\frac{3}{2}}\,\mu_3$ **(Incorrect)**

(3) $\mu_1 = \sqrt{2}\,\mu_2 = \sqrt{3}\,\mu_3$ **(Incorrect)**

(4) $\mu_1 \leq \frac{\mu_1}{\sqrt{2}};\ \mu_1 = \sqrt{\frac{2}{3}}\,\mu_3$ **(Incorrect)**

Correct answer: (1)

Solution:

Total internal reflection occurs at point B. Therefore $\mu_2 < \mu_1$ and $i_c < 45^0$. Hence $(i_c)_{max} = 45^0$. Therefore,

$$\mu_1 \sin 45^0 = (\mu_2)_{max} \sin 90^0$$

$$\therefore \quad (\mu_2)_{max} = \frac{\mu_1}{\sqrt{2}}$$

The ray BC is incident on face QR at an angle of 45^0. It emerges out at an angle of refraction of 60^0. Therefore,

$$\mu_1 \sin 45^0 = \mu_3 \times \sin 60^0$$

or $$\frac{\mu_1}{\sqrt{2}} = \mu_3 \frac{\sqrt{3}}{2}$$

$$\therefore \quad \mu_1 = \sqrt{\frac{3}{2}}\,\mu_3$$

Unit–16 : Optics

Average *Total Internal Reflection*

35. A point source S is placed at the bottom of a transparent block, of height 10 mm, and refractive index 2.72. It is immersed in a liquid of lower refractive index as shown in Fig. 38. It is found that the light, emerging from block to liquid, forms a circular bright spot of diameter 11.54 mm on top of the block. The refractive index of the liquid, is

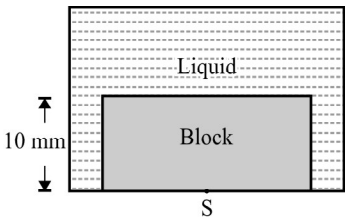


Fig. 38

(1) 1.21 **(Incorrect)**

(2) 1.30 **(Incorrect)**

(3) 1.36 **(Correct)**

(4) 1.42 **(Incorrect)**

(I.I.T Adv. 2014)

Correct answer: (3)

Solution:

In Fig. 39 A_1A_2 is the diameter of bright circular spot. For incident ray SA_1; total internal reflection occurs at A_1. The angle of incidence $= i_c$. We are given that $\mu_1 = 2.72$. Let μ_2 be the refractive index of the liquid. From Snell's law, we have

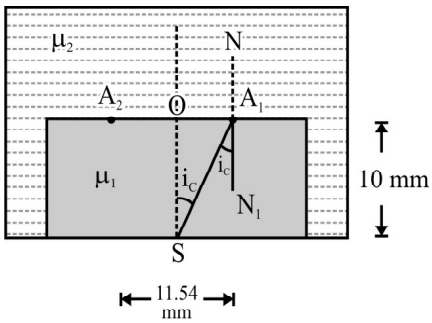


Fig. 39

$$2.72 \sin i_c = \mu_2 \times \sin \frac{\pi}{2} = \mu_2 \qquad .....(1)$$

From right–angled ΔOA_1S

$$\sin i_c = \frac{OA_1}{SA_1} = \frac{5.77}{\left[(10)^2 + (5.77)^2\right]^{1/2}} = \frac{5.77}{11.54} = \frac{1}{2} .....(2)$$

$$\therefore \quad \mu_2 = \frac{2.72}{2} = 1.36$$

Unit–16 : Optics

Average *Refraction Through a Prism*

36. Three identical equilateral prisms P, Q and R are arranged as shown in Fig. 40. An incident ray passes through all three and undergoes deviations δ_P, δ_Q and δ_R respectively, in each of the three prisms. δ is the net deviation, due to the whole arrangement. Then

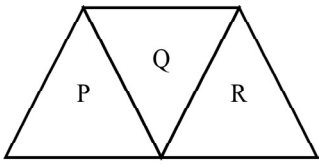


Fig. 40

(1) $|\delta_P| = |\delta_Q| = |\delta_R| = \delta_1 (\text{say})$ **and** $\delta = 3\delta_1$ **(Incorrect)**

(2) $|\delta_P| = |\delta_Q| = |\delta_R| = \delta_1 (\text{say})$ **and** $\delta = \delta_1$ **(Correct)**

(3) $\delta_P = (-\delta_Q) = \delta_R$; $\delta = 2\delta_P$ **(Incorrect)**

(4) $\delta_P = (-\delta_Q)\ ^1\ \delta_R$; $\delta = 2\delta_P + \delta_Q$ **(Incorrect)**

Correct answer: (2)

Solution:

Since the three prisms are equilateral and identical; we would have

$$|\delta_P| = |\delta_Q| = |\delta_R| = \delta_1$$

However δ_Q is in a direction opposite to δ_P. Hence the net deviation.

$$\delta = |\delta_P| - |\delta_Q| + |\delta_R| = \delta_1$$

Unit–16 : Optics

Difficult *Passing of Light Through a Prism*

37. A ray of light PQ is incident on an equilateral prism ABC of refarctive index $\sqrt{3}$. It passes through the prism ABC as shown in Fig. 41. CDE is a second prism exactly identical to prism ABC. The emergent ray, from prism ABC, is required undergo minimum deviation in prism CDE. For this the prism CDE should be rotated through

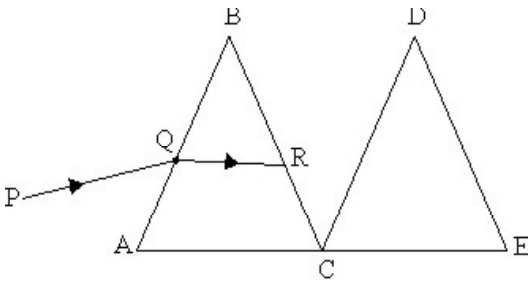


Fig. 41

(1) 30^0, clockwise **(Incorrect)**

(2) 60^0, clockwise **(Incorrect)**

(3) 60^0, anticlockwise **(Correct)**

(4) 30^0, anticlockwise **(Incorrect)**

(I.I.T 2005)

Correct answer: (3)

Solution:

The path, QR, of ray inside prism ABC, is parallel to base AC of prism. The incident ray PQ is therefore, undergoing minimum deviation in this prism. Let i_1 and r_1 denote angle of incidence and refraction, at the face on AB. For minimum deviation,

$$r_1 = r_2 = \frac{A}{2} = 30^0$$

From Snell's law

$$(1)\sin i_1 = \sqrt{3}\sin 30$$

$$\therefore \quad i_1 = 60^0$$

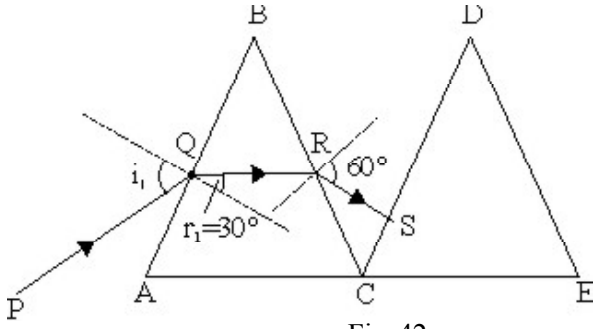


Fig. 42

The ray QR emerges out of prism from face BC; at angle of emergence = 60^0. It is, therefore, incident normally, on face CD, of prism CDE. If this ray is to undergo minimum deviation inside CDE, angle of incidence of RS, on CD, should be again 60^0. This will be so if prism CDE is rotated, through 60^0, in the anticlockwise direction.

Unit–16 : Optics

Average *Total Internal Reflection*

38. PQR is a prism of angle θ (= $\sin^{-1}$ 0.8). The refractive index, μ, of the prism is given by

$$\mu = 1.20 + \frac{8\times10^{-15}}{\lambda^2}$$

λ is wave - length of incident light.

AB is a ray of λ = 400 nm while ray CD has λ = 500 nm. The two rays are incident as shown in Fig. 43. Which of the following statements is correct?

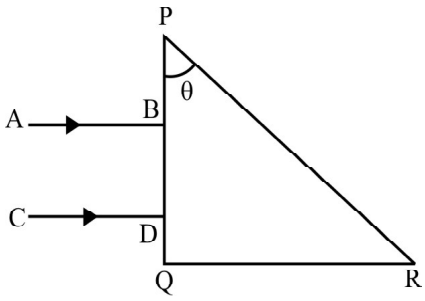


Fig. 43

(1) Both rays AB and CD, emerge out of face PR (Incorrect)

(2) Ray AB does not emerge out, but ray CD emerges out, from face PR (Incorrect)

(3) Ray AB emerges out, but ray CD does not emerge out of face PR (Correct)

(4) Neither ray AB nor ray CD, emerges out of face PR (Incorrect)

Correct answer: (3)

Solution:

Let μ_1 and μ_2 the, respective, refractive indices of the prism for rays AB and CD. Then

$$\mu_1 = 1.20 + \frac{8\times10^{-15}}{\left(4\times10^{-7}\right)^2} = 1.25$$

and $$\mu_2 = 1.20 + \frac{8\times10^{-15}}{\left(5\times10^{-7}\right)^2} = 1.232$$

The angle of incidence of both rays on face PR is θ. The critical angle, i_1 and i_2, for wavelengths λ_1 and λ_2, are

$$i_1 = \sin^{-1}\left(\frac{1}{1.25}\right) = \sin^{-1}(0.85) = \theta$$

$$i_2 = \sin^{-1}\left(\frac{1}{1.232}\right) > \theta$$

Therefore, ray AB undergoes T.I. R on face QR and does not emerge out. For ray CD, angle of incidence θ is less than the critical angle i_2. This ray emerges out of race QR.

Unit–16 : Optics

Average *Refraction Through Prism*

39. PQR is an isoceless right angled prism, made from a material of refractive index μ_1. It is placed inside a block made from a material of refractive index μ_2. The block itself is inside a material of refractive index μ_3. Fig. 44 shows the path of an incident ray through the arrangement. Then

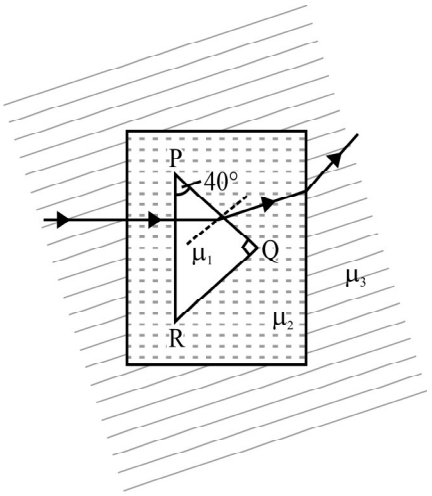


Fig. 44

(1) $\mu_2 > \mu_1; \mu_2 > \mu_3$ **(Correct)**

(2) $\mu_1 < \mu_2, \mu_2 > \mu_3$ **(Correct)**

(3) $\mu_1 = \mu_2 < \mu_3$ **(Incorrect)**

(4) $\mu_1 > \mu_2 = \mu_3$ **(Incorrect)**

Correct answer: (1)

Solution:

On face PQ, of prism, the angle of incidence = 45^0. The ray emerges out. Therefore $i_c > 45$. The ray also bends towards normal. Therefore $\mu_2 > \mu_1$. From the side of the rectangular block the ray emerges out and bends away from normal. Therefore, $\mu_2 > \mu_3$.

Unit–16 : Optics

Average *Total Internal Reflection*

40. AB and CD are two parallel slabs of materials of refractive indices $\mu_1 = \sqrt{2}$ and $\mu_3 = \sqrt{3}$ respectively. The medium between the slabs has a refractive index $\mu_2 = 2$ as shown in Fig. 45. The minimum angle of incidence of ray PQ, so that T.I.R occurs at both the slabs, is

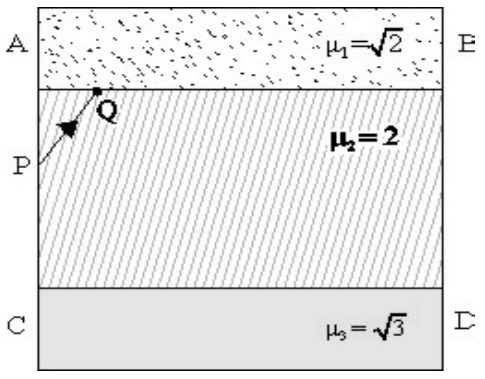


Fig. 45

(1) 30^0 **(Incorrect)**

(2) 45^0 **(Incorrect)**

(3) 60^0 **(Correct)**

(4) 57^0 **(Incorrect)**

Correct answer: (3)

Solution:

Let i_{1c} and i_{2c} be the critical angles of incidence for slabs AB and CD, respectively. Then,

$$2\sin\left(i_{1c}\right)=\sqrt{2}\sin\frac{\pi}{2}$$

or $$i_{1c}=\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=45^0$$

Similarly, $i_{2c}=\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)=60^0$

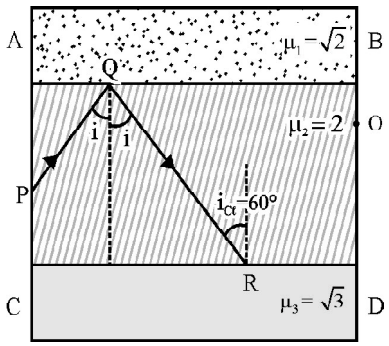


Fig. 46

Let i (> 45^0) be the angle of incidence at AB so that T.I.R occurs at AB. The angle of incidence on CD equals i. For T.I.R to occur at CD; $i_{min}=i_{2c}=60^0$.

Hence the minimum value of i, at AB, must be 60^0.

Unit–16 : Optics

Difficult *Refraction of Light*

41. A glass plate, of thickness t, is made of a material of refractive μ. A ray of light is incident on one face at an angle of incidence equal to the critical angle of incidence, for the glass–air interface. The lateral displacement, $\ell\left(=\frac{t}{\cos r}\sin\left(i-r\right)\right)$ of the ray, as it emerges out of the glass plate, is given by

(1) $\frac{t}{\mu}\left[1-\left(\frac{1}{\sqrt{\mu^2+1}}\right)\right]$ **(Correct)**

(2) $\frac{t}{\mu}\left[1+\left(\frac{1}{\sqrt{\mu^2+1}}\right)\right]$ **(Incorrect)**

(3) $\frac{t}{\mu}\left[1-\sqrt{\frac{\mu^2-1}{\mu^2+1}}\right]$ **(Incorrect)**

(4) $\frac{t}{\mu}\left[1-\sqrt{\frac{\mu^2+1}{\mu^2-1}}\right]$ **(Incorrect)**

Correct answer: (1)

Solution:

The angle of incidence, i, is

$$i = i_c = \sin^{-1}\left(\frac{1}{\mu}\right) \quad \text{or} \quad \sin i_c = \frac{1}{\mu} \qquad .....(1)$$

Let r be angle of refraction into glass plate. From Snell's law,

$$(1)\sin i_c = \mu \sin r$$

or $$\sin r = \frac{\sin i_c}{\mu} = \frac{1}{\mu^2} \qquad .....(2)$$

The lateral displacement, ℓ, is given by

$$\ell = \frac{t}{\cos r}\left[\sin(i-r)\right] = \frac{t}{\cos r}\left[\sin i\cos r - \cos i \sin r\right]$$

$$= t\left[\sin i - \cos i \tan r\right] \qquad .....(3)$$

Now,

$$\cos i = \left(1-\sin^2 i\right)^{1/2} = \left(1-\frac{1}{\mu^2}\right)^{1/2}$$

$$\therefore \quad \tan r = \frac{\left(\frac{1}{\mu^2}\right)}{\left[1-\frac{1}{\mu^4}\right]^{1/2}} = \frac{\mu^2}{\left[\mu^4-1\right]^{1/2}}$$

Substituting these values, in Eqn. (3) and simplifying, we get

$$\ell = \frac{t}{\mu}\left[1-\left(\frac{1}{\sqrt{\mu^2+1}}\right)\right]$$

Unit–16 : Optics

Average *Refraction of Light*

42. A monochromatic light ray, OP, is incident on face AB, near the vertex, B, at an angle of 60^0, as shown in Fig. 47. Refractive index of the material is $\sqrt{3}$. Which of the following statements is incorrect?

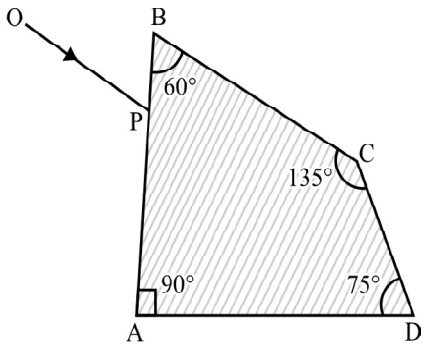


Fig. 47

(1) The incident ray gets totally internally reflected at face CD **(Correct)**

(2) The ray emerges out of face AD **(Correct)**

(3) The angle between the incident ray and the emergent ray is 90^0 **(Correct)**

(4) The angle between the incident ray and the emergent ray is 120^0 **(Incorrect)**

(I.I.T 2010)

Correct answer: (4)

Solution:

Fig. 48 shows the path of the incident ray OP inside block ABCD. For refraction at P

$$(1)\sin 60 = \left(\sqrt{3}\right)\sin r_1$$

or $$\frac{1}{2} = \sin r_1$$

$$\therefore \quad r_1 = 30^0$$

The critical angle of incidence i_c for air – material of ABCD, interface is

$$i_c = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right) = 35^0$$

The angle of incidence, at Q, is 45^0. This is more than i_c; therefore T.I.R, occurs at Q. For ray incident at point R, angle of incidence $r_2 = 30^0$. This is less than i_c; the ray, therefore, emerges out of face AD. Let i_2 be angle of emergence. From Snell's law

$$\left(\sqrt{3}\right)\sin 30^0 = (1)\times \sin i_2$$

$$\therefore \quad i_2 = 60^0$$

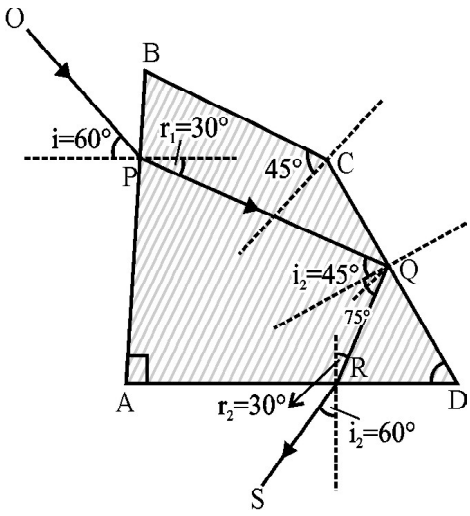


Fig. 48

The angle between the incident ray OP and emergent ray RS is 90^0.

Hence only statement (4) is an incorrect statement.

Unit–16 : Optics

Difficult *Refraction Through a Prism*

43. ABC and BCD are two prisms joined together as shown in Fig. 49 Given

$$\mu_1 = 1.20 + \frac{10.8\times 10^4}{\lambda^2}$$

and $$\mu_2 = 1.45 + \frac{1.80\times 10^4}{\lambda^2}$$

(λ is in nanometer)

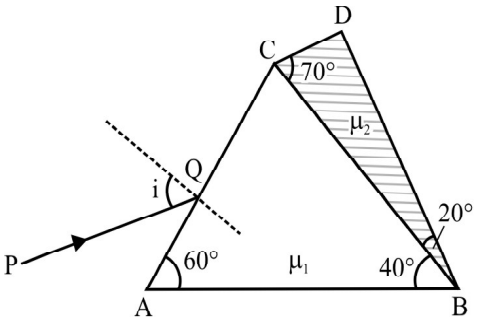


Fig. 49

PQ is an incident ray. It undergoes minimum deviation through the combination, suffereing no deviation at BC. The wave length, λ, and the angle of incidence of ray, PQ, are equal, respectively, to

(1) λ = 600 nm; i = $\sin^{-1}$ (0.5) **(Incorrect)**

(2) λ = 450 nm; i = $\sin^{-1}$ (0.75) **(Incorrect)**

(3) λ = 450 nm; i = $\sin^{-1}$ (0.75) **(Incorrect)**

(4) $\lambda = 600$ nm; $i = \sin^{-1}(0.75)$ (Correct)

Correct answer: (4)

Solution:

The will be no deviation at face BC it $\mu_1 = \mu_2$; i.e., if

$$1.20 + \frac{10.8 \times 10^4}{\lambda^2} = 1.45 + \frac{1.80 \times 10^4}{\lambda^2}$$

This gives $\lambda = 600$ nm

For $\mu_1 = \mu_2$; the given arrangement is a part of a truncated prism of angle 60^0 as shown in Fig. 50. Also $\mu = 1.5$.

[Value of μ is obtained by putting $\lambda = 600$ nm in the expression for either μ_1 or μ_2]. For minimum deviation the angle of refraction r, at face AC, is given by

$$r = \frac{A}{2} = 30^0$$

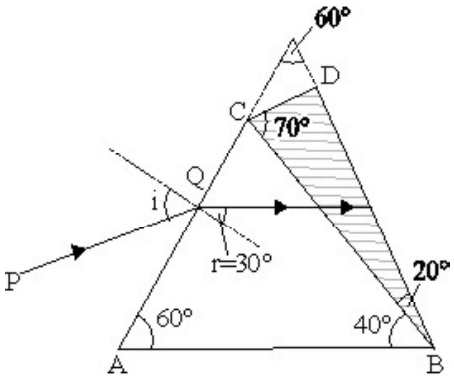


Fig. 50

Applying Snell's law at point Q, we have

$$(1)\sin(i) = (1.5)\sin 30^0 = 0.75$$

$\therefore \quad i = \sin^{-1}(0.75)$

Unit–16 : Optics

Difficult *Dispersion Through a Prism*

44. A prism P, of angle 4^0, is made from a material of refractive index 1.54 and dispersive power 0.05. It is combined with another prism Q of angle α. The refractive index and dispersive power of material of prism Q are 1.72 and 0.06, respectively. There is no net deviation due to combination, but there is a net angular dispersion, Δ. Then

(1) $\alpha = 3^0$; $\Delta = 0.4^0$ **(Incorrect)**

(2) $\alpha = 3^0$; $\Delta = 0.3^0$ **(Incorrect)**

(3) $\alpha = 3^0$; Δ; 0.022^0 **(Correct)**

(4) $\alpha = 3^0$; Δ ; 0.24^0 **(Incorrect)**

Correct answer: (3)

Solution:

Let α_1; μ_1 and ω_1 denote angle of prism; refractive index and dispersive power for prism P. α_2; μ_2 and ω_2 are the corresponding quantities for prism Q. The combination produces not net deviation; therefore

$$\delta = (\mu_1 - 1)\alpha_1 + (\mu_2 - 1)\alpha_2 = 0$$

$$\therefore \quad \alpha_2 = -\left[\frac{(1.54-1)4^0}{(1.72-1)}\right] = -3^0$$

Negative sign of α_2 of indicates that the base of P and Q are oppositely directed to each other. Then net angular dispersion, Δ ; is

$$\Delta = (\omega_2 - \omega_1)(\mu_1 - 1)\ \alpha_1$$

$$= (0.06 - 0.05) \times 0.54 \times 4^0 \simeq 0.022^0$$

Unit–16 : Optics

Average *Refraction at a Spherical Surface*

45. A spherical glass surface, of radius of curvature R, seperates air (refractive index 1.0) from glass (refractive index 1.5). The centre of curvature of the glass surface lies in glass. A point object, P, placed in air, is found to form a real image, Q, in the glass. The line PQ cuts the surface at O and OP = PQ. The distance, OP, is

(1) 5 R **(Correct)**

(2) 3 R **(Incorrect)**

(3) 2 R **(Incorrect)**

(4) 1.5 R **(Incorrect)**

Correct answer: (1)

Solution:

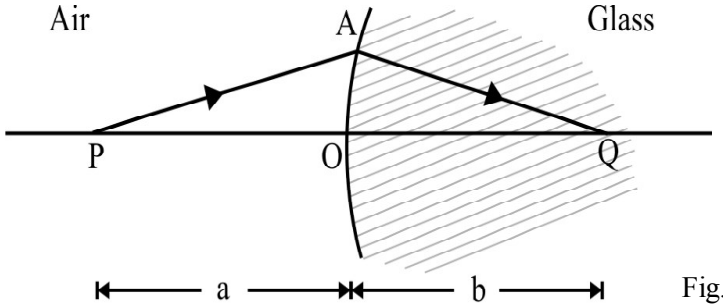


Fig. 51

Fig. 51 shows the object P, and its real image, Q, formed inside the glass. Let a and b be the object and image distances. Given a = b. Therefore,

$u = -a,\ v = +a,\ R = +C,\ \mu_1 = 1.0,\ \mu_2 = 1.5$

Using refraction formula $\frac{\mu_2}{v} - \left(\frac{\mu_1}{u}\right) = \left(\frac{\mu_2 - \mu_1}{R}\right)$; we have

$$\frac{1.5}{a} - \left(\frac{1}{-a}\right) = \frac{1.5-1}{C},\ \frac{2.5}{a} = \frac{1}{2C}$$

$\therefore\ a = 5C = 5R$

Unit–16 : Optics

Difficult *Refraction at a Sperical Surface*

46. A transparent thin film, of uniform thickness and refractive index n_1 (= 1.4), is coated on the convex spherical surface (of radius R), at the one end of a long solid glass cylinder, of refractive index, n_2 (= 1.5), as shown in Fig. 52. Rays of light, parallel to the axis of cylinder, going through the film, from air to glass, get focused at a distance (the opposite way) f_1, from the film; while the rays going from glass to air, get focused out a distance f_2 from the film. We would then have:

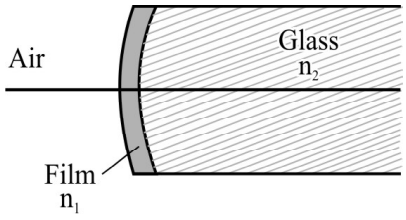


Fig. 52

(1) $|f_1| = 3R$; $|f_2| = 2R$ **(Correct)**

(2) $|f_1| = 3R$; $|f_2| = 3R$ **(Incorrect)**

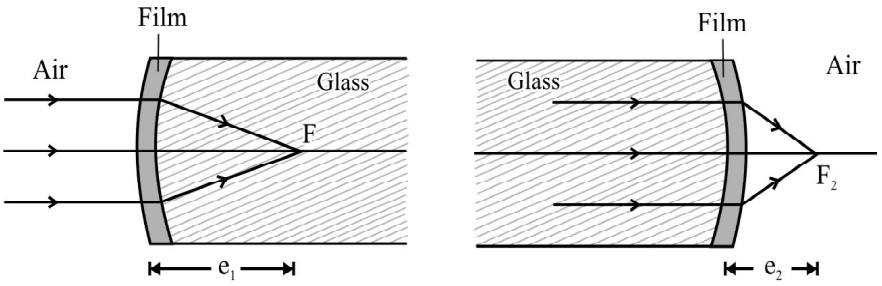
(3) $|f_1| = 2R$; $|f_2| = 2R$ **(Incorrect)**

(4) $|f_1| = 2R$; $|f_2| = 3R$ **(Incorrect)**

(I.I.T Adv. 2012)

Correct answer: (2)

Solution:



(a) Fig. 53 (b)

Let |R| = c, be the radius of each surface of film, of thickness t. The focal length, f, of the thin film, according to lens maker's formula, is

$$\frac{1}{f} = (n_1 - 1)\left(\frac{1}{c} - \frac{1}{c}\right) = 0 \qquad \therefore \quad f = \infty$$

The spherical film, therefore, behaves like a parallel slab; hence there is no deviation in a ray due to its passage through the film. Fig. 53 (a) shows parallel rays, going from air, getting focused at F_1, inside the glass. For refraction, from film to glass, we have

$$u_1 = -\infty; \; v_1 = +e_1; \; R = +c$$

Using the refraction formula, at a spherical surface;

$$\frac{n_2}{v_1} - \left(\frac{n_1}{u_1}\right) = \frac{n_2 - n_1}{R}$$

We get

$$\frac{1.5}{e_1} - \left(\frac{1.0}{\infty}\right) = \frac{1.5 - 1}{c}$$

$\therefore \quad e_1 = 3c \quad$ or $\quad |f_1| = 3R$

Fig. 53 (b)shows the parallel rays, going from glass to air. Now

$u_2 = -\infty$, $v_2 = +e_2$, $R = -c$.

Using the refraction formula:

$$\frac{1}{e_2}-\left[\frac{1.5}{(-\infty)}\right]=\frac{1-1.5}{(-c)} \quad \text{or} \quad e_2 = 2c$$

$\therefore \quad |f_2| = e_2 = 2R$

Unit–16 : Optics

Difficult *Refraction a Spherical Surface*

47. A quarter cylinder, of radius R and refractive index 1.5, is placed on a table. A point object, P, is kept at a distance, mR, from it as shown in Fig. 54. The ray, after refraction through the cylindrical part, emerges parallel to the table, as shown in Fig. 54. Then

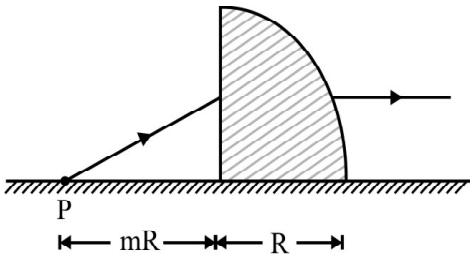


Fig. 54

(1) $\mathbf{m = \frac{3}{4}}$ **(Incorrect)**

(2) $\mathbf{m = 1}$ **(Incorrect)**

(3) $\mathbf{m = \frac{3}{2}}$ **(Incorrect)**

(4) $\mathbf{m = \frac{4}{3}}$ **(Correct)**

(I.I.T 1999)

Correct answer: (2)

Solution:

Fig. 55 shows the refraction of the incident ray PQ; on the plane surface AB. The refracted ray, QR, inside the glass appears to come from P_1. For refraction at this plane surface, we have

$$\frac{1}{1.5}=\frac{AP}{AP_1}$$

$\therefore \quad AP_1 = 1.5\ mR$

The ray QR is incident inside the glass on the curved surface, BO. The emergent ray RS is parallel to the table. For refraction at this spherical surface, we have, $n_1 = 1.5$, $n_2 = 1.0$; $u = -[1.5mR + R]$ and $v = \infty$. Also 'R' needs to be taken with a negative sign, as per the sign convention. Using refraction formula

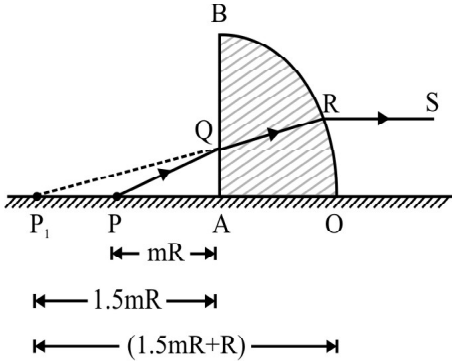


Fig. 55

$$\frac{1}{\infty} - \left[\frac{1.5}{-(1.5mR + R)}\right] = \frac{1-1.5}{(-R)}$$

$$\therefore \quad \frac{1.5}{(1.5m+1)} = \frac{1}{2}$$

or $\quad m = \frac{4}{3}$

Unit–16 : Optics

Difficult *Refraction at Spherical Surface*

48. Two glass rods; S_1 and S_2 (refractive index = 1.5) have one of their ends as convex surface of radius of curvature, 10 cm. They are placed with the curved surfaces, at a distance d, as shown in Fig. 56. When a point object, P, is placed inside S, on their common axis, at distance of 50 cm from the curved surface, the light rays emerging from it, are formed to be parallel to the axis inside S_2. The distance 'd' equals

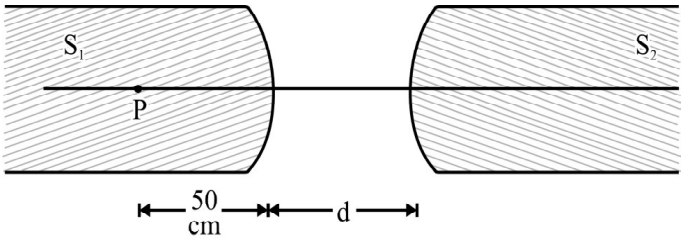


Fig. 56

(1) 60 cm **(Incorrect)**

(2) 70 cm **(Correct)**

(3) 90 cm **(Incorrect)**

(4) 90 cm **(Incorrect)**

(I.I.T Adv. 2015)

Correct answer: (2)

Solution:

For refraction of light, at the curved surface of rod S_1, we have

$u_1 = -50$ cm, $v_1 = ?$, $R_1 = -10$ cm, $\mu_1 = 1.5$, $\mu_2 = 1.0$

From refraction formula, we get

$$\frac{1}{v_1} - \left[\frac{1.5}{(-50)}\right] = \frac{1-1.5}{(-10)} \qquad \therefore \quad v_1 = 50\,\text{cm}$$

Since v_1 is positive, the image Q_1, formed by S_1, is real as shown in the Fig. 57. Q_1 acts as the object for rod S_2. For refraction, at the curved surface of S_2, we have

$u_2 = -(d-50)$ cm; $v_2 = +\infty$; $R_2 = +10$ cm

$\mu_1 = 1.0$ and $\mu_2 = 1.5$

Using refraction formula, we get

$$\frac{1.5}{\infty} - \left[\frac{1}{-(d-50)}\right] = \frac{1.5-1}{+10}$$

$\therefore \quad d = 70\,\text{cm}$

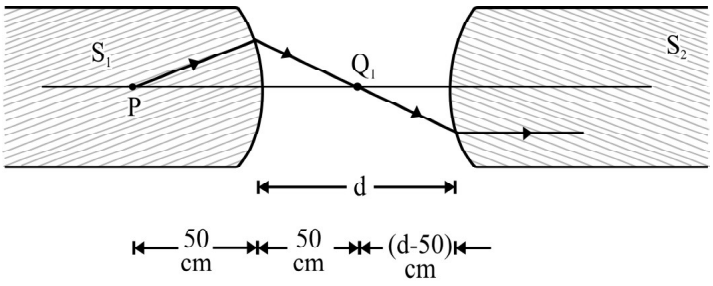


Fig. 57

Unit–16 : Optics

Difficult *Refracted a Spherical Surface*

49. A cylinderical rod of length L is curved at one end. The rod is divided into two equal parts as shown in Fig. 58. A small dot, at the mid–point P, is viewed from the plane, and the curved side, one by one. The position of image, from the respective surface used, is found to be the same. The radius of curvature of curved surface equals L. Then the refractive index, μ_2, of the right half of the cylindrical rod, is

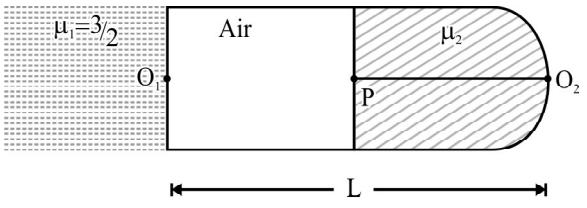


Fig. 58

(1) 1.4 **(Incorrect)**

(2) 1.5 **(Incorrect)**

(3) 2.0 **(Incorrect)**

(4) 3.0 **(Correct)**

Correct answer: (4)

Solution:

In Fig. 59, E_1 is the position of eye when the object P is being viewed from the plane surface of the rod. Let P′ be the apparent position of the image.
Then

$$O_1P' = OP \times \frac{3}{2} = \frac{3L}{4}$$

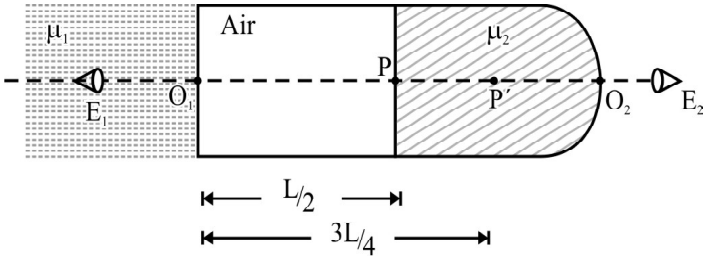


Fig. 59

E_2 is position of eye when P is viewed from the curved surface. For curved surface; the image is again getting formed at P′. Hence

$$u = -\frac{L}{2},\ v = -\frac{L}{4},\ R = -L$$

From refraction formula, for a spherical surface, we get

$$\frac{1}{\left(-\frac{L}{4}\right)} - \left[\frac{\mu_2}{\left(-\frac{L}{2}\right)}\right] = \frac{1-\mu_2}{(L)}$$

$\therefore\ \mu_2 = 3$

Unit–16 : Optics

Average *Lenses*

50. A spherical lens forms an image, twice the length of the object, on a screen. The distance between the object and the screen is 120 cm. The power of the lens, is (nearly)

(1) +3.75 D **(Correct)**

(2) +2.50 D **(Incorrect)**

(3) +2.0 D **(Incorrect)**

(4) +1.25 D **(Incorrect)**

Correct answer: (1)

Solution:

Let a and b the distance of the distance of the object and screen from the lens. Since image is projected on the screen; it is real and, therefore, inverted. The magnification

$$m = -2 = \frac{v}{u} = \frac{(+b)}{(-a)}$$

$\therefore \quad b = 2a$ .....(1)

The distance between screen and object $= 120$ cm (Given)

$\therefore \quad b + a = 120$ .....(2)

From Eqn.s (1) and (2) we have

$a = 40$ cm, $b = 80$ cm

$\therefore \quad u = -40$ cm, $v = +80$ cm, $f = ?$

Using lens formula, we get

$$\frac{1}{80} - \left[\frac{1}{-40}\right] = \frac{1}{f} \quad \text{or} \quad f = \frac{80}{3}\text{cm} = \frac{0.8}{3}\text{m}$$

$$P = \text{The power of lens} = \frac{1}{\left(\frac{0.8}{3}\right)} D \simeq +3.75\,D$$

Unit–16 : Optics

Difficult *Lens*

51. For a given lens, the graph, between object position, u, and the image position, v, is shown in Fig. 60. The focal length of the lens (in cm) is

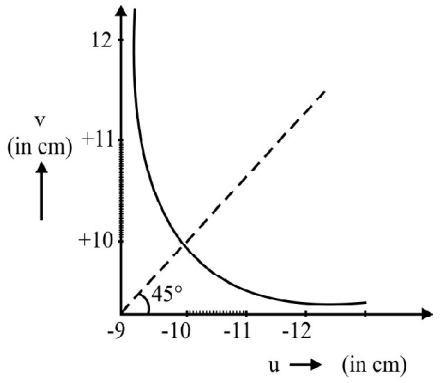


Fig. 60

(1) **(50 ± 0.1)** **(Incorrect)**

(2) **$(5 \pm .0.05)$** **(Correct)**

(3) **(0.5 ± 0.1)** **(Incorrect)**

(4) **(0.5 ± 0.05)** **(Incorrect)**

(I.I.T 2006)

Correct answer: (2)

Solution:

From graph, for $u = -10$ cm, $v = +10$ cm. The focal length, f, is given by

$$\frac{1}{10} - \left[\frac{1}{-10}\right] = \frac{1}{f} \qquad \therefore \; f = 5\text{cm}$$

Also, $\Delta u = \pm\ 0.1$ cm, $\Delta v = \pm = 0.1$ cm;

Now $\frac{1}{v} - \left(\frac{1}{u}\right) = \frac{1}{f}$

$\therefore$ Differentiating, we get

$$-\left(\frac{\Delta f}{f^2}\right) = -\left(\frac{\Delta v}{v^2}\right) + \frac{\Delta u}{u^2}$$

or $$\Delta f = \pm \left[\frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}\right] f^2$$

$$= \pm \left[\frac{0.1}{100} + \frac{0.1}{100}\right] \times (5)^2 = 0.05\,\text{cm}$$

Therefore, the focal length, f, is

$$f = [5 \pm 0.05]\,\text{cm}$$

Unit–16 : Optics

Easy *Lens & Mirror*

52. In Fig. 61 (a) and (b), the same object, PQ, placed, in front of a convex lens and a concave mirror, one by one, forms the image P'Q'. Which of the following statements, for the position of the object PQ, is correct?

	Lens (L)	Mirror (M)	
(1)	In between F and 2F	Beyond C	(Incorrect)
(2)	Beyond F	In between F and C	(Incorrect)
(3)	In between F and 2F	In between F and C	(Correct)
(4)	Beyond 2F	In between F and C	(Incorrect)

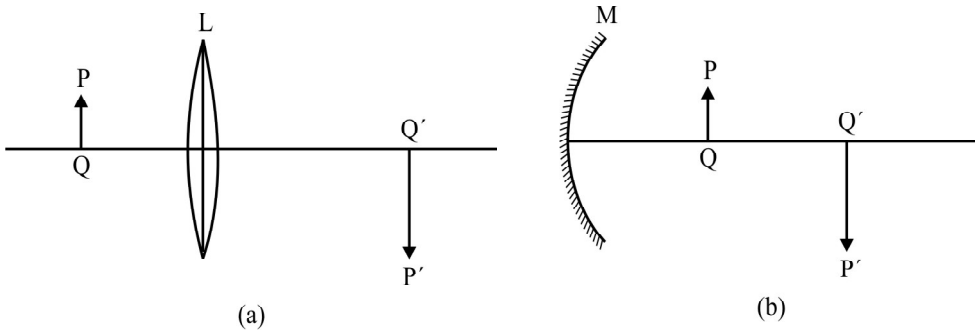


Fig. 61

Correct answer: (3)

Solution:

Both the convex lens and the concave mirror form a real, inverted and enlarged image, when the object is placed in between their f and 2f points for the lens and between F and C points for the mirror.

Unit–16 : Optics

Average *Refraction*

53. The focal length, of a thin biconvex lens is 20 cm. When an object is moved from a distance of 25 cm in front of it to 50 cm, the magnification of its image changes is m_1 to m_2. The ratio $\left(\frac{m_1}{m_2}\right)$ is

(1) 4 **(Incorrect)**

(2) 6 **(Correct)**

(3) 1 **(Incorrect)**

(4) 3 **(Incorrect)**

(I.I.T 2010)

Correct answer: (2)

Solution:

For the first position of the object, let us calculate m_1. Here

$u = -25$ cm; $f = 20$ cm. Using lens formule $\frac{1}{f} = -\left(\frac{1}{u}\right) + \frac{1}{v}$, we have

$$\frac{1}{20} = -\left(\frac{1}{-25}\right) + \frac{1}{v}$$

or $$\frac{1}{v} = \frac{1}{20} - \left(\frac{1}{25}\right) = \frac{1}{100}$$

$\therefore$ $v = 100$ cm

The magnification m_1 is

$$m_1 = -\left(\frac{v}{u}\right) = \left(\frac{100}{-25}\right) = -4$$

Let us now calculate magnification m_2 for the second position of object. We have

$u = -50$ cm, and $f = 20$ cm

$\therefore$ $$\frac{1}{20} = -\left(\frac{1}{-50}\right) + \frac{1}{v}$$

or $$\frac{1}{v} = \frac{1}{20} - \left(\frac{1}{50}\right) = \frac{3}{100}$$

$\therefore$ $$v = \frac{100}{3}\text{cm}$$

The magnification m_2 is

$$m_2 = \frac{v_2}{u_2} = \frac{\left(100/3\right)}{-50} = -\left(\frac{2}{3}\right)$$

$\therefore \quad \frac{m_1}{m_2} = \frac{4 \times 3}{2} = 6$

Hence option (2) is correct

Unit–16 : Optics

Easy *Lenses*

54. A convex lens has a circular aperture of radius r and focal length f. The lens form a real image of on object. Let v and I denote the position and intensity of image formed. The central part, of the lens, of radius $\left(\frac{r}{2}\right)$ is now painted black. Let v' and I' denote the position and intensity of image formed now. We would than have

(1) $v' = v,\ I' = I$ **(Incorrect)**

(2) $v' = v;\ I' = \frac{I}{2}$ **(Incorrect)**

(3) $v' = 2v;\ I' = \frac{I}{2}$ **(Incorrect)**

(4) $v' = v;\ I' = \frac{3I}{4}$ **(Correct)**

Correct answer: (4)

Solution:

Painting the central part, of radius $r/2$, does not change the focal length of the lens. Hence $v' = v$. The area of the aperture of the lens from which light passes, when central part has been painted black, is now $= \pi r^2 - \pi \left(r/2\right)^2 = \left(3/4\right)\pi r^2 = \left(3/4\right) A$. The intensity I', of the image, is therefore $\left(3/4\right)$ of its earlier value, i.e., $I' = \left(3/4\right) I$.

Unit–16 : Optics

Difficult *Lenses*

55. An equiconvex lens has surfaces of radius of curvature 40 cm each. It is made of two parts as shown in Fig. 62. An object is kept at a distance of 80 cm from the lens. Which of the

following statements is correct?

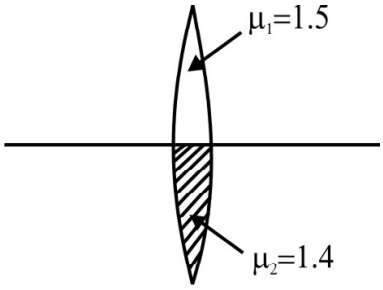


Fig. 62

(1) There is one image at a distance of 80 cm only. (Incorrect)

(2) There is one image at a distance of 133.3 cm only. (Incorrect)

(3) There is one image at a distance of 80 cm and another image that 20 cm away from the first image. (Incorrect)

(4) There are two images that are 53.3 cm away from each other. (Correct)

Correct answer: (4)

Solution:

Let f_1 and f_2 respectively be the focal length of the two parts of lens, made from material of $\mu_1 = 1.5$ and $\mu_2 = 1.4$. Then from Lens Maker's formula,

$$\frac{1}{f_1} = (1.5-1)\left(\frac{1}{40}+\frac{1}{40}\right) \quad \text{or} \quad f_1 = 40\,\text{cm}$$

and $$\frac{1}{f_2} = (1.4-1)\left(\frac{1}{40}+\frac{1}{40}\right) \quad \text{or} \quad f_2 = 50\,\text{cm}$$

For both the parts of lens, of focal length f_1 and f_2, u = –80 cm. Le v_1 and v_2 be the position of the image due to two parts of the lenser. Then

$$\frac{1}{v_1} - \left[\frac{1}{-80}\right] = \frac{1}{40} \quad \text{or} \quad v_1 = 80\,\text{cm}$$

and $$\frac{1}{v_2} - \left[\frac{1}{-80}\right] = \frac{1}{50} \quad \text{or} \quad v_2 = 133.3\,\text{cm}$$

The distance between the two images = (133.3 – 80) cm = 53.3 cm

Unit–16 : Optics

Average *Lens*

56. A thin convex lens made from crown glass $\left(\mu = \frac{3}{2}\right)$ has a focal length f. When immersed (one by one) in two different liquids, having refractive indices $\frac{4}{3}$ and $\frac{5}{3}$; it has focal length f_1 and f_2, respectively. The correct relation, between the three lengths, is

(1) $f_1 = f_2 < f$ **(Incorrect)**

(2) $f_1 > f_2$; f_2 **becomes negative** **(Correct)**

(3) $f_2 > f$; **and** f_1 **becomes negative** **(Incorrect)**

(4) f_1 **and** f_2 **both become negative** **(Incorrect)**

(I.I.T Mains 2014)

Correct answer: (2)

Solution:

From lens maker's formula

$$\frac{1}{f} = (1.5-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= 0.5\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad .....(1)$$

$$\frac{1}{f_1} = \left(\frac{1.5}{4/3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= 0.125\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad .....(2)$$

$$\frac{1}{f_2} = \left(\frac{1.5}{5/3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (-0.1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{.....(3)}$$

From Eqn. (1) and (2)

$$\frac{1}{f_1} < \frac{1}{f} \qquad \therefore \quad f_1 > f$$

From Eqn. (3), f_2, has a negative value.

Unit–16 : Optics

Difficult *Refraction at a Lens*

57. The image of an object formed by a plano–convex lens, is real at a distance of 8m behind the lens and ⅓ the size of the object. The wave length of (a given mon–chromatic) light; inside the lens, is ⅔ times its wave length in free space.The radius of curvature of the curved surface, of the plano–convex lens, is

(1) 1 m **(Incorrect)**

(2) 2 m **(Incorrect)**

(3) 3 m **(Correct)**

(4) 6 m **(Incorrect)**

Correct answer: (3)

Solution:

Let f be the focal length of lens and u the object position. Given

$$v = +8 \text{ m}; \; m = -\left(\frac{1}{3}\right)$$

$$\therefore \quad \frac{v}{u} = -\left(\frac{1}{3}\right) \quad \text{or} \quad u = 3v = -24\text{m}$$

Using lens formula $\frac{1}{v} - \left(\frac{1}{u}\right) = \frac{1}{f}$ we have

$$\frac{1}{8} - \left[\frac{1}{-24}\right] = \frac{1}{f} \qquad \therefore \quad f = 6\,\text{m}$$

The refractive index, μ of the lens material, is

$$\mu = \frac{\text{wavelength in air}}{\text{wavelength in side lens}} = \frac{1}{(2/3)} = \frac{3}{2}$$

Let R be the radius of curvature of the curved surface of the lens. From Lens Maker's formula,

$$\frac{1}{6} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R} - \frac{1}{\infty}\right)$$

$\therefore \quad R = 3\,m$

Unit–16 : Optics

Difficult *Lens*

58. A biconvex lens is formed by joining two thin plano–convex lenses, as shown in Fig. 63. Refractive index of the material of the first lens, n_1 = 1.5 and that of the material of the second lens, is n_2 = 1.2. Both curved surfaces have the same radius of curvature, 14 cm each. For this bi–convex lens, for an object distance of 40 cm; the image distance will be

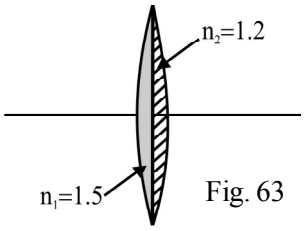


Fig. 63

(1) –280.0 cm **(Incorrect)**

(2) + 40.0 cm **(Correct)**

(3) +21.5 cm **(Incorrect)**

(4) +13.3 cm **(Incorrect)**

(I.I.T 2012)

Correct answer: (2)

Solution:

Let f_1 and f_2 be the focal length of the two plano–convex lenses. From lens maker's formula

$$\frac{1}{f_1} = (1.5 - 1)\left(\frac{1}{14} - \frac{1}{\infty}\right) \qquad \therefore \quad \frac{1}{f_1} = \frac{1}{28}\ cm^{-1}$$

and $$\frac{1}{f_2} = (1.2 - 1)\left[\frac{1}{\infty} - \left(\frac{1}{-14}\right)\right] = \frac{1}{70}\ cm^{-1}$$

Let f be the focal length of the bi–convex lens i.e the combination of two plano convex lens in contact. Then

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{28}+\frac{1}{70}$$

$\therefore \quad f = 20\,cm$

Given u = – 40cm, v = ? Using the thin lens formula, we get

$$\frac{1}{v}-\left[\frac{1}{-40}\right]=\frac{1}{20}$$

$\therefore \quad v = 40\,cm$

Unit–16 : Optics

Difficult *Lenses*

59. Fig. 64 shows a thin lens for which the radii of curvature of two surfaces are equal. Also $\mu_1 = 1.2$; $\mu_2 = 1.5$, $\mu_3 = 1.7$. A parallel beam of light, incident in μ_1, is bright to focus, in medium μ_3, at a distance of 34 cm from the lens. The radius of curvature of the surfaces of lens is

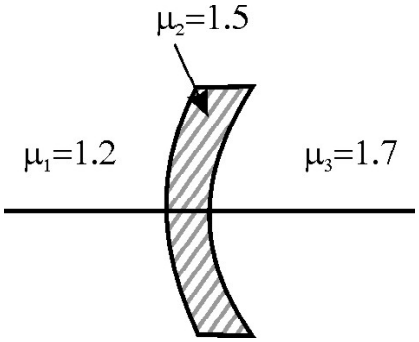


Fig. 64

(1) 10 cm **(Correct)**

(2) 12 cm **(Incorrect)**

(3) 11.33 cm **(Incorrect)**

(4) 15 cm **(Incorrect)**

Correct answer: (2)

Solution:

Let c be the magnitude of the radius of each surface of lens. Had there been only one surface; the incident parallel beam, after refraction would have met the principal axis at Q_1, at a distance b_1 from this surface.

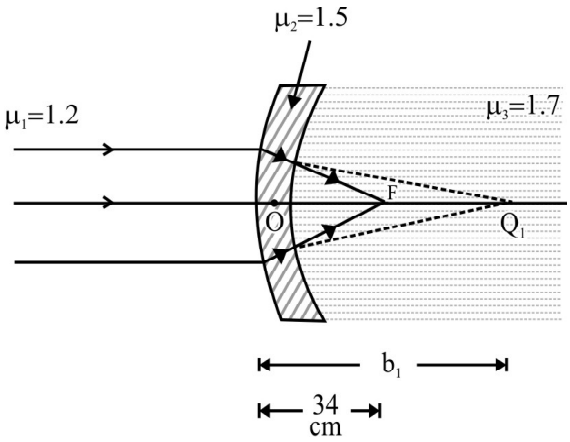


Fig. 65

Form refraction at first surface of lens we have

$$\frac{1.5}{+b_1}-\left[\frac{1.2}{-\infty}\right]=\frac{1.5-1.2}{c}$$

$\therefore \quad b_1 = 5c$ .....(1)

For refraction at second surface, Q_1 acts as a virtual object, forming the real image at F. Therefore,

$u_2 = +b_1$; $v_2 = +34$ cm; $R_2 = +c$

From refraction formula; we now get

$$\frac{1.7}{34}-\left[\frac{1.5}{b_1}\right]=\frac{1.7-1.5}{c}$$

$$\frac{1}{20}-\left[\frac{3}{2\times 5c}\right]=\frac{2}{10c}$$

$\therefore \quad c = 10\,cm$

Unit–16 : Optics

Average *Lenses*

60. A lens, having a focal length f, is made of a material of dispersive power ω. A parallel beam of white light parallel to the principal axis, is incident on the lens. The violet and red coloured rays are brought to focus at distances *x* and y from the lens. Then

(1) $x = y$ **(Incorrect)**

(2) $x < y$ **(Incorrect)**

(3) $y - x = \omega f$ **(Correct)**

(4) $y - x = \frac{\omega}{f}$ **(Incorrect)**

Correct answer: (3)

Solution:

Let f_V and f_R be the focal length of lens, for violet and red colour, respectively. We know $\mu_V > \mu_R$ [Cauchy's relation]. From lens maker's formula, we get

$$\frac{1}{f_V} > \frac{1}{f_R} \qquad \text{or} \qquad f_V < f_R$$

$\therefore \quad x < y$

Also $y - x = |df| = (f_R - f_V)$. From lens maker's formula, we have

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Differentiating, we get

$$-\left(\frac{df}{f^2}\right) = (d\mu)\left[\frac{1}{R_1} - \left(\frac{1}{R_2}\right)\right]$$

$$= \left[\frac{d\mu}{\mu - 1}\right]\left[(\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\right] = \frac{\omega}{f}$$

$\therefore \quad |df| = \omega f$

Unit–16 : Optics

Average *Lenses*

61. A convex lens is in contact with a concave lens. The magnitude, of the ratio of their focal lengths, is $\left(\frac{2}{3}\right)$. Their equivalent focal length is 30 cm. The individual focal lengths (in cm), of the two lenses, are, respectively,

(1) –75; 50 **(Incorrect)**

(2) –10; 15 **(Incorrect)**

(3) 75; 50 **(Incorrect)**

(4) 10; –15 **(Correct)**

Correct answer: (4)

Solution:

Let f_1 and f_2 be the, respective, focal lengths of the convex and concave lens. Given

$$\frac{f_1}{|f_2|} = \frac{2}{3}$$

$$\therefore \quad |f_2| = \frac{3}{2} f_1$$

The equivalent focal length, F, of combination is +30 cm. Therefore,

$$\frac{1}{30} = \frac{1}{f_1} + \frac{1}{\left(-3f/2\right)}$$

$$= \frac{1}{f_1} - \left(\frac{2}{3f_1}\right) \quad \therefore \quad f_1 = +10\,\text{cm}$$

Hence $f_2 = -\frac{3}{2} \times 10\,\text{cm} = -15\,\text{cm}$

Unit–16 : Optics

Difficult *Lenses*

62. A plano convex lens, if silvered on its plane surface, is equivalent to a concave mirror of focal length 28 cm. If, however, its curved surface (only) is silvered it is equivalent to a concave mirror of focal length 10 cm. The refractive index, of the material of lens, is

(1) $\frac{3}{2}$ **(Incorrect)**

(2) $\frac{14}{9}$ **(Correct)**

(3) $\frac{17}{9}$ **(Incorrect)**

(4) $\frac{14}{5}$ **(Incorrect)**

Correct answer: (2)

Solution:

Let c be the radius of curvature of curved surface of plano convex lens of focal length f (say), made from a material of refractive index μ. From lens maker's formula, we then have

$$\frac{1}{f} = \frac{(\mu - 1)}{c} \qquad .....(1)$$

The focal length, F, of the equivalent mirror, when one of the two surfaces of the lens is silvered is

$$\frac{1}{F} = \frac{2}{f} + \frac{1}{f_m} \qquad .....(2)$$

Here f_m is the focal length of the mirror formed due to silvered surface of lens. When the plane surface of the plano convex lens is silvered, $f_m = \infty$ and F = 28 cm given. Therefore

$$\frac{1}{28} = \frac{2(\mu - 1)}{c} + \frac{1}{\infty}$$

$$\therefore \quad \frac{1}{28} = \frac{2(\mu - 1)}{c} \qquad .....(3)$$

When the curved surface of lens is silvered $f_m = \frac{c}{2}$ and F = 10 cm (given). Therefore,

$$\frac{1}{10} = \frac{2(\mu - 1)}{c} + \frac{2}{c} \qquad .....(4)$$

From Eqns. (3) and (4), we have

$$\frac{1}{10} - \left(\frac{1}{28}\right) = \frac{2}{c} \quad \text{or} \quad \frac{2}{c} = \frac{18}{280} \qquad .....(5)$$

From Eqns. (3) and (5), we have

$$\frac{1}{28}=\frac{18}{280}\left[(\mu-1)\right]$$

or $$\frac{10}{18}=\mu-1$$

$$\therefore \quad \mu=\frac{28}{18}=\frac{14}{9}$$

Unit–16 : Optics

Difficult *Lenses*

63. The refractive indices of material of an equi–concave lens, of radius of curvature c, for wave lengths λ_1 and λ_2 are n_1 and n_2, respectively. The corresponding refractive indices of an equi–concave lens, having radius of curvature, d, are m_1 and m_2. The two lens are placed co–axially in contact with each other. A parallel beam, containing light of wave lengths λ_1 and λ_2 is incident on the combination. It is observed that both the wave lengths are brought to focus at the same point on the principle axis. We have

(1) $\frac{n_1-n_2}{m_1-m_2}=\frac{c}{d}$ **(Correct)**

(2) $\frac{n_1}{m_1}=\frac{c+d}{2d}$ **(Incorrect)**

(3) $\frac{n_2}{m_2}=\frac{c}{d}$ **(Incorrect)**

(4) $\frac{n_1-n_2}{m_1-m_2}=\frac{c-d}{2d}$ **(Incorrect)**

Correct answer: (1)

Solution:

Let n and m be the mean values of the refractive indices of the materials of the convex and concave lenses; Let f_1 and f_2 the values of their mean focal lengths. From lens maker's formula:

$$\frac{1}{f_1}=(n-1)\left[\frac{1}{c}+\frac{1}{c}\right] \quad \text{or} \quad f_1=\frac{c}{2(n-1)} \qquad .....(1)$$

Similarly

$$\frac{1}{f_2}=(m-1)\left[\left(\frac{1}{-d}\right)-\frac{1}{d}\right] \qquad \text{or} \qquad f_2=\frac{d}{2(m-1)} \qquad .....(2)$$

Let ω_1 and ω_2 be the dispersive powers of the materials of the convex and concave lenses. Then

$$\omega_1=\frac{(n_1-n_2)}{(n-1)};\quad \omega_2=\frac{(m_1-m_2)}{(m-1)}$$

For the combination, the given condition is the condition for no net dispersion. It is satisfied, if

$$\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}=0$$

$$\therefore \quad \frac{(n_1-n_2)}{(n-1)}\frac{2(n-1)}{c}=\frac{(m_1-m_2)}{(m-1)}\times\frac{2(m-1)}{d}$$

$$\text{or} \qquad \frac{n_1-n_2}{m_1-m_2}=\frac{c}{d}$$

Unit–16 : Optics

Average *Refraction*

64. A convex lens is placed above an empty tank of depth 45 cm. The lens forms an image of a point, on the bottom of the tank 36 cm above the lens. A transparent liquid is now poured into tank upto a depth of 40 cm. The distance of the image of the same point, is now 12 cm above its earlier position. The refractive index, of the liquid is

(1) 1.30 **(Incorrect)**

(2) 1.36 **(Correct)**

(3) 1.40 **(Incorrect)**

(4) 1.414 **(Incorrect)**

Correct answer: (2)

Solution:

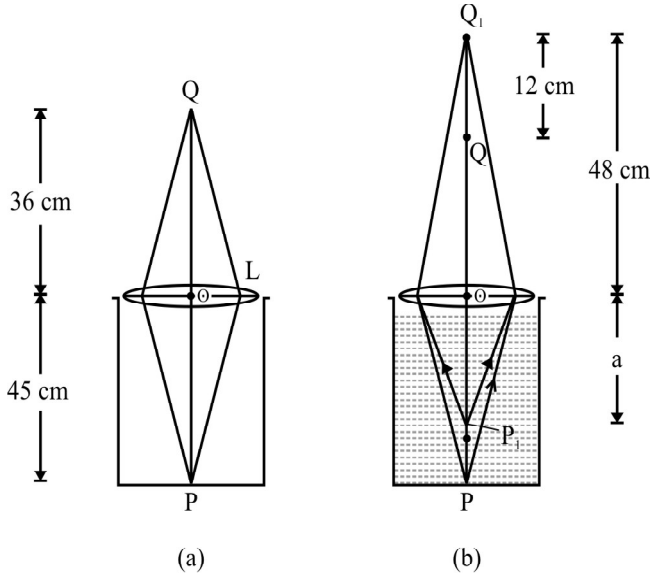


Fig. 66

From Fig. 66 (a); $u_1 = -45$ cm, $v_1 = +36$ cm, $f = ?$

$$\therefore \quad \frac{1}{36} - \left[\frac{1}{-45}\right] = \frac{1}{f} \text{ or } f = 20 \text{ cm}$$

In Fig. 65 (b) liquid has been poured into the tank. Due to refraction, inside the liquid, P appears to have shifted to portion P_1. For refraction from lens now, we have

$u_2 = -a$, $v_2 = +48$ cm; $f = 20$ cm

$$\therefore \quad \frac{1}{48} - \left[\frac{1}{-a}\right] = \frac{1}{20} \qquad \text{or} \qquad a = \frac{240}{7} \text{ cm}$$

$$\therefore \quad \mu = \text{refractive index of liquid} = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{45}{a}$$

$$= \frac{45 \times 7}{240} \simeq 1.36$$

Unit–16 : Optics

Difficult *Lens*

65. The aperture, of a given plano convex lens, is 6 cm and its thickness, at the center, is 3 mm. If speed of light, in the material of lens, is 2×10^8 m/s, the focal length of the lens, is

(1) 10 cm (Incorrect)

(2) 15 cm (Incorrect)

(3) 20 cm (Incorrect)

(4) 30 cm (Correct)

Correct answer: (4)

Solution:

Fig. 67 shows the plano convex lens AO_1BOA. Given

$AB = 2\ OB = 6$ cm

$\therefore$ $OB = 3$ cm

Also, $OO_1 = 3$ mm $= 0.3$ cm

Let BC (= R) be the radius of curvature of the curved surface of lens. From right angled triangle OBC, we have

$$R^2 = (3)^2 + (R-0.3)^2$$

$\therefore$ $R = 15$ cm

μ = The refractive index of the material of lens

$$= \frac{c}{v} = \frac{3\times10^8}{2\times10^8} = 1.5$$

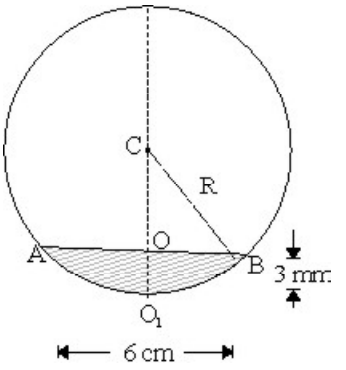


Fig. 67

The focal lenth, f of the lens is given by

$$\frac{1}{f} = (1.5-1)\left[\frac{1}{15} - \left(\frac{1}{\infty}\right)\right]$$

$\therefore$ $f = 30$ cm

Unit–16 : Optics

Difficult *Lens*

66. A tank is filled with water $\left(\mu = \frac{4}{3}\right)$ up to 18 cm. An oil $\left(\mu = \frac{7}{4}\right)$, spread on the surface of water, forms a convex surface of radius 6 cm. Consider the oil as a thin lens. An object, S, is placed 24 cm above the surface of water. The distance of the image formed, from the

bottom of the tank, is

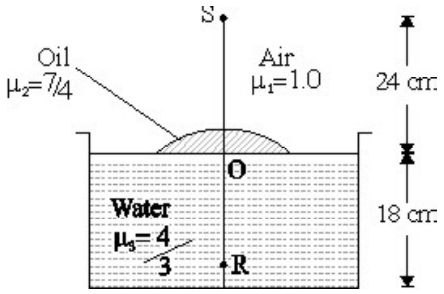


Fig. 68

(1) 16 cm **(Incorrect)**

(2) 2 cm **(Correct)**

(3) 12 cm **(Incorrect)**

(4) 4 cm **(Incorrect)**

Correct answer: (2)

Solution:

Let f be focal length of the oil lens. Here

μ_1 = refractive index of object space = 1.00

μ_3 = refractive index of image space = $4/3$

μ_2 = refractive index of the material of lnes = $7/4$

For the plano–convex oil–lens, we then have

$$\frac{\mu_3}{f}=\frac{\left(\frac{7}{4}-1\right)}{(+6)}+\frac{\left(\frac{4}{3}-\frac{7}{4}\right)}{\infty}=\frac{3}{24}\text{cm} \qquad .....(1)$$

Also u = – 24 cm, v = ? Using the refraction formula, $\left(\frac{\mu_3}{v}-\frac{\mu_1}{u}=\frac{\mu_3}{f}\right)$

We have

$$\frac{(4/3)}{v}-\left[\frac{1}{-24}\right]=\frac{3}{24}; \qquad \therefore \quad v=16\,\text{cm}$$

The distance of image (R) formed from the oil lens, is 16 cm. Hence its distance, from the botom of the tank = (18–16) cm = 2 cm

Unit–16 : Optics

Average *Lenses*

67. An equiconvex lens is made of a material of refractive index μ_1. The power of lens is P_1. A plano–concave lens is put in contact with this convex lens, co–axially, as shown in Fig. 69. If the power of combination is P, the ratio, $\frac{P}{P_1}$, is:

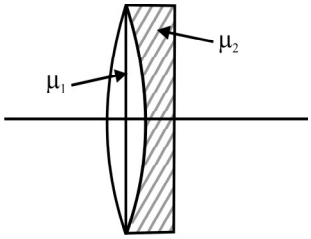


Fig. 69

(1) $\left[\frac{2(\mu_1-\mu_2)-1}{(\mu_1-1)}\right]$ **(Incorrect)**

(2) $\left[\frac{(2\mu_1-\mu_2)-1}{2(\mu_1-1)}\right]$ **(Correct)**

(3) $\left[\frac{2(\mu_1+\mu_2)-1}{(\mu_1-1)}\right]$ **(Incorrect)**

(4) $\left[\frac{2(\mu_1+\mu_2)-1}{2(\mu_1-1)}\right]$ **(Incorrect)**

Correct answer: (2)

Solution:

Let (in meter) c be the radius of curvature of each surface of the equi–convex lens. Then

$$P_1=\frac{1}{f_1}=(\mu_1-1)\left(\frac{1}{c}+\frac{1}{c}\right)=\frac{2(\mu_1-1)}{c} \qquad .....(1)$$

The power, P_2, of the plano–concave lens, is

$$P_2=\frac{1}{f_2}=(\mu_2-1)\left[\frac{1}{(-c)}+\frac{1}{\infty}\right]=-\left[\frac{\mu_2-1}{c}\right] \qquad .....(2)$$

The power, P, of the combination is

$$P = P_1 + P_2 = \frac{2(\mu_1 - 1)}{c} - \left(\frac{\mu_2 - 1}{c}\right)$$

$$= \frac{(2\mu_1 - \mu_2 - 1)}{c} \qquad .....(3)$$

$$\therefore \quad \frac{P}{P_1} = \frac{\left[(2\mu_1 - \mu_2) - 1\right]}{\left[2(\mu_1 - 1)\right]}$$

Unit–16 : Optics

Average *Lens*

68. An equiconvex lens, of focal length 20 cm, is made of glass $(\mu = 1.5)$. The lens is placed so that its one surface is in contact with air and its other, with a transparent medium, of refractive index 1.3. A parallel beam is incident on the lens from the air side. The emergent beam, would then meets the principle axis, at a distance *x* from lens, where *x* is

(1) 14.3 cm (Incorrect)

(2) 18.6 cm (Correct)

(3) 20.0 cm (Incorrect)

(4) 30.0 cm (Incorrect)

Correct answer: (2)

Solution:

Let c be the radius of curvature of each surface of the lens. From lens maker's formula, we have

$$\frac{1}{20} = (1.5 - 1)\left[\frac{1}{c} - \left(\frac{1}{-c}\right)\right] = \frac{1}{c}$$

$\therefore \quad c = 20\,\text{cm}$

Fig. 70 shows lens placed with its second surface in contact with the medium of refractive index, $\mu_3 = 1.3$.

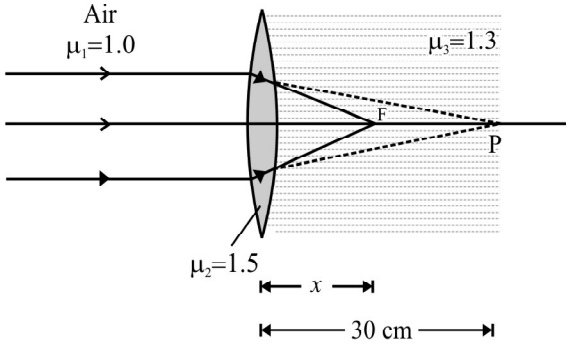


Fig. 70

Consider the refraction at the first surface of the lens.

Here $\mu_1 = 1.0,\ \mu_2 = 1.5,\ u_1 = -\infty,\ v_1 = ?$

$$\therefore \quad \frac{1.5}{v_1} - \left(\frac{1}{-\infty}\right) = \frac{1.5-1}{10}$$

$$\therefore \quad v_1 = 30\,\text{cm}$$

Had there been only this refracting surface, the refracted raywould have meet the principal axis at P. For the second surface of the lens, P acts as a virtual object; which forms the final image at F. Now

$$\frac{\mu_3}{v_2} - \left(\frac{\mu_2}{u_2}\right) = \frac{\mu_3 - \mu_2}{R_2}$$

Here $\mu_3 = 1.3,\ \mu_2 = 1.5,\ \upsilon_2 = +30\text{cm},\ v_2 = +x,\ R_2 = -20\,\text{cm}$

$$\therefore \quad \frac{1.3}{x} - \left(\frac{1.5}{30}\right) = \frac{1.3-1.5}{(-10)}$$

$$\therefore \quad x = 18.6\,\text{cm}$$

Unit–16 : Optics

Difficult *Thin Lenses*

69. A planoconvex lens is made of a material of refractive index n. When a small object is placed 30 cm away, in front of the curved surface of the lens, an image, of double its size, is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm from the lens. The refractive index, n, and the focal length f of lens are

(1) n = 2.5; f = 20 cm **(Correct)**

(2) n = 2.0; f = 20 cm **(Incorrect)**

(3) n = 2.5; f = 10 cm **(Incorrect)**

(4) n = 1.5; f = 20 cm **(Incorrect)**

(I.I.T Adv. 2016)

Correct answer: (1)

Solution:

For image, due to refraction by the convex lens, we have

$$u = -30\,cm;\ \ m = -2 = \frac{v}{u}$$

$\therefore \quad v = +60\,cm$

Using the lens formula $\frac{1}{v} - \left(\frac{1}{u}\right) = \frac{1}{f}$, we have

$$\frac{1}{60} - \left(\frac{1}{-30}\right) = \frac{1}{f}$$

$\therefore \quad f = +20\,cm$ .....(1)

Let R be the radius, of curvature of the curved suface of the lens. From lens–maker's formula;

$$\frac{1}{20} = (n-1)\left(\frac{1}{R}\right)$$ .....(2)

The faint image formed is due to reflection from the curved surface of the lens. We now have to use the 'mirror formula'. Here

$$u' = -30\,cm,\ \ v' = +10\,cm$$

$$\therefore \quad \frac{1}{10} + \left(\frac{1}{-30}\right) = \frac{2}{R} \quad \therefore \quad R = 30$$ .....(3)

From Eqns. (2) and (3), we have

$$\frac{1}{20} = (n-1) \times \frac{1}{30} \qquad \therefore \quad n = 2.5$$

Unit–16 : Optics

Difficult *Lenses*

70. A plano convex lens is placed on a plane mirror as shown in Fig. 71. For a pin, placed at a distance *x*, there is no parallax between the pin and its image formed by the system. The space between the plane mirror, and the curved surface of lens is now filled with a transparent

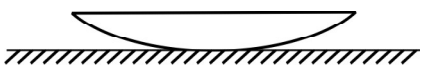


Fig. 71

liquid, forming a liquid lens. The pin has to be raised through a distance y so that again there is no parallax between the pin and its image formed by the (new) system. The focal length of the plano–conves lens, and the liquid lens, are, respectively,

(1) $\mathbf{x;\ -\left[\frac{xy}{x-y}\right]}$ **(Incorrect)**

(2) $\mathbf{y;\ -\left(\frac{y}{x}\right)}$ **(Incorrect)**

(3) $\mathbf{(x-y);\ -\left(\frac{x+y}{y}\right)}$ **(Incorrect)**

(4) $\mathbf{x;\ -\left[\frac{x(x+y)}{y}\right]}$ **(Correct)**

Correct answer: (4)

Solution:

Fig. 72 shows the ray diagram for no parallx between the image and the object pin. The rays, after refraction from the lens, must fall normally on the plane mirror so that they retrace their path. The object pin is therefore, at focus, F_1, of the plano–convex lens. Hence

$$f_1 = x$$

Similarly, for the combination of plano–convex lens and liquid lens, of focal length f_2 (say), the focal length of the combination, $F = x+y$. Now

$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$ therefore

$$\frac{1}{(x+y)} = \frac{1}{x} + \frac{1}{f_2}$$

or $\frac{1}{f_2} = \frac{1}{(x+y)} - \left(\frac{1}{x}\right)$

$\therefore\ f_2 = -\left[\frac{x(x+y)}{y}\right]$

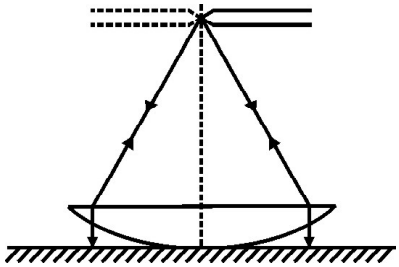


Fig. 72

Unit–16 : Optics

Difficult *Lenses*

71. Two identical equiconvex lens L_1 and L_2, made of a material of refractive index $\mu_1 = 1.5$ and having a focal length, f, each, are placed in contact, co–axially, as shown in Fig. 73 (a). The space between the two lenses, is sealed at the two ends, after being filled with a transparent liquid of referactive index, $\mu_2 = 1.8$, as shown in Fig. 73 (b). The change, in the equivalent focal length, of the arrangment, is

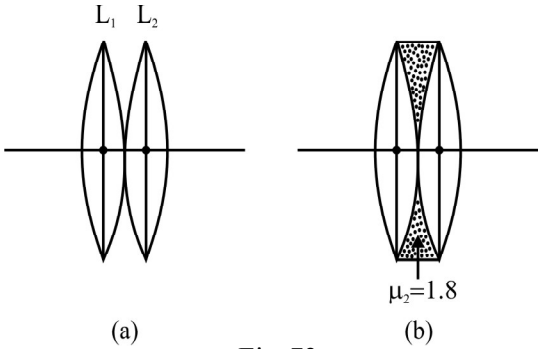


Fig. 73

(1) + f **(Incorrect)**

(2) – 1.5 f **(Incorrect)**

(3) + 2.0 f **(Correct)**

(4) – 2.0 f **(Incorrect)**

Correct answer: (3)

Solution:

The focal length, of each lens, in terms of the equal radii of curvature, c, of its two surfaces, is

$$\frac{1}{f} = (1.5-1)\left(\frac{1}{c}+\frac{1}{c}\right) \quad \text{or} \quad f = c \qquad .....(1)$$

The equivalent focal length, F_1, of the combination shown in Fig. 72 (a), is

$$\frac{1}{F_1} = \left(\frac{1}{f}+\frac{1}{f}\right)$$

$$\therefore \quad F_1 = \frac{f}{2} = 0.5\,f \qquad .....(2)$$

The arrangment, shown in Fig. 72 (b), is a combination of three thin lenses. The additional lens is an equiconcave liquid lens. Let f_3 be the focal length of liquid lens. Then

$$\frac{1}{f_3} = (1.8-1)\left(-\frac{1}{c}-\frac{1}{c}\right)$$

$$= -\left(\frac{1.6}{c}\right) \qquad .....(3)$$

The equivalent focal length, F_1, of the arrangment shown in Fig. (b), is

$$\frac{1}{F_2} = \frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3} = \frac{1}{c}+\frac{1}{c}+\left(-\frac{1.6}{c}\right)$$

$$= \frac{04}{c}$$

$\therefore$ $F_2 = 2.5c = 2.5\ f$ .....(4)

The change in focal length $= F_2 - F_1 = 2.5\ f - 0.5\ f$

$= +\ 2.0\ f$

Unit–16 : Optics

Difficult *Lens*

72. Two thin lenses, of focal length, f_1, and, f_2, are made of materials of dispersive powers, ω_1 and ω_2, respectively. A beam of white light, parallel to their common principal axis, is incident on the combination. If the violet and red coloured rays meet at same point on the principal axis, we must have

(1) $\omega_1 f_1 + \omega_2 f_2 = 0$ **(Incorrect)**

(2) $\omega_1 f_1 \sim \omega_2 f_2 = 0$ **(Incorrect)**

(3) $\frac{\omega_1}{f_1} - \frac{\omega_2}{f_2} = 0$ **(Incorrect)**

(4) $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$ **(Correct)**

Correct answer: (4)

Solution:

The equivalent focal length, F, of the combination, is

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2} \qquad .....(1)$$

Since violet and red rays after refraction, meet at the same point on the principal axis, $|dF| = F_V - F_R = 0$

Differentiating Eqn. (1), we have

$$\frac{dF}{F_1^2}=\frac{df_2}{f_2^2}+\frac{df_1}{f_1^2}=0$$

Also $(df_1)=\omega_1 f_1$ and $(df_2)=\omega_2 f_2$. Therefore, we must have

$$\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}=0$$

Unit–16 : Optics

Difficult *Lenses*

73. A thin convex lens of focal length 20 cm is placed co–axially at a distance, ***x*, from a concave lens of focal length 40 cm. The combination assumed to behave like a thin lens, forms a real image, of same size as the object when an object is placed at a distance of 60 cm from it. The distance *x* is**

(1) 5.0 cm **(Incorrect)**

(2) 6.67 cm **(Correct)**

(3) 7.5 cm **(Incorrect)**

(4) 10 cm **(Incorrect)**

Correct answer: (2)

Solution:

Let f_1 and f_2 be the focal lengths of the two lenses, placed co–axially, a distance *x* apart. The combination behaves like a lens of focal length F, where

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\left(\frac{x}{f_1 f_2}\right)$$

Given $f_1 = +20$ cm; $f_2 = -40$ cm

$$\therefore \quad \frac{1}{F} = \frac{1}{20} - \left(\frac{1}{40}\right) + \frac{x}{20 \times 40}$$

$$= \frac{1}{40} + \frac{x}{20 \times 40} \qquad \text{.....(1)}$$

For the equivalent thin lens; $u = -60$ cm, $v = +60$cm, $f = F$

$$\therefore \quad \frac{1}{60} + \frac{1}{60} = \frac{1}{F} \qquad \text{or} \qquad \frac{1}{F} = \frac{1}{30} \qquad \text{.....(2)}$$

From Eqn. (1) and (2), we have

$$\frac{1}{40} + \frac{x}{20 \times 40} = \frac{1}{30}$$

$$\therefore \quad x = \frac{20}{3} \text{ cm} = 6.67 \text{ cm}$$

Unit–16 : Optics

Difficult *Lenses*

74. A point object, P, is placed at a distance of 20 cm from a thin plano–convex lens of (original) focal length 15 cm, whose plane surface has been silvered. The image, formed by the system, is at

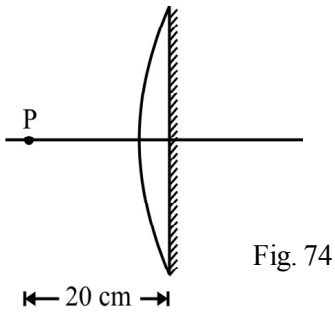


Fig. 74

(1) 60 cm to left of the system **(Incorrect)**

(2) 60 cm to right of the system **(Incorrect)**

(3) 12 cm to the left of the system **(Correct)**

(4) 12 cm to the right of the system **(Incorrect)**

(I.I.T 2006)

Correct answer: (3)

Solution:

Let f be focal length of plano–convex lens, and f_m the focal length of the mirror, formed due to silvering of the plane surface of the lens. The equivalent focal length, F, of the system is given by

$$\frac{1}{|F|} = \frac{2}{f} + \frac{1}{f_m}$$

Given f = 15 cm; $f_m = \infty$; therefore

$$\frac{1}{|F|} = \frac{2}{15} + \frac{1}{\infty} \quad \text{or} \quad |F| = 7.5\,\text{cm}$$

For the equivalent system,

u = –20 cm; F = –7.5 cm, v = ?

$$\therefore \quad \frac{1}{v} + \left(\frac{1}{-20}\right) = -\left(\frac{2}{15}\right)$$

or $v = -12\,\text{cm}$

The –ve sign indicates that the image is real, i.e. to the left of the system.

Unit–16 : Optics

Difficult *Lenses*

75.

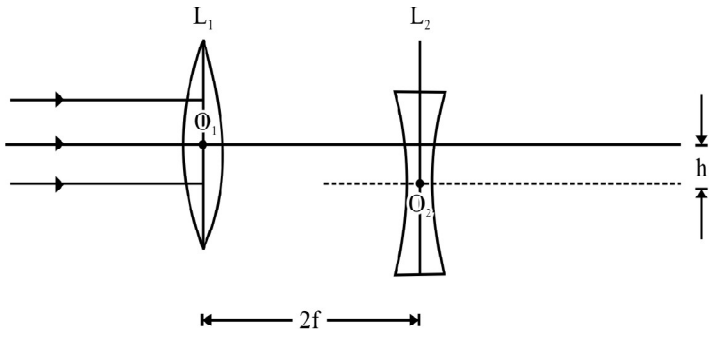


Fig. 75

L_1 is a convex lens of focal length 3f and L_2 is a concave lens of focal length f. The two are placed a distance 2f apart. The principle axis of two lens are parallel, but a distance h apart, as shown in Fig. 75. A parallel beam is incident on L_1. The emergent beam, after refraction at both L_1 and L_2, is

(1) A parallel beam parallel to axis of L_1 and L_2 (Incorrect)

(2) A parallel beam, making an angle of $\tan^{-1}\left(\frac{h}{y}\right)$, in the anticlockwise direction, with the principal axis (Correct)

(3) A parallel beam, making an angle of $\tan^{-1}\left(\frac{h}{y}\right)$, in the clockwise direction, with the principle axis (Incorrect)

(4) A divergent beam (Incorrect)

Correct answer: (2)

Solution:

Fig. 76 shows the ray diagram. The incident parallel beam, after refraction at L_1 would have meet its principle axis at point P. The point, P, acts as a virtual object for the concave lens, L_2, and is lying off the axis of L_2, in its focal plane. The emergent beam is, therefore, a parallel beam, inclined to the principle axis of L_2, at an angle θ. From right angled triangle O_2PF_2;

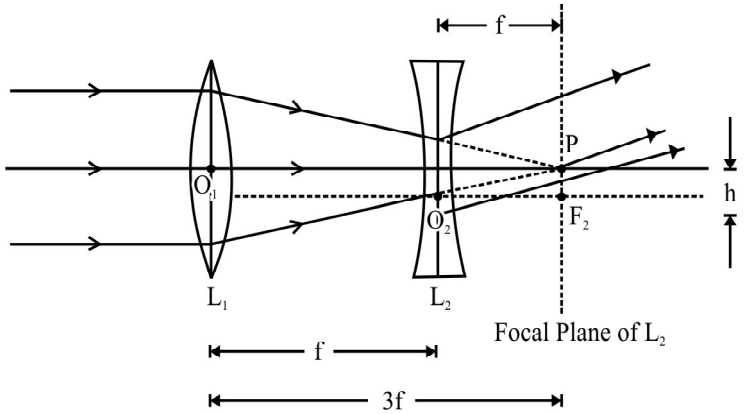


Fig. 76

$$\tan\theta = \frac{F_2P}{O_2F_2} = \frac{h}{f}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{h}{f}\right)$$

Unit–16 : Optics

Average *Use of Lenses*

76. A lens, of power P, forms a virtual image of an object, (kept at a distance of 25 cm) at a distance of 40 cm from it. The same lens forms a virtual image, of another object (kept at a distance z) at a distance of 250 cm from the lens.

The values of P and z are, respectively,

(1) 1.5 D and 250 cm **(Incorrect)**

(2) 0.5 D and 53 cm **(Incorrect)**

(3) 1.5 D and 53 cm **(Correct)**

(4) 1.5 D and 40 cm **(Incorrect)**

Correct answer: (3)

Solution:

We have here u = –25 cm and v = – 40 cm. Hence,

$$\frac{1}{f} = -\left(\frac{1}{40}\right) + \frac{1}{25} = \frac{3}{200}$$

or $$f = \frac{200}{3}\text{cm} = \frac{2}{3}\text{m}$$

The power, P, of the lens used is $\frac{1}{f}$ (f, in meter) $= \frac{3}{2}$ D = 1.5 D

We also have; v = – 250 cm for u = – z cm. Hence,

$$\left(\frac{1}{-250}\right) - \left(\frac{1}{z}\right) = \frac{3}{200}$$

Solving we obtain

$$z \simeq -53\,\text{cm}$$

Hence option (3) is the correct option.

Unit–16 : Optics

Difficult *Lenses*

77. A convex lens of focal length 15 cm, has a point object P placed at a distance of 25 cm on its principle axis, X'OX. The lens is rotated about point O through an angle $\theta = 2.90^0$ as shown in Fig, 77 which of the following statements is correct for the image formed by the lens in its rotated position?

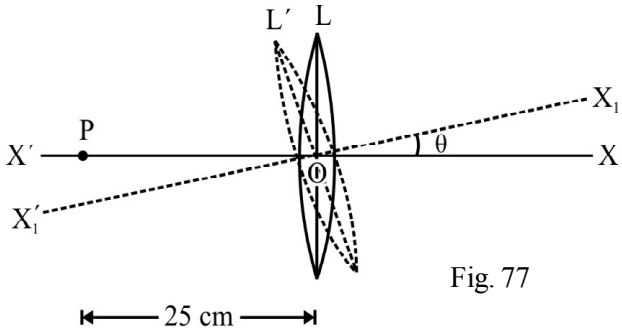


Fig. 77

(1) Point image, at a distance 37.5 cm on $X_1'OX_1$ (Incorrect)

(2) Point image, at a distance 37.5 cm on $X'OX$ (Incorrect)

(3) Point image, at a distance 37.5 cm on $X_1'OX_1$, at a perpendicular distance of 1.5 cm, above $X_1'OX_1$ (Incorrect)

(4) Point image, at a distance 37.5 cm below $X_1'OX_1$, at a perpendicular distance of 1.5 cm (Correct)

Correct answer: (4)

Solution:

For rotated position L' of lens $X_1'OX_1$ is the principal axis. Point object, P on XOX', appears as an object of length P'P for lens L'. The object distance $OP' = OP\cos\theta$. Given $OP = 25$ cm; $\theta = 2.29^0$

$\therefore \quad OP' \simeq OP \simeq 25\,\text{cm}$ [For small angles, $\cos\theta \approx 1$]

Also, $PP' = OP\sin\theta \simeq OP \times \theta$

$$= 25\left(\frac{\pi}{180}\right) \times 2.29\,\text{cm} \simeq 1\,\text{cm}$$

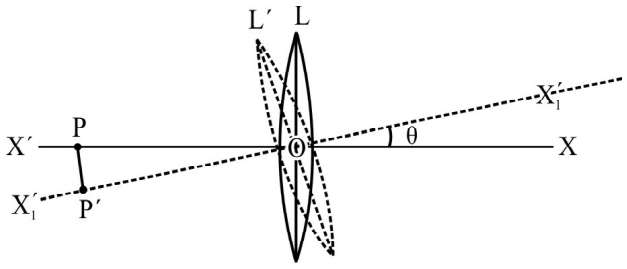


Fig. 78

$\therefore$ For the lens, in position L', we have

$u = -25$ cm, $v = ?$ $f = +15$ cm

$$\therefore \quad \frac{1}{v} - \left(\frac{1}{-25}\right) = \frac{1}{15}$$

$\therefore \; v = 37.5$ cm

Let m be the magnification. Then

$$m = -\left(\frac{v}{u}\right) = -\left(\frac{37.5}{25}\right) = -1.5$$

Negative sign, indicates image is inverted with respect to PP′, i.e., image formed is below $X_1' OX_1$.

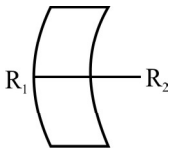
Also length of image $= |m| \times PP'$

$$= 1.5 \times 1\,\text{cm} = 1.5\,\text{cm}$$

Unit–16 : Optics

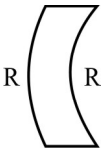
Average *Lenses*

78. Which of the following spherical lens does not exhibit a net dispersion? The radii of curvature of lens surface are as given in the Fig. 78

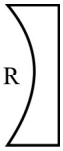
(1) 

(Incorrect)

(2)
(Incorrect)

(3) 

(Correct)

(4) 

(Incorrect)

Correct answer: (3)

Solution:

For case (3), the radii of curvature, of the two lens surfaces, have the same magnitude and same sign. The focal length, f, of this arrangement is therefore, infinity. The arrangement behaves like a parallel slab and there is no dispersion as white light passes through it.

For all the other arrangments, f is finite; also the value of f will be different for different colours due to variation of μ with λ. All these arrangements will therefore, cause a dispersion of the white light incident on them.

Unit–16 : Optics

Difficult *Lenses*

79. The size of the image of an object; (which is at infinity) as formed by a convex lens, of focal length 30 cm, is 2 cm. If a concave lens, of focal length 20 cm, is placed between the convex

lens and the image, at a distance of 26 cm from the convex lens, the new size of the image would be

(1) 1.05 cm (Incorrect)

(2) 1.25 cm (Incorrect)

(3) 2.0 cm (Incorrect)

(4) 2.5 cm (Correct)

(I.I.T 2003)

Correct answer: (4)

Solution:

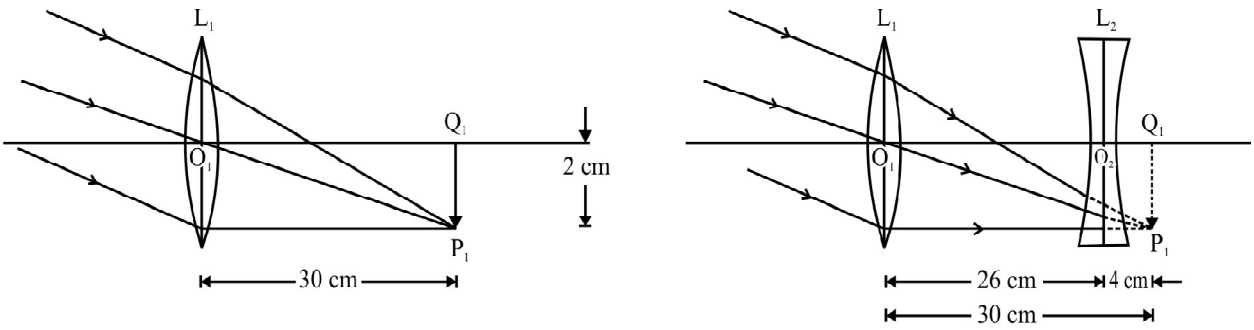


Fig. 79

Fig. 79 (a) shows the image P_1Q_1, being formed due to the convex lens alone. The image is formed in the focal plane of L_1. Fig. 79 (b) shows the concave lens, L_2, placed between L_1 and image P_1Q_1. For the concave, P_1Q_1, acts as a virtual object. For concave lens

$u = +4$ cm; $f = -20$ cm, $v = ?$

From refraction formula

$$\frac{1}{v}-\left(\frac{1}{+4}\right)=\frac{1}{(-20)}$$

$\therefore \quad v = 5\,\text{cm}$

$m =$ magnification due to concave lens $= \frac{v}{u} = \frac{5}{4} = 1.25$

$\therefore$ The size of final image $= |m| \times P_1Q_1$

$= 1.25 \times 2$ cm $= 2.5$ cm

Unit–16 : Optics

Easy *Telescape*

80. An observer looks at a distant tree of height 10 cm, with a telescope of magnifying power 20. To the observer the tree would appears.

(1) 10 times taller (Incorrect)

(2) 10 times nearer (Incorrect)

(3) 20 time taller (Incorrect)

(4) 20 time nearer (Correct)

(I.I.T Mains 2016)

Correct answer: (4)

Solution:

The magnitying power of a telescope, is the ratio of the angular size of image to the angular size of object. For M.P = 20; the angular size of image is 20 times that of object. This will be so if the image formed is 20 times nearer than the object.

Unit–16 : Optics

Average *Huygen's Principle*

81. In Fig. 80, ab, cd and ef represent the wave–fronts of incident rays that are incident on the device XY. a'b', c'd', e'f' represent the corresponding wave fronts for the rays refracted by 'XY'. The device is a

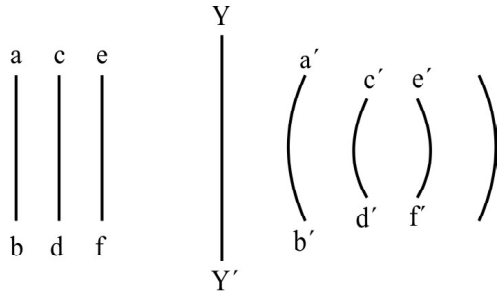


Fig. 80

(1) convex lens (Correct)

(2) concave lens (Incorrect)

(3) plane glass–slab (Incorrect)

(4) prism (Incorrect)

Correct answer: (1)

Solution:

The incident wave fronts are plane and parallel to one another. The incident beam is, therefore, a parallel ray beam. The refracted wave fronts are spherical. The refracted beam is first a convergent beam and is then seen as divergent. This is because for XY, a convex lens, the refracted rays first converge towards the focus and then diverge away.

Unit–16 : Optics

Average *Huygen's Principle*

82. Fig. 81 shows a plane boundary XX', seperating two media. LM is the incident wave front in medium 1 and NO is the refracted wavefront in medium 2. If V_1 and V_2 are the speeds of light, in medium 1 and medium 2, respectively the ratio $\frac{V_1}{V_2}$ is

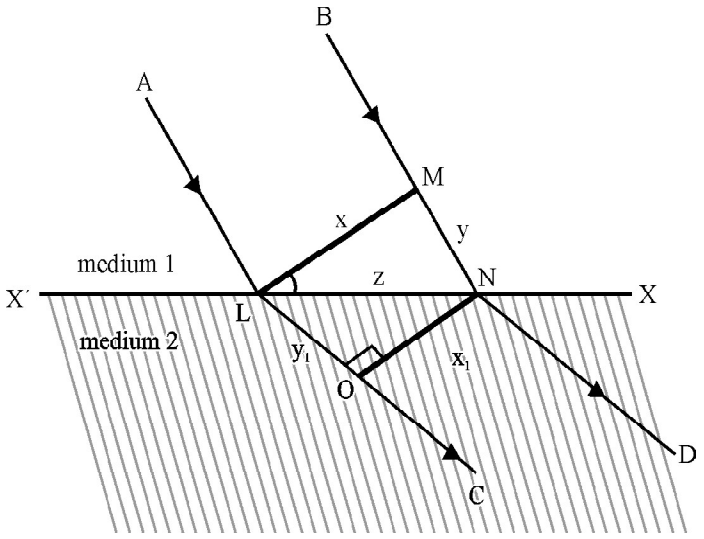


Fig. 81

(1) $\frac{x}{x_1}$ **(Incorrect)**

(2) $\frac{y_1}{y}$ **(Incorrect)**

(3) $\frac{y}{y_1}$ **(Correct)**

(4) $\frac{xy}{x_1y_1}$ **(Incorrect)**

Correct answer: (3)

Solution:

The incident and the refracted wave fronts, LM, and PQ, are plane, wave front. Hence the incident rays and the refracted rays are parallel beam, as shown in the Fig. 82. Let μ_1 and μ_2 be the refractive indices, of medium 1 and medium 2, respectively. From Snell's law

$$\mu_1 \sin i = \mu_2 \sin r$$

$$\therefore \quad \frac{\mu_1}{\mu_2} = \frac{\sin r}{\sin i} = \frac{y_1/z}{y/z} = \frac{y_1}{y}$$

$$\therefore \quad \frac{c/v_1}{c/v_2} = \frac{y_1}{y} \quad \text{or} \quad \frac{V_1}{V_2} = \frac{y}{y_1}$$

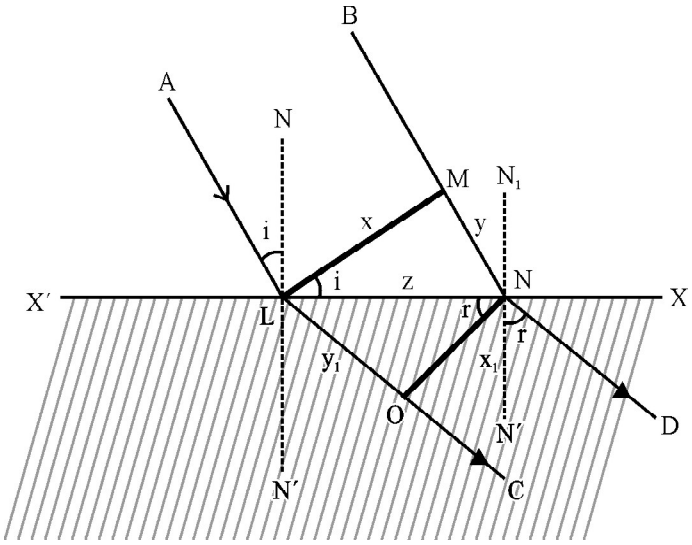


Fig. 82

Unit–16 : Optics

Difficult *Interference*

83. In Young's double slit experiment intensity at a point is $\left(\frac{1}{4}\right)$ of the maximum intensity. The angular position, of this point, is:

(1) $\sin^{-1}\left(\lambda/d\right)$ **(Incorrect)**

(2) $\sin^{-1}\left(\lambda/2d\right)$ **(Incorrect)**

(3) $\sin^{-1}\left(\lambda/3d\right)$ **(Correct)**

(4) $\sin^{-1}\left(\lambda/4d\right)$ **(Incorrect)**

Correct answer: (3)

Solution:

In Fig. 83, P is the point where intensity = $I = \frac{I_0}{4}$ (I_0 maximum intensity). Let δ be the phase difference,

between the two disturbances, at P. Then

$$I = \frac{I_0}{4} = I_0 \cos^2\left(\frac{\delta}{2}\right)$$

$$\therefore \quad \cos\left(\frac{\delta}{2}\right) = \frac{1}{2} \quad \text{or} \quad \frac{\delta}{2} = \frac{\pi}{3}$$

$$\therefore \quad \delta = \frac{2\pi}{3}$$

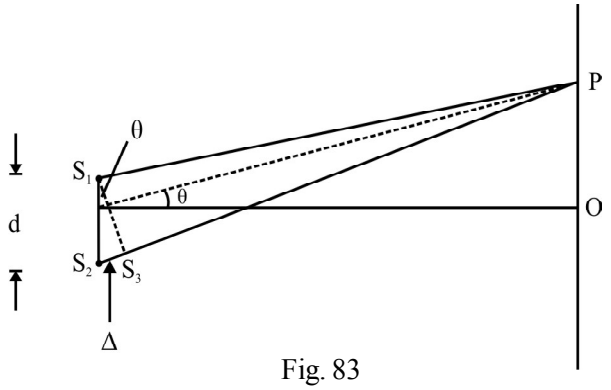


Fig. 83

Let Δ be the path difference between the two disturbances, at P. We know $\delta = \frac{2\pi}{\lambda}(\Delta)$.

$$\therefore \quad \frac{2\pi}{3} = \frac{2\pi}{\lambda}(\Delta) \text{ or } \Delta = \frac{\lambda}{3}$$

From right angled triangle $S_1S_2S_3$,

$$\Delta = S_2S_3 = d\sin\theta = \frac{\lambda}{3}$$

$$\therefore \quad \theta = \sin^{-1}\left(\frac{\lambda}{3d}\right)$$

Unit–16 : Optics

Difficult *Interference*

84. A point source S, emits light of λ = 600 nm. It is placed a small distance h above a plane minor, AB, as shown in Fig. 84. The intensity of reflected light is 36% of the incident light. Interference fringes are observed on a screen placed parallel to AB, a large distance D away from it as shown in Fig. 84. The point P, on the screen, corresponds to a maximum. The minimum distance, through which AB be moved, so that intensity at, P, is again maximum, is

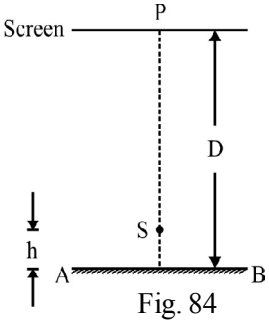


Fig. 84

(1) 150 nm **(Incorrect)**

(2) 300 nm **(Correct)**

(3) 450 nm **(Incorrect)**

(4) 600 nm **(Incorrect)**

Correct answer: (2)

Solution:

Interference occurs at P between the direct waves and the reflected waves. We can imagine reflected waves as coming from a virtual save S', where S' is the image of S due to reflection in the mirror. The path difference, Δ_0, of the waves, at P, is

$$\Delta_0 = S'P - SP = 2h$$

Since P is the position of a maximum, we have

$$2h = (2n-1)\frac{\lambda}{2}$$

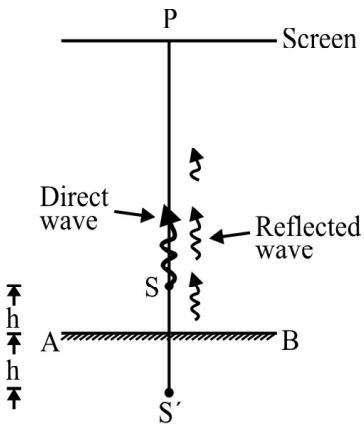


Fig. 85

(Note that there is a phase change of π in reflected wave from AB).

Let x_{min} be minimum distance through which AB is moved, upwards or downwards, so that P is again a maxima. This will so if

$$2x_{min} = \lambda \qquad \text{or} \qquad x_{min} = \frac{\lambda}{2} = 300\,\text{nm}$$

Unit–16 : Optics

Difficult *Interference*

85. In Young's double slit experiment, instead of taking slits of equal width; one slit is made twice is wide as the other. Then; in the interference pattern,

(1) The intensities of both the maxima and the minima increases (Correct)

(2) The intensity of the maxima increases and the minmum has zero intensity (Incorrect)

(3) The intensity of the maxima decreases and that of the minima increases (Incorrect)

(4) The intensity of maxima decreases and the minima has zero intensity (Incorrect)

Correct answer: (1)

Solution:

When both slit have same width, intensities of S_1 and S_2 are equal, say I_0. Then

$I_{max} = 4\ I_0$ and $I_{min} = 0$

When width of one slit, say S_2, increases its intensity I_2' is more than I_0. Let $I_0' = \eta I_0 \ (\eta > 1)$

Now

$$I'_{max} = \left[\sqrt{I_0} + \sqrt{I_0'}\right]^2$$

$$= I_0\left[1 + \sqrt{\eta}\right]^2 > 4I_0$$

and $$I'_{min} = \left[\sqrt{I_0} \sim \sqrt{I_0'}\right]^2$$

$$= I_0\left[\sqrt{\eta} - 1\right]^2 > 0$$

Hence intensities at both the maxima and the minima, become more than their earlier values.

Unit–16 : Optics

Average *Interference*

86. In the Young's double slit experimental set up, shown in Fig. 86 here, $S_1S_2 = d = m\lambda$ where λ is wavelength of incident light. The intensity, at the maximum in the interference pattern, observed on screen, equal I_0. The intensity, at the centeral point, O, is

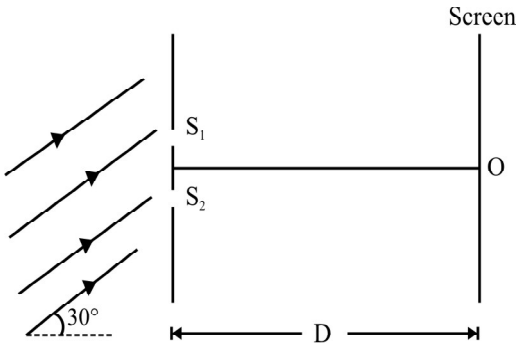


Fig. 86

(1) I_0 **(Incorrect)**

(2) $I_0\cos^2\left[\frac{m\pi}{4}\right]$ **(Correct)**

(3) $I_0\cos^2\left[\frac{m\pi}{2}\right]$ **(Incorrect)**

(4) $\mathbf{I_0 \sin^2\left[\frac{m\pi}{2}\right]}$ **(Incorrect)**

Correct answer: (2)

Solution:

There is an initial path difference Δ_i, between S_1 and S_2. We have

$$\Delta_i = d \sin i = m\lambda \sin 30^0 = \frac{m\lambda}{2}$$

The disturbances, from S_1 and S_2, arrive at, O, with a phase difference, $\delta = \frac{2\pi}{\lambda}(\Delta_i) = \frac{m\pi}{2}$

Hence, the intensity, at O, is

$$I = I_0 \cos^2\left(\frac{\delta}{2}\right) = I_0 \cos^2\left[\frac{m\pi}{4}\right]$$

Unit–16 : Optics

Difficult *Interference*

87. In a Young's double slit experimental set up; the wave length of incident light is λ. The distance between the two interferring sources S_1S_2 =d The distance, of the screen, from the interferring sources, is D. The minimum distance from the central fringe, of the point on the screen, where the intensity is half due to either sources, is

(1) $\mathbf{\frac{\lambda D}{d} \cos^{-1}\left(\frac{1}{2\sqrt{2}}\right)}$ **(Incorrect)**

(2) $\mathbf{\frac{\lambda D}{d} \sin^{-1}\left(\frac{1}{2\sqrt{2}}\right)}$ **(Incorrect)**

(3) $\mathbf{\frac{\lambda D}{2\pi d} \cos^{-1}\left(\frac{1}{2\sqrt{2}}\right)}$ **(Correct)**

(4) $\mathbf{\frac{\lambda D}{2\pi d} \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)}$ **(Incorrect)**

Correct answer: (3)

Solution:

Let I_0 be the intensity of each of the sources S_1 and S_2. We know that $I_{max} = 4I_0$. For the given point, P we want $I = \frac{I_0}{2}$. We also know that

$$I = I_{max} \cos^2(\delta)$$

where $\delta \left[= \frac{2\pi}{\lambda}\left(\frac{yd}{D}\right)\right]$ is the phase difference, between the two interferring rays, at P. Let y be the distance of P from central point, O. We then have

$$\frac{I_0}{2} = 4I_0 \cos^2(\delta)$$

$$\therefore \quad \delta = \cos^{-1}\left(\frac{1}{2\sqrt{2}}\right)$$

$$\therefore \quad \frac{2\pi}{\lambda}\left(\frac{yd}{D}\right) = \cos^{-1}\left(\frac{1}{2\sqrt{2}}\right)$$

$$\therefore \quad y = \frac{\lambda D}{2\pi d} \cos^{-1}\frac{1}{2\sqrt{2}}$$

Unit–16 : Optics

Easy *Diffraction*

88. A single slit is illuminated with a parallel beam of light of wavelength λ = 600 nm. The angular width of the diffraction pattern, is measured. When the wavelength of the incident light is changed to λ', the width of the central maximum decreases by 30%. The difference $\Delta\lambda = (\lambda - \lambda')$, is

(1) 1500 A **(Incorrect)**

(2) 1800 A **(Correct)**

(3) 2000 A **(Incorrect)**

(4) 2500 A **(Incorrect)**

Correct answer: (2)

Solution:

The angular width, $= 2\theta$ of the centeral maximum, in a single slit diffraction pattern, is

$$\omega = 2\theta \simeq \frac{\lambda}{d}$$

Let ω' be the angular width of the central maximum when the wavelength of incident light is λ'. Given

$$\omega' = 0.7\,\omega = \frac{\lambda'}{d}$$

$$\therefore \quad \frac{\lambda'}{\lambda} = 0.7 \qquad \text{or} \qquad \lambda' = 0.7 \times 600 = 420 \text{ nm}$$

$$\Delta\lambda = \lambda - \lambda' = (600 - 420)\,\text{nm} = 180\,\text{nm}$$

$$= 1800 \text{ A}$$

Unit–16 : Optics

Difficult *Interference*

89. A coherent beam of microwaves, of wavelength λ (= 0.5 mm), falls on a Young's double slit arrangement. The separation, between slits, is 1 mm and the screen is at a distance of 1m. The total number of minima; observed on the screen, equals

(1) 1 **(Incorrect)**

(2) 2 **(Incorrect)**

(3) 3 **(Incorrect)**

(4) 4 **(Correct)**

Correct answer: (4)

Solution:

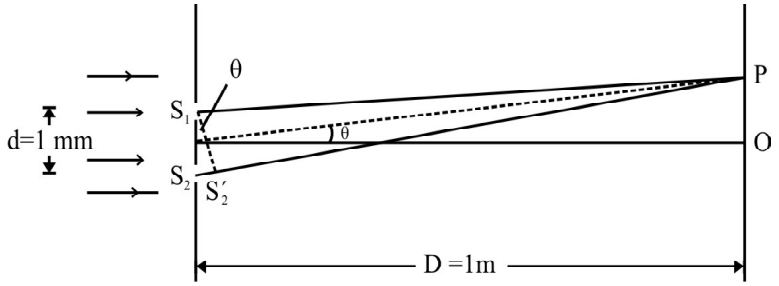


Fig. 87

Fig. 87 shows the experiemntal set up. For any point, P, on the screen, along the direction θ; the path difference Δ, between the two interferring beams, is

$$\Delta = S_2P - S_1P \simeq S_2S_2' = d\sin\theta$$

For minima, we have

$$\Delta = d\sin\theta = (2n-1)\frac{\lambda}{2};\quad 2n-1 = \frac{2d\sin\theta}{\lambda} = \frac{2x1\sin\theta}{0.5} = 4\sin\theta$$

We know that the maximum value of $\sin\theta = 1$. Hence n can be either ± 1 or ± 2. Hence, there are two minima, observed above O, and two below O. Hence the total number of minima, observed, on the screen, is four.

Unit–16 : Optics

Average *Interference*

90. In a Young's double slit experiment, using monochromatic light of wavelength λ, the path difference (in terms of integer n), corresponding to a point, having half the peak intensity, is

(1) $\left(2n+1\right)\frac{\lambda}{2}$ **(Incorrect)**

(2) $\left(2n+1\right)\frac{\lambda}{4}$ **(Incorrect)**

(3) $\left(2n+1\right)\frac{\lambda}{8}$ **(Correct)**

(4) $\left(2n+1\right)\frac{\lambda}{16}$ **(Incorrect)**

(I.I.T Adv. 2013)

Correct answer: (2)

Solution:

The intensity, I, at a point where two waves arrive with a phase difference $\Delta\phi$ is

$$I = I_0\cos^2\left[\frac{\Delta\phi}{2}\right]$$

where I_0 is the peak (or maximum) value of intensity. Also,

$$\Delta\phi = \frac{2\pi}{\lambda}(\Delta)$$

Here Δ is the path difference, at the point considered. Given

$$I = \frac{I_0}{2}$$

$$\therefore \quad \frac{1}{2} = \cos^2\left(\frac{\Delta\phi}{2}\right) \qquad \text{or} \qquad \Delta\phi = (2n+1)\frac{\pi}{4} \qquad (n = 0,1,2,3.....)$$

$$\therefore \quad \Delta\phi = (2n+1)\,\frac{\pi}{4} = \frac{2\pi}{\lambda}\Delta$$

$$\text{or} \qquad \Delta = (2n+1)\,\frac{\lambda}{8}$$

Unit–16 : Optics

Average *Interference*

91. A monochromatic beam, of intensity I_0, is incident on a parallel film. BR and B_1R_1 are the two parallel rays produced due to reflection and refraction, at the two susfaces of the film. If the reflection and transmission coefficient are r and t, the ratio. $\left(\frac{I_{max}}{I_{min}}\right)$, in the interference pattern, produced due to interference between BR and B_1R_1, would be

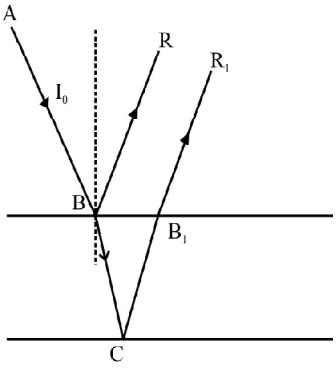


Fig. 88

(1) $\left(\frac{1+r}{1-r}\right)^2$ **(Incorrect)**

(2) $\left(\frac{1+t}{1-t}\right)^2$ **(Correct)**

(3) $\left(\frac{1+r}{1-t}\right)^2$ **(Incorrect)**

(4) $\left(\frac{1+t}{1-r}\right)^2$ **(Incorrect)**

Correct answer: (2)

Solution:

Using the definition of reflection and transmission coefficents, the intensity I_1, of BR, and I_2, of B_1R_1, are as shown in Fig. 89. Now

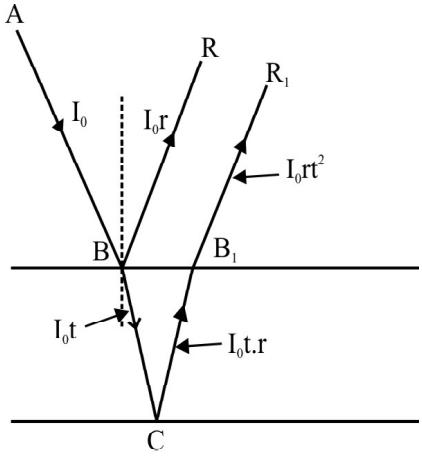


Fig. 89

$$\frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2 = \frac{(I_0 r)}{(I_0 r t^2)}$$

$$\therefore \quad \frac{a_1}{a_2} = \frac{1}{t}$$

$$\therefore \quad \frac{I_{max}}{I_{min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 = \left(\frac{1+t}{1-t}\right)^2$$

Unit–16 : Optics

Difficult *Interference*

92. In a Young's double experiment, bi–chromatic light, having wavelengths 400 nm and 560 nm in it, is being used. The distance, between the plane of the slits and the screen, is 1m. The minimum distance, between two successive regions of complete darkness, is

(1) 4.0 mm (Incorrect)

(2) 5.6 mm (Incorrect)

(3) 14 mm (Correct)

(4) 28 mm (Incorrect)

(I.I.T 2004)

Correct answer: (3)

Solution:

Complete darkness is observed at a point where we have dark fringes due to both the wavelentghs λ_1 and λ_2. Let the m^{th} dark fringe, of λ_1 coincide with n^{th} dark fringe, of λ_2 at any such point. Then, we must have

$$(2m-1)\frac{\lambda_1}{2} = (2n-1)\frac{\lambda_2}{2}$$

or $\frac{2m-1}{2n-1}=\frac{\lambda_2}{\lambda_1}=\frac{560}{400}=\frac{14}{10}=\frac{7}{5}$ .....(1)

The minimum values, of m and n, satisfying Eqn (1) are:

m = 4 and n = 3

The position co–ordinate, y_1, of this point, is

$$y_1=\left[(2m-1)\frac{\lambda}{2}\right]\frac{D}{d}=\frac{(2\times4-1)\times4\times10^{-7}\times1}{2\times10^{-4}}m$$

$$=14\times10^{-3}\ m=14\,mm$$

The next set of values, of m and n, satisfying Enq. (1), are

m = 11, n = 8

The position co–ordinate, y_2, of this point, is

$$y_2=\frac{(2\times11-1)\times4\times10^{-7}\times1}{2\times10^{-4}}m=28\ mm$$

Therefore, the minimum distance, between two consecutive positions of complete darkness on the screen is = (28–14) mm = 14 mm

Unit–16 : Optics

Difficult *Interference*

93. A light source, which emits two wave length $\lambda_1=400$ nm; $\lambda_2=600$ nm, is used in a Young's double slit experiment. Let β_1 and β_2 be the fringe widths, for the wave lengths λ_1 and λ_2, respectively at a point, distant, y, from the central fringe. The number of fringes, observed for λ_1 and λ_2, are m_1 and m_2; respectively. Which of the following statements is incorrect?

(1) $\beta_2>\beta_1$ **(Correct)**

(2) $m_1>m_2$ **(Correct)**

(3) **Third maximum of λ_1, overlaps the fifth minimum of λ_2** **(Correct)**

(4) **Angular seperation of fringes of λ_1, is greater than that for λ_2** **(Incorrect)**

Correct answer: (4)

Solution:

(a) The fringe width;

$$\beta_1 = \frac{\lambda_1 D}{d};\ \beta_2 = \frac{\lambda_2 D}{d}$$

Since $\lambda_1 < \lambda_2;\ \beta_1 < \beta_2$. Statement (1), is therefore, a correct statement.

(b) $m_1 = \text{number of fringes of } \lambda_1 \text{ observed} = \frac{y}{\beta_1}$

$m_2 = \text{number of fringes of } \lambda_2 \text{ observed} = \frac{y}{\beta_2}$

Since $\beta_1 < \beta_2\ ; m_1 > m_2$. Hence statement (2) is also correct statement.

(c) The distance, of 3rd maximum of λ_2, from the centeral fringe $= 3\beta_2 = \frac{3\lambda_2 D}{d} = \left(\frac{1800\,D}{d}\right)\text{nm}$

The distance of 5th minimum λ_1 of from the centeral fringe $= \left(2\times 5 - \frac{1}{2}\right)\beta_1 = \frac{9\lambda_1 D}{2d} = \frac{1800\,D}{d}\text{nm}$

Have statement (3) is also a correct statement.

(d) Angular width of fringe $= \frac{\lambda}{d}$. Since $\lambda_1 < \lambda_2$, angular width of fringes of λ_1 is less than for λ_2. Hence statements (4) is the (required) incorrect statement.

Unit–16 : Optics

Difficult *Interference*

94. In a Young's double slit experiment, light of wave length λ is used. A thin film of refractive index, μ_1, and having a thickness $t_1 (= \lambda)$ is introduced in the path of light coming out of one of the slits, S_1. Another thin film, of refractive index, μ_2 and thickness $t_2\left(=\frac{3\lambda}{2}\right)$, is placed in the path of light coming out of the other slit, S_2. The intensity at the central point, then becomes 25% of its earlier value. The relation, between μ_1 and μ_2, is

(1) $\mu_2 = \frac{2\mu_1}{3}$ **(Incorrect)**

(2) $\mu_2 = \frac{2(\mu_1 + 1)}{3}$ **(Correct)**

(3) $\mu_2 = \frac{2(\mu_1 - 1)}{3}$ **(Incorrect)**

(4) $\mu_2 = \left(\frac{3\mu_1}{2} + 1\right)$ **(Incorrect)**

Correct answer: (2)

Solution:

The additional path differences, Δ_1 and Δ_2, in the path of the two interferring beams, are

$$\Delta_1 = (\mu_1 - 1)\, t_1 = (\mu_1 - 1)\lambda$$

$$\Delta_2 = (\mu_2 - 1)\, t_2 = (\mu_2 - 1)\frac{3\lambda}{2}$$

Let ϕ be the phase difference, between the two interferring beams, at the central point. We have

$$\phi = \frac{2\pi}{\lambda}(\Delta_1 - \Delta_2) \qquad \text{.....(1)}$$

Since $I = \frac{I_0}{4}$, we have

$$\frac{I_0}{4} = I_0 \cos^2(\phi) \Rightarrow \frac{1}{4} = \cos^2(\phi)$$

$$\therefore \quad \phi = \frac{\pi}{3} \qquad \text{.....(2)}$$

From Eqns. (1) and (2) we get

$$\frac{\pi}{3} = \frac{2\pi}{\lambda}\left[(\mu_1 - 1)\lambda - (\mu_2 - 1)\frac{3\lambda}{2}\right]$$

or $\frac{1}{6}=\left[\left(\mu_1-\frac{3\mu_2}{2}\right)+\frac{1}{2}\right]$

$\therefore \quad \mu_2=\left[\frac{2(\mu_1+1)}{3}\right]$

Unit–16 : Optics

Difficult *Interference*

95. In a Young's double slit experiment, the seperation between the two slits is d and the wave length of light used in λ. The intensity of light from slit 1 is four times the intensity from S_2. Which of the following statements is correct?

(1) For $d=\lambda$, no fringe is observed on the screen (Incorrect)

(2) For $\lambda<d<2d$, the total number of fringes, observed on the screen, is three (Correct)

(3) If intensity of light, from S_1, is made equals to the intensity from S_2; the intensities of the observed dark and bright fringes, would increases (Incorrect)

(4) If intensity of light, from S_2, is increases so that if becomes equal to the intensity from S_1, the intensities of the observed bright and dark fringes, would increase (Incorrect)

Correct answer: (2)

Solution:

(i) For $d=\lambda$; only the centeral maximum would be observed on screen. Hence statement (1) is incorrect.

(ii) For $\lambda<d<2\lambda$; there would be three fringes observed on the screen. These are the centeral maximum, and one dark fringe (corresponding to path difference $\Delta=\pm\lambda$), on either side of the central maximum. Hence statement (2) is a correct statement.

(iii) When I_1 and I_2 become equal; the intensity, at the position of dark fringes, is zero; whereas it was finite when $I_1\neq I_2$. The intensity of dark fringes therefore, decreases (when $I_1=I_2$), from what it was when $I_1\neq I_2$. Hence statement (3) and (4) are both incorrect statements.

Unit–16 : Optics

Difficult *Interference*

96. In Fig. 90, CP represents a wave front and AO and BP are two rays corresponding to this wave front. The condition, for constructive interference, at P, between ray BP and ray OP, is

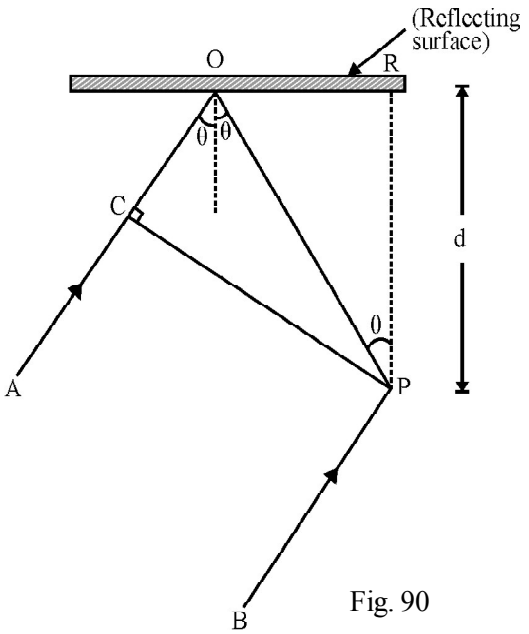


Fig. 90

(1) $\cos\theta = \frac{3\lambda}{2d}$ **(Incorrect)**

(2) $\cos\theta = \frac{\lambda}{4d}$ **(Correct)**

(3) $(\sec\theta - \cos\theta) = \frac{\lambda}{d}$ **(Incorrect)**

(4) $(\sec\theta - \cos\theta) = \frac{4\lambda}{d}$ **(Incorrect)**

(I.I.T 2003)

Correct answer: (2)

Solution:

From right angled triangle OPR,

$$\frac{PR}{OP} = \cos\theta \qquad \text{or} \qquad OP = d\sec\theta \qquad .....(1)$$

From right angled triangle COP;

$$\frac{CO}{OP} = \cos 2\theta \qquad \text{or} \qquad CO = OP\cos 2\theta$$

$$\therefore \quad CO = (d\sec\theta)(\cos 2\theta) \qquad .....(2)$$

Let Δ be the path difference, at P, between the two rays. Obviously

$$\Delta = CO + OP$$

$$= d[\sec\theta\cos 2\theta + \sec\theta]$$

There is an additional path difference, $\frac{\lambda}{2}$ between the two rays due to reflection at the boundary of a denser medium. For constructive interference, at P, we, therefore, have

$$\left(\Delta + \frac{\lambda}{2}\right) = n\lambda \qquad (n = 0,1,2,3,.....)$$

or $\Delta = (2n-1)\frac{\lambda}{2}$

Taking n =1, we get

$$d\sec\theta[1+\cos 2\theta] = \frac{\lambda}{2}$$

$$\left(\frac{d}{\cos\theta}\right)\left[1+\left(2\cos^2\theta - 1\right)\right] = \frac{\lambda}{2}$$

or $2d\cos\theta = \frac{\lambda}{2}$

$$\therefore \quad \cos\theta = \frac{\lambda}{4d}$$

Unit–16 : Optics

Difficult *Interference*

97. A glass plate, of refractive index 1.5, is coated with a thin layer of thickness t and refractive index $\mu_1 = 1.8$. Light, of wave length λ, travelling in air is incident (nearly) normally an the layer. It is partly reflected at its upper and lower ends and the two reflected rays interfere. If $\lambda = 648$ nm, the minimum value of t, for which constructive interference would take place, is

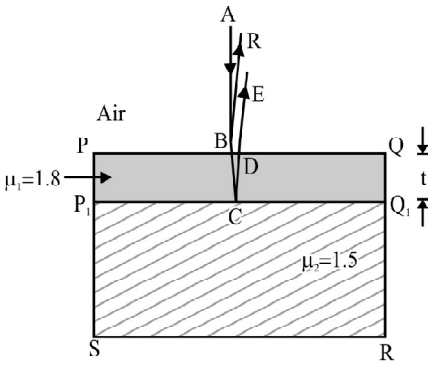


Fig. 91

(1) 45 nm **(Incorrect)**

(2) 60 nm **(Incorrect)**

(3) 90 nm **(Correct)**

(4) 108 nm **(Incorrect)**

Correct answer: (2)

Solution:

In Fig. 91, PQRS is the glass plate and PQQ_1P_1 is the thin fim. AB is the (nearly normally) incident ray. BR is the reflected ray from the upper end and CD is the reflected ray at the lower and (P_1Q_1). DE is the corresponding emergent ray. Interference occurs between BR and DE. The path difference

$$\Delta \simeq 2\mu_1 t$$

There is an additional path difference of $\frac{\lambda}{2}$ due to reflection at PQ (reflection from the boundary of a denser medium). There is no extra path difference due to reflection at P_1Q_1, as this reflection is from the boundary of a rarer medium. For constructive interference, we therefore, have

$$2\mu_1 t + \frac{\lambda}{2} = n\lambda; \qquad (n = 1, 2, 3, ...)$$

$$\therefore \quad t = \frac{\left(n - \frac{1}{2}\right)\lambda}{2\mu_1}$$

For minimum value of t, n = 1. Therefore

$$t_{min} = \frac{\lambda}{4\mu_1} = \frac{648}{4\times 1.8}\,nm \simeq 90\,nm$$

Unit–16 : Optics

Difficult *Interference*

98. In an ideal double–slit experiment, when a glass–plate (refractive index 1.5) of thickness t is introduced in the path of one of the interferring beams (wave length λ), the intensity, at the position where the central fringe occured previously, remains unchanged. The minimum thickness, of the glass plate, is

(1) 2λ **(Correct)**

(2) λ **(Incorrect)**

(3) $\frac{2\lambda}{3}$ **(Incorrect)**

(4) $\frac{\lambda}{3}$ **(Incorrect)**

(I.I.T 2002)

Correct answer: (1)

Solution:

The additional path difference, introduced due to the glass plate, is

$$= (\mu - 1)t$$

Since intensity at position where central maximum occured eailer, remains same; position of one maximum must have been taken over by another maximum. This will be so if the additional path difference is an integral multiple λ. i.e. if

$$(\mu - 1)t = n\lambda \qquad (n = 1, 2, 3, ...)$$

$$\therefore \quad t = \frac{n\lambda}{(\mu - 1)}$$

For minimum value of t; n = 1. Therefore,

$$t_{mn} = \frac{\lambda}{(1.5 - 1)} = 2\lambda$$

Unit–16 : Optics

Difficult *Interference*

99. In a given set up, imagine the Young's double slit experimental set up to be immersed completely in a liquid of $\mu = 1.33$. In the given set up:

d = distance between two slits = 1mm

D = distance of screen from slits = 1.33 m

λ = wave length of light = 630 nm

If a thin film, of $\mu = 1.53$ and thickness t, were to be now inserted in the path of one of the two interferring sources, the position of the central fringe gets occupied by the adjacent dark fringe. The thickness, t, of the film is (nearly)

(1) 2.37 µm (Correct)

(2) 1.58 µm (Incorrect)

(3) 1.19 µm (Incorrect)

(4) 0.74 µm (Incorrect)

Correct answer: (1)

Solution:

λ' = wave length of light inside liquid

$$= \frac{\lambda}{\mu} = \frac{630\times10^{-9}}{1.33}\,\text{m} \simeq 473.7\times10^{-9}\,\text{m}$$

β = The fringe width of fringes

$$= \frac{\lambda' D}{d} = \frac{473.7\times10^{-9}\times1.33}{10^{-4}}\,\text{m} \simeq 0.63\,\text{mm}$$

Let t be the thickness of the film introduced. The additional path difference = (1.53-1) × t

Since position of centeral fringe is now the position of first minimum additional path difference is $\frac{\lambda}{2}$

$$\therefore \quad (1.53-1.33)\,t = \frac{473.9\times10^{-9}}{2}\,\text{m}$$

or $t \simeq 2.37\,\mu\text{m}$

Unit–16 : Optics

Average *Diffraction*

100. In a single slit diffraction experimental set up, the angular width, of the central maximum, changes by (–25%) when the incident wave length (λ = 600 nm) is changed to a new value, say, λ'. It is also observed that while keeping λ = 600 nm, the arrangement is immersed is a transparent liquid of refractive index μ, the change in angular width observed now is the same as that due to a change of λ to its new value λ'. The value of μ is

(1) 1.2 (Incorrect)

(2) 1.33 (Correct)

(3) 1.4 (Incorrect)

(4) 1.5 (Incorrect)

Correct answer: (2)

Solution:

Let ω and ω' be the angular widths of central maximum for wave lengths λ and λ', respectively. Then

$$\omega = \frac{\lambda}{d} \quad \text{and} \quad \omega' = \frac{\lambda'}{d}$$

Given $\omega' = (1 - 0.25)\,\omega = 0.75\,\omega$

$$\therefore \quad \frac{\omega'}{\omega} = 0.75 = \frac{\lambda'}{\lambda}$$

or $\lambda' = 0.75\,\lambda$

When the arrangement, using λ is immersed inside the liquid, the wave length becomes $\lambda''\left(= \frac{\lambda}{\mu}\right)$.

Since change in angular width is the same, $\lambda' = \lambda''$.

$$\frac{\lambda}{\mu} = 0.75\lambda$$

or $$\mu = \frac{1}{0.75} = 1.33$$

Unit–16 : Optics

Difficult *Interference*

101. A thin film of thickness t; refractive index μ_2 is placed inside a medium of refractive index $\mu_1 (< \mu_2)$ as shown in Fig. 92. AB is a monochromatic ray of wave length λ incident almost normally on uper surface of film. It undergoes partial reflection as shown in Fig. 92. The condition for constructive interference between BC and GH is

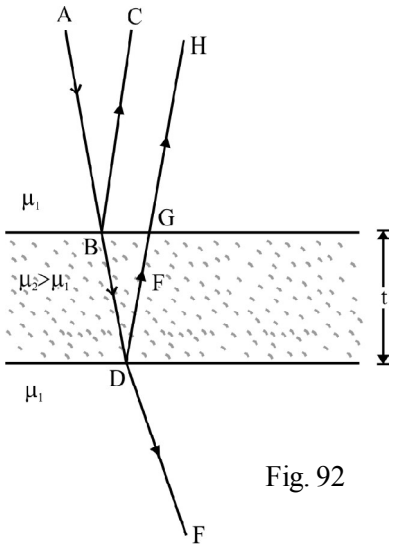


Fig. 92

(1) $\frac{2\mu_2 t}{\lambda} = m + \frac{1}{2}$ **(Incorrect)**

(2) $\frac{2\mu_2 t}{\mu_1 \lambda} = m + \frac{1}{2}$ **(Correct)**

(3) $\frac{2\mu_2 t}{\lambda} = m + \frac{3}{4}$ **(Incorrect)**

(4) $\frac{2\mu_1 t}{\mu_2 \lambda} = m + \frac{1}{2}$ **(Incorrect)**

Correct answer: (2)

Solution:

Ray GH travels an extra distance 2t in a medium of refractive index μ_2 (for near normal incidence). This is equal to a path $2\left(\frac{\mu_2}{\mu_1}\right)t$ in medium of refractive index μ_1. Since $\mu_2 > \mu_1$, there is an additional path

difference of $\frac{\lambda}{2}$ introduced due to reflection at point B. The total path difference

$$\Delta = \frac{2\mu_2 t}{\mu_1} + \frac{\lambda}{2}$$

For constructive interference

$$\Delta = m'\,\lambda \qquad (m' = 0,1,2,3,.....)$$

or $$\frac{2\mu_2 t}{\mu_1} + \frac{\lambda}{2} = m'\,\lambda$$

or $$\frac{2\mu_2 t}{\mu_1 \lambda} = m' - \frac{1}{2} = m + \frac{1}{2}$$

(where $m = m' + 1 = 1, 2, 3, ...$)

Unit–16 : Optics

Average *Interference*

102. In the Fig. 93, S_1 and S_2, are two monochromatic coherent sources, both emitting light of $\lambda = 2\mu m$. It is given that $S_1S_2 = d = 6\mu m$ and R, the radius of circle with centre O is $20\mu m$. Which of the following statements is incorrect?

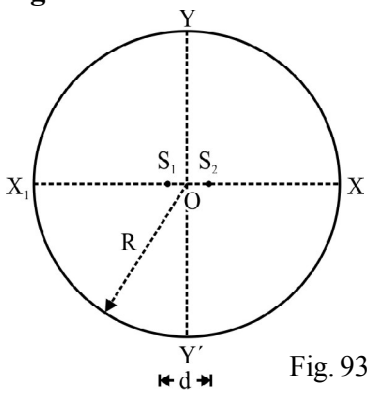


Fig. 93

(1) X and Y are both positions of bright fringes (Correct)

(2) X and Y are both positions of dark fringes (Incorrect)

(3) The total number of bright fringes, observed, are 12 (Correct)

(4) The total number of dark fringes, observed, are 12 **(Correct)**

Correct answer: (2)

Solution:

At point X, the two disturbances arrive having a path difference Δx .

$$\Delta_x = S_1X - S_2X = d = 6\,\mu m$$

Δ_x is an integral multiple $\lambda = 2\,\mu m$ and equals 3λ . Therefore X is the position of the 3rd bright fringe.

At point Y; the path difference, Δy , between the two disturbances , is

$$\Delta_y = S_2Y - S_1Y = \text{zero}$$

Hence Y is the position of the zeroth order maximum. Hence X and Y are both positions of bright fringes. Further, there are 3 bright fringes in the quddrant XY. Therefore total number of bright fringes observed equals $3\times4 = 12$. This would also be the total number of dark fringes observed.

Unit–16 : Optics

Average *Diffraction*

103. Using the expression

$$\mathbf{2d\sin\theta = \lambda}$$

One calculates the values of d by measuring θ, over range 0^0 to 90^0. (λ is known exactly). The error, in θ, is constant for all values of θ. As θ increaes from 0^0 to 90^0;

(1) the absolute error in, 'd', remains constant **(Incorrect)**

(2) the absolute error in, 'd', increaes **(Incorrect)**

(3) the fractional error in, 'd', remains constant **(Incorrect)**

(4) the fractional error in, 'd', decreases **(Correct)**

(I.I.T Adv. 2013)

Correct answer: (4)

Solution:

Given $d = \dfrac{\lambda}{2\sin\theta}$

$\therefore \quad \ell n\, d = \ell n\, \lambda - \ell n 2 - \ell n(\sin\theta)$

Differentiating, we get

$$\frac{\Delta(d)}{d} = \left(\frac{\Delta\lambda}{\lambda}\right) - \left(\frac{\cos\theta}{\sin\theta}\right)\Delta\theta$$

Since there is no error is λ (given); $\Delta\lambda = 0$. Therefore,

$$\left|\frac{\Delta(d)}{d}\right| = |\cot\theta|(\Delta\theta) \qquad .....(1)$$

The absolute error in d is $|\Delta(d)| = (d\cot\theta)(\Delta\theta)$ .....(2)

As θ increases from 0^0 to 90^0; $\cot\theta$ decreases.

From equations (1) and (2), it follows that both the fractional error, $\left|\frac{\Delta(d)}{d}\right|$, as well as the absolute error, $|\Delta(d)|$, in d, decrease.

Unit–16 : Optics

Average *Diffraction*

104. The head lights of a car are 1.2 m apart. They emit light of λ = 600 nm. The diameter of the pupil of the observers eye is 3 mm. The maximum distance of car from observer so that the two head lights are just resolved is

(1) 4 km (Incorrect)

(2) 5 km (Incorrect)

(3) 6 km (Correct)

(4) 8 km (Incorrect)

Correct answer: (4)

Solution:

The two head lights are just resolved, according to Rayleigh's criterion; if angular width of the two lines equals half angular width of diffraction pattern at the eye of the observer. If D is maximum distance of car from observer;

$\theta = \frac{1.2}{D} = \frac{\lambda}{d}$; $d = 3\,mm = 10^{-3} \times 3\,m$

or $D = \frac{1.2 \times 3 \times 10^{-3}}{6 \times 10^{-7}}\,m$

$= 6 \times 10^3\,m = 6\,km$

Unit–16 : Optics

Average *Polarisation*

105. A beam of unpolarised light, of intensity I_0, is passed through a polariod A. It is next passed through another polariod, B, which is oriented so that its principal plane makes an angle of 45^0, relative to that of A. The intensity of emergent light, is

(1) $\mathbf{I_0}$ **(Incorrect)**

(2) $\frac{\mathbf{I_0}}{\mathbf{2}}$ **(Incorrect)**

(3) $\frac{\mathbf{I_0}}{\mathbf{4}}$ **(Correct)**

(4) $\frac{\mathbf{I_0}}{\mathbf{8}}$ **(Incorrect)**

(I.I.T Main 2013)

Correct answer: (3)

Solution:

Since light, incident on A, is unpolarised, the intensity, I_A, of light emerging from A, is

$$I_A = \frac{I_0}{2}$$

Light incident on B is plane polarised. Also the angle, between the axis of A and B is $\frac{\pi}{4}$. Using Malus law;

I_B = Intensity of light transmitted by B

$$= I_A \cos^2\left(\frac{\pi}{4}\right) = \frac{I_0}{2}\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{I_0}{4}$$

Unit–16 : Optics

Average *Polarisation*

106. Two beams A and B, of plane polarised light, (having mutually perpendicular planes of polarisation) are seen through a polariser. From the position, when beam A has maximum intensity (and beam B has zero intensity), a rotation of the polaroid, through 30^0, makes the two beam appear equally bright. The initial intensities, of beam A and B, are I_A and I_B, respectively. The ratio $\frac{I_A}{I_B}$, is

(1) 3 **(Incorrect)**

(2) $\frac{3}{2}$ **(Incorrect)**

(3) 1 **(Incorrect)**

(4) $\frac{1}{3}$ **(Correct)**

(I.I.T Mains 2014)

Correct answer: (4)

Solution:

When polariser is rotated by 30^0, the angle between the axis of the polariser and the analyser, for beam A, is 30^0. The intensity, I'_A, of transmitted light, is

$$I'_A = I_A \cos^2(30) \qquad .....(1)$$

When intensity of beam A is maximum, that of beam B is zero. This means that the plane of polarisation of A and B, are mutually perpendicular. Hence, when the axis of the polaroid makes an angle of 30^0 with the plane of polarisation of A, the angle between the axis of the polared and that of beam B, is $(90–30)^0 = 60^0$. Therefore, I'_B; the intensity of transmitted beam B, is

$$I'_B = I_B \cos^2(60) \qquad .....(2)$$

Given $I'_A = I'_B$, therefore

$$\frac{I_A}{I_B} = \frac{\cos^2(60^0)}{\cos^2(30^0)} = \frac{1}{3}$$

Unit–16 : Optics

Difficult *Reflection*

107.

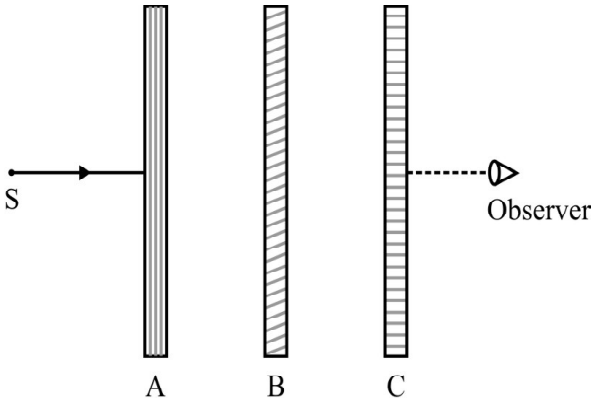
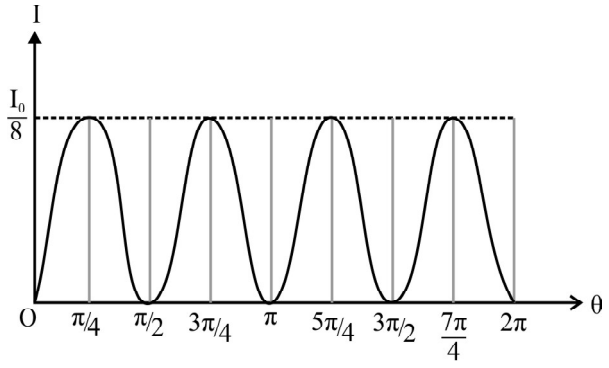


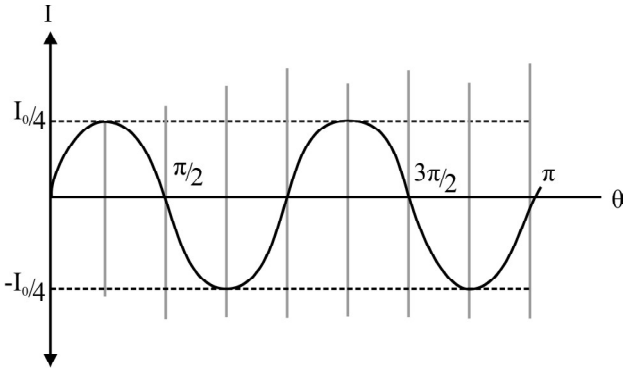
Fig. 94

A, B and C are three polariods, arranged as shown in Fig. 94. The pass axis of C, is perpendicular to the pass axis of A. S is a monochromatic source of intensity I_0. Polarised B is rotated through one full circle. Let θ be the instantaneous angle between the axis of A and B. If I is the intensity of light, as observed by observer O, the graph between 'I' and θ is the graph labelled as graph

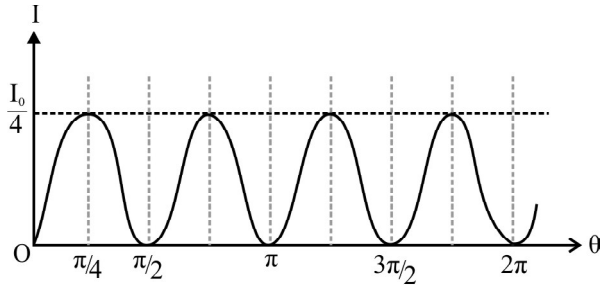
(1) P **(Correct)**



(2) Q **(Incorrect)**

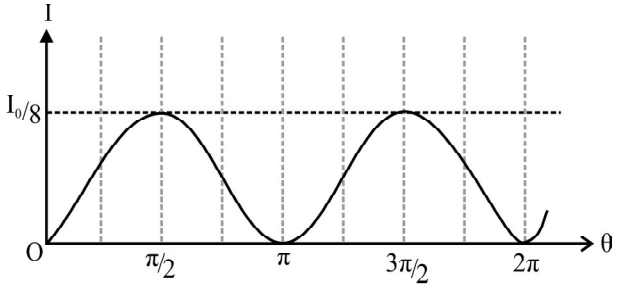


(3) R



(Incorrect)

(4) S



(Incorrect)

Correct answer: (1)

Solution:

Light, incident on polariod A, is unpolarised; therefore, intensity, I_A, of light transmitted by A, is

$$I_A = \frac{I_0}{2} \qquad .....(1)$$

Let θ be the instantaneous angle between the pass axes of A and B. Light, incident on B, is plane polarised. The intensity, I_B, of light transmitted by B, is

$$I_B = I_A \cos^2\theta = \left(\frac{I_0}{2}\right)\cos^2\theta \qquad .....(2)$$

The instantaneous angle between the pass axes of B and C is $\left(\frac{\pi}{2}-\theta\right)$. The intensity, $I_C = I$, of light transmitted by polariod C, is

$$I = I_B \cos^2\left(\frac{\pi}{2}-\theta\right)$$

$$= \left(\frac{I_0}{2}\right)\cos^2\theta \sin^2\theta$$

$$= \left(\frac{I_0}{8}\right)\left[\sin^2\theta\right]^2 \qquad .....(3)$$

It follows that I varies as $\sin^2\theta$ and has a peak value of $\left(\frac{I_0}{8}\right)$. These features have been correctly depicted only in the graph labelled as graph (P).

Unit–16 : Optics

Average *Polarisation*

108. A ray, of monochromatic light, is in incident, at an angle θ, on the interface between a parallel slab (of refractive index μ) and another medium (of refractive index μ_1). The emergent ray just grazes along the interface of the slab and the adjacent medium. An unpolarised ray of the same light is next incident (from air) on the other face of the slab and the reflected light gets plane polarised. The angle of incidence, of this unpolarised ray, on other face of the slab, is

(1) $\tan^{-1}(\mu)$ **(Incorrect)**

(2) $\tan^{-1}(\mu_1)$ **(Incorrect)**

(3) $\tan^{-1}\left(\frac{\mu}{\sin\theta}\right)$ **(Incorrect)**

(4) $\tan^{-1}\left(\frac{\mu_1}{\sin\theta}\right)$ **(Correct)**

Correct answer: (4)

Solution:

Fig. 95 shows the ray, AB, incident inside the slab on its face RS. BC is the emergent ray. Applying Snell's law

$$\mu \sin\theta = \mu_1 \times \sin\frac{\pi}{2}$$

$$\therefore \quad \mu = \left(\frac{\mu_1}{\sin\theta}\right)$$

Let i_p be the angle of incidence, of the unpolarised light, on face PQ. Since the reflected light is now plane polarised, we get, from, Brewster's law.

$$\tan i_P = \mu = \left(\frac{\mu_1}{\sin\theta}\right)$$

$$\therefore \quad i_P = \tan^{-1}\left(\frac{\mu_1}{\sin\theta}\right)$$

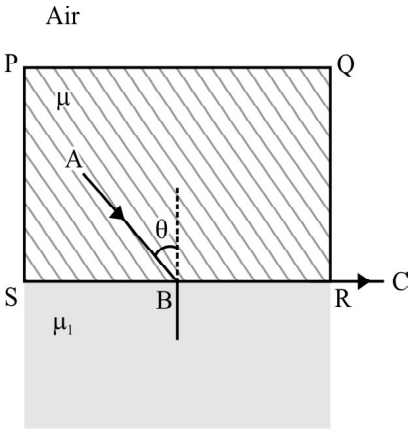


Fig. 95

Unit–16 : Optics

Difficult *Polarisation*

109. A beam has two components, A and B, of plane polarised light, that are polarised in mutually perpendicular directions. Intensity of A is I_0 and that of B is $2\ I_0$. The composite beam passes through a polariod. If I is the total intensity of light transmitted by this polariod, when its axis makes an angle θ with the axis of the polariser, used for beam A, we wouldhave,

(1) $\mathbf{I = 2\,I_0 \sin^2\theta}$ **(Incorrect)**

(2) $\mathbf{I = I_0 \cos^2(\theta)}$ **(Incorrect)**

(3) $\mathbf{I = I_0\left[1+\cos^2\theta\right]}$ **(Incorrect)**

(4) $\mathbf{I = I_0\left[1+\sin^2\theta\right]}$ **(Correct)**

Correct answer: (4)

Solution:

Let I'_A and I'_B be the intensities, of the components A and B, after passing through the poloroid. From Malus law, we have

$$I'_A = I_0 \cos^2(\theta)$$

$$I'_B = 2I_0 \cos^2\left(\frac{\pi}{2}-\theta\right) = 2\,I_0 \sin^2\theta$$

I = The total intensity of light transmitted by the polaroid

$$= I'_A + I'_B = I_0 \cos^2\theta + 2I_0 \sin^2\theta$$

$$= I_0\left[\left(\cos^2\theta + \sin^2\theta\right) + \sin^2\theta\right]$$

$$= I_0\left[1 + \sin^2\theta\right]$$

Unit–16 : Optics

Average *Polarisation*

110. A beam consists of an unpolarised and a plane polarised part. The beam is viewed through a polarised. On rotating the polaroid through one full circle, about the direction of propagation of the beam as axis, the intensity of the transmitted light.

(1) Remains same throughout (Incorrect)

(2) Varies and becomes zero, twice in one full rotation (Incorrect)

(3) Varies, between a maximum and minimum value, but the minimum value is not zero (Correct)

(4) Varies, and becomes maximum, three times in one full revolution (Incorrect)

Correct answer: (3)

Solution:

Since incident beam has a unpolarised part, a part, of constant intensity, passes through in all positions of the polariod. For the plane polarised component; the intensity transmitted varies between a maximum and zero. The resultant intensity, therefore, varies but does not become zero, at any stage.